Entanglement Decay in Open Quantum Systems

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Research Objective

Simulate and analyze two models that describe entanglement behavior in quantum systems:

- **Jaynes–Cummings (JC) model:** Interaction between a two-level atom and a cavity field
- **Two-qubit thermal memory model:** Evolution of entanglement under thermal noise

Both models were implemented using **QuTiP** (Quantum Toolbox in Python), a numerical simulation framework for open quantum systems.

Outline

Motivation

Quantum Computers

Error Mitigation

Error Mitigation in Low-Level Pulses

Implementation

Technical Terms

Jaynes–Cummings Model

Two-qubit System in Thermal noise

Conclusion

What do the results mean?

Next Steps

Motivation

- Quantum Computers
- Error Mitigation
- Error Mitigation in Low-Level Pulses

Quantum Computers

- Qubits are the basic unit of quantum information.
- Quantum bits can be 0, 1, or both at once.
- Quantum computers operate using these quantum bits or qubits.

Quantum Speed-up

- Shor’s Factoring Algorithm: Finds the prime factors of a big number.
- Grover’s Search Algorithm: Finds the “right” item in a list faster than classical search.
- HHL: Solves systems of linear equations using quantum circuits.

Quantum Speed-up

Algorithm	Problem	Classical Complexity	Quantum Complexity	Speedup Type
Shor's Algorithm	Integer factoring	$O(e^{(\log N)^{1/3}(\log\log N)^{2/3}})$ (sub-exponential)	$O((\log N)^3)$	**Exponential**
Grover's Algorithm	Unstructured search (size N)	$O(N)$	$O(\sqrt{N})$	**Quadratic**
HHL Algorithm	Solve $A\vec{x} = \vec{b}$	$O(N \log N)$ to $O(N^3)$	$O(\log N)$ *	**Exponential***

Biggest Challenge: NOISE

Why Do Qubit Operations Go Wrong?

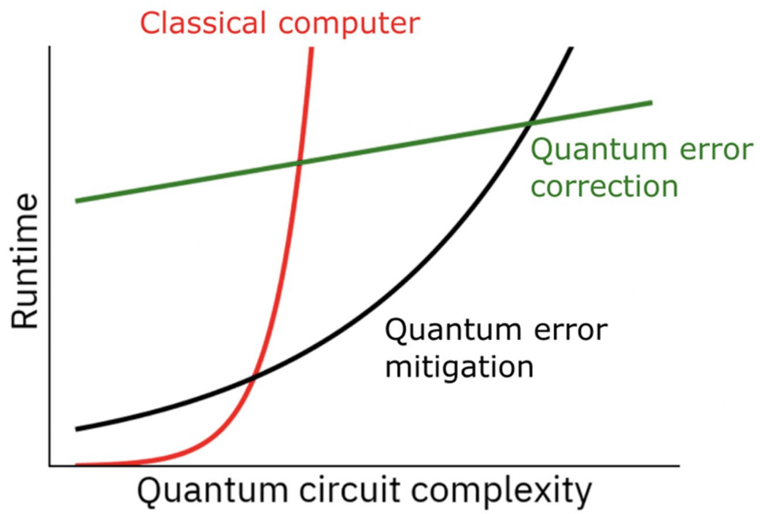
- Qubits are extremely sensitive to their surroundings.
- Even tiny interactions with the environment, like **thermal noise or stray electromagnetic fields**, disturb their state.
- Over time, this leads to loss of coherence or unwanted entanglement with external systems.
- Two-qubit gates are more vulnerable:

  Any mismatch in frequency, timing, or control pulses introduces additional noise.

As a result, quantum gates often drift from their ideal behavior, causing computational errors.

