Chapter 6: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts
- Efficient Pattern Mining Methods
- Pattern Evaluation
- Summary

1

What Is Pattern Discovery?

- What are patterns?
  - Patterns: A set of items, subsequences, or substructures that occur frequently together (or strongly correlated) in a data set
  - Patterns represent intrinsic and important properties of datasets
- Pattern discovery: Uncovering patterns from massive data sets
- Motivation examples:
  - What products were often purchased together?
  - What are the subsequent purchases after buying an iPad?
  - What code segments likely contain copy-and-paste bugs?
  - What word sequences likely form phrases in this corpus?

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Basic Concepts: k-Itemsets and Their Supports

- Itemset: A set of one or more items
- k-itemset: $X = \{x_1, \ldots, x_k\}$
  - Ex. {Beer, Nuts, Diaper} is a 3-itemset
- *(absolute) support* (*count*) of X, sup{X}: Frequency or the number of occurrences of an itemset X
  - Ex. sup{Beer} = 3
  - Ex. sup{Diaper} = 4
  - Ex. sup{Beer, Diaper} = 3
  - Ex. sup{Beer, Eggs} = 1

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk

- *(relative) support*, *s{X}*: The fraction of transactions that contains X (i.e., the probability that a transaction contains X)
  - Ex. s{Beer} = 3/5 = 60%
  - Ex. s{Diaper} = 4/5 = 80%
  - Ex. s{Beer, Eggs} = 1/5 = 20%

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Basic Concepts: Frequent Itemsets (Patterns)

- An itemset (or a pattern) X is *frequent* if the support of X is no less than a *minsup* threshold σ
- Let *σ = 50%* (σ: *minsup* threshold)

  For the given 5-transaction dataset
  - All the frequent 1-itemsets:
    - Beer: 3/5 (60%); Nuts: 3/5 (60%)
    - Diaper: 4/5 (80%); Eggs: 3/5 (60%)
  - All the frequent 2-itemsets:
    - {Beer, Diaper}: 3/5 (60%)
  - All the frequent 3-itemsets?
    - None

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk

- Why do these itemsets (shown on the left) form the complete set of frequent *k*-itemsets (patterns) for any *k*?
- **Observation**: We may need an efficient method to mine a complete set of frequent patterns

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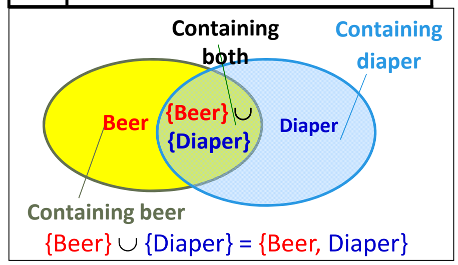
From Frequent Itemsets to Association Rules

- Compared with itemsets, rules can be more telling
  - Ex. *Diaper* → *Beer*
    - *Buying diapers may likely lead to buying beers*
- How strong is this rule? (support, confidence)
  - Measuring association rules: *X* → *Y* (s, c)
    - Both *X* and *Y* are itemsets
  - Support, *s*: The probability that a transaction contains X ∪ Y
    - Ex. s{Diaper, Beer} = 3/5 = 0.6 (i.e., 60%)
  - Confidence, *c: The conditional probability* that a transaction containing X also contains *Y*
    - Calculation: *c* = sup(X ∪ Y) / sup(X)
    - Ex. *c* = sup{Diaper, Beer}/sup{Diaper} = ¾ = 0.75

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk



Note: X ∪ Y: the union of two itemsets
- The set contains both X and Y

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Mining Frequent Itemsets and Association Rules

- **Association rule mining**
  - Given two thresholds: *minsup, minconf*
  - Find all of the rules, *X* → *Y* (s, c)
    - such that, s ≥ *minsup* and *c* ≥ *minconf*
- Let *minsup = 50%*
  - Freq. 1-itemsets: Beer: 3, Nuts: 3, Diaper: 4, Eggs: 3
  - Freq. 2-itemsets: {Beer, Diaper}: 3
- Let *minconf = 50%*
  - *Beer* → *Diaper* (60%, 100%)
  - *Diaper* → *Beer* (60%, 75%)

(Q: Are these all the rules?)

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk

- **Observations:**
  - Mining association rules and mining frequent patterns are very close problems
  - Scalable methods are needed for mining large datasets

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Challenge: There Are Too Many Frequent Patterns!

- A long pattern contains a combinatorial number of sub-patterns
- How many frequent itemsets does the following TDB_1 contain?
  - TDB_1: T_1: $\{a_1, ..., a_{50}\}$; T_2: $\{a_1, ..., a_{100}\}$
  - Assuming (absolute) *minsup* = 1
  - Let's have a try

  1-itemsets: $\{a_1\}$: 2, $\{a_2\}$: 2, ..., $\{a_{50}\}$: 2, $\{a_{51}\}$: 1, ..., $\{a_{100}\}$: 1,

  2-itemsets: $\{a_1, a_2\}$: 2, ..., $\{a_1, a_{50}\}$: 2, $\{a_1, a_{51}\}$: 1 ..., ..., $\{a_{99}, a_{100}\}$: 1,

  ..., ..., ..., ...

  99-itemsets: $\{a_1, a_2, ..., a_{99}\}$: 1, ..., $\{a_2, a_3, ..., a_{100}\}$: 1

  100-itemset: $\{a_1, a_2, ..., a_{100}\}$: 1
- The total number of frequent itemsets:

$$\binom{100}{1} + \binom{100}{2} + \binom{100}{3} + \cdots + \binom{100}{100} = 2^{100} - 1$$

A too huge set for any one to compute or store!

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Expressing Patterns in Compressed Form: Closed Patterns

- How to handle such a challenge?
- Solution 1: **Closed patterns**: A pattern (itemset) X is closed if X is *<u>frequent</u>*, and there exists *no super-pattern* Y ɔ X, *with the <u>same</u> support* as X
  - Let Transaction DB TDB_1: T_1: $\{a_1, ..., a_{50}\}$; T_2: $\{a_1, ..., a_{100}\}$
  - Suppose *minsup* = 1. How many closed patterns does TDB_1 contain?
    - Two: P_1: "$\{a_1, ..., a_{50}\}$: 2"; P_2: "$\{a_1, ..., a_{100}\}$: 1"
- Closed pattern is a lossless compression of frequent patterns
  - Reduces the # of patterns but does not lose the support information!
  - You will still be able to say: "$\{a_2, ..., a_{40}\}$: 2", "$\{a_5, a_{51}\}$: 1"

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Expressing Patterns in Compressed Form: Max-Patterns

- Solution 2: **Max-patterns**: A pattern X is a max-pattern if X is frequent and there exists no frequent super-pattern Y ɔ X
- Difference from close-patterns?
  - Do not care the real support of the sub-patterns of a max-pattern
  - Let Transaction DB TDB_1: T_1: $\{a_1, …, a_{50}\}$; T_2: $\{a_1, …, a_{100}\}$
  - Suppose *minsup* = 1. How many max-patterns does TDB_1 contain?
    - One: P: "$\{a_1, …, a_{100}\}$: 1"
- Max-pattern is a lossy compression!
  - We only know $\{a_1, …, a_{40}\}$ is frequent
  - But we do not know the real support of $\{a_1, …, a_{40}\}$, …, any more!
- Thus in many applications, mining close-patterns is more desirable than mining max-patterns

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Efficient Pattern Mining Methods

- **The Downward Closure Property of Frequent Patterns**
- The **Apriori** Algorithm
- Extensions or Improvements of Apriori
- Mining Frequent Patterns by Exploring Vertical Data Format
- **FPGrowth**: A Frequent Pattern-Growth Approach
- Mining Closed Patterns

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The Downward Closure Property of Frequent Patterns

- Observation: From $TDB_{1:}$ T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - We get a frequent itemset: $\{a_1, \ldots, a_{50}\}$
  - Also, its subsets are all frequent: $\{a_1\}, \{a_2\}, \ldots, \{a_{50}\}, \{a_1, a_2\}, \ldots, \{a_1, \ldots, a_{49}\}, \ldots$
  - There must be some hidden relationships among frequent patterns!
- The downward closure (also called "Apriori") property of frequent patterns
  - If **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**
  - Every transaction containing {beer, diaper, nuts} also contains {beer, diaper}
  - Apriori: Any subset of a frequent itemset must be frequent
- Efficient mining methodology
  - If any subset of an itemset S is infrequent, then there is no chance for S to be frequent—why do we even have to consider S!? A sharp knife for pruning!

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Apriori Pruning and Scalable Mining Methods

- Apriori pruning principle: If there is any itemset which is infrequent, its superset should not even be generated! (Agrawal & Srikant @VLDB'94, Mannila, et al. @ KDD' 94)
- Scalable mining Methods: Three major approaches
  - Level-wise, join-based approach: **Apriori** (Agrawal & Srikant@VLDB'94)
  - Vertical data format approach: Eclat (Zaki, Parthasarathy, Ogihara, Li @KDD'97)
  - Frequent pattern projection and growth: **FPgrowth** (Han, Pei, Yin @SIGMOD'00)

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Apriori: A Candidate Generation & Test Approach

- Outline of Apriori (level-wise, candidate generation and test)
  - Initially, scan DB once to get frequent 1-itemset
  - Repeat
    - Generate length-(k+1) candidate itemsets from length-k frequent itemsets
    - Test the candidates against DB to find frequent (k+1)-itemsets
    - Set k := k +1
  - Until no frequent or candidate set can be generated
  - Return all the frequent itemsets derived

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The Apriori Algorithm (Pseudo-Code)

C_k: Candidate itemset of size k

F_k : Frequent itemset of size k

K := 1;

F_k := {frequent items}; // frequent 1-itemset

While (F_k != ∅) **do {** // when F_k is non-empty

  C_{k+1} := candidates generated from F_k; // candidate generation

  Derive F_{k+1} by counting candidates in C_{k+1} with respect to *TDB* at minsup;

  k := k + 1

  }

return $\cup_k F_k$ // return F_k generated at each level

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The Apriori Algorithm—An Example

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

minsup = 2

1^{st} scan → C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

→ F_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2^{nd} scan → C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

→ F_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

3^{rd} scan → F_3

Itemset	sup
{B, C, E}	2

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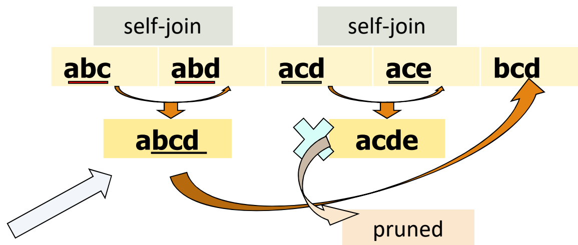
Apriori: Implementation Tricks

- How to generate candidates?
  - Step 1: self-joining F_k
  - Step 2: pruning
- Example of candidate-generation
  - $F_3 = \{abc, abd, acd, ace, bcd\}$
  - Self-joining: $F_3 * F_3$
    - *abcd* from *abc* and *abd*
    - *acde* from *acd* and *ace*
  - Pruning:
    - *acde* is removed because *ade* is not in F_3
  - $C_4 = \{abcd\}$



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Apriori: Improvements and Alternatives

- Reduce passes of transaction database scans
  - **Partitioning** (e.g., Savasere, et al., 1995)
  - Dynamic itemset counting (Brin, et al., 1997)
- Shrink the number of candidates
  - **Hashing** (e.g., DHP: Park, et al., 1995)
  - Pruning by support lower bounding (e.g., Bayardo 1998)
  - Sampling (e.g., Toivonen, 1996)
- Exploring special data structures
  - Tree projection (Agarwal, et al., 2001)
  - H-miner (Pei, et al., 2001)
  - Hypecube decomposition (e.g., LCM: Uno, et al., 2004)

To be discussed in subsequent slides

To be discussed in subsequent slides

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Partitioning: Scan Database Only Twice

- **Theorem**: *Any itemset that is potentially frequent in TDB must be frequent in at least one of the partitions of TDB*

Here is the proof!

TDB_1 + TDB_2 + . . . + TDB_k = TDB

$sup_1(X) < \sigma|TDB_1|$ . . . $sup_2(X) < \sigma|TDB_2|$ . . . $sup_k(X) < \sigma|TDB_k|$ $sup(X) < \sigma|TDB|$

- Method: Scan DB twice (A. Savasere, E. Omiecinski and S. Navathe, *VLDB'95)*
  - Scan 1: Partition database so that each partition can fit in main memory (why?)
    - Mine local frequent patterns in this partition
  - Scan 2: Consolidate global frequent patterns
    - Find global frequent itemset candidates (those frequent in at least one partition)
    - Find the true frequency of those candidates, by scanning TDB_i one more time

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Direct Hashing and Pruning (DHP)

- DHP (Direct Hashing and Pruning): (J. Park, M. Chen, and P. Yu, SIGMOD'95)
- Hashing: Different itemsets may have the same hash value: v = *hash*(itemset)
- 1st scan: When counting the 1-itemset, hash 2-itemset to calculate the bucket count
- Observation: A *k*-itemset cannot be frequent if its corresponding hashing bucket count is below the *minsup* threshold
- Example: At the 1st scan of TDB, count 1-itemset, and
  - Hash 2-itemsets in the transaction to its bucket
    - {ab, ad, ce}
    - {bd, be, de}
    - …
  - At the end of the first scan,
    - *if minsup = 80, remove ab, ad, ce, since count*{*ab, ad, ce*} < 80 (same for {yz, qs, wt})

