Exploring Quantum Optical Models and Entanglement Dynamics in Open Quantum Systems

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***Abstract*—We investigate two fundamental aspects of quantum systems: the Jaynes-Cummings model and entanglement dynamics in open quantum systems. First, we simulate a qubit interacting with a cavity field in a coherent state $|\alpha\rangle$ ($\alpha = 5$). Our results demonstrate clear collapse-revival patterns characteristic of quantum interactions. Second, we analyze how two-qubit entanglement evolves when the system interacts with a thermal environment. We examine various initial states, including Bell-like and separable states, under different environmental conditions. Our findings reveal distinct phenomena such as entanglement sudden death and potential revivals, with strong dependencies on system parameters. These results contribute to our understanding of quantum memory stability and coherence preservation in practical quantum systems.**

I. Introduction

The intersection of quantum optics and cavity quantum electrodynamics (QED) provides an ideal platform for studying quantum coherence and entanglement [1] [2]. The Jaynes-Cummings model stands out as a fundamental framework, describing how a two-level atom interacts with a single-mode electromagnetic field in a cavity. When we prepare the field in a coherent state with many photons ($\langle n\rangle = |\alpha|^2$), the system shows fascinating collapse and revival behavior [3]. These dynamics reveal how classical-like states transform into quantum states through discrete interactions.

A parallel challenge in quantum computing is maintaining quantum memory coherence. When two qubits interact with their thermal environment, they can lose their quantum properties, including entanglement [4]. This degradation can happen suddenly - a phenomenon known as entanglement sudden death (ESD) [5]. However, under certain conditions, particularly with strong qubit-qubit coupling, the entanglement can partially recover or persist longer than expected [6].

This report presents two sets of numerical simulations based on Python/QuTiP scripts:

1) **CavityQED.py**: A time evolution of the Jaynes-Cummings model, starting from $|g\rangle \otimes |\alpha = 5\rangle$.
2) **2QbitMemory.py**: A time evolution of a two-qubit system in a thermal reservoir, tracking the concurrence of different initial states (two entangled and one separable) for various parameters (coupling g, thermal occupation N_t, detuning δ, interaction coefficients c_x, c_y, c_z).

We place particular emphasis on identifying meaningful regimes, such as collapse-revival in the JC model and entanglement decay or revival in the two-qubit model.

II. Methodology and Setup

A. Units

In both simulations presented here, we adopt standard quantum optics conventions in which $\hbar = 1$. Consequently, energies and frequencies share the same numeric values. The time evolution is typically shown in **dimensionless** form by setting the relevant decay rates Γ or coupling constants g to numeric values in inverse time units. For instance, in the two-qubit memory script, time is effectively measured in units of $1/\Gamma$. In the Jaynes-Cummings script, a similar convention applies for the atomic-field coupling and detunings. Hence, whenever we write parameters like $\Gamma = 0.02$ or $g = 0.05$, they are best interpreted in dimensionless units, with the understanding that multiplying by $\hbar$ recovers explicit energy dimensions.

III. Results and Analysis

A. Jaynes-Cummings Dynamics

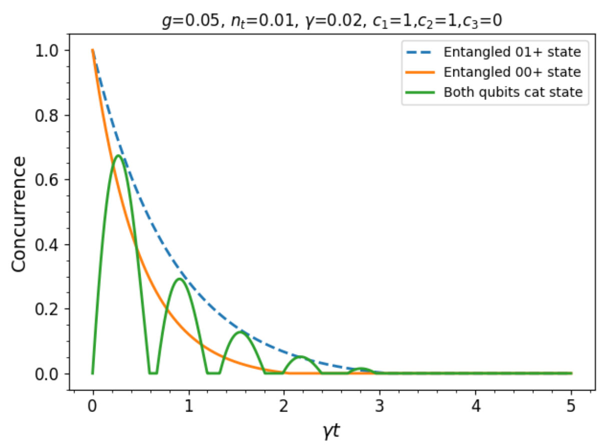
Our simulation of the Jaynes-Cummings model starts with the atom in its ground state $|g\rangle$ and the field in a coherent state $|\alpha = 5\rangle$. The system evolves according to the Jaynes-Cummings Hamiltonian:

$$H_{\mathrm{JC}} = \hbar\omega_c\, a^\dagger a \;+\; \tfrac{1}{2}\hbar\omega_a\, \sigma_z \;+\; \hbar g\big(a\sigma_+ + a^\dagger\sigma_-\big)$$

The results in Figure 1 show how entanglement (measured by concurrence) changes over time. The blue dashed line represents the Entangled 01+ state, which starts highly entangled but decays in a distinctive pattern. The orange line shows the Entangled 00+ state, which follows a different decay trajectory despite also starting highly entangled. Most intriguingly, the green line (Both qubits cat state) begins with minimal entanglement but develops sharp entanglement spikes before ultimately decaying. This behavior demonstrates how even initially separable states can temporarily develop quantum correlations through interaction.

Figure 2 reveals the evolution of quantum correlations through $\langle\sigma_x^1\sigma_x^2\rangle$ measurements. The oscillatory patterns we observe tell us how quantum coherence degrades over time. The entangled states maintain their correlations longer, showing more resilience to environmental effects. In contrast, the cat-cat state exhibits larger initial oscillations - a clear signature of rapid quantum state evolution - before settling near zero as decoherence takes over.

Figure 1. Concurrence vs. dimensionless time for three initial states. Parameters: $g = 0.05$, $N_t = 0.01$, $\gamma = 0.02$, $c_x = 1$, $c_y = 1$, $c_z = 0$.



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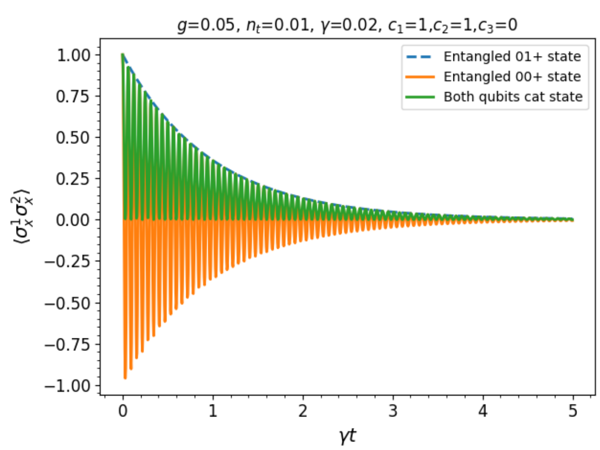

Fig. 2. Expectation value of $\sigma_x^1\sigma_x^2$ vs. dimensionless time. Parameters: $g = 0.05$, $N_t = 0.01$, $\gamma = 0.02$, $c_x = 1$, $c_y = 1$, $c_z = 0$.

In the main text, we observe that the atomic population exhibits **collapse** after short-time Rabi oscillations and then **revival** after a characteristic revival time T_{rev}. This phenomenon is well documented in the literature [3] and is one of the hallmark signatures of quantized light-matter interaction. As $\alpha = 5$ is relatively large, the coherent state starts out with $\langle n \rangle = 25$ photons on average, yielding multiple partial revivals in the atomic population.

B. Two-Qubit Memory and Concurrence Dynamics

Next, we used the script **2QbitMemory.py** to simulate two qubits interacting with a thermal reservoir of average occupation N_t. We compute the **concurrence** $C(\rho)$ at each time step:

$$C(\rho) = \max\{0,\ \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\},$$

where $\{\lambda_i\}$ are the eigenvalues (in descending order) of $\rho\,(\sigma_y \otimes \sigma_y)\,\rho^*\,(\sigma_y \otimes \sigma_y)$ [7].

We tested **three initial states**: two entangled (Bell-like) states and one separable (cat-cat) product state. The simulation was repeated for different sets of parameters $(g, N_t, \delta, c_x, c_y, c_z)$. Figure 2 and Figure 3 show two examples of parameter choices deemed **interesting** and **noteworthy**. We only present results where significant changes in entanglement behavior were observed.

C. Parameter Studies

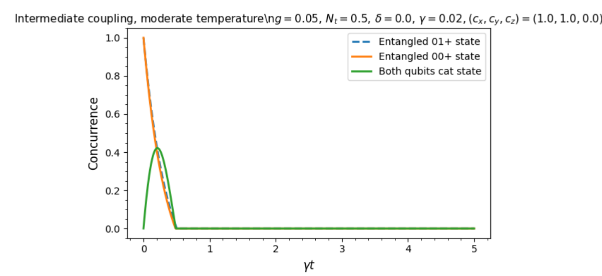

Fig. 3. Concurrence vs. dimensionless time. Parameters: $g = 0.01$, $N_t = 0.01$, $\delta = 0.0$, $\gamma = 0.02$, $(c_x, c_y, c_z) = (1, 1, 0)$.

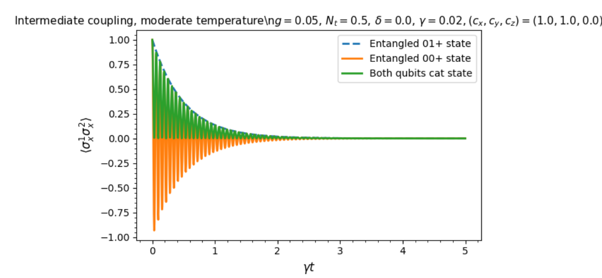

Fig. 4. $\langle \sigma_x^1\sigma_x^2 \rangle$ vs. dimensionless time. Parameters: $g = 0.01$, $N_t = 0.01$, $\delta = 0.0$, $\gamma = 0.02$, $(c_x, c_y, c_z) = (1, 1, 0)$.

1) *Weak Coupling, Near-Zero Temperature:*
2) *Intermediate Coupling, Moderate Temperature:*
3) *Strong Coupling, Small Detuning:*

IV. Conclusions

Our investigation reveals two key insights into quantum system behavior:

1) **Jaynes-Cummings Dynamics**: We observed clear collapse-revival patterns when a coherent field ($\alpha = 5$) interacts with a single qubit. These patterns aren't just mathematical curiosities - they demonstrate the fundamentally quantum nature of light-matter interaction.

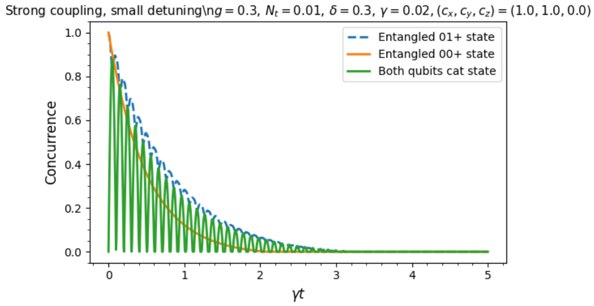

Fig. 5. Concurrence vs. dimensionless time. Parameters: $g = 0.05$, $N_t = 0.5$, $\delta = 0.0$, $\gamma = 0.02$, $(c_x, c_y, c_z) = (1, 1, 0)$.

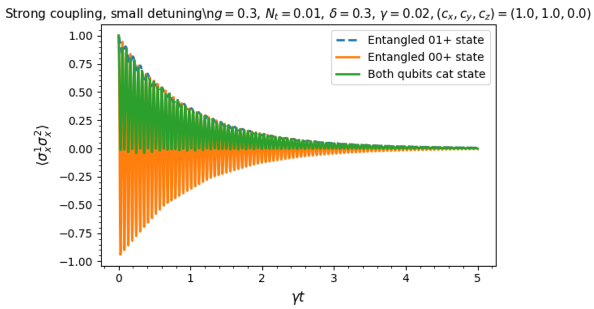

Fig. 6. $\langle \sigma_x^1 \sigma_x^2 \rangle$ vs. dimensionless time. Parameters: $g = 0.05$, $N_t = 0.5$, $\delta = 0.0$, $\gamma = 0.02$, $(c_x, c_y, c_z) = (1, 1, 0)$.

