Assignment - 1

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Linear Algebra (BTCSH-107).

1. Find the value of determinant of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$$

Sol. let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ be a given matrix.

Find : Determination of A $|A| = ?$

So, $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{vmatrix}$

$R_1 \to R_1 + R_2$

$$|A| = \begin{vmatrix} 2 & 6 & 5 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{vmatrix}$$

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Since elements of R_1 are identical to R_3.

$\therefore$, $|A| = 0$ [By property of determinant]

Q2. Show that $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$

Sol. To prove : $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$

L.H.S : let $|A| = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

$R_2 \to R_2 + (-1)R_1$, $R_3 \to R_3 + (-1)R_3$

$$|A| = \begin{vmatrix} 1 & a & b+c \\ 0 & (b-a) & (c+a-c-b) \\ 0 & (c-a) & (a+b-b-c) \end{vmatrix}$$

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$$|A| = \begin{vmatrix} 1 & a & b+c \\ 0 & (b-a) & -(b-a) \\ 0 & (c-a) & -(c-a) \end{vmatrix}$$

Taking '$(b-a)$' common from R_2 and '$(c-a)$' common from R_3

$$|A| = (b-a)(c-a) \begin{vmatrix} 1 & a & b+c \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{vmatrix}$$

Expanding along R_1 :

$$|A| = (b-a)(c-a)\left[1(-1+1) - a(0-0) + (b+c)(0-0)\right]$$

$$= (b-a)(c-a)\left[1(0) - a(0) + (b+c)(0)\right]$$

$$= (b-a)(c-a)[0]$$

$$= 0 = R.H.S.$$

Hence proved

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Q3. If $\begin{bmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{bmatrix} = 0$ in which

a, b, c are different, Show that $abc = 1$

Sol. Given : $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$

To prove : $abc = 1$, where $a \neq b \neq c$

Sol : $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$

By property of determinant :

$$\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} + \begin{vmatrix} a & a^2 & -1 \\ b & b^2 & -1 \\ c & c^2 & -1 \end{vmatrix} = 0$$

Taking 'a' from R_1, 'b' from R_2 and 'c' from R_3 common from Δ_1

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$$abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} (abc - 1) = 0$$

$\therefore$ $abc = 1$

Hence proved

Q4. Apply Cramer's Rule to solve the following equations :

$x + y + z = 4$
$x - y + z = 0$
$2x + y + z = 5$

Sol. The determinant formed by given system of equations :

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

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$= 1(-1-1) - 1(1-2) + 1(1+2)$

$= 1(-2) - 1(-1) + 1(3)$

$= -2 + 1 + 3$

$= 2$

$$D_1 = \begin{vmatrix} 4 & 1 & 1 \\ 0 & -1 & 1 \\ 5 & 1 & 1 \end{vmatrix}$$

$= 4(-1-1) - 1(0-5) + 1(0+5)$

$= 4(-2) + 5 + 5$

$= -8 + 10$

$= 2$

$$D_2 = \begin{vmatrix} 1 & 4 & 1 \\ 1 & 0 & 1 \\ 2 & 5 & 1 \end{vmatrix}$$

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$$= 1(0-5) - 4(1-2) + 1(5-0)$$

$$= -5 + 4 + 5$$

$$= 9 - 5$$

$$= 4.$$

$$D_3 = \begin{vmatrix} 1 & 1 & 4 \\ 1 & -1 & 0 \\ 2 & 1 & 5 \end{vmatrix}$$

$$= 1(-5-0) - 1(5-0) + 4(1+2)$$

$$= -5 - 5 + 4(3)$$

$$= -10 + 12$$

$$= 2.$$

By applying Cramer's Rule:

$$x = \frac{D_1}{D} = \frac{2}{2} = 1 \Rightarrow \boxed{x = 1}$$

$$y = \frac{D_2}{D} = \frac{4}{2} = 2 \Rightarrow \boxed{y = 2}$$

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$$z = \frac{D_3}{D} = \frac{2}{2} = 1 \Rightarrow \boxed{z = 1}$$

Q.5. Using Cramer's rule, find the solution of the equations

$$2x + y + z = 10$$
$$x - y + z = 3$$
$$x + y + z = 9$$

Sol. The determinant formed by given system of equations:

$$D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 2(-1-1) - 1(1-1) + 1(1+1)$$

$$= 2(-2) - 1(0) + 1(2)$$

$$= -4 + 2$$

$$= -2$$

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$$D_1 = \begin{vmatrix} 10 & 1 & 1 \\ 3 & -1 & 1 \\ 9 & 1 & 1 \end{vmatrix}$$