Itemsets	Count
{*ab, ad, ce*}	35
{*bd, be, de*}	298
……	…
{*yz, qs, wt*}	58

Hash Table

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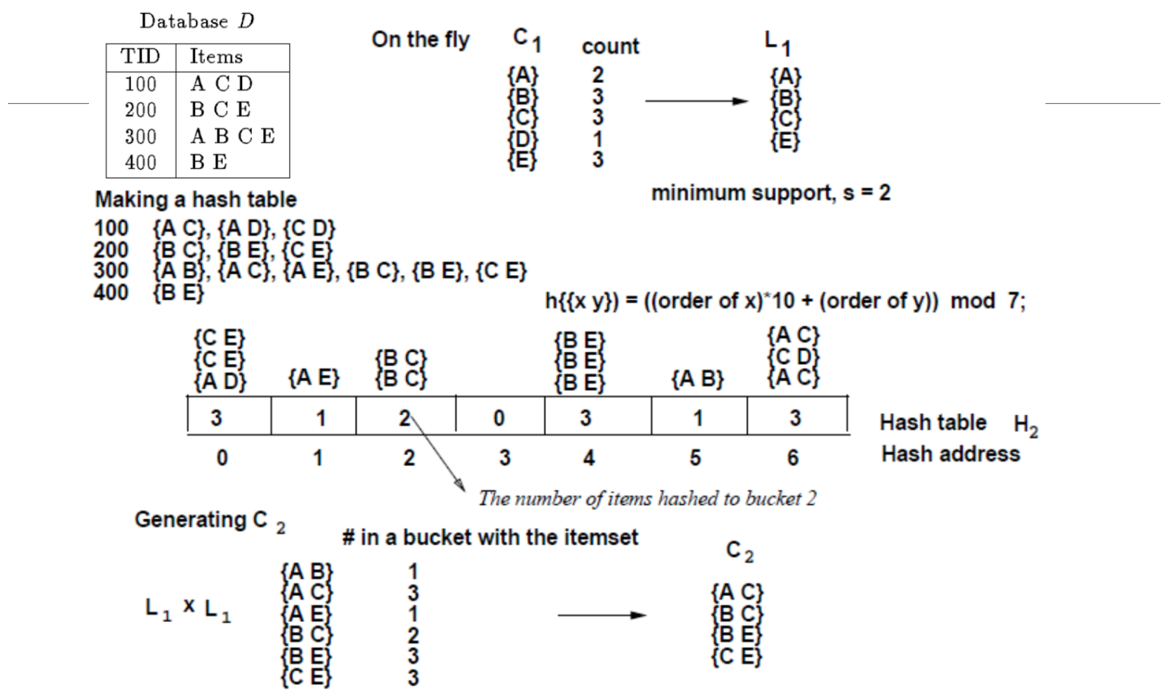

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Exploring Vertical Data Format: ECLAT

- ECLAT (Equivalence Class Transformation): A **depth-first** search algorithm using set intersection [Zaki et al. @KDD'97]
- Tid-List: List of transaction-ids containing an itemset
- Vertical format: $t(e) = \{T_{10}, T_{20}, T_{30}\}$; $t(a) = \{T_{10}, T_{20}\}$; $t(ae) = \{T_{10}, T_{20}\}$
- Properties of Tid-Lists
  - $t(X) = t(Y)$: X and Y always happen together (e.g., $t(ac) = t(d)$)
  - $t(X) \subset t(Y)$: transaction having X always has Y (e.g., $t(ac) \subset t(ce)$)
- Deriving frequent patterns based on vertical intersections
- Using diffset to accelerate mining
  - Only keep track of differences of tids
  - $t(e) = \{T_{10}, T_{20}, T_{30}\}$, $t(ce) = \{T_{10}, T_{30}\}$ → Diffset $(ce, e) = \{T_{20}\}$

A transaction DB in Horizontal Data Format

Tid	Itemset
10	a, c, d, e
20	a, b, e
30	b, c, e

The transaction DB in Vertical Data Format

Item	TidList
a	10, 20
b	20, 30
c	10, 30
d	10
e	10, 20, 30

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Why Mining Frequent Patterns by Pattern Growth?

- Apriori: A ***breadth-first*** *search* mining algorithm
  - First find the complete set of frequent k-itemsets
  - Then derive frequent (k+1)-itemset candidates
  - Scan DB again to find true frequent (k+1)-itemsets
- Motivation for a different mining methodology
  - Can we develop a ***depth-first*** *search* mining algorithm?
  - For a frequent itemset ρ, can subsequent search be confined to only those transactions containing ρ?
- Such thinking leads to a frequent pattern growth approach:
  - FPGrowth (J. Han, J. Pei, Y. Yin, "Mining Frequent Patterns without Candidate Generation," SIGMOD 2000)

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Example: Construct FP-tree from a Transaction DB

TID	Items in the Transaction	Ordered, frequent itemlist
100	{*f, a, c, d, g, i, m, p*}	*f, c, a, m, p*
200	{*a, b, c, f, l, m, o*}	*f, c, a, b, m*
300	{*b, f, h, j, o, w*}	*f, b*
400	{*b, c, k, s, p*}	*c, b, p*
500	{*a, f, c, e, l, p, m, n*}	*f, c, a, m, p*

1. Scan DB once, find single item frequent pattern:

   Let min_support = 3

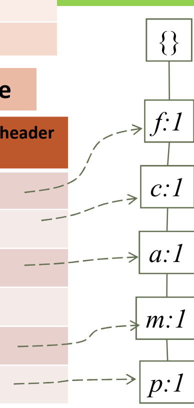
   f:4, a:3, c:4, b:3, m:3, p:3
2. Sort frequent items in frequency descending order, f-list

   F-list = f-c-a-b-m-p
3. Scan DB again, construct FP-tree
   - The frequent itemlist of each transaction is inserted as a branch, with shared sub-branches merged, counts accumulated

Header Table

Item	Frequency	header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	

After inserting the 1st frequent Itemlist: "*f, c, a, m, p*"



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Example: Construct FP-tree from a Transaction DB

TID	Items in the Transaction	Ordered, frequent itemlist
100	*{f, a, c, d, g, i, m, p}*	*f, c, a, m, p*
200	*{a, b, c, f, l, m, o}*	*f, c, a, b, m*
300	*{b, f, h, j, o, w}*	*f, b*
400	*{b, c, k, s, p}*	*c, b, p*
500	*{a, f, c, e, l, p, m, n}*	*f, c, a, m, p*

After inserting the 2nd frequent itemlist *"f, c, a, b, m"*

1. Scan DB once, find single item frequent pattern:
   Let min_support = 3
   f:4, a:3, c:4, b:3, m:3, p:3
2. Sort frequent items in frequency descending order, f-list
   F-list = f-c-a-b-m-p
3. Scan DB again, construct FP-tree
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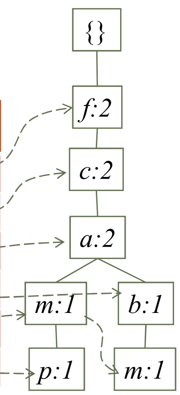

Example: Construct FP-tree from a Transaction DB

TID	Items in the Transaction	Ordered, frequent itemlist
100	*{f, a, c, d, g, i, m, p}*	*f, c, a, m, p*
200	*{a, b, c, f, l, m, o}*	*f, c, a, b, m*
300	*{b, f, h, j, o, w}*	*f, b*
400	*{b, c, k, s, p}*	*c, b, p*
500	*{a, f, c, e, l, p, m, n}*	*f, c, a, m, p*

After inserting all the frequent itemlists

1. Scan DB once, find single item frequent pattern:
   Let min_support = 3
   f:4, a:3, c:4, b:3, m:3, p:3
2. Sort frequent items in frequency descending order, f-list
   F-list = f-c-a-b-m-p
3. Scan DB again, construct FP-tree
   - The frequent itemlist of each transaction is inserted as a branch, with shared sub-branches merged, counts accumulated

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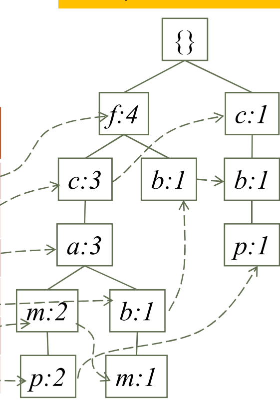
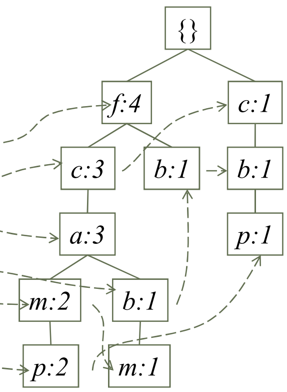


Mining FP-Tree: Divide and Conquer Based on Patterns and Data

- Pattern mining can be partitioned according to current patterns
  - Patterns containing *p*: *p*'s conditional database: *fcam:2, cb:1*
    - *p*'s conditional database (i.e., the database under the condition that *p* exists):
      - *transformed prefix paths* of item *p*
  - Patterns having m but no p: m's conditional database: *fca:2, fcab:1*
  - …… ……

min_support = 3

Item	Frequency	Header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



Conditional database of each pattern

Item	*Conditional database*
c	*f:3*
a	*fc:3*
b	*fca:1, f:1, c:1*
m	*fca:2, fcab:1*
p	*fcam:2, cb:1*

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Mine Each Conditional Database Recursively

min_support = 3

Conditional Data Bases

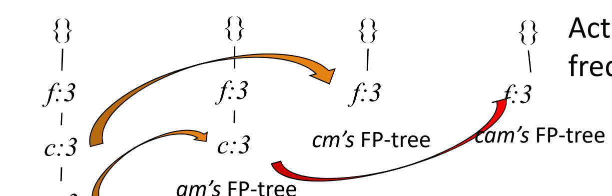
item	*cond. data base*
c	*f:3*
a	*fc:3*
b	*fca:1, f:1, c:1*
m	*fca:2, fcab:1*
p	*fcam:2, cb:1*

- For each conditional database
  - Mine single-item patterns
  - Construct its FP-tree & mine it

p's conditional DB: ***fcam:2, cb:1 → c: 3***

m's conditional DB: ***fca:2, fcab:1 → fca: 3***

b's conditional DB: ***fca:1, f:1, c:1 → ϕ***



m's FP-tree

Then, mining m's FP-tree: fca:3

Actually, for single branch FP-tree, all the frequent patterns can be generated in one shot

m: 3
fm: 3, cm: 3, am: 3
fcm: 3, fam:3, cam: 3
fcam: 3

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Practice (what is the DB and FP-tree of f?)

TID	Items
1	acdef
2	abe
3	cefg
4	acdf

Let minsupport = 2

a:3 c:3 e:3 f:3 d:2
F-List: a-c-e-f-d

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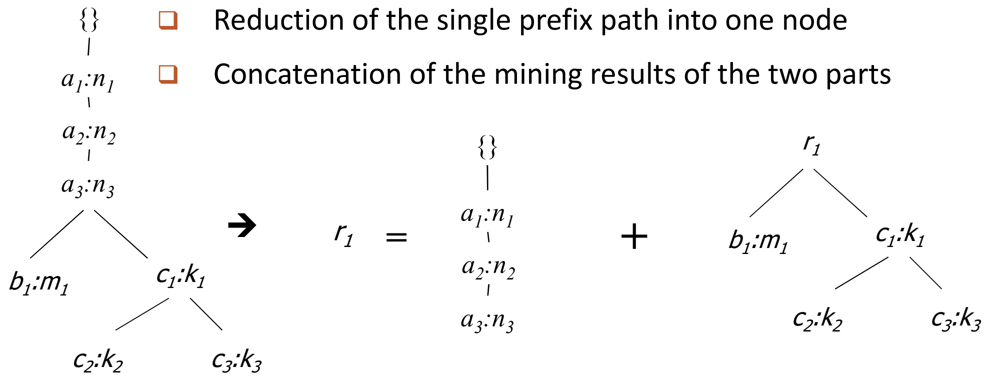
A Special Case: Single Prefix Path in FP-tree

- Suppose a (conditional) FP-tree T has a shared single prefix-path P
- Mining can be decomposed into two parts
  - Reduction of the single prefix path into one node
  - Concatenation of the mining results of the two parts



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FPGrowth: Mining Frequent Patterns by Pattern Growth

- Essence of frequent pattern growth (FPGrowth) methodology
  - Find frequent single items and partition the database based on each such single item pattern
  - Recursively grow frequent patterns by doing the above for each *partitioned database* (also called the pattern's *conditional database*)
  - To facilitate efficient processing, an efficient data structure, FP-tree, can be constructed
- Mining becomes
  - Recursively construct and mine (conditional) FP-trees
  - **Until the resulting FP-tree is empty, or until it contains only one path—single path will generate all the combinations of its sub-paths, each of which is a frequent pattern**

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Pattern Evaluation

- Limitation of the Support-Confidence Framework
- Interestingness Measures: Lift and χ^2
- Null-Invariant Measures
- Comparison of Interestingness Measures

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How to Judge if a Rule/Pattern Is Interesting?

- Pattern-mining will generate a large set of patterns/rules
  - Not all the generated patterns/rules are interesting
- Interestingness measures: Objective vs. subjective
  - Objective interestingness measures
    - Support, confidence, correlation, …
  - Subjective interestingness measures:
      - Different users may judge interestingness differently
    - Let a user specify
      - Query-based: Relevant to a user's particular request
    - Judge against one's knowledge-base
      - unexpected, freshness, timeliness

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Limitation of the Support-Confidence Framework

- Are *s* and *c* interesting in association rules: "A $\Rightarrow$ B" [*s*, *c*]? Be careful!
- Example: Suppose one school may have the following statistics on # of students who may play basketball and/or eat cereal:

	play-basketball	not play-basketball	sum (row)
eat-cereal	400	350	750
not eat-cereal	200	50	250
sum(col.)	600	400	1000

2-way contingency table

- Association rule mining may generate the following:
  - *play-basketball* $\Rightarrow$ *eat-cereal* [40%, 66.7%] (higher s & c)
- But this strong association rule is misleading: The overall % of students eating cereal is 75% > 66.7%, a more telling rule:
  - ¬ *play-basketball* $\Rightarrow$ *eat-cereal* [35%, 87.5%] (high s & c)

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Interestingness Measure: Lift

- Measure of dependent/correlated events: **lift**

$$lift(B, C) = \frac{c(B \rightarrow C)}{s(C)} = \frac{s(B \cup C)}{s(B) \times s(C)}$$

