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10 Solved (with steps)

1)

Premises:

1. P→QP \to Q
2. PP
   Goal: ?
   Steps: MP on (1),(2) ⇒ QQ.
   Conclusion: QQ.

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2)

Premises:

1. P→QP \to Q
2. ¬Q\lnot Q
   Goal: ?
   Steps: MT on (1),(2) ⇒ ¬P\lnot P.
   Conclusion: ¬P\lnot P.

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3)

Premises:

1. P∨RP \lor R
2. ¬P\lnot P
   Goal: ?
   Steps: DS on (1),(2) ⇒ RR.
   Conclusion: RR.

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4)

Premises:

1. P→QP \to Q
2. Q→RQ \to R
3. PP
   Goal: ?
   Steps: HS on (1),(2) ⇒ P→RP \to R; MP with (3) ⇒ RR.
   Conclusion: RR.

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5)

Premises:

1. P∧QP \land Q
   Goal: ?
   Steps: Simplification ⇒ from (1) get PP (also QQ).
   Conclusion: PP (and separately QQ).

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6)

Premises:

1. PP
2. QQ
   Goal: P∧QP \land Q
   Steps: Conjunction (∧-Intro) on (1),(2).
   Conclusion: P∧QP \land Q.

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7)

Premises:

1. P→(Q∨R)P \to (Q \lor R)
2. ¬Q\lnot Q
3. PP
   Goal: ?
   Steps: MP on (1),(3) ⇒ Q∨RQ \lor R; DS with (2) ⇒ RR.
   Conclusion: RR.

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8)

Premises:

1. (P∧Q)→R(P \land Q) \to R
2. R→SR \to S
3. PP
4. QQ
   Goal: ?
   Steps: From (3),(4) via ∧-Intro ⇒ P∧QP \land Q; MP with (1) ⇒ RR; MP with (2) ⇒ SS.
   Conclusion: SS.

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9)

Premises:

1. P→QP \to Q
2. R→¬QR \to \lnot Q
3. P∨RP \lor R
   Goal: ?
   Steps: Case analysis:

- If PP: MP (1) ⇒ QQ.
- If RR: MP (2) ⇒ ¬Q\lnot Q.
  Since cases yield QQ **or** ¬Q\lnot Q, **no single determined conclusion** follows.
  Conclusion: No valid single conclusion.

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10)

Premises:

1. ¬(P∧Q)\lnot(P \land Q)
   Goal: ¬P∨¬Q\lnot P \lor \lnot Q
   Steps: By De Morgan's Law, ¬(P∧Q)≡¬P∨¬Q\lnot(P \land Q) \equiv \lnot P \lor \lnot Q.
   Conclusion: ¬P∨¬Q\lnot P \lor \lnot Q.

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40 Unsolved (you try!)

For each set of premises, derive the strongest valid conclusion (or state "No valid conclusion").

Direct inference & DS/MP/MT

1. P→Q, PP \to Q,\; P ⇒ ?
2. P→Q, ¬QP \to Q,\; \lnot Q ⇒ ?
3. P∨Q, ¬PP \lor Q,\; \lnot P ⇒ ?
4. P∨Q, ¬QP \lor Q,\; \lnot Q ⇒ ?
5. P→Q, QP \to Q,\; Q ⇒ ?
6. P→(Q∧R), PP \to (Q \land R),\; P ⇒ ?
7. P→(Q∨R), ¬Q, PP \to (Q \lor R),\; \lnot Q,\; P ⇒ ?
8. (P∧Q)→R, ¬R(P \land Q) \to R,\; \lnot R ⇒ ?
9. P→Q, R→S, P∧RP \to Q,\; R \to S,\; P \land R ⇒ ?
10. ¬(P∨Q)\lnot(P \lor Q) ⇒ ?

Chain reasoning / HS

11. P→Q, Q→R, PP \to Q,\; Q \to R,\; P ⇒ ?
12. P→Q, Q→RP \to Q,\; Q \to R ⇒ ? (derivable conditional)
13. (P∨Q)→R, P(P \lor Q) \to R,\; P ⇒ ?
14. (P∧Q)→R, P, Q(P \land Q) \to R,\; P,\; Q ⇒ ?
15. P→Q, R→S, P∨RP \to Q,\; R \to S,\; P \lor R ⇒ ?