$= 10(-1-1) - 1(3-9) + 1(3+9)$

$= 10(-2) - 1(-6) + 1(12)$

$= -20 + 6 + 12$

$= -20 + 18$

$= -2$

$$D_2 = \begin{vmatrix} 2 & 10 & 1 \\ 1 & 3 & 1 \\ 1 & 9 & 1 \end{vmatrix}$$

$= 2(3-9) - 10(1-1) + 1(9-3)$

$= 2(-6) - 10(0) + 6$

$= -12 + 6$

$= -6$

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$$D_3 = \begin{vmatrix} 2 & 1 & 10 \\ 1 & -1 & 3 \\ 1 & 1 & 9 \end{vmatrix}$$

$$= 2(-9-3) - 1(9-3) + 10(1+1)$$

$$= 2(-12) - 1(6) + 10(2)$$

$$= -24 - 6 + 20$$

$$= -30 + 20$$

$$= -10.$$

By applying Cramer's Rule:

$$x = \frac{D_1}{D} = \frac{-2}{-2} = 1$$

$$y = \frac{D_2}{D} = \frac{-6}{-2} = 3$$

$$z = \frac{D_3}{D} = \frac{-10}{-2} = 5$$

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Q.6. Define (i) Transpose of a matrix

(ii) Symmetric matrix

Ans (1) Transpose of a matrix :-

The matrix obtained from any given matrix A, by interchanging rows and columns is called the transpose of A and is denoted by A' or A^T.

For example :- The transposed matrix of

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix} \text{ is } A' = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \end{bmatrix}$$

The transpose of an $m \times n$ matrix is an $n \times m$ matrix. Also the transpose of the transpose of a matrix coincides with itself, i.e, $(A')' = A$.

(2) Symmetric Matrix :-

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A Square matrix $A = [a_{ij}]$ is said to be Symmetric when $a_{ij} = a_{ji}$ for all i and j.

It means $A = A^T$.

Q.1. Define : (i) Adjoint of a Square matrix

(ii) Orthogonal Matrix.

Ans. (i). Adjoint of a Square matrix :

The determinant of the Square matrix

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \text{ is } \Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

The matrix formed by the cofactors of the elements in Δ is

$$\begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{bmatrix}$$. Then the transpose of this

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matrix i.e. $\begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix}$

is called the adjoint of the matrix A and is written as Adj. A.

Thus, the adjoint of A is the transposed matrix of cofactors of A.

(ii) Orthogonal Matrix :-

A square matrix with real no. of elements is said to be an orthogonal matrix if its transpose is equal to its inverse matrix. Or we can say that, when the product of a square matrix and its transpose gives an identity matrix, then the square matrix is known as an orthogonal matrix.

i.e. $A^T = A^{-1}$

or

$AA^T = I$.

Assignment - 2

Q.1. Find the rank of the following matrices :

(i) $\begin{bmatrix} 3 & 4 & 5 & 7 \\ 1 & 2 & -1 & 6 \\ 2 & 3 & 1 & 4 \end{bmatrix}$

Sol. Given matrix $A = \begin{bmatrix} 3 & 4 & 5 & 7 \\ 1 & 2 & -1 & 6 \\ 2 & 3 & 1 & 4 \end{bmatrix}$

Applying row operations to the given matrix :

$R_1 \longleftrightarrow R_2$

$$A \sim \begin{bmatrix} \cancel{3}\,1 & 2 & -1 & 6 \\ 3 & 4 & 5 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$$

$R_2 \to R_2 + (-3)R_1 \quad , R_3 \to R_3 + (-2)R_1$

$$A \sim \begin{bmatrix} 1 & 2 & -1 & 6 \\ 0 & -2 & 8 & -9 \\ 0 & -1 & 3 & -8 \end{bmatrix}$$

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$R_2 \leftrightarrow R_3$

$$A \sim \begin{bmatrix} 1 & 2 & -1 & 6 \\ 0 & -1 & 3 & -8 \\ 0 & -2 & 8 & -9 \end{bmatrix}$$

$R_3 \rightarrow R_3 + (-2)R_2$

$$A \sim \begin{bmatrix} 1 & 2 & -1 & 6 \\ 0 & -1 & 3 & -8 \\ 0 & 0 & 2 & 7 \end{bmatrix}$$

Since, matrix A has 3 non-zero independent rows.

So, $\rho(A) = 3$.

(ii) $$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

Sol. Applying row operations to given matrix

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$R_4 \leftrightarrow R_3$

$$A \sim \begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & -2 & 0 \\ 3 & 1 & 0 & 2 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$A \sim \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \end{bmatrix}$$

$R_2 \leftrightarrow R_1$

$$A \sim \begin{bmatrix} 1 & 1 & -2 & 0 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \end{bmatrix}$$

$R_4 \rightarrow R_4 + (-3)R_1$

$$A \sim \begin{bmatrix} 1 & 1 & -2 & 0 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & -2 & 6 & 2 \end{bmatrix}$$

$R_4 \rightarrow R_4 + 2R_2$

$$A \sim \begin{bmatrix} 1 & 1 & -2 & 0 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since, matrix A has '3' non-zero independent rows ∴

So, $\rho(A) = 3$.

Q.2. Find the inverse of the matrices given below:

(i) $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

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Sol. Given matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

Find: Inverse of matrix $A = ?$

Forming an augmented matrix $[A:I]$

$$[A:I] = \left[\begin{array}{ccc:ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 4 & 2 & 0 & 1 & 0 \\ 2 & 6 & 5 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 + (-1) R_1$, $R_3 \to R_3 + (-2) R_1$

$$[A:I] \sim \left[\begin{array}{ccc:ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & -2 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 + (-1) R_2$, $R_3 \to R_3 + (-1) R_2$

$$[A:I] \sim \left[\begin{array}{ccc:ccc} 1 & 0 & 4 & 2 & -1 & 0 \\ 0 & 2 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{array}\right]$$

Since, the determinant of matrix A is 0 and rank of matrix A is less than it's order, So it's inverse is not possible.

(ii) $\begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$.

Sol. Given matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$

Find : Inverse of matrix A = ?

Forming an augmented matrix [A:I] :

$$[A:I] = \left[\begin{array}{ccc:ccc} 3 & 2 & 4 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 3 & 5 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 + \left(\frac{-2}{3}\right) R_1$, $R_3 \to R_3 + \left(\frac{-1}{3}\right) R_1$

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$$[A:I] \sim \left[\begin{array}{ccc:ccc} 3 & 2 & 4 & 1 & 0 & 0 \\ 0 & -1/3 & -5/3 & -2/3 & 1 & 0 \\ 0 & 7/3 & 1/3 & -1/3 & 0 & 1 \end{array}\right]$$

$R_3 \to R_3 + 7R_2$

$$[A:I] \sim \left[\begin{array}{ccc:ccc} 3 & 2 & 4 & 1 & 0 & 0 \\ 0 & -1/3 & -5/3 & -2/3 & 1 & 0 \\ 0 & 0 & -8 & -1/3 & 0 & 1 \end{array}\right]$$

$R_1 \to \frac{1}{3}R_1$

$$[A:I] \sim \left[\begin{array}{ccc:ccc} 3\times 1/3 & 2/3 & 4/3 & 1/3 & 0 & 0 \\ 0 & -1/3 & -5/3 & -2/3 & 1 & 0 \\ 0 & 0 & -8 & -5 & 7 & 1 \end{array}\right]$$

$R_1 \to R_1 + 2R_2$

$$[A:I] \sim \left[\begin{array}{ccc:ccc} 1 & 0 & -2 & -1 & 2 & 0 \\ 0 & -1/3 & -5/3 & -2/3 & 1 & 0 \\ 0 & 0 & -8 & -5 & 7 & 1 \end{array}\right]$$

$R_2 \to (-3)R_2$

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$$[A:I] \sim \begin{bmatrix} 1 & 0 & -2 & : & -1 & 2 & 0 \\ 0 & 1 & 5 & : & 2 & -3 & 0 \\ 0 & 0 & -8 & : & -5 & 7 & 1 \end{bmatrix}$$

$R_1 \to R_1 + \left(\frac{-1}{4}\right) R_3$, $R_2 \to R_2 + \left(\frac{5}{8}\right) R_3$

$$[A:I] \sim \begin{bmatrix} 1 & 0 & 0 & : & 1/4 & 1/4 & -1/4 \\ 0 & 1 & 0 & : & -9/8 & 11/8 & 5/8 \\ 0 & 0 & -8 & : & -5 & 7 & 1 \end{bmatrix}$$

$R_3 \to \left(\frac{-1}{8}\right) R_3$

$$[A:I] \sim \begin{bmatrix} 1 & 0 & 0 & : & 1/4 & 1/4 & -1/4 \\ 0 & 1 & 0 & : & -9/8 & 11/8 & 5/8 \\ 0 & 0 & 1 & : & +5/8 & -7/8 & -1/8 \end{bmatrix}$$

So, $$A^{-1} = \begin{bmatrix} 1/4 & 1/4 & -1/4 \\ -9/8 & 11/8 & 5/8 \\ 5/8 & -7/8 & -1/8 \end{bmatrix}$$

$$= \frac{1}{8} \begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix}$$

Q3. Define Linearly dependent and linearly independent vectors ?

