GL Applied Data Science Program

Network Analysis

Overview

Overview of this week / module:

- Data collection and visualization for exploratory data analysis
- Network analysis
- Unsupervised learning - clustering

Overview of this lecture:

- Examples of networks and representing networks
- Summary statistics of a network
- Centrality measures - finding important nodes in a network

Network

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Network research:

- In recent years network research witnessed a big change:
  - From study of a single graph on 10-100 nodes to the statistical properties of large networks on millions of nodes
  - Characterize the structure of networks
  - Identify important nodes / edges in a network
  - Identify missing links in a network

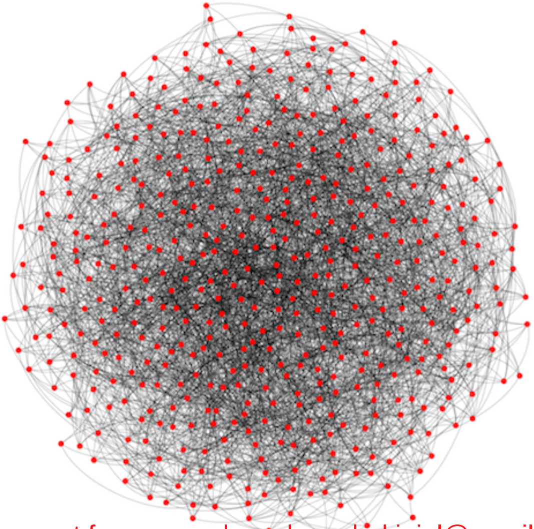
Examples of networks

Network	Vertex	Edge
World Wide Web	web page	hyperlink
Internet	computer	network protocol interaction
power grid	generating station / substation	transmission line
friendship network	person	friendship
gene regulatory network	gene	regulatory effect
neural network	neuron	synapse
transportation	airport	direct flight
Netflix	person / movie	rating

Different kinds of networks

- simple network: undirected network with at most one edge between any pair of vertices and no self-loops
  - e.g. Internet, power grid, telephone network
- multigraph: self-loops and multiple links between vertices possible
  - e.g. neural network, road network
- directed network: $i \rightarrow j$ does not imply $j \rightarrow i$
  - e.g. World Wide Web, food web, citation network
- weighted network: with edge weights or vertex attributes
  - e.g. transportation networks
- bipartite network: edges between but not within classes
  - e.g. recommender systems such as Netflix
- hypergraph: generalized 'edges' for interaction between > 2 nodes
  - e.g. protein-protein interaction network

Large networks look like hairballs



Representation of a network

Two common representations of a network $G = (V, E)$:

- **adjacency list**
  - undirected graph $1 - 2 - 3$: $E = \{\{1,2\},\{2,3\}\}$
  - directed graph $1 \rightarrow 2 \leftarrow 3$: $E = \{(1,2),(3,2)\}$

- **adjacency matrix** of size $n \times n$ (where $n = |V|$) with

$$A_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{otherwise} \end{cases}$$

  - For weighted graph, A_{ij} can be non-binary

How does the adjacency matrix of an undirected graph look like? How to count the number of friends or suggest new friends in a social network?

Representation of a network

Quantitative measures of networks

Some quantitative measures of networks to describe structural patterns of a network and to compare networks:

- connected components
- degree distribution
- diameter and average path length
- homophily or assortative mixing

Connected Components

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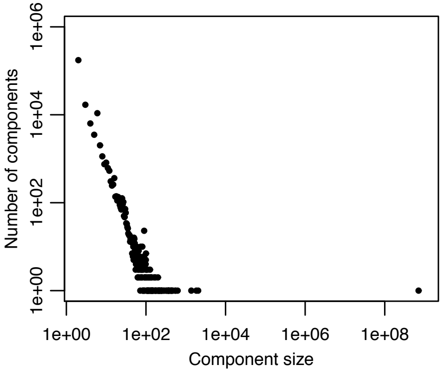
- Many networks consist of one large component and many small ones



Component size distribution in the 2011 Facebook network on a log-log scale. Most vertices (99.91%) are in the largest component.

Degree distribution of the Internet

Degree of a node: number of edges connected to a node

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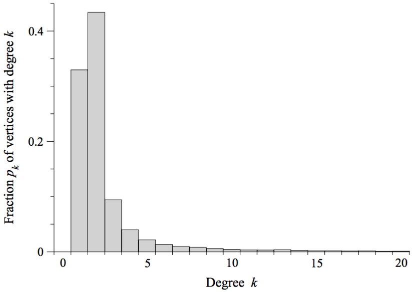
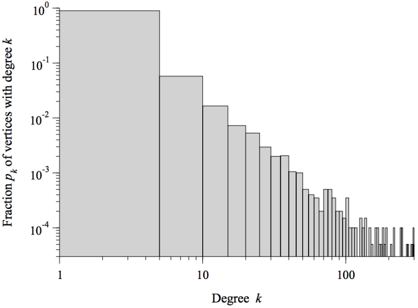
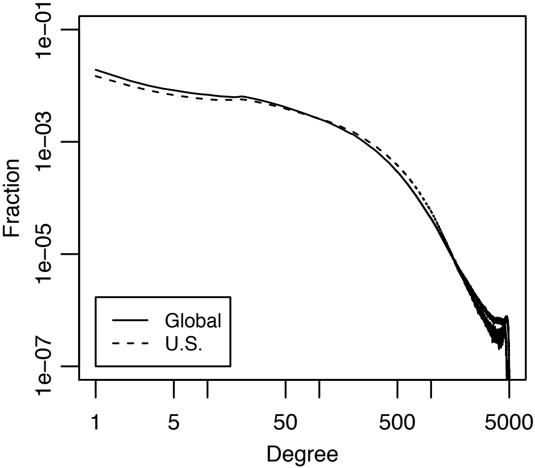
- Many networks show a power-law degree distribution (i.e., distribution that is linear in log-log plot)





Figures from Chapter 8 in "Networks: An Introduction" by M.E.J. Newman (2010)

Degree distribution of Facebook network



From "The Anatomy of the Facebook Social Graph" by Ugander et al. (2011)

Diameter of a graph

- Let d_{ij} denote the length of the geodesic path (or shortest path) between node i and j
- The diameter of a network is the largest distance between any two nodes in the network:

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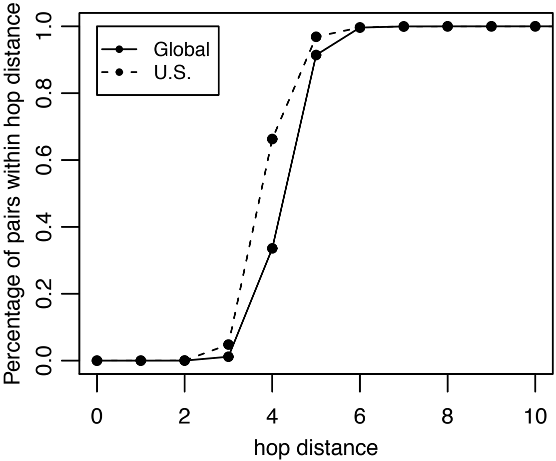
- If network is not connected, one often computes the diameter in the largest component.

- Algorithms for finding shortest paths: breadth-first search for unweighted graph, Dijkstra's algorithm for weighted graphs

Small-world and 6 degrees of separation

- Concept of 6 degrees of separation was made famous by sociologist Stanley Milgram and his study "The Small World Problem" (1967)
- In his experiment participants from a particular town were asked to get a letter to a particular person in a different town by passing it from acquaintance to acquaintance.
- 18 out of 96 letters made it in an average of 5.9 steps, suggesting that the diameter of the social network in the US is 6
- Any reasons why we should take the conclusion of 6 degrees of separation with a grain of salt?

Diameter of Facebook (2011)



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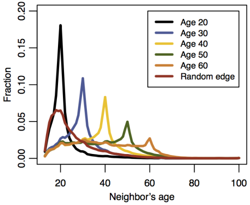
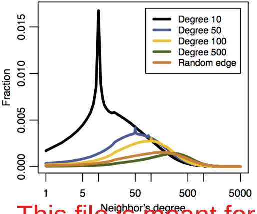
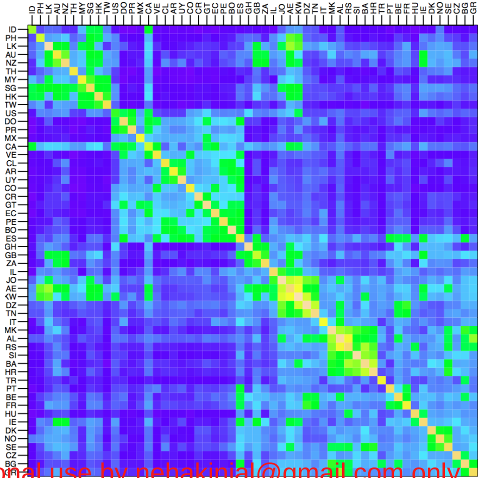
Homophily

Homophily (or assortative mixing): tendency of people to associate with others that are similar







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  - tracks how close a node is to any other node
- **betweenness centrality**
  - measures the extent to which a node lies on paths between other nodes

Which centrality measure to use

Choice of centrality measure depends on application!

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In a friendship network:

- high degree centrality: most popular person
- high eigenvector centrality: most popular person that is friends with popular people
- high closeness centrality: person that could best inform the group
- high betweenness centrality: person whose removal could best break the network apart

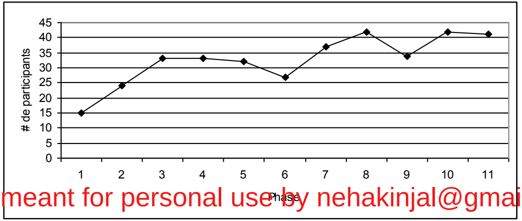
Case study: CAVIAR (criminal network in Montreal)

- Data based on 11 wiretap warrants from 1994-1996 $\rightarrow$ 11 periods
- Mandate of CAVIAR project: Seize drugs, arrests only in period 11
- 11 seizures total with monetary losses for traffickers of $32 mio
  - phase 4: 1 seizure $ 2.5mio, 300kg of marijuana
  - phase 6: 3 seizures $ 1.3mio, 2 x 15kg of marijuana, 1 x 2 kg of cocaine
  - phase 7: 1 seizure $ 3.5mio, 401kg of marijuana
  - phase 8: 1 seizure $ 0.4mio, 9kg of cocaine
  - phase 9: 2 seizures $ 4.3mio, 2kg of cocaine + 1 x 500kg marijuana
  - phase 10: 1 seizure $ 18.7mio, 2200kg of marijuana
  - phase 11: 2 seizures $ 1.3mio, 12kg of cocaine + 11kg of cocaine

Unique opportunity to study changes in the structure of a criminal network in upheaval by police forces

Case study: CAVIAR (criminal network in Montreal)

- network consists of 110 (numbered) players: 1-82 are traffickers, 83-110 are non-traffickers (financial investors, accountants, owners of various importation businesses, etc.)
- initially, investigation targeted Daniel Serero, alleged mastermind of drug network in downtown Montreal
- initially marijuana was imported to Canada from Morocco
- after first seizure in phase 4, traffickers reoriented to cocaine import from Colombia, transiting through the United States