Contraposition & contradictions

16. P→Q, ¬QP \to Q,\; \lnot Q ⇒ ?
17. (P∧Q)→R, ¬R(P \land Q) \to R,\; \lnot R ⇒ ?
18. P→¬Q, QP \to \lnot Q,\; Q ⇒ ?
19. ¬¬P\lnot\lnot P ⇒ ?
20. P→Q, P→¬QP \to Q,\; P \to \lnot Q ⇒ ?

Mixed connectives

21. P∨(Q∧R), ¬PP \lor (Q \land R),\; \lnot P ⇒ ?
22. (P→Q)∧(P→R), P(P \to Q) \land (P \to R),\; P ⇒ ?
23. (P→Q)∧(R→S), P∨R(P \to Q) \land (R \to S),\; P \lor R ⇒ ?
24. P↔Q, PP \leftrightarrow Q,\; P ⇒ ?
25. P↔Q, ¬PP \leftrightarrow Q,\; \lnot P ⇒ ?

Validity / No-conclusion traps

26. P→Q, QP \to Q,\; Q ⇒ ?
27. P→Q, ¬PP \to Q,\; \lnot P ⇒ ?
28. P∨Q, PP \lor Q,\; P ⇒ ?
29. P∨Q, QP \lor Q,\; Q ⇒ ?
30. P→Q, ¬Q→R, ¬RP \to Q,\; \lnot Q \to R,\; \lnot R ⇒ ?

Derivations with equivalences

31. ¬(P∨Q)\lnot(P \lor Q) ⇒ rewrite to CNF form
32. ¬(P∧Q)\lnot(P \land Q) ⇒ rewrite to DNF-style disjunction
33. P→QP \to Q ⇒ rewrite with only ∨,¬\lor,\lnot
34. P↔QP \leftrightarrow Q ⇒ rewrite as conjunction of two conditionals

Small truth/evaluation tasks (treat variables as booleans)

35. Is P∨¬PP \lor \lnot P a tautology? ⇒ True/False
36. Is P∧¬PP \land \lnot P satisfiable? ⇒ Yes/No

37. If PP is False and QQ is True, evaluate P→QP \to Q.

38. If PP is True and QQ is False, evaluate P↔QP \leftrightarrow Q.

39. If PP is False, evaluate (P→Q)∧(Q→P)(P \to Q) \land (Q \to P) when QQ is True.

40. If PP is True and QQ is True, evaluate (P∧Q)→(P∨Q)(P \land Q) \to (P \lor Q).

Answer Key (for the 40 unsolved)

1) QQ | **2)** ¬P\lnot P | **3)** QQ | **4)** PP | **5)** No valid conclusion | **6)** Q∧RQ \land R | **7)** RR | **8)** ¬(P∧Q)\lnot(P \land Q) | **9)** Q∧SQ \land S | **10)** ¬P∧¬Q\lnot P \land \lnot Q

11) RR | **12)** P→RP \to R | **13)** RR | **14)** RR | **15)** No valid single conclusion (either QQ or SS; not fixed)

16) ¬P\lnot P | **17)** ¬(P∧Q)\lnot(P \land Q) (i.e., ¬R\lnot R gives at least one of ¬P\lnot P or ¬Q\lnot Q, but not which) | **18)** ¬P\lnot P | **19)** PP | **20)** ¬P\lnot P (from P→QP\to Q and P→¬QP\to \lnot Q ⇒ P→(Q∧¬Q)P\to(Q\land \lnot Q) ⇒ ¬P\lnot P)

21) Q∧RQ \land R | **22)** Q∧RQ \land R | **23)** No valid single conclusion (either QQ or SS, case-dependent) | **24)** QQ | **25)** ¬Q\lnot Q

26) No valid conclusion (affirming the consequent) | **27)** No valid conclusion | **28)** PP | **29)** QQ | **30)** QQ (since ¬R\lnot R forces ¬¬Q\lnot\lnot Q ⇒ QQ)

31) ¬P∧¬Q\lnot P \land \lnot Q | **32)** ¬P∨¬Q\lnot P \lor \lnot Q | **33)** ¬P∨Q\lnot P \lor Q | **34)** (P→Q)∧(Q→P)(P \to Q) \land (Q \to P)

35) True (tautology) | **36)** No | **37)** True | **38)** False | **39)** False (True∧False) | **40)** True

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