CSCE 5150 – Analysis of Computer Algorithms

Homework No. 1 - Introduction & Growth of Functions

Due on Friday, Sebtember 16, 2022

Q1. [*5 points*] Suppose we are comparing implementations of insertion sort and merge sort on the same machine. For inputs of size n, insertion sort runs in $8n^2$ steps, while merge sort runs in $64n \lg n$ steps. For which values of n does insertion sort beat merge sort? (Exercise 1.2-2)

Q2. [*5 points*] We can express insertion sort as a recursive procedure as follows. In order to sort **A [1 .. *n*]**, we recursively sort **A [1 .. *n*-1]** and then insert **A [*n*]** into the sorted array **A [1 .. *n*-1]**. Write a recurrence $T(n)$ for the running time of this recursive version of insertion sort. (Exercise 2.3-4)

Q3. [*5 points*] Describe a $\Theta(n \lg n)$ - time algorithm that, given a set S of n integers and another integer x, determines whether there exist two elements in S whose sum is exactly x. (Exercise 2.3-7)

Q4. [*5 points*] Sort all the functions below in increasing order of asymptotic (**big-O**) growth. If some have the same asymptotic growth, then be sure to indicate that. As usual, **lg** means base 2.

1. $5n$
2. n^4
3. $4 \lg n$
4. $n^{n/4}$
5. $n^{1/2} \lg^4 n$

Q5. [*5 points*] Prove that $2^{n+1} = O(2^n)$. (Exercise 3.1-4) (Hint: Try to satisfy the definition of ***O*-notation** with some constants $c, n_0 > 0$)