PR

$$A = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}$$

① $\lambda I = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

② $A - \lambda I = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 2-\lambda & -12 \\ 1 & -5-\lambda \end{bmatrix}$

③ $\det \begin{bmatrix} 2-\lambda & -12 \\ 1 & -5-\lambda \end{bmatrix} = (2-\lambda)(-5-\lambda) - (-12)(1)$

$= -10 - 2\lambda + 5\lambda + \lambda^2 + 12$

$= \lambda^2 + 3\lambda + 2$

$= \lambda^2 + 2\lambda + \lambda + 2$

④ $= \lambda(\lambda+2) + 1(\lambda+2)$

$= (\lambda+2)(\lambda+1) = 0$

$\boxed{\lambda = -2}$ $\boxed{\lambda = -1}$

Eigen values are -2 and -1

⑤ $\begin{bmatrix} 2-\lambda & -12 \\ 1 & -5-\lambda \end{bmatrix}$

$\lambda = -2$

$$\begin{bmatrix} 2+2 & -12 \\ 1 & -5+2 \end{bmatrix} = \begin{bmatrix} 4 & -12 \\ 1 & -3 \end{bmatrix} = B$$

Solve

$B\bar{x} = \bar{0}$

$$\begin{bmatrix} 4 & -12 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4x_1 - 12x_2 \\ x_1 - 3x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$4x_1 - 12x_2 = 0$ $\qquad$ $x_1 - 3x_2 = 0$

So $x_1 = 3x_2$

Let $\boxed{x_2 = 1}$

$3(1) = \boxed{x_1 = 3}$

$\therefore$ Eigen vector for $\lambda = 8$ is $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$\lambda = -1$

$$\begin{bmatrix} 2+1 & -12 \\ 1 & -5+1 \end{bmatrix} = \begin{bmatrix} 3 & -12 \\ 1 & -4 \end{bmatrix} = C$$

Solve

$C\bar{x} = \bar{0}$

$$\begin{bmatrix} 3 & -12 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3x_1 - 12x_2 \\ x_1 - 4x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$3x_1 - 12x_2 = 0$ $\qquad$ $x_1 - 4x_2 = 0$

So $x_1 = 4x_2$ $\qquad$ let $\boxed{x_2 = 1}$ $\qquad$ $\boxed{x_1 = 4}$

So

Eigen vector for $\lambda = -1$ is $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

```python
import numpy as np
a = np.array([[2, -12], [1, -5]])
w, v = np.linalg.eig(a)

print(w)
print(v)
```

main

```
C:\Users\nirav\anaconda3\python.exe C:/Users/nirav/PycharmProjects/pythonProject/main.py
[-1. -2.]
[[0.9701425  0.9486833 ]
 [0.24253563 0.31622777]]

Process finished with exit code 0
```