Program Structures and Algorithms
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GITHUB LINK: https://github.com/nehadevarapalli/INFO6205

Task: To implement the code for the Random Walk experiment and deduce the relationship between the mean distance (d) and the number of steps taken (m).

Relationship Conclusion: $d \approx \sqrt{m}$

1. With each step, the man can move only +1 or -1 in the 'x' or 'y' direction.
2. Since there is no bias in the walk, the average value of each individual displacement in '$\langle x_i \rangle$' and '$\langle y_i \rangle$' is equal to 0.
3. Also since there is no correlation between the steps (stochastic), the dot product of 'x' and 'y' is 0 i.e.,

$$\sum x_i x_j = 0 \text{ and } \sum y_i y_j = 0$$

With these assumptions we will calculate the mean displacement of a walk after 'm' steps and then the root mean square displacement.

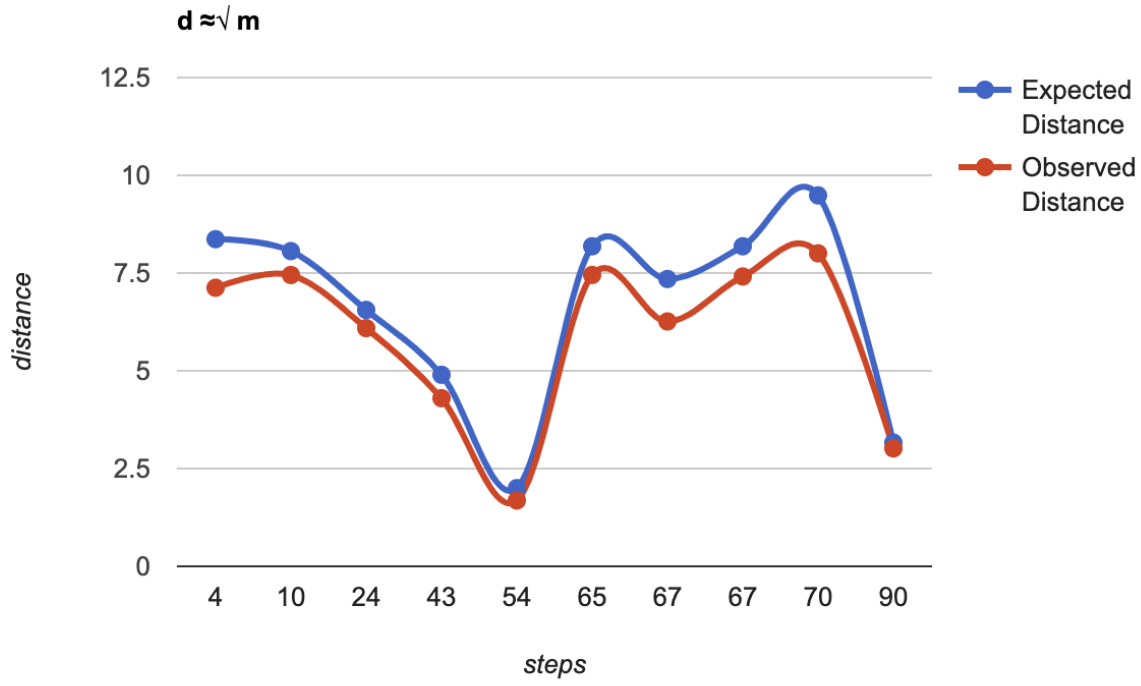
1. The mean displacement can be calculated by calculating the mean / average of the sum of individual displacements ($\sum x$ and $\sum y$ from 1 to m). But we know that $\langle x_i \rangle$ and $\langle y_i \rangle$ are 0 and hence, the mean displacement also is 0.

2. Now, to calculate the root mean square displacement, we have to take the sum of the displacements, square them and calculate the average of each ($\langle (\sum x)^2 \rangle$ and $\langle (\sum y)^2 \rangle$ from 1 to m. When we perform the above calculation there will be terms like:
   $\langle \sum x_i^2 + \sum x_i x_j \rangle$ and $\langle \sum y_i^2 + \sum y_i y_j \rangle$ where $i \neq j$ and i,j = 1 to m
   Since there are no correlations between the steps, $\sum x_i x_j$ and $\sum y_i y_j$ will be 0.
   And the terms $\sum x_i^2$ and $\sum y_i^2$ will have 'm' such terms of the length equal to 1 each and hence the average will be 'm'. SO, $d^2 \approx m$
   Hence, the root mean square distance (d) $\approx \sqrt{m}$

Evidence to support that conclusion:
This is the data that I have got after running my own unit test where I calculate the expected distance with the relation proved above and compare it with the observed distances.

	# Expected	# Observed
1	8.3666	7.1237
2	8.0623	7.4504
3	6.5574	6.0919
4	4.8990	4.2999
5	2.0000	1.6841
6	8.1854	7.4518
7	7.3485	6.2625
8	8.1854	7.4129
9	9.4868	8.0047
10	3.1623	3.0155

I have also plotted a graph to observe the relation between the two,



Unit Test Screenshots: