**Indiana University Bloomington**

**Fall-2023**
**CSCI-B505 / INFO-I500**
**APPLIED ALGORITHMS**
**Examination – I**

**October 10, 2023, Tuesday, 6:30 p.m. – 7:30 p.m.**

| | |
|---|---|
| Name & Surname | |
| University ID | |
| Signature | |

**Rules:**

1. There are 24 questions in this examination.
2. Duration of the exam is 60 minutes.
3. Write your name and surname on every page at the designated positions.
4. Put your ID card on your desk so that the proctors can check your identity.
5. The use of lecture notes, books, and any other resources, calculators, computers, mobile phones, and any digital equipment is prohibited.
6. Every student taking this examination is subject to the university discipline code. Any act or attempt of cheating, including helping others, will be considered a violation of the code.
7. Whenever you need, the cases of Master's Theorem are given below.
   (a) If $f(n) = O(n^{\log_b a - \varepsilon})$ for some constant $\varepsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$.
   (b) If $f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} lgn)$.
   (c) If $f(n) = \Omega(n^{\log_b a + \varepsilon})$ for some constant $\varepsilon > 0$, and if $af(n/b) \leq cf(n)$ for some $c < 1$, then $T(n) = \Theta(f(n))$.

| **Name & Surname:** |
|---|

1. In the **Accounting** method, the total amortized cost should always be (4 points)

   (a) Equal to the total actual cost

   (b) Greater than the total actual cost

   (c) Greater than or equal to the total actual cost ✓

   (d) Less than or equal to the total actual cost

2. Given the implementation of a queue using two empty stacks. While using the **accounting method** of analysis, consider the actual cost of `enqueue` and `dequeue` operations to be 1 and the amortized cost of `enqueue` and `dequeue` operations are 4 and 0 (free) respectively. What will be the result of your analysis using the accounting method for the following operations performed in order? (5 points)

   - Enqueue(15)
   - Enqueue(16)
   - Enqueue(8)
   - Dequeue()
   - Dequeue()
   - Enqueue(10)
   - Dequeue()
   - Enqueue(5)
   - Dequeue()
   - Dequeue()

   I) Bankruptcy

   II) No bankruptcy

   III) Remaining balance 1

   IV) Remaining balance 0

   (a) Only I is correct

   (b) Only II is correct

   (c) II and III are correct

   (d) II and IV are correct ✓

3. Consider a dynamic array D that doubles its capacity when full. Suppose the actual cost of appending an element into the dynamic array is 1. The potential function is given as $\phi = k \cdot (2 \cdot size(D) - capacity(D))$, where $k >= 0$ and `size` refers to the number of elements the array currently contains and `capacity` refers to the total space of the array. Using the potential method what is the amortized cost of appending a new element into the array when the size and the capacity of the array D are 5 and 8 respectively? (4 points)

   (a) 3
   (b) 4
   (c) 2k
   (d) 2k + 1 ✓

4. Consider a dynamic array that doubles its capacity when full and no change in the capacity when elements are removed. Now, we introduce two operations, `PushBack` (adds the element at the end of the array) and `PopBack` (removes the element from the end of the array). What would be the minimum and maximum capacity of the array if **28 PushBack** and **28 PopBack** operations are performed on the empty array (not necessarily in that order)? (Hint: Consider the order in which both the operations are performed to find the minimum and maximum capacity of the array) (4 points)

   (a) Minimum 0, Maximum 28
   (b) Minimum 1, Maximum 32 ✓
   (c) Minimum 0, Maximum 32
   (d) Minimum 1, Maximum 56

5. For n elements, how many arrays are maintained in the amortized dictionary? (4 points)

   (a) $\lceil log(n+1) \rceil$ ✓
   (b) $\lfloor log(n+1) \rfloor$
   (c) $\left\lfloor \frac{log(n+1)}{2} \right\rfloor$
   (d) $\left\lceil \frac{log(n+1)}{2} \right\rceil$

**Answer the next two questions based on the below information**
Consider an amortized dictionary starting at level `A0`. Insert the following elements in order.

```
Insert(1)
Insert(12)
Insert(5)
Insert(4)
Insert(3)
Insert(6)
Insert(7)
Insert(10)
Insert(8)
Insert(11)
Insert(9)
Insert(2)
Insert(13)
```

6. Which level of the amortized dictionary is empty after inserting the given elements in the specified order? (4 points)

   (a) `A0`
   (b) `A1` ✓
   (c) `A2`
   (d) `A3`

7. Which level is returned when searching for the element **2**? (5 points)

   (a) `A0`
   (b) `A1`
   (c) `A2` ✓
   (d) `A3`

8. In a recursive function, which option represents the concept of the **base case**? (4 points)

   (a) The case where the function calls itself
   (b) The condition that terminates the recursion ✓
   (c) The final result of the recursion
   (d) None of the above

9. What is the **time complexity** of the following recursive function? (4 points)

```
def quux(n):
    if n <= 1:
        return 1
    for i in range(5):
        quux(n/2)
```

(a) $O((\log_2 n)^5)$

(b) $O(5^{\log_2 n})$ ✓

(c) $O(n^5)$

(d) $O(\log_2 n)$

10. Consider a sequence F defined as $F(0) = 1$, $F(1) = 1$ and $F(n) = 10 \cdot F(n–1) + 100 \cdot F(n–2)$ for $n \geq 2$. Then, what shall be the initial 4 values of the sequence F? (4 points)

(a) 1, 1, 110, 1200 ✓

(b) 1, 1, 110, 600

(c) 1, 1, 2, 55, 110

(d) 1, 1, 55, 110

11. What does the below recursive function do? (4 points)

```
    def recursive_function(x,y):
        if y==0:
            return x
        return (x + recursive_function(x,y-1))
```

(a) $x \cdot y$

(b) $(x+1) \cdot y$

(c) $x \cdot (y+1)$ ✓

(d) $x^y$

12. In the Master's Theorem, $T(n) = aT(n/b) + f(n)$, what does the **a** and **(n/b)** represents respectively? (4 points)

   I) Number of subproblems in the recursion

   II) Height of the recursion tree

   III) Size of each subproblem

   (a) a: I, (n/b): II

   (b) a: II, (n/b): III

   (c) a: I, (n/b): III ✓

   (d) a: III, (n/b): I

13. Suppose you have a divide-and-conquer algorithm that recursively divides the input data into two equal parts, processes each partition independently, and then combines the results using a linear-time operation. For instance, when the input size is n, the algorithm divides it into two partitions of size n/2, processes each partition, and then combines the results with a linear operation that takes n steps. What is the recurrence relation for this algorithm? (4 points)

   (a) $T(n) = (1/2) \cdot T(n) + O(n)$

   (b) $T(n) = 2 \cdot T(n/2) + O(n)$ ✓

   (c) $T(n) = T(n/2) + O(n)$

   (d) $T(n) = 2 \cdot T(n) + O(n)$

14. Using the master's theorem, what is the time complexity of the following recurrence relation $T(n) = T(n/12) + \log n$? (4 points)

   (a) $\Theta(n^{log(n)})$

   (b) $\Theta(n)$

   (c) $\Theta(log(n))$ ✓

   (d) $\Theta(n \cdot log(n))$

15. Using the master's theorem, what is the time complexity of the following recurrence relation $T(n) = 4T(\frac{n}{4}) + 1$? (4 points)

   (a) $\Theta(\log n)$

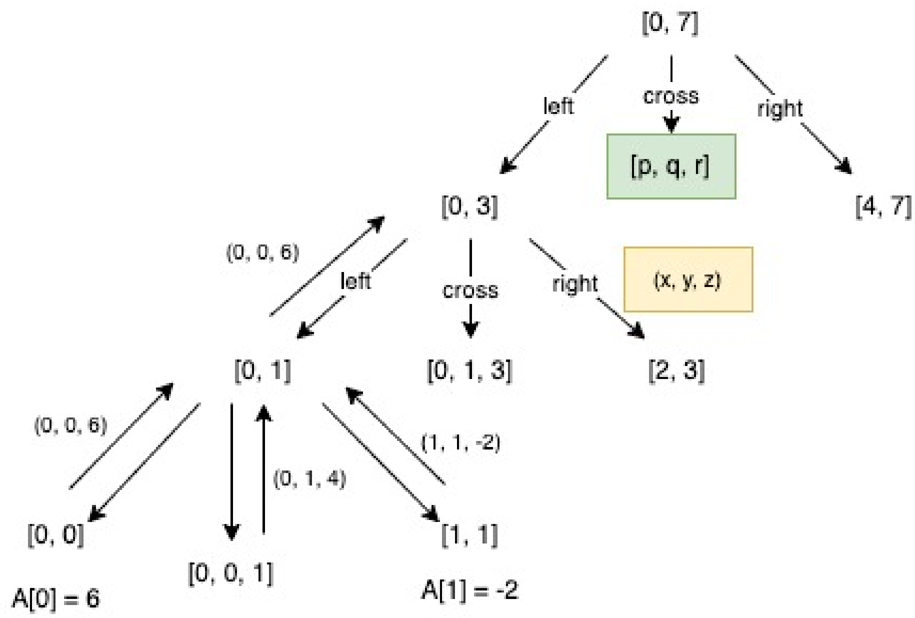
   (b) $\Theta(1)$

   (c) $\Theta(n \log n)$

   (d) $\Theta(n)$ ✓

**Answer the next two questions based on the information below**

Given an array `A = [6, -2, 4, -3, 15, -4, 1, 11]`, as discussed in the lecture we are using the divide and conquer algorithm to find the sub-array with maximum sum. The recursion tree is shown below.




