# ITCS 6150 – Intelligent Systems Fall 2021 A Project report On

"Solving 8-puzzle using A\* algorithm"

## **Team details**

Mohith Raju mraju2@uncc.edu 801254279 Nehal Kathale nkathale@uncc.edu 801205316

Renu Karule rkarule@uncc.edu 801261337

# A\* search Algorithm:

The A\* search algorithm is an informed search algorithm that is guaranteed to find the most optimal path from a start node to the goal node if there exists one. It is a tree traversal technique that searches for an optimal path based on the path cost and heuristic functions that are associated with each node.

The A\* search Algorithm is terminated when it finds a least-cost path from start node to the goal or if no such path can be determined. This search is also to be terminated when an infinite loop is formed while traversing between nodes. This happens when the node that is being expanded to has the same path costs and heuristics as the current node's parent. This infinite loop can be avoided by avoiding such nodes and expanding to the next most optimal node.

Initially, from the start node, A\* search algorithm traverses through the tree and expands to the child node with the least path cost. This process is repeated until we reach the goal node by expanding to the most optimal node. The path cost for each of the nodes is given by f(n) which comprises two factors, g(n) and h(n).

$$f(n) = g(n) + h(n)$$

Where.

n = the current node on the path.

g(n) = the path cost from the start node to the current node.

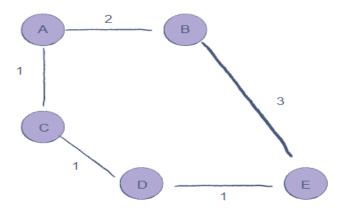
h(n) = the estimated cost for reaching the goal node from the current node

f(n) = the estimated path cost from the current node to the goal.

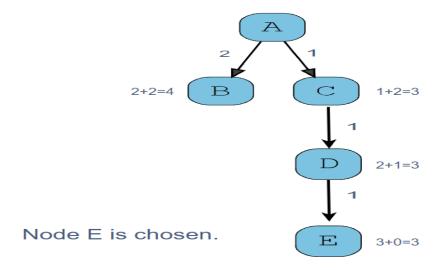
For A\* search algorithm to find the most optimistic path, the given heuristics must be admissible, i.e. it most never overestimate the cost to reach the goal. A heuristic h(n) of a node n is admissible when  $h(n) < h^*(n)$ , where  $h^*(n)$  is the actual cost from n to the goal. If such a heuristic can be estimated, then the path cost we formulate will always be the most optimal.

# Example:

Here is a graph with 5 nodes and how A \* algorithm solve this graph traversal. The start state is A and the goal node is E.



A\* algorithm makes decisions based on f-value by adding together g and h values. The node with minimum value of f will be selected next on a path to the goal node.





# Properties of A\* search Algorithm:

## Completeness:

A\* search Algorithm is guaranteed to terminate and it is complete. i.e. It will always return the most optimal solution if there exists one. In order to ensure this optimality, We need to make sure the algorithm does not get caught in infinite loops (The successor node is the same as the parent node for n).

## Time Complexity:

The time complexity of A\* search is given by O(b^d).

Where,

b = the number of nodes in the tree.

d = the depth of the goal in the tree.

## **Space Complexity:**

The Space complexity of A\* search is given by O(b\*d), since all the nodes are stored in the memory.

## Optimality:

A\* search is optimal. It is guaranteed to find the shortest path from start to goal if such a path exists. This is because A\* search does expand to the nodes where  $f(n) > C^*$ , i.e. It does not expand to nodes whose f(n) is greater than the current node's f(n). It only expands if  $f(n) < C^*$  or  $f(n) = C^*$ .

#### The 8-Puzzle Problem:

The 8-Puzzle Problem consists of a 3x3 grid and each tile in the grid is numbered from 1 to 8 and the 9th tile is a blank. Our goal is to solve this puzzle by arranging the blocks in a desired order by following the most optimal path. This can be achieved by moving either the blank tile's adjacent tiles to its place, and continuing until we reach our goal state.

#### **Heuristic Functions:**

The heuristics provide some preliminary information using which we can decide which successor tile we can expand to. For the 8-puzzle problem, We are going to use one of two heuristics, Misplaced tile heuristic or The Manhattan distance heuristic.

## - Misplaced Tile:

The Misplaced Tile heuristic is calculated by comparing the current state to the goal state and determining the number of tiles that are misplaced.

#### Manhattan Distance:

The Manhattan Distance of the current state is calculated by comparing each tile's position numbered 1 to 8 to each tile in the goal state. Suppose the coordinates of tile numbered '1' are (I, m) in the current state and the coordinates are (i, j) in the goal state, then the Manhattan distance between these two tiles is given by : |i - I| + |j - m|. i.e. The difference in the absolute values between in the (x, y) coordinates from both the states.

# Program Design:

The functions used in this program are as follows:

## checkRepetitiveState():

It is important to check whether the current state generated may have already been generated. If the current state has already been generated, then it may cause an infinite loop. If any of the n states have already been generated, they will be ignored.

## fetchEmptyPuzzleTileLocation():

When starting to solve this puzzle we must find the blank tile. After locating the blank tile, we can determine what tiles to move and at what positions. The blank tile uses the number "0" as the specification.

## calculateHeuristicValue():

It calculates the distance by comparing a node to the goal state. The result is stored as a heuristic cost. It identifies the same value tile in both the state and computes the absolute difference between the two tile indexes. In the example above, there are two tiles, 1 is at location [0,1] in initial state and [1,2] as a goal state, thus [abs(0-1)+abs(1-2)] = 2 is calculated and is repeated for all tiles.

## swapEmptyPuzzleTile():

In the 8 Puzzle, the tiles must be shifted into blank spaces. Shifting is accomplished by providing the array location from the tile arrangement to this function.

## checkSimilarPuzzleGridState():

While moving the tiles to an empty space, There may be instances when the new state formed is similar to the previous state. This must be avoided as it will cause infinite loops in the algorithm. This can be avoided by constantly checking if the new state that was formed is the same as the previous state and not expanding onto these nodes.

## findAdjacentNode():

This function is used to find the position of the blank space in the currently generated state.

## getProcessLocation():

This function is used to find the position of the blank space while the determineHeuristicValue() function is being executed.

## displayPathFollowed():

This function is used to print the 8-puzzle grid values after each successful expansion. This makes it easier for the user to understand the order of expansions of our algorithm to reach the goal state.

## Source Code:

Source code of implementation is available at below path:

https://drive.google.com/drive/folders/1C2jkogeBCHTO8GJRJD3HrJ4nDblCooe5?usp=sharing

# Output:

## Using Manhattan Distance Heuristic

#### Case1:

Select a Heuristic function Press

- 1. Misplaced Tile
- 2. Manhattan Distance

```
Please provide initial 8 Puzzle grid. Enter 0 for blank tile Please enter [0] [0] location of initial 8 Puzzle 1
Please enter [0] [1] location of initial 8 Puzzle 2
Please enter [0] [2] location of initial 8 Puzzle 3
Please enter [1] [0] location of initial 8 Puzzle 7
Please enter [1] [1] location of initial 8 Puzzle 4
Please enter [1] [2] location of initial 8 Puzzle 5
Please enter [2] [0] location of initial 8 Puzzle 6
Please enter [2] [1] location of initial 8 Puzzle 8
```

```
Please enter [2] [2] location of initial 8 Puzzle
Please provide goal 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of final 8 Puzzle
Please enter [0] [1] location of final 8 Puzzle
Please enter [0] [2] location of final 8 Puzzle
Please enter [1] [0] location of final 8 Puzzle
Please enter [1] [1] location of final 8 Puzzle
Please enter [1] [2] location of final 8 Puzzle
Please enter [2] [0] location of final 8 Puzzle
Please enter [2] [1] location of final 8 Puzzle
Please enter [2] [2] location of final 8 Puzzle
Transition States
123
745
680
123
740
685
123
704
685
123
784
605
123
784
065
123
084
```

```
765
123
804
765
123
864
705
123
864
750
Total moves required: 8
Number of nodes expanded: 9
Number of nodes generated: 19
Case2:
Select a Heuristic function
Press
1. Misplaced Tile
2. Manhattan Distance
2
Please provide initial 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of initial 8 Puzzle
Please enter [0] [1] location of initial 8 Puzzle
Please enter [0] [2] location of initial 8 Puzzle
Please enter [1] [0] location of initial 8 Puzzle
Please enter [1] [1] location of initial 8 Puzzle
Please enter [1] [2] location of initial 8 Puzzle
Please enter [2] [0] location of initial 8 Puzzle
```

```
Please enter [2] [1] location of initial 8 Puzzle
Please enter [2] [2] location of initial 8 Puzzle
Please provide goal 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of final 8 Puzzle
Please enter [0] [1] location of final 8 Puzzle
Please enter [0] [2] location of final 8 Puzzle
Please enter [1] [0] location of final 8 Puzzle
Please enter [1] [1] location of final 8 Puzzle
Please enter [1] [2] location of final 8 Puzzle
Please enter [2] [0] location of final 8 Puzzle
Please enter [2] [1] location of final 8 Puzzle
Please enter [2] [2] location of final 8 Puzzle
Transition States
724
506
831
724
536
801
724
536
8 1 0
724
530
8 1 6
724
503
8 1 6
```

