Hashing

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Based on slides by Hector Garcia-Molina

Hashing: hash function

- Let K be a domain of key values (K can be very large)
- Let R = [0,m) (usually m << |K|) be a range of values
- A hash function h maps K to R

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h: K -> R
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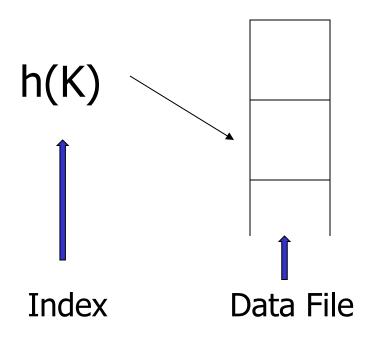
Example: $h(k) = k \mod m$

- Given a record r with key value k, h(k) provides an address (name) of a bucket in which to store r
- A bucket is stored in secondary memory as a block or a list of blocks
- Retrieving a bucket (and the records therein) can be done in O(n) where n is the number of blocks that store the bucket.

Hash function: collision

- Let r1 and r2 be two records with key values k1 and k2
- We permit that k1 = k2
- We say that h has a collision for r1 and r2 if h(k1) = h(k2)
 Consequence: r1 and r2 will be stored in the same bucket
- If K is the domain of a primary key, a collision will store records with different key values in same bucket (not desirable)
- If K is not the domain of a primary key, different records with the same key values will be placed in the same bucket (desirable). Partitioning.
- The latter property is exploited in key-value stores since records with the same key value will be sent to the same reducer

Next: example to illustrate inserts, overflows, deletes



EXAMPLE 2 records/bucket

(For simplicity, we identify a record with its key value)

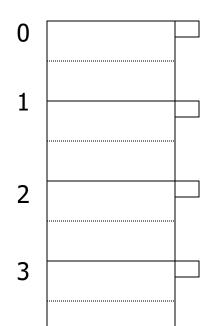
INSERT:

$$h(a) = 1$$

$$h(b) = 2$$

$$h(c) = 1$$

$$h(d) = 0$$



EXAMPLE 2 records/bucket

INSERT:

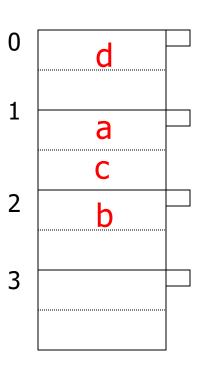
$$h(a) = 1$$

$$h(b) = 2$$

$$h(c) = 1$$

$$h(d) = 0$$

$$h(e) = 1$$



EXAMPLE 2 records/bucket

INSERT:

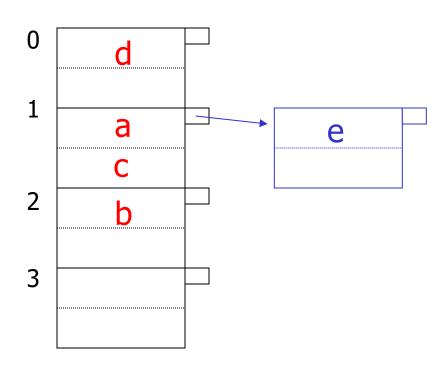
$$h(a) = 1$$

$$h(b) = 2$$

$$h(c) = 1$$

$$h(d) = 0$$

$$h(e) = 1$$

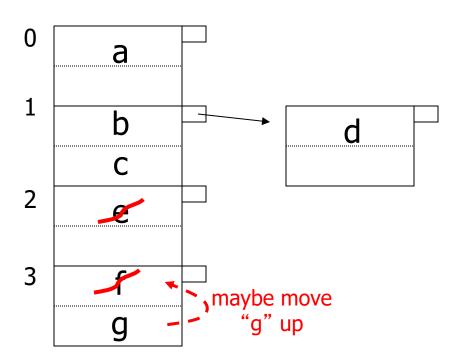


EXAMPLE: deletion

Delete: o a f c a d d f f

EXAMPLE: deletion

Delete: e f



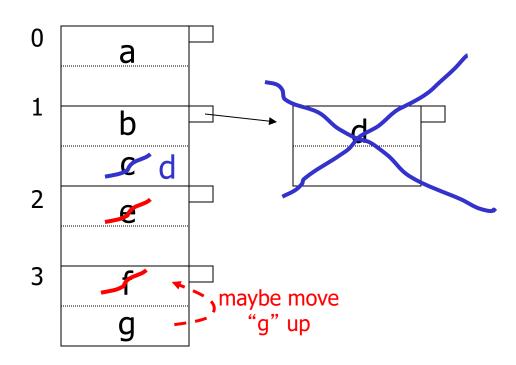
EXAMPLE: deletion

Delete:

e

f

C



How do we cope with growth?