(i). Linearly Dependent :-

Let $V(F)$ be a vector space and $S = \{(\alpha_1, \alpha_2 \ldots \alpha_n)\}$ be a finite subset of V, then the set S is called linearly dependent if :

$$a_1\alpha_1 + a_2\alpha_2 + \ldots\ldots + \alpha_n a_n = 0$$

where, $a_1, a_2 \ldots a_n$ not all zero.

It means the vectors $\{\alpha_1, \alpha_2 \ldots \alpha_n\}$ are called linearly dependent if :

$\exists\ a_1, a_2 \ldots a_n \in F$ not all zero :

$$a_1\alpha_1 + a_2\alpha_2 + \ldots a_n\alpha_n = 0$$

(ii) Linearly independent vectors :

The vectors $\alpha_1, \alpha_2 \ldots \alpha_n$ are called linearly independent vectors if :

$$a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 \ldots\ldots + \alpha_n a_n = 0$$

where $a_1 = a_2 = a_3 = 0$

Q.5. Apply Gauss elimination method to solve the following equations :

$$x + 4y - z = -5$$
$$x + y - 6z = -12$$
$$3x - y - z = 4$$

Sol. The matrix $[A:B]$ formed from given equations is :

$$[A:B] = \begin{bmatrix} 1 & 4 & -1 & : & -5 \\ 1 & 1 & -6 & : & -12 \\ 3 & -1 & -1 & : & 4 \end{bmatrix}$$

$$R_2 \to R_2 + (-1)R_1, \quad R_3 \to R_3 + (-3)R_1$$

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$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 4 & -1 & -5 \\ 0 & -3 & -5 & -7 \\ 0 & -13 & 2 & 19 \end{array}\right]$$

$$R_3 \to R_3 + \left(\frac{-13}{3}\right) R_2$$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 4 & -1 & -5 \\ 0 & -3 & -5 & -7 \\ 0 & 0 & 71/3 & 148/3 \end{array}\right]$$

So, corresponding System of equations will be:

$$\frac{71}{3} z = \frac{148}{3}$$

$$\boxed{z = \frac{148}{71}}$$

$$-3y - 5z = -7$$

$$-3y - 5\left(\frac{148}{71}\right) = -7$$

$$-3y = 5\left(\frac{148}{71}\right) - 7$$

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$$-3y = \frac{740}{71} - 7$$

$$-3y = \frac{740 - 497}{71}$$

$$-3y = \frac{243}{71}$$

$$y = \frac{243}{71} \times \left(\frac{1}{-3}\right)$$

$$\boxed{y = \frac{-81}{71}}$$

$$x + 4y - z = -5$$

$$x + 4\left(\frac{-81}{71}\right) - \left(\frac{148}{71}\right) = -5$$

$$x = 4\left(\frac{81}{71}\right) + \frac{148}{71} - 5$$

$$x = \frac{324}{71} + \frac{148}{71} - 5$$

$$= \frac{324 + 148 - 355}{71}$$

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$$= \frac{472 - 355}{71}$$

$$\boxed{x = \frac{117}{71}}$$

Q.6 Investigate the consistency and find the solutions, if exist of the following system of equations:

$$4x - 2y + 6z = 8$$
$$x + y - 3z = -1$$
$$15x - 3y + 9z = 21$$

Sol. The given equations are of the form:

$$AX = B$$

So, the augmented matrix [A:B] formed is:

$$[A:B] = \begin{bmatrix} 4 & -2 & 6 & : & 8 \\ 1 & 1 & -3 & : & -1 \\ 15 & -3 & 9 & : & 21 \end{bmatrix}$$

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$R_1 \leftrightarrow R_2$

$$[A:B] \sim \begin{bmatrix} 1 & 1 & -3 & : & -1 \\ 4 & -2 & 6 & : & 8 \\ 15 & -3 & 9 & : & 21 \end{bmatrix}$$

$R_2 \to R_2 + (-4)R_1$, $R_3 \to R_3 + (-15)R_1$

$$[A:B] \sim \begin{bmatrix} 1 & 1 & -3 & : & -1 \\ 0 & -6 & 18 & : & 12 \\ 0 & -18 & 54 & : & 36 \end{bmatrix}$$

$R_3 \to R_3 + (-3)R_2$

$$[A:B] \sim \begin{bmatrix} 1 & 1 & -3 & : & -1 \\ 0 & -6 & 18 & : & 12 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$\therefore$, $\rho[A:B] = 2 = \rho[A] < n$

and $n = 3$

Here, $r < n$.

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So, this system is consistent and have infinitely many solutions.

So, corresponding system of equation from the augmented [A:B] matrix is:

$x + y - 3z = -1$ —— (1)
$-6y + 18z = 12$ —— (2)

let $z = k$

Putting value of z in eqn (2)

$-6y + 18k = 12$

$18k - 12 = 6y$

$\boxed{3k - 2 = y}$

Putting value of y and z in eqn (1)

$x + 3k - 2 - 3k = -1$

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$x = 2 - 1$

$\boxed{x = 1}$

Hence, $\boxed{x = 1}$, $\boxed{y = 3k - 2}$ and $\boxed{z = k}$

Independent Solution

Q.7. Find the values of k, so that the system of equations becomes consistent and find the corresponding solutions:

$$x + y + z = 1$$
$$2x + y + 4z = k$$
$$4x + y + 10z = k^2$$

Sol. The augmented matrix $[A:B]$ formed is:

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 2 & 1 & 4 & k \\ 4 & 1 & 10 & k^2 \end{array}\right]$$

$R_2 \to R_2 + (-2)R_1$, $R_3 \to R_3 + (-4)R_3$

$$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & -1 & 2 & : & k-2 \\ 0 & -3 & 6 & : & k^2-4 \end{bmatrix}$$

$R_3 \rightarrow R_3 + (-3)R_2$

$$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & -1 & 2 & : & k-2 \\ 0 & 0 & 0 & : & (k^2-4-3k+6) \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & -1 & 2 & : & (k-2) \\ 0 & 0 & 0 & : & (k-1)(k-2) \end{bmatrix}$$

So, we have two possible values of k, for which System has Solutions.

(i) $k=1$ (ii) $k=2$

(i). When, $k=1$, Corresponding System of eq^n will be :

$x+y+z=1$ —— ①.

$-y+2z = 1-2$

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$-y + 2z = -1$ —— (2)

Let $z = k_1$

Putting value of z in eq^n (2).

$-y + 2k_1 = -1$

$\boxed{2k_1 + 1 = y}$

Putting value of y and z in eq^n (1)

$x + 2k_1 + 1 + k_1 = 1$

$\boxed{x = -3k_1}$

Hence, for $k = 1$, the system of eq^n has one independent solution which is

$\boxed{z = k_1}$ and two dependent solutions

which are $\boxed{x = -3k_1}$ $\boxed{y = 2k_1 + 1}$

(ii) When $k = 2$, corresponding system of eq^n will be :

$$x + y + z = 1 \quad \text{--- (3)}$$

$$-y + 2z = 2 - 2$$

$$-y + 2z = 0 \quad \text{--- (4)}$$

let $\boxed{z = k_2}$

$$\boxed{y = 2k_2}$$

$$x + 2k_2 + k_2 = 1$$

$$\boxed{x = 1 - 3k_2}$$

Hence, for $k = 2$, the system of eq^n has one independent solution which is $\boxed{z = k_2}$ and two dependent solutions which are :

$\boxed{y = 2k_2}$ and $\boxed{x = 1 - 3k_2}$

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Q.8. Find the values of λ and μ so that the system of equations :

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

has (i) unique solution
(ii) infinite solutions
(iii) no solution.

Sol. The augmented matrix [A:B] formed from given equations is :

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & 3 & : & 10 \\ 1 & 2 & \lambda & : & \mu \end{bmatrix}$$

$$R_2 \to R_2 + (-1)R_1 \quad , R_3 \to R_3 + (-1)R_1$$

$$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 1 & \lambda-1 & : & \mu-6 \end{bmatrix}$$

$R_3 \to R_3 + (-1)R_2$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

(i) For unique solution :

$\lambda \neq 3$, μ = any value.

(ii). For infinite solution :

$\lambda = 3$, $\mu = 10$

(iii). For no solution :

$\lambda = 3$, $\mu \neq 10$

Q.9 Solve the equations :

$$4x + 2y + z + 3w = 0$$
$$6x + 3y + 4z + 7w = 0$$
$$2x + y + w = 0.$$

Sol. The matrix A formed from given eq^n & is :

$$A = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

$R_3 \leftrightarrow R_2$

$$A \sim \begin{bmatrix} 4 & 2 & 1 & 3 \\ 2 & 1 & 0 & 1 \\ 6 & 3 & 4 & 7 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$A \sim \begin{bmatrix} 2 & 1 & 0 & 1 \\ 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \end{bmatrix}$$

$R_2 \to R_2 + (-2)R_1$, $R_3 \to R_3 + (-3)R_1$

$$A \sim \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 4 & 4 \end{bmatrix}$$

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$R_3 \to R_3 + (-4) R_2$

$$A \sim \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

So, $\rho(A) = 2 < n = 4 \rightarrow$ System having infinite solⁿ

Corresponding system of eqⁿ will be

$2x + y + \cancel{z} w = 0$

$\cancel{y} \quad z + w = 0$

$\boxed{z = -w}$

$\boxed{y = -w - 2x}$

which gives an infinite no. of non-trivial solutions, x and w being the parameters

Assignment - 3

Q.1. Define Vector Space ?

Ans Let $F / F(+, \cdot)$, be a field whose elements are called scalar. Let 'V' be a non-empty set whose elements are called vectors, then $(V, +, \cdot) / V(F) / V$ is called vector space, if the following properties are satisfied.

1. Closure property : V is closed with respect to operation '+'
   i.e., $\alpha + \beta \in V \quad \forall \; \alpha, \beta \in V$.

2. Associative property : V follows associative property, if
   $$(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma) \quad \forall \; \alpha, \beta, \gamma \in V$$

3. Commutative property : V is commutative w.r.t. '+' if
   $$\alpha + \beta = \beta + \alpha \quad \forall \; \alpha, \beta \in V$$

4. Identity property : There exist ($\exists$) a unit vector $\vec{0} \in V$, such that, $\alpha + 0 = 0 + \alpha = \alpha \quad \forall \; \alpha \in V$

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5. Inverse property: For any element, $\alpha \in V$, $\exists$ a unit vector $(-\alpha) \in V$. Such that

$$\alpha + (-\alpha) = (-\alpha) + \alpha = 0 \quad \forall \alpha \in V$$

Then '$-\alpha$' is called inverse of α

The set $(V, +)$ is abelian group, if it satisfies all the above properties

6. There is define an external composition in V over F
"V is closed under Scalar multiplication"

i.e, $\alpha . a \in V \ \forall \ \alpha \in V, a \in F$

7. Distributive Law Satisfied in V over F

(i) $a(\alpha + \beta) = a\alpha + \beta a \ \forall \ \alpha, \beta \in V, a \in F$

(ii) $(a+b)\alpha = a\alpha + b\alpha \ \forall \ a, b \in F, \alpha \in V$

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Define
Q2. (i) Orthogonal vectors
(ii) Basis of a vector Space.

Ans. (i) Orthogonal vectors :-

Two vectors are orthogonal if they are perpendicular to each other, i.e. the dot product of the two vectors is zero.

If two elements α and β of a vector Space $V(F)$ with bilinear form V are Said to be orthogonal if $B(U, V) = 0$.

(ii). Basis of a vector Space :-

Let $V(F)$ be a vector Space over the field F, then a Set S of V is called basis of V if,

(i) S is linearly independent.
(ii) Each element of V can be expressed as the linear combination of element of S.

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Q.3. Show that the set of 3 vectors $(1,1,1)$, $(1,2,3)$ and $(1,4,2)$ is a basis for the vector space R^3.

Sol. Let $\alpha_1 = (1,1,1)$
$\alpha_2 = (1,2,3)$
$\alpha_3 = (1,4,2)$.

$a_1, a_2, a_3 \in F$

So, $a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 = 0$

$a_1(1,1,1) + a_2(1,2,3) + a_3(1,4,2) = 0$.

$(a_1, a_2 + a_3, a_1 + 2a_2 + 4a_3, a_1 + 3a_2 + 2a_3) = 0$

Matrix formed is :

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 2 \end{bmatrix}$$

$R_2 \to R_2 + (-1)R_1$, $R_3 \to R_3 + (-1)R_1$

$$A \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 2 & 1 \end{bmatrix}$$

$R_3 \to R_3 + (-2) R_2$

$$A \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & -5 \end{bmatrix}$$

$\because \rho(A) = 3 =$ no. of variables

Hence, $a_1 = a_2 = a_3 = 0$ [Trivial Sol^n].

$\therefore$, given vectors are linearly independent and is a basis for vector space R^3.

Q.4. Define vector Subspace ?

Ans. Let V(F) be a vector space then the subset ω of V is called vector subspace of V if ω itself a vector space over field F.

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Theorem: The necessary and sufficient conditions for non-empty subset W of a vector space $V(F)$ to be a subspace of V, is that W is closed under vector addition and scalar multiplication.

i.e. $\alpha + \beta \in W \; \forall \; \alpha, \beta \in W$

$a\alpha \in W \; \forall \; a \in F, \alpha \in W$

Q5. Show that the vectors $(2,1,1)$, $(1,-1,2)$ and $(3,1,2)$ is linearly dependent

Sol. let $\alpha_1 = (2,1,1)$
$\alpha_2 = (1,-1,2)$
$\alpha_3 = (3,1,2)$

$a_1, a_2, a_3 \in F$

and $a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 = \vec{0}$

$a_1(2,1,1) + a_2(1,-1,2) + a_3(3,1,2) = \vec{0}$

$(2a_1 + a_2 + 3a_3, \; a_1 - a_2 + a_3, \; a_1 + 2a_2 + 2a_3) = \vec{0}$

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Equations are :

$$2a_1 + a_2 + 3a_3 = 0$$
$$a_1 - a_2 + a_3 = 0$$
$$a_1 + 2a_2 + 2a_3 = 0$$

The matrix formed is :

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$R_2 \leftrightarrow R_1$

$$A \sim \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 2 \end{bmatrix}$$

$R_2 \to R_2 + (-2)R_1 \; , \; R_3 \to R_3 + (-1)R_1$

$$A \sim \begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & 1 \\ 0 & 3 & 1 \end{bmatrix}$$

$R_3 \to R_3 + (-1) R_2$

$$A \sim \begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Since, $|A| = 0$.

$\therefore$ Given vectors are linearly dependendent.

Q.6 Show that the intersection of any two Subspace of vector Space $V(F)$ is again a subspace.

Sol. Let W_1 and W_2 are two vector Subspace of vector Space $V(F)$.

To prove : $W_1 \cap W_2 \subseteq V$

Given : $W_1 \subseteq V$, $W_2 \subseteq V$.

Let $\alpha, \beta \in W_1 \cap W_2$ and $a, b \in F$

Since, $\alpha, \beta \in W_1 \cap W_2 \Rightarrow \alpha, \beta \in W_1$ and

$\alpha, \beta \in W_2$

Now, if $\alpha, \beta \in W_1$ and $a, b \in F$

then by applying theorem :

$a\alpha + b\beta \in W_1$ [$\because$, W_1 is subspace of V].

Similarly, $\alpha, \beta \in W_2$, $a, b \in F$

$\Rightarrow a\alpha + b\beta \in W_2$ [$\because$, W_2 is also subspace of V]

$\therefore$, $a\alpha + b\beta \in W_1$ and $a\alpha + b\beta \in W_2$

$\Rightarrow a\alpha + b\beta \in W_1 \cap W_2$

Thus, $W_1 \cap W_2$ is a subspace of $V(F)$

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Assignment - 4.

Q.1 Define (i) Hermitian matrix (ii) Unitary matrix

Ans. (i) A Square matrix 'A' is said to be Hermetian matrix if and only if the transpose of A is equal to conjugate of A.

i.e., $$A^T = \overline{A}$$

$$A = (\overline{A})^T = A^\theta \rightarrow \text{conjugate transpose}$$

(ii) A Square matrix 'A' is said to be an Unitary matrix, if and only if, conjugate transpose of A (A^θ) is equal to inverse of A.

i.e. $$(\overline{A})^T = A^{-1}$$

$$A^\theta = A^{-1}.$$

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Q2 Show that any square matrix A and its transpose A^T have the same eigen values.

Proof. Recall that the eigenvalues of a matrix are roots of its characteristic polynomial.

Hence, if the matrices A and A^T have the same characteristic polynomial, then they have the same eigenvalues.

So, we show that, the characteristic polynomial

$p_A(\lambda) = \det(A - \lambda I)$ of A is same as

the characteristic polynomial $p_{A^T}(\lambda) =$

$\det(A^T - \lambda I)$ of the transpose A^T.

