CSCE 5310 - Empirical Analysis
Assignment 2

1. In a box there are 3 red cards and 5 blue cards. The red cards are marked with the numbers 1, 2, and 3, and the blue cards are marked with the numbers 1, 2, 3, 4, and 5. The cards are well-shuffled. You reach into the box (you cannot see into it) and draw one card.

   Let R = red card is drawn, B = blue card is drawn, E = even-numbered card is drawn. The sample space S = R1, R2, R3, B1, B2, B3, B4, B5. S has 8 outcomes.

   a) P(R) = 3/8
   b) P(B) = 5/8
   c) P (R AND B) = 0
   d) P(E) = 3/8
   e) P(E|B) = 2/5
   f) P(B|E) = 2/3
   g) The events R and B are mutually exclusive because P (R AND B) = 0
   h) Let G = card with a number greater than 3. G = {B4, B5}, P(G) = 1/4
   i) Let H = blue card numbered between 1 and 4, inclusive. H = {B1, B2, B3, B4}. P(G|H) = 1/4

1) a) $P(R)$ = Probability of drawing a red card = $\frac{\text{Favourable outcomes}}{\text{Total no. of outcomes}}$
$= \frac{3}{8}$

b) $P(B)$ = Probability of drawing a blue card $= \frac{5}{8}$

c) $P(R \cap B) = 0$ (There are no cards that are both red and blue)

d) $P(E)$ = Probability of drawing a even card = $\frac{\text{Red 2 + Blue 2 + Blue 4}}{\text{Total Cards}}$
$= \frac{3}{8}$

e) $P(E/B)$ = Probability of getting a even card and blue is drawn
$= \frac{P(E \cap B)}{P(B)} = \frac{\text{Blue 2 + Blue 4/Total}}{\text{Total Blue/Total}} = \frac{2}{5} \left(\frac{2/4}{5/4}\right)$

f) $P(B/E)$ = Probability of getting a blue card when even is drawn
$= \frac{P(B \cap E)}{P(E)} = \frac{\text{Blue 2 + Blue 4/Total}}{\text{Total Even/Total}} \frac{2/8}{3/8} = \frac{2}{3}$

g) The events R and B are mutually exclusive because P(R and B) = 0

h) Let G = card with number greater than 3, $G = \{B4, B5\}$,
$P(G) = \frac{2}{8} = \frac{1}{4}$

i) Let H = blue card numbered between 1 and 4, inclusive
$H = \{B1, B2, B3, B4\}$
$P(G/H) = \frac{P(G \cap H)}{P(H)} = \frac{B4}{\text{Total } H} = \frac{1}{4}$

2. In a particular college class, there are male and female students. Some students have long hair and some students have short hair. Write the symbols for the probabilities of the events for parts (a) through (j) below. (Note that you can't find numerical answers here. You were not given enough information to find any probability values yet; concentrate on understanding the symbols.)
   • Let F be the event that a student is female.
   • Let M be the event that a student is male.
   • Let S be the event that a student has short hair.
   • Let L be the event that a student has long hair.

   a. The probability that a student does not have long hair. =P(S)
   b. The probability that a student is male or has short hair. =P(M or S)

c. The probability that a student is a female and has long hair. P(F and L)
d. The probability that a student is male, given that the student has long hair. P(M|L)
e. The probability that a student has long hair, given that the student is male. P(L|M)
f. Of all the female students, the probability is that a student has short hair. P(S|F)
g. Of all students with long hair, the probability is that a student is female. P(F|L)
h. The probability that a student is female or has long hair. P(F or L)
i. The probability that a randomly selected student is a male student with short hair. P(M and S)
j. The probability that a student is female. P(F)

2) a) The probability that the student does not have long hair
$= P(L') = P(S) =$ Probability of having short hair

b) The probability that a student is male or has short hair
$= P(M \text{ or } S) = P(M \cup S)$

c) The probability that a student is a female and has long hair $= P(F \text{ and } L) = P(F \cap L)$

d) The probability that a student is male, given that the student has long hair $= P(M/L)$

e) The probability that a student has long hair, given that the student is male $= P(L/M)$

f) Of all the female studentss, the probability that student has short hair $= P(S/F)$

g) Of all students with long hair, the probability that a student is female $= P(F/L)$

h) The probability that a student is female or has long hair $= P(F \text{ or } L) = P(F \cup L)$

i) The probality that a randomly selected student is a male student with short hair $= P(M \text{ and } S)$

j) The probability that a student is female $= P(F)$

3. Suppose that you have 8 cards. 5 are blue and 3 are red. The 5 Blue cards are numbered 1, 2, 3, 4, and 5. The 3 Red cards are numbered 1, 2, and 3. The cards are well-shuffled. You randomly draw one card.

• B = card drawn is blue

- E = card drawn is even-numbered

a) List the sample space. {B1,B2,B3,B4,B5,R1,R2,R3}
b) P (B) = 5/8
c) P (B|E) = 2/3
d) P (B AND E) = 1/4
e) P (B OR E) = 3/4
f) Are B and E mutually exclusive? Justify your answer numerically.
P(B|E) is not equal to zero, we got the value of P(B|E) as 2/3. Hence B and E are not mutually exclusive, because the probability of occurrence of event B when E occurs is 2/3.

3) a) The sample space = {B1, B2, B3, B4, B5, R1, R2, R3}

b) $P(B) = \frac{\text{Favourable outcomes}}{\text{Total cards}} = \boxed{\frac{5}{8}}$

c) $P(B/E) = \frac{P(B \cap E)}{P(E)} = \frac{B2 + B4/\text{Total}}{B2+B4+R2/\text{Total}} \quad \frac{2/8}{3/8} = \boxed{\frac{2}{3}}$

d) $P(B \text{ and } E) = P(B \cap E) = \frac{B2+B4}{\text{Total cards}} = \frac{2}{8} = \boxed{\frac{1}{4}}$

e) $P(B \text{ or } E) = P(B \cup E) = \frac{B1+B2+B3+B4+B5+R2}{\text{Total cards}} = \frac{6}{8} = \boxed{\frac{3}{4}}$

f) Are B & E mutually exclusive? The probability of P(B/E) is not zero. Hence B & E are not mutually exclusive.