- Overflows and reorganizations Reorganization can be done by enlarging the range R and changing the hash function Reorganization requires complete rehashing and is linear in the |Data file|
- Dynamic hashing (extensible hashing)

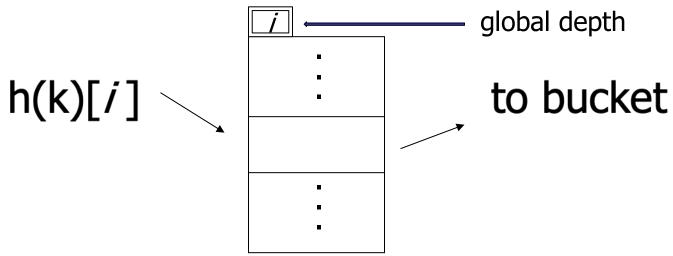
Extensible hashing: two ideas

(a) Use *i* of *b* bits output by hash function

$$h(k) \rightarrow 00110101$$

USE $I \rightarrow$ grows/shrinks over time....

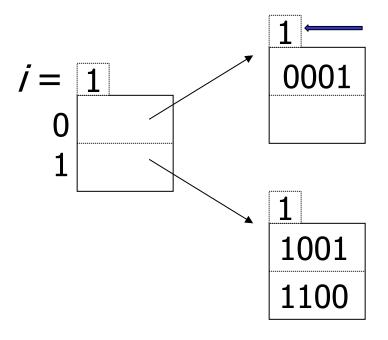
(b) Use directory of size 2ⁱ



h(k) has b bits, but we will only look at its first i bits

h(k)[/] consists of the first / bits of h(k) these / bits specify the position for k the directory

