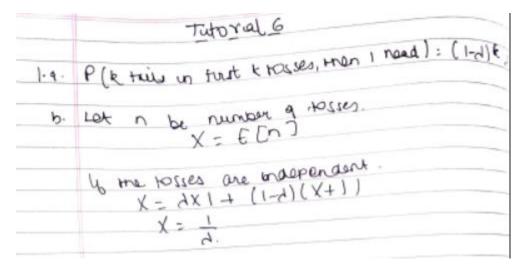
## Data Science, 2022

## Tut 6: Machine Learning 1

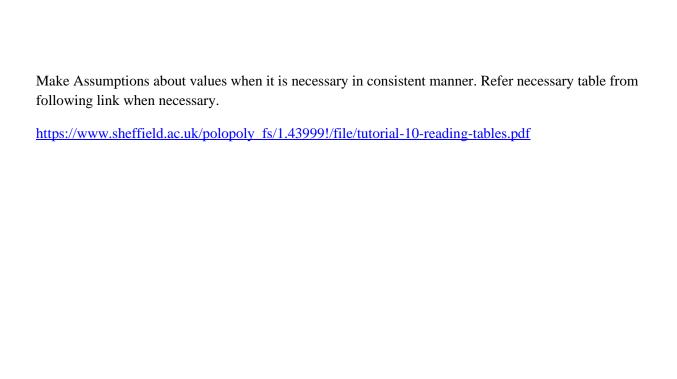
- 1. [Probability] Assume that the probability of obtaining heads when tossing a coin is  $\lambda$ .
  - a. What is the probability of obtaining the first head at the (k + 1)-th toss?
  - b. What is the expected number of tosses needed to get the first head?



- 2. [Probability] Assume X is a random variable.
  - a. We define the variance of X as:  $Var(X) = E[(X E[X])^2]$ . Prove that  $Var(X) = E[X^2] E[X]^2$ .
  - b. If E[X] = 0 and  $E[X^2] = 1$ , what is the variance of X? If Y = a + bX, what is the variance of Y?

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X-1 random variable
      Yar CX]: E[(X-E[X])2]
 var(y) = var(q+bx)
van(y) = 92+62 (9) = 62
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- 3. [Probability] Your friend Aku is a great predictor about winning horse race. Assume that we know three facts: 1) If Aku tells you that a horse name black beauty will win, it will win with probability 0.99. 2) If Aku tells you that a black beauty will not win, it will not win with probability 0.99999. 3) With probability  $10^{-5}$ , Aku predicts that a black beauty is a winning horse. This also means that with probability  $1 10^{-5}$ , Aku predicts that a black beauty will not win.
- a. Given a horse, what is the probability that it wins?
- b. What is the probability that Aku correctly predicts a black beauty is winning?



ans.	Let A be the overt "Aren preasing that me move
	is a warring home."
	Lot -A be the event "Aren product, that the horse
	is vot a sound parti.
	Let -0 be the event that the notes is a winning horse.
	Lot I've be the crest that horse is not a warry home
	Curen P( w   A ) > 097 P(-0) -1): 0.99999
	P(A): 10-5
_	

9) P(w): P(w;A) + P(w, ¬A) 1 : P(w | A) (PA) + P(w| ¬A) P(¬∀) 1 : P(w | A) (PA) + P(w| ¬A) P(¬∀) 2 : P(w | A) (PA) + P(w, ¬A) 2 : P(w | A) (PA) + P(w, ¬A) 2 : P(w | A) (PA) + P(w, ¬A) 2 : P(w | A) (PA) + P(w, ¬A) 2 : P(w | A) (PA) + P(w, ¬A) 2 : P(w | A) (PA) + P(w, ¬A) 2 : P(w | A) (PA) + P(w, ¬A) 2 : P(w | A) (PA) + P(w, ¬A) 2 : P(w | A) (PA) + P(w, ¬A) 2 : P(w | A) (PA) + P(w, ¬A) 3 : P(w | A) (PA) + P(w, ¬A) 3 : P(w | A) (PA) + P(w, ¬A) 4 : P(w | A) (PA) + P(w, ¬A) 3 : P(w | A) (PA) + P(w, ¬A) 4 : P(w | A) (PA) + P(w, ¬A) 4 : P(w | A) (PA) + P(w, ¬A) 4 : P(w | A) (PA) + P(w, ¬A) 4 : P(w | A) (PA) + P(w, ¬A) 4 : P(w | A) (PA) + P(w, ¬A) 4 : P(w | A) (PA) + P(w, ¬A) 4 : P(w | A) (PA) + P(w, ¬A) 4 : P(w | A) (PA) + P(w, ¬A) 5 : P(w | A) (PA) + P(w, ¬A) 6 : P(w | A) (PA) + P(w, ¬A) 6 : P(w | A) (PA) + P(w, ¬A) 7 : P(w | A) (PA) + P(w, ¬A) 8 : P(w | A) (PA) + P(w, ¬A) 8 : P(w | A) (PA) + P(w, ¬A) 8 : P(w | A) (PA) + P(w, ¬A) 8 : P(w | A) (PA) + P(w, ¬A) 8 : P(w | A) (PA) + P(w, ¬A) 8 : P(w | A) (PA) + P(w, ¬A) 8 : P(w | A) (PA) + P(w, ¬A) 8 : P(w | A) (PA) + P(w, ¬A) 8 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w, ¬A) 9 : P(w | A) (PA) + P(w,

b) Prob that very produst winney correctly.

P(A/W) = P(A, W)

P(W/A) P(A)

1-99 x 10-7 7-01 x 199.1