- Lift(B, C) may tell how B and C are correlated
  - Lift(B, C) = 1: B and C are independent
  - > 1: positively correlated
  - < 1: negatively correlated
- For our example,

$$lift(B,C) = \frac{400/1000}{600/1000 \times 750/1000} = 0.89$$

$$lift(B,\neg C) = \frac{200/1000}{600/1000 \times 250/1000} = 1.33$$

- Thus, B and C are negatively correlated since lift(B, C) < 1;
  - B and ¬C are positively correlated since lift(B, ¬C) > 1

Lift is more telling than s & c

	B	¬B	Σ_{row}
C	400	350	750
¬C	200	50	250
$\Sigma_{col.}$	600	400	1000

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Interestingness Measure: χ^2

- Another measure to test correlated events: χ^2

$$\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected}$$

- For the table on the right,

$$\chi^2 = \frac{(400-450)^2}{450} + \frac{(350-300)^2}{300} + \frac{(200-150)^2}{150} + \frac{(50-100)^2}{100} = 55.56$$

	B	¬B	Σ_{row}
C	400 (450)	350 (300)	750
¬C	200 (150)	50 (100)	250
Σ_{col}	600	400	1000

Expected value

Observed value

- By consulting a table of critical values of the χ^2 distribution, one can conclude that the chance for B and C to be independent is very low (< 0.01) https://www.medcalc.org/manual/chi-square-table.php
- χ^2-test shows B and C are negatively correlated since the expected value is 450 but the observed is only 400
- Thus, χ^2 is also more telling than the support-confidence framework

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Lift and χ^2 : Are They Always Good Measures?

- **Null** transactions: Transactions that contain neither B nor C
- Let's examine the new dataset D
  - BC (100) is much rarer than B¬C (1000) and ¬BC (1000), but there are many ¬B¬C (100000)
  - Unlikely B & C will happen together!
- But, Lift(B, C) = 8.44 >> 1 (Lift shows B and C are strongly positively correlated!)
- χ^2 = 670: Observed(BC) >> expected value (11.85)
- *Too many null transactions may "spoil the soup"!*

	B	¬B	Σ_{row}
C	100	1000	1100
¬C	1000	100000	101000
$\Sigma_{col.}$	1100	101000	102100

null transactions

Contingency table with expected values added

	B	¬B	Σ_{row}
C	100 (11.85)	1000	1100
¬C	1000 (988.15)	100000	101000
$\Sigma_{col.}$	1100	101000	102100

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Interestingness Measures & Null-Invariance

- *Null invariance:* Value does not change with the # of null-transactions
- A few interestingness measures: Some are null invariant

Measure	Definition	Range	Null-Invariant?
$\chi^2(A,B)$	$\sum_{i,j} \frac{(e(a_i,b_j)-o(a_i,b_j))^2}{e(a_i,b_j)}$	$[0, \infty]$	No
$Lift(A,B)$	$\frac{s(A \cup B)}{s(A) \times s(B)}$	$[0, \infty]$	No
$Allconf(A,B)$	$\frac{s(A \cup B)}{max\{s(A), s(B)\}}$	$[0, 1]$	Yes
$Jaccard(A,B)$	$\frac{s(A \cup B)}{s(A)+s(B)-s(A \cup B)}$	$[0, 1]$	Yes
$Cosine(A,B)$	$\frac{s(A \cup B)}{\sqrt{s(A) \times s(B)}}$	$[0, 1]$	Yes
$Kulczynski(A,B)$	$\frac{1}{2}\left(\frac{s(A \cup B)}{s(A)} + \frac{s(A \cup B)}{s(B)}\right)$	$[0, 1]$	Yes
$MaxConf(A,B)$	$max\{\frac{s(A \cup B)}{s(A)}, \frac{s(A \cup B)}{s(B)}\}$	$[0, 1]$	Yes

X^2 *and lift are not null-invariant*

Jaccard, consine, AllConf, MaxConf, and *Kulczynski* are *null-invariant measures*

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Null Invariance: An Important Property

- Why is null invariance crucial for the analysis of massive transaction data?
  - Many transactions may contain neither milk nor coffee!

milk vs. coffee contingency table

	milk	$\neg$*milk*	Σ_{row}
coffee	mc	$\neg mc$	c
$\neg$*coffee*	$m\neg c$	$\neg m \neg c$	$\neg c$
Σ_{col}	m	$\neg m$	Σ

- Lift and χ^2 are not null-invariant: not good to evaluate data that contain too many or too few null transactions!
- Many measures are not null-invariant!

Null-transactions w.r.t. m and c

Data set	mc	$\neg mc$	$m\neg c$	$\neg m \neg c$	χ^2	*Lift*
D_1	10,000	1,000	1,000	100,000	90557	9.26
D_2	10,000	1,000	1,000	100	0	1
D_3	100	1,000	1,000	100,000	670	8.44
D_4	1,000	1,000	1,000	100,000	24740	25.75
D_5	1,000	100	10,000	100,000	8173	9.18
D_6	1,000	10	100,000	100,000	965	1.97

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What Measures to Choose for Effective Pattern Evaluation?

