Modeling Roster Construction as an Optimization Problem

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1 Hard Cap Model

1.1 Informal Description

This model seeks to answer the following question:

Given a set of constraints derived from financial considerations and league rules, what roster yields the highest expected regular season win percentage?

To answer this question, I took a set of players who are eligible to be added to a roster (2024 free agents, in particular, but they can be swapped for any set of available players), as well as a set of already rostered players. I then approximated the percent of the salary cap each of these players would likely demand, and tried to craft a roster that yields the highest expected win percentage. I required that these rosters satisfy the following constraints:

- **(Salary cap constraint)** The sum of the proportions of the salary cap demanded by the players on the roster must be less than or equal to some predefined limit. This limit can be 100 percent of the salary cap, or higher, or lower.
- **(Roster size constraint)** I required that the total number of players on the roster be no greater than 15 and no less than 12. I allowed the size of the roster to shrink to as few as 12 players to account for two-way and 10 day deals, given that my model only considers players seeking to sign a uniform player contract.
- **(Rostered players constraint)** I assumed that the team has no option to release the players currently signed, requiring that currently rostered players be included on any potentially optimal rosters.
- **(Play time constraint)** To account for the desire of players to have substantial playing time, I required that the sum of the average possessions per game for all players on the roster not exceed $5 \times$ [the league average possessions per game], ensuring that the team does not promise more possessions of play time than are available in a game.

To test out this model, visit my my website.

1.2 Variables

Let $\{p_i\}$, i $\in \mathbb{Z}_n$, $p_i \in \mathbb{Z}_2$ be a collection of n players either currently on the roster or available to be added to the roster. If $p_i = 1$, then we consider player i to be on the roster. If $p_i = 0$, then they are not on the team roster.

For each i $\in \mathbb{Z}_n$, let c_i be the proportion of the salary cap (i.e. the cost) of adding player p_i to the roster. To gauge such a cost, I "ranked" players in the NBA by estimated plus-minus and found the average proportion of the salary cap paid out to players less than or equal to three ranks away from p_i.

To assess the points added by a given player, let e_i^o be p_i's estimated offensive rating divided by 100 to approximate their points added per possession. Similarly, let e_i^d be p_i estimated defensive rating divided by 100, representing the points given up by a player per-possession.

Let n_p^i represent p_i's average possessions per game, and let n_p be the team average possessions per game in the 2023-24 season.

1.3 Objective Function

The function I seek to maximize is the ordinary Pythagorean win percent function, with exponent k=13.91. For p $\in \mathbb{Z}_2^n$ (i.e. a vector p where p_i is 1 if player p_i is in the roster, 0 if not), we have the following objective function:

$$\text{F: } \mathbb{Z}_2^n \to [0,\, 1]$$
$$\text{F(p)} = \frac{[\sum_{i=0}^{n} e_i^0 n_i^p p_i]^k}{[\sum_{i=0}^{n} e_i^0 n_i^p p_i]^k + [\sum_{i=0}^{n} e_i^d n_i^p p_i]^k}$$

The $[e_i^0 n_i^p p_i]$ term represents the points added by a given player, given by their estimated points per possession, e_i^0, times their average number of possessions per game, n_i^p, times p_i to ensure that the term disappears when the player is not included in the roster. Similarly, the $[e_i^d n_i^p p_i]$ term represents points given up, with e_i^d representing the points given up per possession by player p_i.

1.4 Rostered Players Constraint

Let J $\subseteq \mathbb{Z}_n$ represent the players already on the roster of the team being augmented. For instance, if j $\in$ J, then player p_j is on the team's roster, and cannot be replaced (i.e. they are a mandatory part of any salary cap or expected win percent calculations).

To ensure the optimization respects rostered players, I introduced the following constraint, requiring any p in the parameter space $\mathbb{Z}_2^n$ satisfy:

$$p_i = 1 \text{ if } i \in J$$

1.5 Salary cap constraint

In this model, the salary cap is modeled as a hard limit. This limit is not necessarily equal to the actual 2023-24 salary cap. Rather, it is represented as a percentage of the cap, allowing for teams to spend up to 2x the cap. Although simplistic, in practice, this model of the CBA's salary system may be sufficient, given that GMs face caps imposed by ownership related to their willingness to enter the luxury tax.

Given p_i and c_i as above, as well as some $t \in [.95, 2]$, I imposed the following constraint on the optimization:

$$\sum_{i=0}^{n} c_i p_i \leq t$$

For instance, if t=1.5, then the maximum salary able to be paid out is equal to 1.5 times the 2023-24 salary cap.

1.6 Roster size constraint

The above problem considers only players on a uniform contract. To account for the fact that two-way and 10-day players could fill slots on rosters, some variability in the size of the team roster is permitted. The following constraint is imposed, requiring that the roster include at least 12 and no more than 15 players:

$$12 \leq \sum_{i=0}^{n} p_i \leq 15$$

1.7 Play time constraint

Players, when considering teams, often express the desire to have a defined role and play time. To ensure that the team does not over-promise playing time, I introduced the following constraint, equivalent to requiring that the sum of the average possessions per game for all players on the roster be no greater than 5 times the league-average team possessions per game.

$$\sum_{i=0}^{n} n_p^i p_i \leq 5 n_p$$

Because players may be willing to accept a smaller role in exchange for a higher chance of winning, I excluded this constraint from the optimization by default.

1.8 Optimization

I performed the resulting binary, non-linear optimization using the GEKKO package.

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