8 1 6

8 1 6

8 1 6

8 1 6

8 1 6

8 1 6

0 1 5

```
023
```

786

123

045

786

123

405

786

123

450

786

123

456

780

Total moves required: 20

Number of nodes expanded: 169 Number of nodes generated: 452

\_\_\_\_\_\_

#### Case 3:

Select a Heuristic function

Press

- 1. Misplaced Tile
- 2. Manhattan Distance

2

Please provide initial 8 Puzzle grid. Enter 0 for blank tile

Please enter [0] [0] location of initial 8 Puzzle

1

Please enter [0] [1] location of initial 8 Puzzle

2

Please enter [0] [2] location of initial 8 Puzzle

3

Please enter [1] [0] location of initial 8 Puzzle

```
4
Please enter [1] [1] location of initial 8 Puzzle
Please enter [1] [2] location of initial 8 Puzzle
Please enter [2] [0] location of initial 8 Puzzle
Please enter [2] [1] location of initial 8 Puzzle
Please enter [2] [2] location of initial 8 Puzzle
Please provide goal 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of final 8 Puzzle
Please enter [0] [1] location of final 8 Puzzle
Please enter [0] [2] location of final 8 Puzzle
Please enter [1] [0] location of final 8 Puzzle
Please enter [1] [1] location of final 8 Puzzle
Please enter [1] [2] location of final 8 Puzzle
Please enter [2] [0] location of final 8 Puzzle
Please enter [2] [1] location of final 8 Puzzle
Please enter [2] [2] location of final 8 Puzzle
Transition States
123
480
765
123
485
760
123
485
706
123
```

```
4 0 5
7 8 6

1 2 3
4 5 0
7 8 6

1 2 3
4 5 6
7 8 0

Total moves required: 5
Number of nodes expanded: 6
```

Number of nodes generated: 12

\_\_\_\_\_\_

#### Case 4:

```
Select a Heuristic function
Press
1. Misplaced Tile
2. Manhattan Distance
Please provide initial 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of initial 8 Puzzle
Please enter [0] [1] location of initial 8 Puzzle
Please enter [0] [2] location of initial 8 Puzzle
Please enter [1] [0] location of initial 8 Puzzle
Please enter [1] [1] location of initial 8 Puzzle
Please enter [1] [2] location of initial 8 Puzzle
Please enter [2] [0] location of initial 8 Puzzle
Please enter [2] [1] location of initial 8 Puzzle
Please enter [2] [2] location of initial 8 Puzzle
5
Please provide goal 8 Puzzle grid. Enter 0 for blank tile
```