- Null value cases are predominant in many large datasets
  - Neither milk nor coffee is in most of the baskets; neither Mike nor Jim is an author in most of the papers; ……
- *Null-invariance* is an important property
- Lift, χ^2 and cosine are good measures if null transactions are not predominant
  - Otherwise, *Kulczynski* + *Imbalance Ratio* should be used to judge the interestingness of a pattern
- Exercise: Mining research collaborations from research bibliographic data
  - Find a group of frequent collaborators from research bibliographic data (e.g., DBLP)
  - Can you find the likely advisor-advisee relationship and during which years such a relationship existed?
  - Ref.: C. Wang, J. Han, Y. Jia, J. Tang, D. Zhang, Y. Yu, and J. Guo, "Mining Advisor-Advisee Relationships from Research Publication Networks", KDD'10

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Chapter 6: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts
- Efficient Pattern Mining Methods
- Pattern Evaluation
- Summary

Summary

- Basic Concepts
  - What Is Pattern Discovery? Why Is It Important?
  - Basic Concepts: Frequent Patterns and Association Rules
  - Compressed Representation: Closed Patterns and Max-Patterns
- Efficient Pattern Mining Methods
  - The Downward Closure Property of Frequent Patterns
  - The Apriori Algorithm
  - Extensions or Improvements of Apriori
  - Mining Frequent Patterns by Exploring Vertical Data Format
  - FPGrowth: A Frequent Pattern-Growth Approach
  - Mining Closed Patterns
- Pattern Evaluation
  - Interestingness Measures in Pattern Mining
  - Interestingness Measures: Lift and χ^2
  - Null-Invariant Measures
  - Comparison of Interestingness Measures

Recommended Readings (Basic Concepts)

- R. Agrawal, T. Imielinski, and A. Swami, "Mining association rules between sets of items in large databases", in Proc. of SIGMOD'93
- R. J. Bayardo, "Efficiently mining long patterns from databases", in Proc. of SIGMOD'98
- N. Pasquier, Y. Bastide, R. Taouil, and L. Lakhal, "Discovering frequent closed itemsets for association rules", in Proc. of ICDT'99
- J. Han, H. Cheng, D. Xin, and X. Yan, "Frequent Pattern Mining: Current Status and Future Directions", Data Mining and Knowledge Discovery, 15(1): 55-86, 2007

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Recommended Readings (Efficient Pattern Mining Methods)

- R. Agrawal and R. Srikant, "Fast algorithms for mining association rules", VLDB'94
- A. Savasere, E. Omiecinski, and S. Navathe, "An efficient algorithm for mining association rules in large databases", VLDB'95
- J. S. Park, M. S. Chen, and P. S. Yu, "An effective hash-based algorithm for mining association rules", SIGMOD'95
- S. Sarawagi, S. Thomas, and R. Agrawal, "Integrating association rule mining with relational database systems: Alternatives and implications", SIGMOD'98
- M. J. Zaki, S. Parthasarathy, M. Ogihara, and W. Li, "Parallel algorithm for discovery of association rules", Data Mining and Knowledge Discovery, 1997
- J. Han, J. Pei, and Y. Yin, "Mining frequent patterns without candidate generation", SIGMOD'00
- M. J. Zaki and Hsiao, "CHARM: An Efficient Algorithm for Closed Itemset Mining", SDM'02
- J. Wang, J. Han, and J. Pei, "CLOSET+: Searching for the Best Strategies for Mining Frequent Closed Itemsets", KDD'03
- C. C. Aggarwal, M.A., Bhuiyan, M. A. Hasan, "Frequent Pattern Mining Algorithms: A Survey", in Aggarwal and Han (eds.): Frequent Pattern Mining, Springer, 2014

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Recommended Readings (Pattern Evaluation)

- C. C. Aggarwal and P. S. Yu. A New Framework for Itemset Generation. PODS'98
- S. Brin, R. Motwani, and C. Silverstein. Beyond market basket: Generalizing association rules to correlations. SIGMOD'97
- M. Klemettinen, H. Mannila, P. Ronkainen, H. Toivonen, and A. I. Verkamo. Finding interesting rules from large sets of discovered association rules. CIKM'94
- E. Omiecinski. Alternative Interest Measures for Mining Associations. TKDE'03
- P.-N. Tan, V. Kumar, and J. Srivastava. Selecting the Right Interestingness Measure for Association Patterns. KDD'02
- T. Wu, Y. Chen and J. Han, Re-Examination of Interestingness Measures in Pattern Mining: A Unified Framework, Data Mining and Knowledge Discovery, 21(3):371-397, 2010