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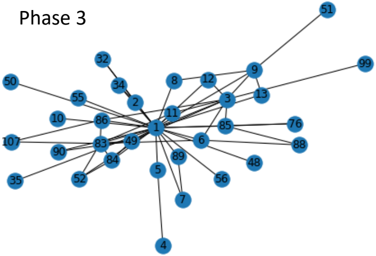
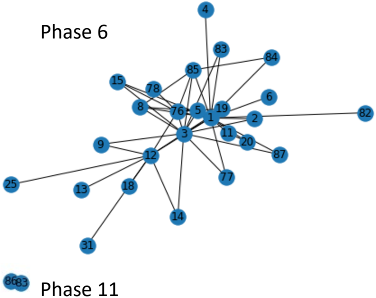
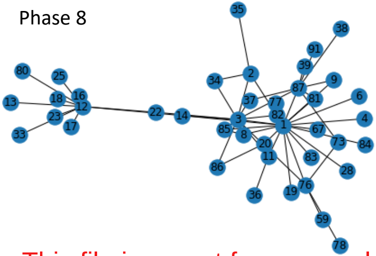
Phase 3



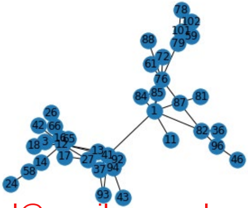
Phase 6



Phase 8



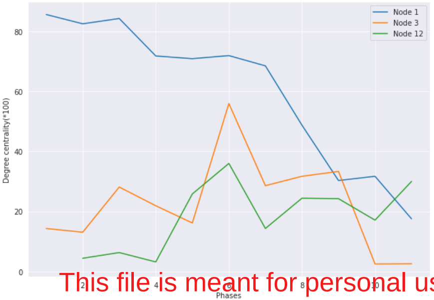
Phase 11

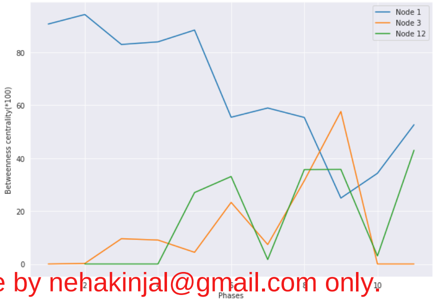


Case study: CAVIAR (criminal network in Montreal)

Role of the different actors:

- Daniel Serero (node 1): mastermind of the network
- Pierre Perlini (node 3): principal lieutenant of Serero (executes his instructions)
- Ernesto Morales (node 12): principal organizer of the cocaine import, intermediary between the Colombians and the Serero organization





Optional: Additional thoughts - Criminal networks

- Given a social network and k criminal suspects, how to determine other suspects?
- Same question is extremely important in biology: given certain genes that are known to cause a certain disease, determine other candidate genes (e.g. based on protein-protein interaction network for determining autism genes: http://dx.doi.org/10.1101/057828)
- How do we identify nodes that are "between" a given set of seed nodes?

Optional: Steiner trees

Determine a small subnetwork that contains the given suspects / genes and connects these nodes

Steiner tree:

- shortest subnetwork that contains a given set of nodes
- NP-complete problem
- there exist polynomial time approximations

⇒ use collection of approximate Steiner trees for further analysis: autism interactome / criminal interactome

For genomics applications, see: http://fraenkel-nsf.csbi.mit.edu/steinernet/tutorial.html

⇒ compute nodes with high betweenness centrality in interactome to obtain candidate genes / suspects

References

- Chapters 1 - 10 (but mostly chapters 6 - 8) in

M. E. J. Newman. *Networks: An Introduction*. 2010.

- For an analysis of the Facebook network:

J. Ugander, B. Karrer, L. Backstrom and C. Marlow. *The Anatomy of the Facebook Social Graph*. 2011.

- For more information on the CAVIAR network:

C. Morselli. Inside Criminal Networks (Springer, New York). Chapter 6: Law-enforcement disruption of a drug-importation network. 2009.