Please enter [0] [0] location of final 8 Puzzle

Please enter [0] [1] location of final 8 Puzzle

Please enter [0] [2] location of final 8 Puzzle

Please enter [1] [0] location of final 8 Puzzle

Please enter [1] [1] location of final 8 Puzzle

Please enter [1] [2] location of final 8 Puzzle

Please enter [2] [0] location of final 8 Puzzle

Please enter [2] [1] location of final 8 Puzzle

Please enter [2] [2] location of final 8 Puzzle 5

**Transition States** 

281

043

765

081

243

765

801

243

765

8 1 0

2 4 3 7 6 5

813

240

765

8 1 3

204

765

8 1 3

765

013

824

765

103

824

765

123

804

765

Total moves required: 9

Number of nodes expanded: 12 Number of nodes generated: 28

\_\_\_\_\_\_

#### Case 5:

Select a Heuristic function

Press

- 1. Misplaced Tile
- 2. Manhattan Distance

```
Please provide initial 8 Puzzle grid. Enter 0 for blank tile Please enter [0] [0] location of initial 8 Puzzle 0
Please enter [0] [1] location of initial 8 Puzzle 2
Please enter [0] [2] location of initial 8 Puzzle 4
Please enter [1] [0] location of initial 8 Puzzle 5
Please enter [1] [1] location of initial 8 Puzzle 6
Please enter [1] [2] location of initial 8 Puzzle 1
```

```
Please enter [2] [0] location of initial 8 Puzzle
Please enter [2] [1] location of initial 8 Puzzle
Please enter [2] [2] location of initial 8 Puzzle
Please provide goal 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of final 8 Puzzle
Please enter [0] [1] location of final 8 Puzzle
Please enter [0] [2] location of final 8 Puzzle
Please enter [1] [0] location of final 8 Puzzle
Please enter [1] [1] location of final 8 Puzzle
Please enter [1] [2] location of final 8 Puzzle
Please enter [2] [0] location of final 8 Puzzle
Please enter [2] [1] location of final 8 Puzzle
Please enter [2] [2] location of final 8 Puzzle
Transition States
024
561
873
524
061
873
524
861
073
524
861
703
524
```

2 1 3

0 1 3

Total moves required: 24

Number of nodes expanded: 1352 Number of nodes generated: 3762

#### Case 6:

Select a Heuristic function Press

- 1. Misplaced Tile
- 2. Manhattan Distance

```
2
Please provide initial 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of initial 8 Puzzle
Please enter [0] [1] location of initial 8 Puzzle
Please enter [0] [2] location of initial 8 Puzzle
Please enter [1] [0] location of initial 8 Puzzle
Please enter [1] [1] location of initial 8 Puzzle
Please enter [1] [2] location of initial 8 Puzzle
Please enter [2] [0] location of initial 8 Puzzle
Please enter [2] [1] location of initial 8 Puzzle
Please enter [2] [2] location of initial 8 Puzzle
Please provide goal 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of final 8 Puzzle
Please enter [0] [1] location of final 8 Puzzle
Please enter [0] [2] location of final 8 Puzzle
Please enter [1] [0] location of final 8 Puzzle
Please enter [1] [1] location of final 8 Puzzle
Please enter [1] [2] location of final 8 Puzzle
Please enter [2] [0] location of final 8 Puzzle
Please enter [2] [1] location of final 8 Puzzle
```

0 1 8

Total moves required: 27

Number of nodes expanded: 817 Number of nodes generated: 2347

## Using Misplaced Tile Heuristic

#### Case1:

Select a Heuristic function

#### Press

- 1. Misplaced Tile
- 2. Manhattan Distance

```
Please provide initial 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of initial 8 Puzzle
Please enter [0] [1] location of initial 8 Puzzle
Please enter [0] [2] location of initial 8 Puzzle
Please enter [1] [0] location of initial 8 Puzzle
Please enter [1] [1] location of initial 8 Puzzle
Please enter [1] [2] location of initial 8 Puzzle
Please enter [2] [0] location of initial 8 Puzzle
Please enter [2] [1] location of initial 8 Puzzle
Please enter [2] [2] location of initial 8 Puzzle
Please provide goal 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of final 8 Puzzle
Please enter [0] [1] location of final 8 Puzzle
Please enter [0] [2] location of final 8 Puzzle
Please enter [1] [0] location of final 8 Puzzle
```

Please enter [1] [1] location of final 8 Puzzle

Please enter [1] [2] location of final 8 Puzzle

Please enter [2] [0] location of final 8 Puzzle

Please enter [2] [1] location of final 8 Puzzle

Please enter [2] [2] location of final 8 Puzzle

**Transition States** 

```
8 6 4
7 0 5
```

864

750

Total moves required: 8

Number of nodes expanded: 20 Number of nodes generated: 42

\_\_\_\_\_\_

#### Case 2:

Select a Heuristic function

Press

- 1. Misplaced Tile
- 2. Manhattan Distance

Please provide initial 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of initial 8 Puzzle