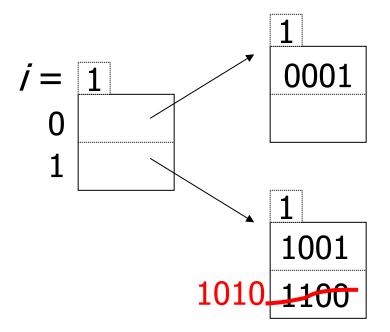
Example: h(k) is 4 bits; 2 keys/bucket



local depth of bucket $\leq i$

Insert 1010

Example: h(k) is 4 bits; 2 keys/bucket

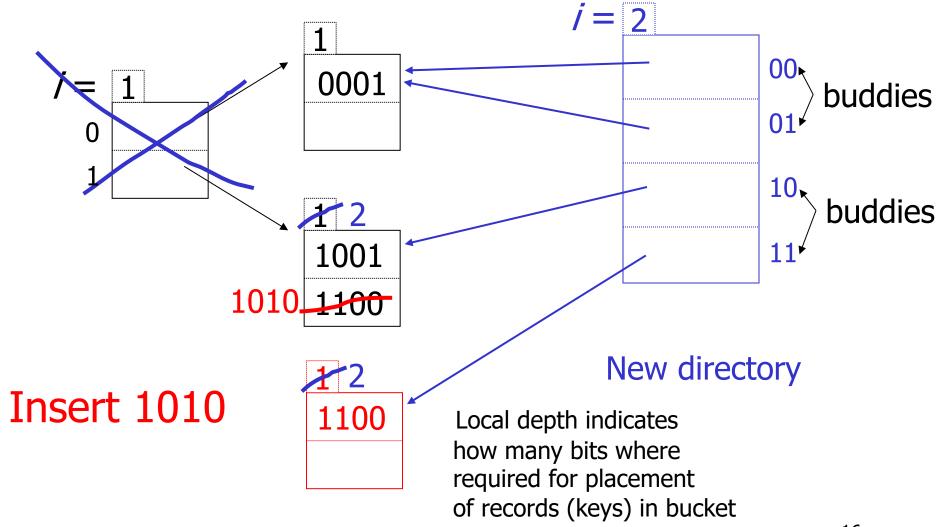


Insert 1010

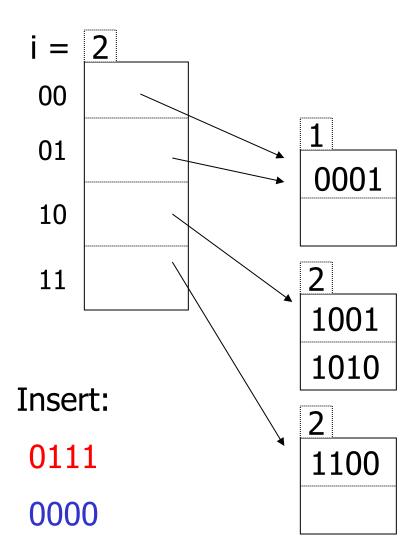
```
1100
```

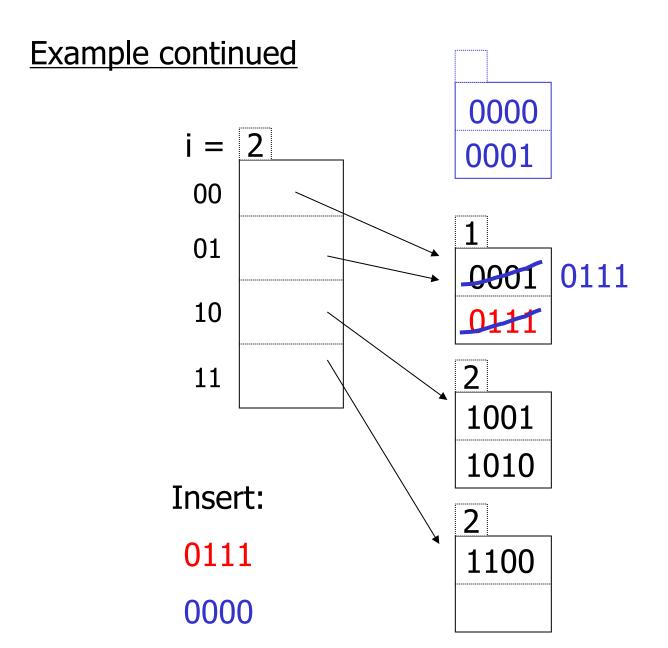
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Main idea:
we need 2 bits to
distinguish
1001
1100
1010
```

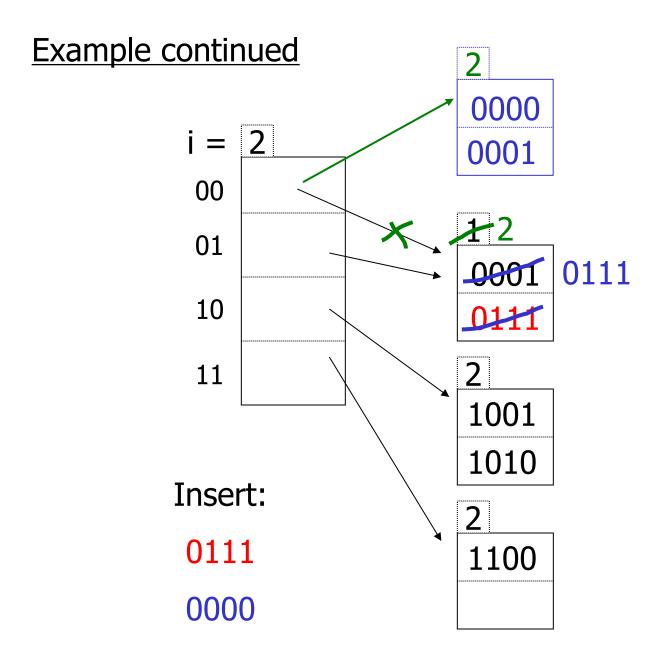
Example: h(k) is 4 bits; 2 keys/bucket



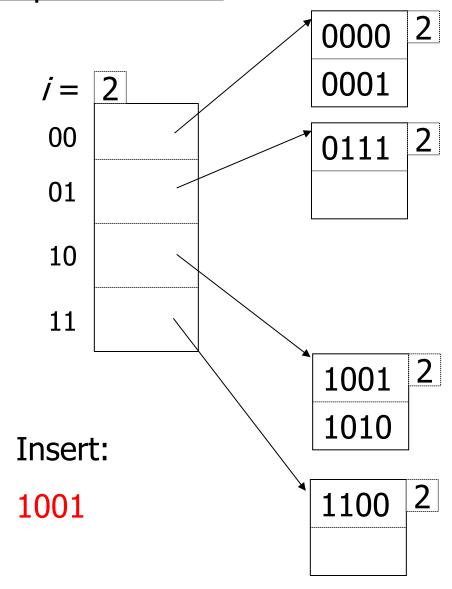
Example continued



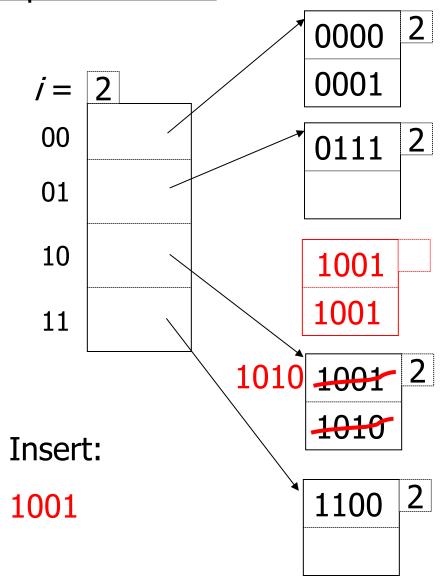


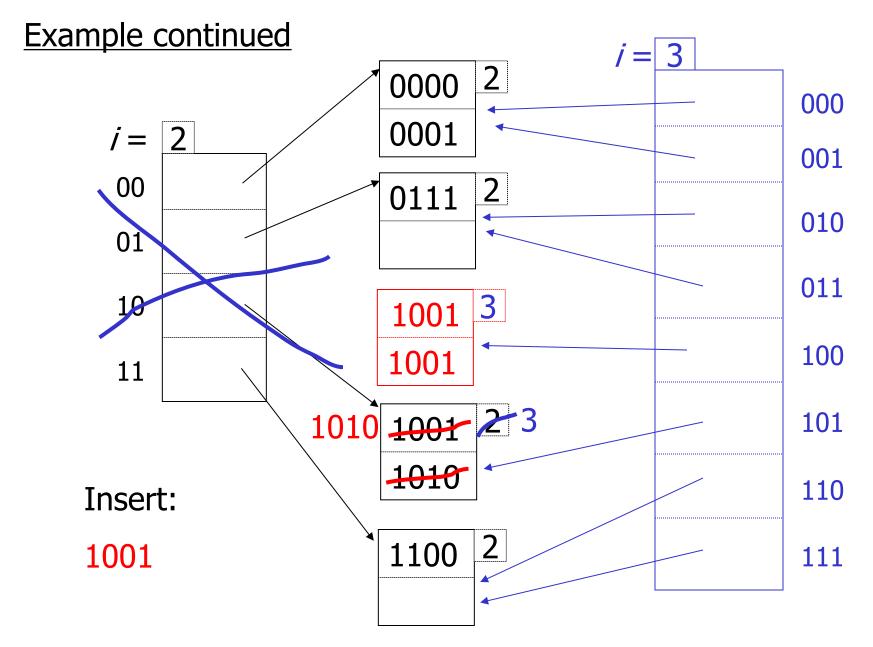


Example continued



Example continued





Extensible hashing: <u>deletion</u>

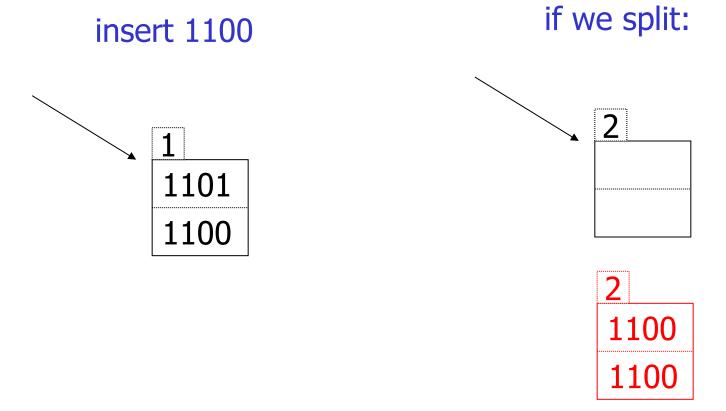
- Merge blocks and cut directory if possible (Reverse insert procedure)
- Two blocks can be merged after a deletion if all records in these blocks can fit in a single block
- One of these two blocks can be removed and the local depth of the other block one can be decreased by 1
- If all local depths are strictly smaller than the global depth i, then the directory can be cut in half and the

Deletion example:

Run through insert example in reverse!

Note: Still need overflow chains

Example: many records with duplicate keys



Solution: overflow chains

insert 1100

add overflow block:

