Data Science, 2022
Tut 5: Evaluation and Measurement- Hypothesis Testing

Make Assumptions about values when it is necessary in consistent manner. Refer necessary table from following link when necessary.

https://www.sheffield.ac.uk/polopoly_fs/1.43999!/file/tutorial-10-reading-tables.pdf

Testing a Proportion of small samples

1. H_0: $p = p_0$

2. One of the alternatives H\: $p < p_n$, $p > p_o$, or $p \neq p_o$

3. Choose a level of significance equal to a.

4. Test statistic: Binomial variable *X* with $p = p_0$.

5. Computations: Find x, the number of successes, and compute the appropriate P-value.

6. Decision: Draw appropriate conclusions based on the P-value

Ex. 1

A builder claims that air-conditions are installed in 70% of all homes being constructed today in the city of Mumbai. Would you agree with this claim
if a random survey of new homes in this city shows that 8 out of 15 had air-conditions installed? Use a 0.10 level of significance

Que 1:

Claim: 70%

Random survey: 8 out of 15 have pumps.

0.1 level of significance

$H_0 : p = 0.7$

$H_1 : p \neq 0.7$

$\alpha = 0.1$

Test statistic:

We have a Binomial variable X with $p = 0.7$ and $n = 15$.

Here, $x = 8$, $n = 15$

$np_0 = (15)(0.7)$

$= 10.5$

Since if,

$P = 2P(X \leq x, \text{ when } p = p_0)$, if $x < np_0$

Since, $x = 8$ & $np_0 = 10.5$

$P = 2P(X \leq 8, \text{ when } p = 0.7)$

$= 2 \sum_{x=0}^{8} (0.7)^x (0.3)^{15-x}$

$= 2 \times 0.1311 = 0.2622$

$\therefore 0.2622 > 0.1$

we don't reject H_0.

We dont have sufficient reason to doubt the claim.

Ex.2

A commonly prescribed drug for relieving nervous tension is believed to be only 60% effective. Experimental results with a new drug administered to a random sample of 100 adults who were suffering from nervous tension show that 70 received relief. Is this sufficient evidence to conclude that the new drug is superior to the one commonly prescribed? Use a 0.05 level of significance.

Ex 2:

Claim:- 0.6 ← commonly prescribed.

New drug:

sample of 100, 70 received relief.

Q. Is it sufficient evidence to conclude that the new drug is superior to commonly prescribed.

Level of significance: $\alpha = 005$

$\therefore$ $H_0 : p = 0.6$

$H_1 : p > 0.6$

$\alpha = 0.05.$

Critical value of $z = 1.645$

$x = 70$ $\quad n = 100$ $\quad p = 0.7$

$$z = \frac{0.7 - 0.6}{\sqrt{(0.6)(0.4)/100}} = 2.04$$

$P = P(Z > 2.04)$

< 0.0207

$< 0.05 = \alpha.$

We reject H_0 & we conclude that the new is superior.

Ex.3

A vote is to be taken among the residents of a Mumbai and the surrounding area to determine whether a proposed Nuclear plant should be constructed. The construction site is within the Mumbai limits, and for this reason many voters in the surrounding area feel that the proposal will pass because of the large proportion of Mumbai voters who favor the construction. To determine if there is a significant difference in the proportion of Mumbai voters and surrounding area voters favoring the proposal, a poll is taken. If 120 of 200 Mumbai voters favor the proposal and 240 of 500 surrounding area residents favor it, would you agree that the proportion of Mumbai voters favoring the proposal is higher than the proportion of surrounding area voters? Use an a = 0.05 level of significance.

Ex 3.

$p_1 \Rightarrow$ Proportion of Mumbai voters favouring the proposal

$p_2 \Rightarrow$ Proportion of surrounding voters favouring the proposal

$\hat{p}_1 \Rightarrow$ Sample proportion of Mumbai voters favouring the proposal

$\hat{p}_2 \Rightarrow$ Sample proportion of surrounding voters favouring the proposal

Mumbai,

$n_1 = 200 \qquad x_1 = 120$

$\hat{p}_1 = \frac{x_1}{n_1} = 0.6 \qquad \hat{q}_1 = 1 - 0.6 = 0.4.$

Surrounding

$n_2 = 500 \qquad x_2 = 240$

$\hat{p}_2 = \frac{x_2}{n_2} = 0.48 \qquad \hat{q}_2 = 1 - 0.48 = 0.52$

Pooled estimate.

$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{360}{700} = 0.51$

$\hat{q} = 1 - 0.51 = 0.49$

Hypothesis is

$H_0: p_1 \leq p_2 \qquad H_1: p_1 > p_2$

$$z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{(0.60 - 0.48)}{\sqrt{0.51 \cdot 0.49\left(\frac{1}{200} + \frac{1}{500}\right)}}$$

$= 2.869$

$p = P(Z > 2.869) = 0.0044.$

Because,

$p < \alpha (0.05)$

we reject H_0.

$p_1 > p_2$ i.e. propotion of Mumbai voters favoring the proposal is more than proportion of surrounding.

Ex.4

State the null and alternative hypotheses to be used in testing the following claims, and determine generally where the critical region is located:

(a) At most, 20% of next year's wheat, crop will be exported to the Russia..
(b) On the average, Indian homemakers drink 3 cups of tea per day.
(c) The proportion of graduates in engineering this year majoring in the computer sciences is at least.
0.15.
(d) The average donation to the Indian Autism Association is no more than 500 INR.
(e) Residents in suburban Mumbai commute, on the average, 15 kilometers to their place of employment.

r.
a) Atmost 20%
Null Hypothesis: H_0 : $p=0.2$
Alternative Hypothesis: $H_1 = p > 0.20$
& critical region is the right tail.

b) On an average 3 cups of tea per day
Null Hypothesis: $H_0 = \mu = 3$
Alternative hypothesis: $H_1 : \mu \neq 3$
since it is $\neq$ there for it is two tailed.

c) Atleast 15%
Null Hypothesis: $H_0 = p = 0.15$
Alternative hypothesis: $H_1 : p < 0.15$
& critical region is in left tail

d) Average no more than 500 INR
Null hypothesis: $H_0 : \mu = 500$
Alternative hypothesis: $H_1 : \mu > 500$
Critical Region is in right tail

e) Average: 15 km
Null Hypothesis: $H_0 : \mu = 15$
Alternate hypothesis: $H_1 : \mu_0 \neq 1.5$
Critical Region is in both tail.

Ex.5

In a study conducted by the Department of computer Engineering and analyzed by the Statistics Consulting Center at SPIT the laptops supplied by
two different companies were compared. Ten sample laptops were made out of the Intel chips supplied by each company and the "robustness" was studied. The
data are as follows:

Company A: 9.3 8.8 6.8, 8.7 8.5 6.7 8.0 6.5 9.2 7.0

Company B: 11.0 9.8 9.9 10.2,10.1 9.7 11.0 11.1 10.2 9.6

Can you conclude that there is virtually no difference in means between the laptops supplied by the two companies? Use a P-value to reach your conclusion. Should variances be pooled here?

Q.5. Null hypothesis H_0 $\mu_1 = \mu_2$
Alternative hypothesis H_1 $\mu_1 \neq \mu_2$

assuming $\alpha = 0.05$

$$\bar{x}_1 = \frac{\Sigma x_i}{n_1} = \frac{79.5}{10} = 7.95$$

$$\bar{x}_2 = \frac{102.6}{10} = 10.26$$

Standard dev.

$$s_1^2 = \frac{1}{n_1 - 1}\left[\Sigma x_1^2 - n\bar{x}_1^2\right]$$

$$= \frac{10.865}{10-1} = 1.2072$$

$$s_2^2 = \frac{1}{n_2 - 1}\left[\Sigma x_2^2 - n_2\bar{x}_2^2\right]$$

$$= \frac{2.924}{10-1} = 0.3248.$$

Now, we need to calculate degree of freedom

$$V = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left(\frac{1}{n_1-1}\right)\left(\frac{s_1^2}{n_1}\right)^2 + \left(\frac{1}{n_2-1}\right)\left(\frac{s_2^2}{n_2}\right)^2}$$

$$= \frac{\left(\frac{1.2072^2}{10} + \frac{0.3248^2}{10}\right)^2}{\frac{1}{9}\left(\frac{1.2072}{10}\right)^2 + \frac{1}{9}\left(\frac{0.3248}{10}\right)^2}$$

$= 10.30$

$v \approx 10$

Test statistic:

$$T = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Considering null hypothesis $\mu_1 - \mu_2 = 0$

$$= \frac{7.95 - 10.26}{\sqrt{\frac{1.2072}{10} + \frac{0.3248}{10}}} = -5.902$$

considering two sided tail

$|t| = |-5.902| = 5.902$

$\therefore$ pvalue $= 2P(T \geq |t|) = 2P(T \geq 5.9)$

$t_{0.0005}(10) = 4.587$

$|t| = 5.9$ is even.

$P(T \geq 5.9) < 0.0005$

$\therefore$ p value < 0.001

$p < \alpha$, $\therefore$ Null hypothesis is rejected.