**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer: B. 0.2676

We have normal distribution with mean=45 minutes, std=8 mins.

Let X be the time required to repair customer’s car. To finish in 1 hour, he has 50 mins

left since he plans to work 10 mins after the car is dropped off.

P(X<=50)= stats.norm.cdf(50,45,8)=0.734

Therefore the probability that service manager cannot meet his commitment is

=1-0.734

=0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Soln: A. Probability of employees older than 44

=P(X>44)=1-P(X<44)=1-stats.norm.cdf(44,38,6)=0.1586=15.86%

Probability of employees having age between 38 and 44

=P(38 <=X =>44)

= stats.norm.cdf(44,38,6)-stats.norm.cdf(38,38,6)

=0.3413

=34.13%

Therefore More employees at the processing center are older than 44 than between

38 and 44 is False.

B.P(X<30)=stats.norm.cdf(30,38,6)=0.0912=9.12%

Number of employees with probability of 9.12% age less than 30

= 0.0912\*400

=36

Therefore a training program for employees under the age of 30 at the center would be

expected to attract about 36 employees is True

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Soln: *X1* =*N*(μ, σ2) and *X2*= *N*(μ, σ2) are two independent random variables.

If X1 and X2 are independent, identically distributed random variables then their sum and

Differences are also normally distributed with their means and variance being added.

Means and variances are parameters of normal distribution.

Then X1+X2=N(μ+ μ, σ2 + σ2 )

X1-X2=N(μ- μ, σ2 - σ2 )

If X1 and X2 are independent, identically distributed random variables, then 2X1 will also

be normally distributed with mean and variance being multiplied by 2.

2X1=N(2μ, ,(2σ)2)

=N(2μ, 4σ2)

Therefore difference between 2X1 and X1+X2

= 2X1-(X1+X2)

= X1+X2

= N(μ+ μ, σ2 + σ2 )

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Soln: Answer D.48.5,151.5

Mean=100,std=20

Probability of the random variable taking a value between them is 0.99 i.e P(a<0.99>b)

Area of the curve is 1. Therefore alpha at both sides of the tail is 0.005

To calculate z score,

Z score for 0.5th percentage = stats.norm.ppf(0.005)=-2.575

Z score for 99.5th percentage = stats.norm.ppf(0.995)=2.575

Therefore P(a<0.99>b) = Z=a-100/20

a=20z+100=20(-2.575)+100=48.5

Z=b-100/20

b=20z+100=20(2.575)+100=151.5

Therefor a=48.5,b=151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Soln: Solved in (AssignmentSet2\_Q5.ipynb)