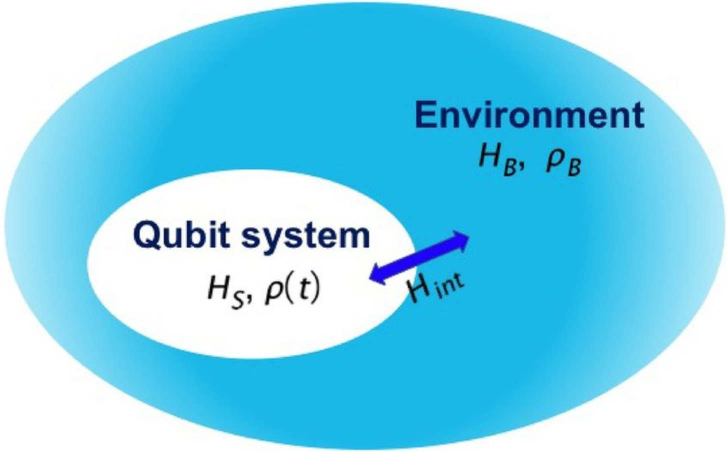
Error Mitigation in Low-Level Pulses

Main Question: How can one perform error mitigation in low-level pulses of a two qubit operation?



Error Mitigation in Low-Level Pulses

- Quantum gates don't operate in isolation -> They're influenced by their environment, just like **open quantum systems**.
- We need to simulate open quantum systems to understand how entanglement fades over time, how sudden death and partial revivals appear.



Implementation

- Technical Terms
- Jaynes–Cummings Model
- Two-qubit System in Thermal noise

Technical Terms

Two-level atom (qubit)	A system with only two energy levels, analogous to a bit in classical computing.
Coherent state $\|\alpha\rangle$	A quantum version of a classical electromagnetic wave. Defined by a complex amplitude α. Used to initialize the field in JC simulations.
Collapse and revival	A pattern in which quantum oscillations (Rabi oscillations) disappear (collapse) and then reappear (revival), due to quantum interference.
Thermal reservoir	An environment with a nonzero temperature that interacts with qubits and introduces decoherence through thermal photons.
Entanglement	A quantum correlation between particles such that the state of one affects the other, even when separated. It's essential for quantum logic and is measured here using **concurrence**.
Cat - Cat State	Baseline: a non-entangled but coherent initial state.

Simulation 1: Jaynes–Cummings Model

- A quantum model describing a **two-level atom interacting with a single-mode electromagnetic field** inside a cavity
- Used to study light–matter interaction in the quantum regime
- Shows non-classical effects like **collapse and revival of atomic population**

Symbol	Meaning
ω_a	Frequency of the atom (energy gap)
ω_c	Frequency of the cavity field
g	Coupling strength between atom and field
$a, a^\dagger$	Field annihilation and creation operators
σ_+, σ_-	Raising/lowering operators for the atom
σ_z	Measures population inversion

Simulation 1: Jaynes–Cummings Model

Hamiltonian (qualitative): Atom + field energy + interaction between them

Initial condition:

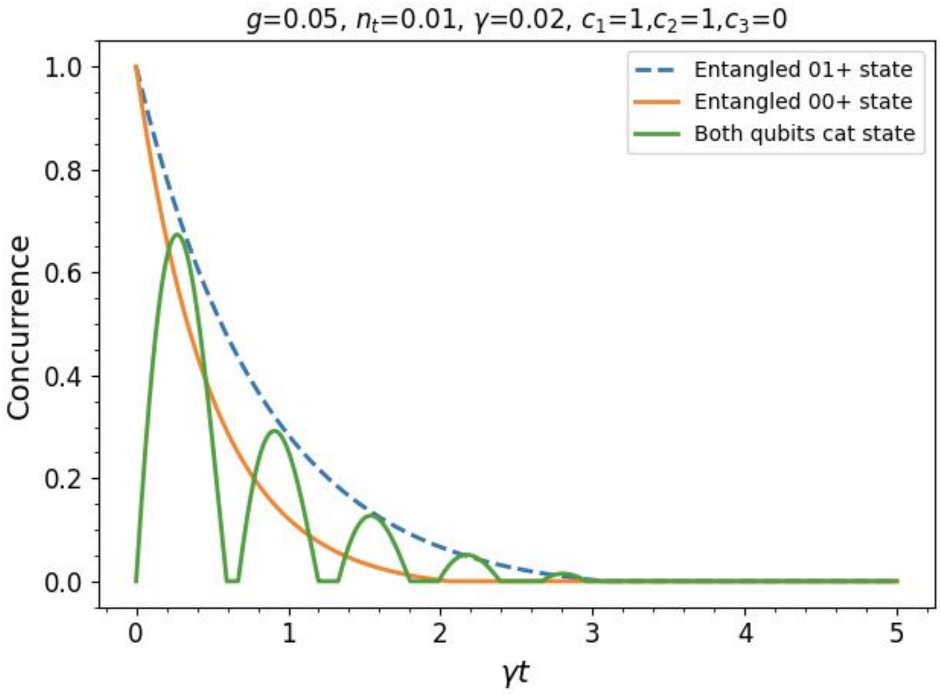
- Atom in ground state: $|g\rangle$
- Cavity in coherent state: $|\alpha = 5\rangle$, Average photon number: $\langle n\rangle = 25$

What we tracked:

- Concurrence: a number from 0 to 1 measuring entanglement
- Expectation value of $\sigma_{1x}\, \sigma_{2x}$: shows correlation between qubits

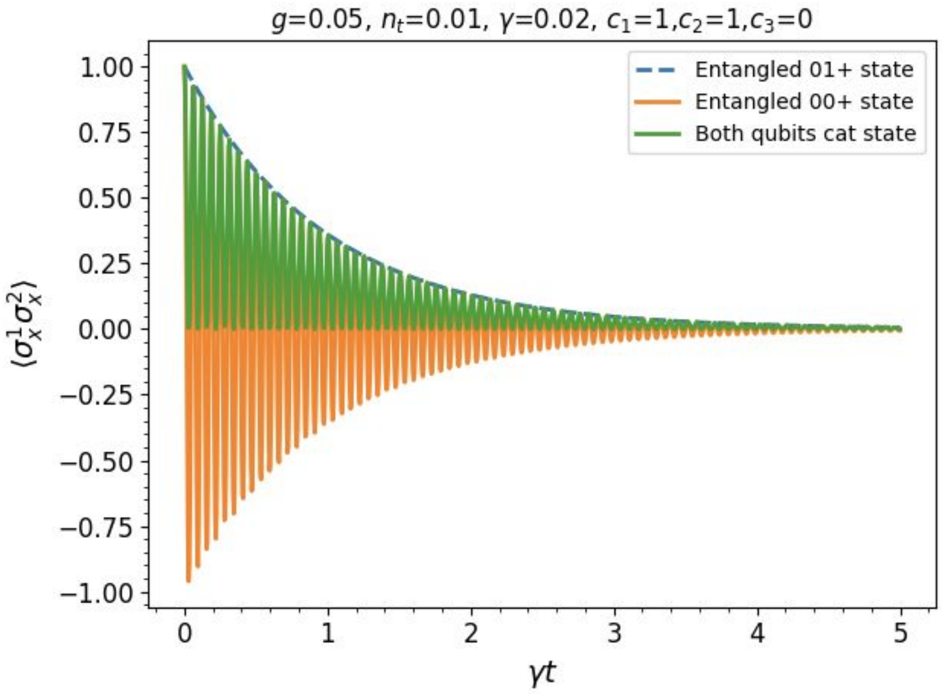
Simulation 1: Jaynes–Cummings Model

- Collapse–revival of entanglement observed
- Entangled states decay differently
- Cat–cat state shows brief entanglement spikes



Simulation 1: Jaynes–Cummings Model

- Oscillations persist across all states
- Entangled states maintain stronger correlations
- Cat–cat state shows fast rotation and decay
- Reflects coherent qubit–field dynamics



Simulation 2: Two-Qubit Thermal Memory

Goal: **Track how two-qubit entanglement changes over time when interacting with a thermal bath.**

Initial states tested:

- Bell-like entangled state: high initial entanglement
- Separable (cat–cat) state: unentangled at start

What we tracked:

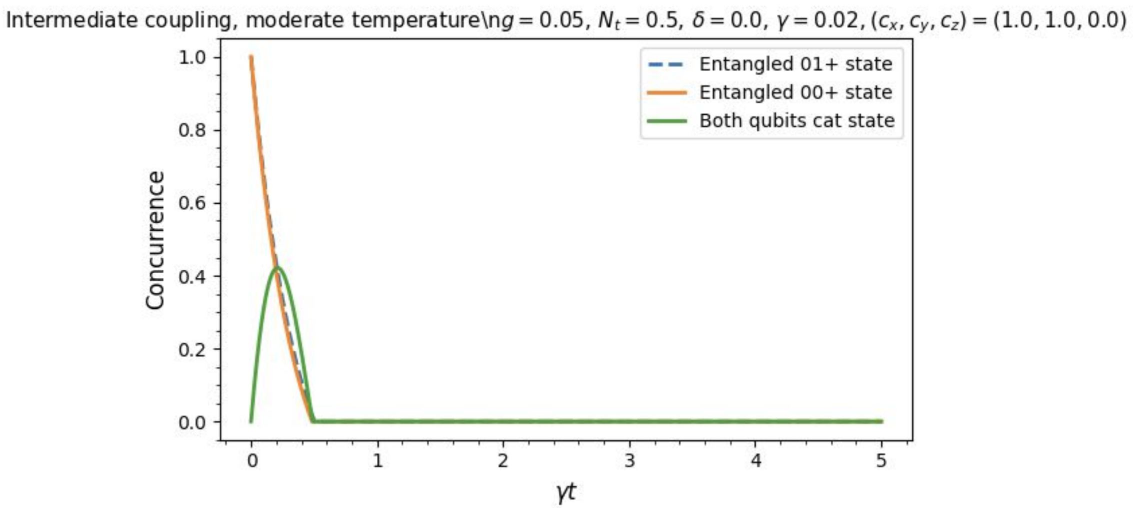
- Concurrence: a number from 0 to 1 measuring entanglement
- Expectation value of $\sigma_{1x}\,\sigma_{2x}$: shows correlation between qubits

Simulation 2: Thermal Noise

- The noise that comes from **heat**.
- Even when no one is touching the system, the surrounding environment at temperature T>0T > 0T>0 can:
  - **Excite** qubits (push them from ground to excited state)
  - **Dephase** them (randomly shift their phase)
  - **Cause decay** (excited → ground state transitions)

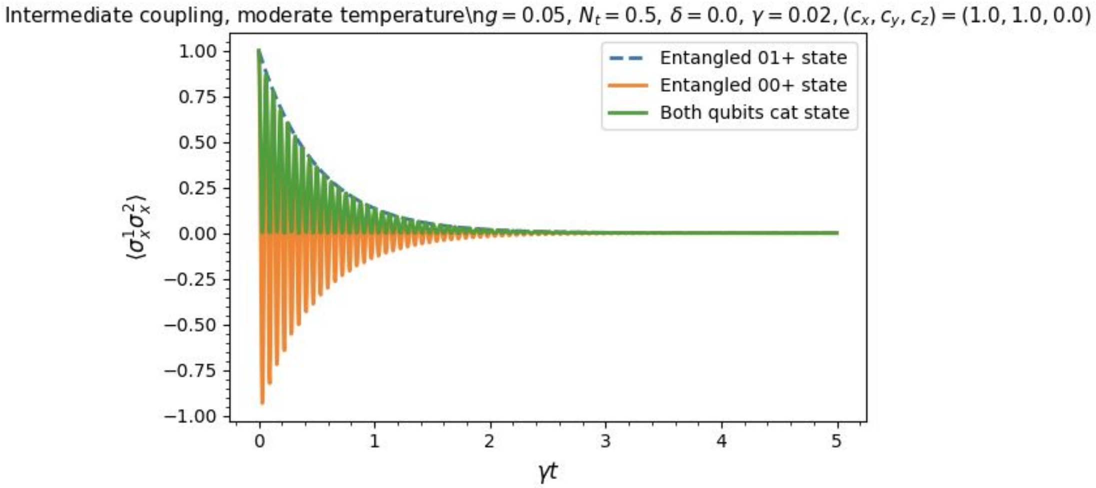
Simulation 2: Two-Qubit Thermal Memory

- Sudden death of entanglement occurs early
- Thermal noise overwhelms interaction
- Cat–cat state remains mostly unentangled
- No revival observed