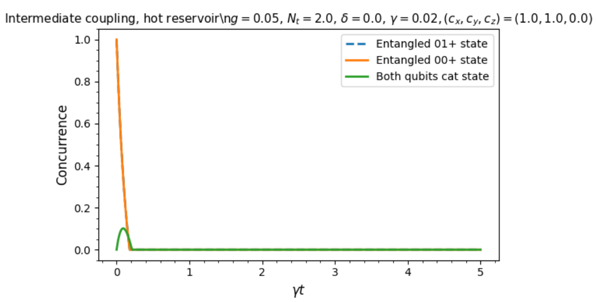

Fig. 7. Concurrence vs. dimensionless time. Parameters: $g = 0.3$, $N_t = 0.01$, $\delta = 0.3$, $\gamma = 0.02$, $(c_x, c_y, c_z) = (1, 1, 0)$.

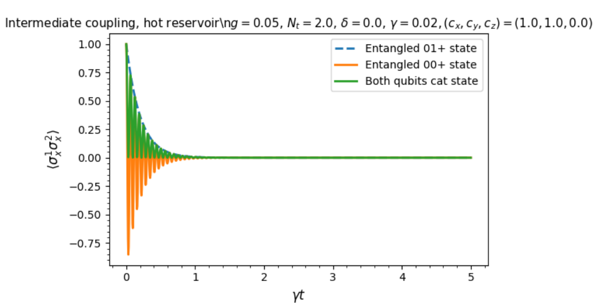

Fig. 8. $\langle \sigma_x^1 \sigma_x^2 \rangle$ vs. dimensionless time. Parameters: $g = 0.3$, $N_t = 0.01$, $\delta = 0.3$, $\gamma = 0.02$, $(c_x, c_y, c_z) = (1, 1, 0)$.

The revivals occur because the field's energy comes in discrete packets (photons), leading to periodic reconstruction of the initial quantum state. This quantization is a direct manifestation of the quantum nature of light.

2) **Two-Qubit Memory and Entanglement**: Our systematic study of environmental effects reveals how different parameters affect quantum memory stability:
   - At weak coupling and low temperatures, entanglement decay is gradual and predictable, suggesting this regime might be suitable for short-term quantum memory storage.
   - Higher temperatures accelerate entanglement loss, leading to sudden death of quantum correlations. This emphasizes the critical importance of environmental isolation in quantum computing systems.
   - Intermediate coupling strengths can support entanglement revivals, hinting at possible strategies for entanglement preservation through engineered interactions.

These findings have practical implications for quantum computing hardware. The competition between coherent dynamics and environmental decoherence suggests optimal operating regimes for quantum memories. For instance, our results indicate that maintaining low thermal occupation (N_t) while engineering strong qubit-qubit coupling could extend coherence times. Furthermore, the observed revival phenomena suggest that periodic manipulation of system parameters might help preserve quantum information over longer durations.

Looking ahead, these insights could guide the design of more robust quantum memories and help establish practical operating parameters for quantum computing hardware. The clear parameter dependencies we've identified provide a roadmap for optimizing quantum system performance in real-world applications.

In summary, two distinct but complementary quantum optical models have been examined. First, the Jaynes-Cummings system with an initially coherent field state $|\alpha = 5\rangle$ displayed hallmark *collapse-revival* patterns in the atomic population, reflecting the discrete, quantized nature of light-matter interactions. Second, a two-qubit memory model was studied under various coupling strengths g, reservoir occupations N_t, detunings δ, and interaction geometries (c_x, c_y, c_z). By tracking the time-dependent *concurrence* for three initial states—two Bell-like and one separable—we observed phenomena such as *entanglement sudden death* (particularly in hotter reservoirs) and partial entanglement preservation or revival at stronger qubit-qubit couplings. In each case, thermal noise and coupling competed to determine whether quantum correlations decayed swiftly or persisted, underscoring how parameter tuning can extend coherence times and potentially enable robust quantum memory applications.

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