Please enter [0] [1] location of initial 8 Puzzle

Please enter [0] [2] location of initial 8 Puzzle

Please enter [1] [0] location of initial 8 Puzzle

Please enter [1] [1] location of initial 8 Puzzle

Please enter [1] [1] location of initial 8 Puzzle

Please enter [1] [2] location of initial 8 Puzzle

Please enter [2] [0] location of initial 8 Puzzle

Please enter [2] [1] location of initial 8 Puzzle

```
Please enter [2] [2] location of initial 8 Puzzle
Please provide goal 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of final 8 Puzzle
Please enter [0] [1] location of final 8 Puzzle
Please enter [0] [2] location of final 8 Puzzle
Please enter [1] [0] location of final 8 Puzzle
Please enter [1] [1] location of final 8 Puzzle
Please enter [1] [2] location of final 8 Puzzle
Please enter [2] [0] location of final 8 Puzzle
Please enter [2] [1] location of final 8 Puzzle
Please enter [2] [2] location of final 8 Puzzle
\Transition States
724
506
831
724
536
801
724
536
8 1 0
724
530
8 1 6
724
```

8 1 6

8 1 6

8 1 6

8 1 6

8 1 6

8 1 6

8 1 6

7 1 5

0 1 5

Total moves required: 20

Number of nodes expanded: 2079 Number of nodes generated: 5732

-----

## Case 3:

Select a Heuristic function

Press

- 1. Misplaced Tile
- 2. Manhattan Distance

```
1
Please provide initial 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of initial 8 Puzzle
Please enter [0] [1] location of initial 8 Puzzle
Please enter [0] [2] location of initial 8 Puzzle
Please enter [1] [0] location of initial 8 Puzzle
Please enter [1] [1] location of initial 8 Puzzle
Please enter [1] [2] location of initial 8 Puzzle
Please enter [2] [0] location of initial 8 Puzzle
Please enter [2] [1] location of initial 8 Puzzle
Please enter [2] [2] location of initial 8 Puzzle
Please provide goal 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of final 8 Puzzle
Please enter [0] [1] location of final 8 Puzzle
Please enter [0] [2] location of final 8 Puzzle
Please enter [1] [0] location of final 8 Puzzle
Please enter [1] [1] location of final 8 Puzzle
Please enter [1] [2] location of final 8 Puzzle
Please enter [2] [0] location of final 8 Puzzle
Please enter [2] [1] location of final 8 Puzzle
Please enter [2] [2] location of final 8 Puzzle
0
```

# **Transition States** Total moves required: 5 Number of nodes expanded: 9 Number of nodes generated: 18

#### Case 4:

Select a Heuristic function

**Press** 

- 1. Misplaced Tile
- 2. Manhattan Distance

```
Please provide initial 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of initial 8 Puzzle
Please enter [0] [1] location of initial 8 Puzzle
Please enter [0] [2] location of initial 8 Puzzle
Please enter [1] [0] location of initial 8 Puzzle
Please enter [1] [1] location of initial 8 Puzzle
Please enter [1] [2] location of initial 8 Puzzle
Please enter [2] [0] location of initial 8 Puzzle
Please enter [2] [1] location of initial 8 Puzzle
Please enter [2] [2] location of initial 8 Puzzle
Please provide goal 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of final 8 Puzzle
Please enter [0] [1] location of final 8 Puzzle
Please enter [0] [2] location of final 8 Puzzle
Please enter [1] [0] location of final 8 Puzzle
Please enter [1] [1] location of final 8 Puzzle
Please enter [1] [2] location of final 8 Puzzle
Please enter [2] [0] location of final 8 Puzzle
Please enter [2] [1] location of final 8 Puzzle
Please enter [2] [2] location of final 8 Puzzle
Transition States
281
```

8 1 0

8 1 3

8 1 3

0 1 3

Total moves required: 9

Number of nodes expanded: 26 Number of nodes generated: 58

------

#### Case 5:

Select a Heuristic function

Press

- 1. Misplaced Tile
- 2. Manhattan Distance

```
1
Please provide initial 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of initial 8 Puzzle
Please enter [0] [1] location of initial 8 Puzzle
Please enter [0] [2] location of initial 8 Puzzle
Please enter [1] [0] location of initial 8 Puzzle
Please enter [1] [1] location of initial 8 Puzzle
Please enter [1] [2] location of initial 8 Puzzle
Please enter [2] [0] location of initial 8 Puzzle
Please enter [2] [1] location of initial 8 Puzzle
Please enter [2] [2] location of initial 8 Puzzle
Please provide goal 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of final 8 Puzzle
Please enter [0] [1] location of final 8 Puzzle
Please enter [0] [2] location of final 8 Puzzle
```

```
Please enter [1] [0] location of final 8 Puzzle
Please enter [1] [1] location of final 8 Puzzle
Please enter [1] [2] location of final 8 Puzzle
Please enter [2] [0] location of final 8 Puzzle
Please enter [2] [1] location of final 8 Puzzle
Please enter [2] [2] location of final 8 Puzzle
Transition States
024
561
873
204
561
873
240
561
873
241
560
873
241
506
873
241
576
803
241
576
```

0 1 6

Total moves required: 24

Number of nodes expanded: 8995 Number of nodes generated: 27302

------

#### Case 6:

```
Select a Heuristic function Press
```

- 1. Misplaced Tile
- 2. Manhattan Distance

```
1
Please provide initial 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of initial 8 Puzzle
Please enter [0] [1] location of initial 8 Puzzle
Please enter [0] [2] location of initial 8 Puzzle
Please enter [1] [0] location of initial 8 Puzzle
Please enter [1] [1] location of initial 8 Puzzle
Please enter [1] [2] location of initial 8 Puzzle
Please enter [2] [0] location of initial 8 Puzzle
Please enter [2] [1] location of initial 8 Puzzle
Please enter [2] [2] location of initial 8 Puzzle
Please provide goal 8 Puzzle grid. Enter 0 for blank tile
Please enter [0] [0] location of final 8 Puzzle
1
Please enter [0] [1] location of final 8 Puzzle
Please enter [0] [2] location of final 8 Puzzle
Please enter [1] [0] location of final 8 Puzzle
Please enter [1] [1] location of final 8 Puzzle
```

Please enter [1] [2] location of final 8 Puzzle 5
Please enter [2] [0] location of final 8 Puzzle 8
Please enter [2] [1] location of final 8 Puzzle 0
Please enter [2] [2] location of final 8 Puzzle 4

#### **Transition States**

3 4 8

3 4 8

3 4 8

0 1 8

1 2 3 7 6 5

804

Total moves required: 27

Number of nodes expanded: 21647 Number of nodes generated: 81409

# Results:

S. No	Initial State	Goal State	Misplace Tile	Manhattan Distance
1.	1 2 3 7 4 5 6 8 0	1 2 3 8 6 4 7 5 0	The operation cost is: 8 Expanded Nodes: 20 Generated Nodes: 42	The operation cost is: 8 Expanded Nodes: 9 Generated Nodes:19
2.	7 2 4 5 0 6 8 3 1	1 2 3 4 5 6 7 8 0	The operation cost is: 20 Expanded Nodes: 2079 Generated Nodes: 5732	The operation cost is: 20 Expanded Nodes: 169 Generated Nodes: 452
3.	1 2 3 4 8 0 7 6 5	1 2 3 4 5 6 7 8 0	The operation cost is: 5 Expanded Nodes: 9 Generated Nodes: 18	The operation cost is: 5 Expanded Nodes: 6 Generated Nodes: 12
4.	2 8 1 0 4 3 7 6 5	1 2 3 8 0 4 7 6 5	The operation cost is: 9 Expanded Nodes: 26 Generated Nodes: 58	The operation cost is: 9 Expanded Nodes: 12 Generated Nodes: 38
5.	0 2 4 5 6 1 8 7 3	134 256 078	The operation cost is: 24 Expanded Nodes: 8995 Generated Nodes: 27302	The operation cost is: 24 Expanded Nodes: 1352 Generated Nodes: 3762
6.	5 6 7 4 0 8 3 2 1	1 2 3 7 6 5 8 0 4	The operation cost is: 27 Expanded Nodes: 21647 Generated Nodes: 81409	The operation cost is: 27 Expanded Nodes: 817 Generated Nodes: 2347