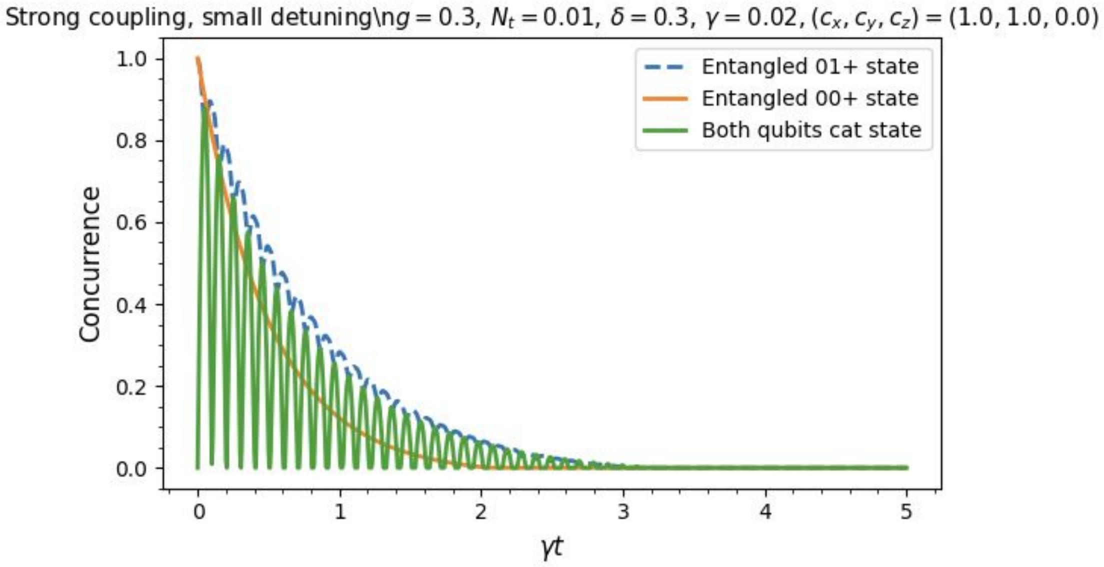
Simulation 2: Two-Qubit Thermal Memory

- Initial coherence fades quickly
- Oscillations disappear by γt ≈ 3
- Correlates directly with sudden death
- Strong damping from the thermal environment



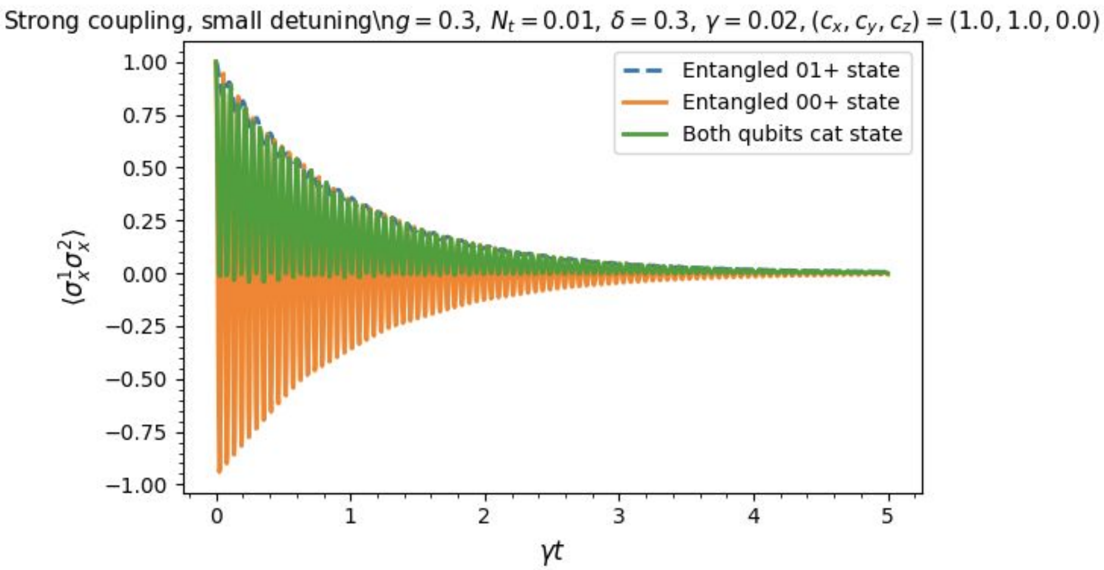
Simulation 2: Two-Qubit Thermal Memory

- Entanglement decays slowly across all states
- Cat–cat state shows strong revival oscillations
- Strong coupling resists decoherence
- Detuning introduces structured dynamics



Simulation 2: Two-Qubit Thermal Memory

- Long-lasting high-frequency oscillations
- All states maintain coherence longer
- Cat–cat state develops strong correlations
- Internal dynamics dominate over noise



Conclusion

Conclusion

Jaynes–Cummings Model:

- Collapse–revival pattern shows pure quantum interference
- Coherence decays slowly, oscillations persist
- Serves as a clean reference: **no external decoherence**, only internal dynamics
- Highlights how a quantized field modifies qubit evolution in a structured way

Conclusion

Two-Qubit Thermal Model:

Intermediate coupling, moderate temperature:
Sudden death of entanglement, no revival
→ Noise overwhelms internal dynamics

Strong coupling, small detuning:
Entanglement survives longer, cat–cat state oscillates
→ System resists decoherence through stronger internal interaction

Next Steps

Relevance to NISQ error mitigation:

These models reflect dominant error sources in real hardware (thermalization, drift, dephasing)

Parameter sweeps help locate conditions where entanglement survives longer

Next steps:

Apply simulation insights to calibrate more robust two-qubit operations

Extend models to include:

- Non-Markovian reservoirs
- Realistic gate noise (Pauli channels, cross-talk)

References

[1] R. Thompson, G. Rempe, and H. Kimble, “Observation of normal-mode splitting for an atom in an optical cavity,” Physical Review Letters, vol. 68, no. 8, pp. 1132–1135, 1992.
[2] H. Walther, B. Varcoe, B. Englert, and T. Becker, “Cavity quantum electrodynamics,” Reports on Progress in Physics, vol. 69, no. 5, pp. 1325–1382, 2006.
[3] E. Jaynes and F. Cummings, “Comparison of quantum and semiclassical radiation theories with application to the beam maser,” Proceedings of the IEEE, vol. 51, no. 1, pp. 89–109, 1963.
[4] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information, 10th ed. Cambridge University Press, 2010.
[5] T. Yu and J. Eberly, “Finite-time disentanglement via spontaneous emission,” Physical Review Letters, vol. 93, p. 140404, 2004.
[6] M. Santos, P. Milman, L. Davidovich, and N. Zagury, “Direct measurement of finite-time disentanglement induced by a reservoir,” Physical Review A, vol. 73, p. 040305(R), 2006.
[7] S. Hill and W. Wootters, “Entanglement of a pair of quantum bits,” Physical Review Letters, vol. 78, pp. 5022–5025, 1997.
[8] P.~W. Shor, “Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer,” 35th Ann. Symp. on Foundations of Computer Science, 1994
[9] L.~K. Grover, “A Fast Quantum Mechanical Algorithm for Database Search,” STOC ’96, 1996.
[10] A.~W. Harrow, A.~Hassidim & S.~Lloyd, “Quantum Algorithm for Linear Systems of Equations,” Phys. Rev. Lett. 103, 150502 (2009)

Thank you!