16. What are the values of `[p, q, r]`, which represent the (low, mid, high) for cross sum? (4 points)

    (a) `[2, 3, 4]`
    (b) `[0, 3, 7]` ✓
    (c) `[0, 4, 7]`
    (d) `[0, 3, 6]`

17. What are the values of `(x, y, z)`, which represent the (low, high, max-sum) returned after solving the sub-problem A[2, 3]? (4 points)

    (a) `[2, 2, 4]` ✓
    (b) `[2, 3, 1]`
    (c) `[2, 2, 5]`
    (d) `[0, 2, 8]`

18. Which scenario among the given options represents the most suitable situation for the application of dynamic programming? (4 points)

    (a) When the solution does not involve any recursive steps.

    (b) When there is a reductive solution with significant overlap between recursive steps. ✓

    (c) When exhaustive search is the only option to find the optimum solution.

    (d) None of the above.

19. Consider the dynamic programming approach used to compute the binomial coefficient $\binom{n}{k}$, which is the number of ways to choose $k$ items out of $n$ items without repetition and without any specific order. In this approach, a two-dimensional table, $C$, is constructed such that $C[i][j]$ stores the value of $\binom{i}{j}$. Given this setup, which of the following expressions are True for any given $i$ and $j$? (4 points)

    I) C[i][j] = C[j][i]

    II) The formula to fill C[i][j] is C[i][j] = C[i-1][j] + C[i-1][j-1]

    III) The formula to fill C[i][j] is C[i][j] = C[i][j-1] + C[i-1][j-1]

    (a) I and II are correct

    (b) I and III are correct

    (c) Only II is correct ✓

    (d) Only III is correct

20. We have two individuals, each with their DNA sequence:

    John's DNA sequence: *ATGATGCCTCAGTAGCAGTTAGCA*
    Emma's DNA sequence: *TATATATA*

    Assume the key to confirming whether John is the father of Emma is to check if any one of the subsequences of John's DNA matches Emma's DNA sequence.

    Is John the father? (4 points)

    (a) Yes, he is the father. ✓

    (b) No, he is not the father.

    (c) It is inconclusive whether John is the father based on the provided information.

21. Given two strings `s1 = SATURDAY, s2 = SUNDAY`. What is the edit distance between s1 and s2? (4 points)

    (a) 2

    (b) 3 ✓

    (c) 4

    (d) 5

22. Consider an ordered partitioning problem where a given sorted array of positive integers $\{2,4,6,7\}$ is split into $k = 3$ partitions such that the cost of partitioning is the minimum. The cost of partitioning is defined as the maximum sum of integers in each of those partitions. Let $M[n,k]$ be the minimum cost of partitioning the given array with n elements into $k$ partitions. While solving this problem using dynamic programming we constructed the below table.

| i | $s_i$ | 1 | 2 | 3 |
|---|---|---|---|---|
| 1 | 2 | 2 | 2 | 2 |
| 2 | 4 | 6 | 4 | 4 |
| 3 | 6 | 12 | 6 | 6 |
| 4 | 7 | 19 | P | Q |

What are the values of P and Q in the above table?
P = **12** (2 points)

Q = **7** (2 points)

23. Given the following sequence of numbers. While calculating the longest increasing subsequence, what are the final values of the array $L$, where $L_i$ is defined as the length of the longest increasing subsequence of $< s_1, s_2, ..., s_i >$ ending at $s_i$? (5 points)

$$S = < 5, 7, 11, 9, 7, 8, 6, 10, 17, 13 >$$

(a) $[1,1,2,2,3,3,4,4,5,5]$

(b) $[1,2,3,3,2,3,2,4,5,5]$ ✓

(c) $[1,2,2,3,2,3,2,3,4,5]$

(d) $[1,2,3,3,3,3,4,4,5,5]$

24. Given a set S = {2, 3, 4, 5, 12, 14}, You are asked to find if there is a subset of S, whose elements sum up to k = 9. You are constructing the below truth table to solve the problem.

| i | $s_i$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | T | F | F | F | F | F | F | F | F | F |
| 1 | 2 | T | F | T | F | F | F | F | F | F | F |
| 2 | 3 | T | | | | | X | | | | |
| 3 | 4 | T | | | | | | | | Y | |
| 4 | 5 | T | | | | | | | | | |
| 5 | 12 | T | | | | | | | | | |
| 6 | 14 | T | | | | | | | | | |

What are the truth values of X and Y respectively? (5 points)

(a) T and T

(b) T and F ✓

(c) F and T

(d) F and F