What is Symbol-Based Logic?

Symbol-based logic uses **symbols** to represent logical statements (propositions) and uses logical operators to connect them. This makes reasoning more clear and systematic.

Basic Symbols in Logic:

- **Propositions:** Usually represented by capital letters like **P, Q, R, S**
  (Each stands for a statement that can be either true or false)

- **Logical Connectives:**

  - $\neg P$ = NOT P (negation)

  - $P \wedge Q$ = P AND Q (conjunction)

  - $P \vee Q$ = P OR Q (disjunction)

  - $P \rightarrow Q$ = If P then Q (implication)

  - $P \leftrightarrow Q$ = P if and only if Q (biconditional)

Example 1: Symbolization and Explanation

Problem:

Consider these statements:

- P: "It is raining."

- Q: "The ground is wet."

Express the statement:
"If it is raining, then the ground is wet."

Symbolization:

- "It is raining" = **P**

- "The ground is wet" = **Q**

- The statement "If it is raining, then the ground is wet." = $P \rightarrow Q$

Explanation:

- The implication $P \rightarrow Q$ means that if P is true, then Q must be true.

- If it is raining (P is true), then the ground will be wet (Q is true).

- If it is not raining, the statement says nothing about whether the ground is wet or not.

Example 2: Using Logical Connectives

Problem:

Express this statement symbolically:
"It is raining and it is cold."

Symbolization:

- P: "It is raining"

- R: "It is cold"

- Statement: "It is raining AND it is cold." = **P ∧ R**

Explanation:

- **P ∧ R** means both P and R must be true for the whole statement to be true.

- If either it is not raining or it is not cold, the entire statement is false.

Example 3: More Complex Statement

Problem:

Express this in symbols:
"Either it is not raining or the ground is wet."

Symbolization:

- P: "It is raining."

- Q: "The ground is wet."

- Statement: "Either it is NOT raining OR the ground is wet." = **¬P ∨ Q**

Explanation:

- **¬P** means "It is NOT raining."

- The disjunction **¬P ∨ Q** means at least one of these is true: either it is not raining or the ground is wet.

- The whole statement is false only if both it is raining AND the ground is NOT wet.

Truth Table Example for P → Q

P (Raining)	Q (Ground Wet)	P → Q (If P then Q)
True	True	True
True	False	False
False	True	True
False	False	True

Explanation:

- The only time **P → Q** is false is when P is true but Q is false.
- If it's raining but the ground is not wet, the implication breaks.

Summary

- Use capital letters for statements.
- **Use logical connectives (¬, ∧, ∨, →, ↔)** to connect statements.
- Analyze using truth tables or logical equivalences.