We have.

$$p_{A^T}(\lambda) = \det(A^T - \lambda I)$$

$$= \det(A^T - \lambda I^T) \qquad [\because I^T = I]$$

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$$= \det\left((A-\lambda I)^T\right)$$

$$= \det(A-\lambda I) \quad [\because \det(B^T) = \det(B)]$$

$$= P_A(\lambda)$$

$\therefore$ we obtain $P_{A^T}(\lambda) = P_A(\lambda)$ and we conclude that the eigenvalues of A and A^T are the same.

Q.3. Find the Eigen values of the matrices given below:

(i) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

Sol. The characteristic eq^n of given matrix A is $|A-\lambda I| = 0$.

i.e. $\begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{vmatrix} = 0$

$R_1 \rightarrow R_1 - R_3$

$$\begin{vmatrix} -2-\lambda & 0 & 2+\lambda \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{vmatrix} = 0$$

$\Rightarrow (-2-\lambda)[(5-\lambda)(1-\lambda)-1] - 0 + (2+\lambda)(1-15+3\lambda) = 0$

$\Rightarrow -(2+\lambda)[5-5\lambda-\lambda+\lambda^2-\cancel{5}1] + (2+\lambda)(-14+\lambda 3) = 0$

$\Rightarrow -(2+\lambda)[\lambda^2-6\lambda+4] + (2+\lambda)(-14+3\lambda) = 0$

$\Rightarrow -(2+\lambda)(\lambda^2-6\lambda+4) + (2+\lambda)(-14+3\lambda) = 0$

$\Rightarrow -(2+\lambda)[\lambda^2-6\lambda+4+[-(-14+3\lambda)]] = 0$

$\cancel{-(2+\lambda)[\lambda^2-3\lambda+18] = 0}$

$\Rightarrow -(2+\lambda)[\lambda^2-6\lambda+4+14-3\lambda] = 0$

$\Rightarrow -(2+\lambda)(\lambda^2-9\lambda+18) = 0$

$\Rightarrow (\lambda+2)(\lambda-3)(\lambda-6) = 0.$

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Thus, the eigen values of A are

$\lambda = -2, 3, 6.$

(ii) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

Sol. The characteristic eq^n of given matrix A is $|A - \lambda I| = 0$.

i.e. $\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & 0-\lambda \end{vmatrix} = 0$

$R_1 \to R_1 + R_3$

$\begin{vmatrix} -3-\lambda & 0 & -3-\lambda \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix} = 0$

$\Rightarrow (-3-\lambda)\left[(1-\lambda)(-\lambda) - 12\right] - 0 + (-3-\lambda)(-4-(1$

$= 0$

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$\Rightarrow (-3-\lambda)[\lambda^2-\lambda-12]+(-3-\lambda)[-3-\lambda]=0$

$\Rightarrow (-3-\lambda)[\lambda^2-\lambda-12-3-\lambda]=0$

$\Rightarrow (-3-\lambda)[\lambda^2-2\lambda-15]=0$

$\Rightarrow -(\lambda+3)[\lambda^2-5\lambda+3\lambda-15]=0$

$\Rightarrow (\lambda+3)[(\lambda(\lambda-5)+3(\lambda-5)]=0$

$\Rightarrow (\lambda+3)(\lambda-5)(\lambda+3)=0$

Hence, the eigen values of A are:

$\lambda=-3,-3,5.$

Q4. Find the Eigen values and Eigen vectors of the following matrices

(i) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ Eigen values are

$\lambda=-2,3,6.$

Sol. Now, if x, y, z be the components of an eigen vector corresponding to the eigen value λ, we have

$$[A-\lambda I]X = \begin{bmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

(i) Putting $\lambda = -2$, we have

$$\begin{bmatrix} 1+2 & 1 & 3 \\ 1 & 5+2 & 1 \\ 3 & 1 & 1+2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 3 & 1 & 3 \\ 1 & 7 & 1 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\cancel{-x + y + 3z = 0}$

$$3x + y + 3z = 0$$
$$x + 7y + z = 0$$
$$3x + y + 3z = 0$$

The first and 3^{rd} eq^n being the same, we have from first two

$$\frac{x}{-20} = \frac{y}{0} = \frac{z}{20} \quad \text{or} \quad \frac{x}{-1} = \frac{y}{0} = \frac{z}{1}$$

Hence, independent eigen vectors for $\lambda = -2$ are

$$X = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

(ii) Putting, $\lambda = 3$, we have

$$\begin{bmatrix} 1-3 & 1 & 3 \\ 1 & 5-3 & 1 \\ 3 & 1 & 1-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$-2x + y + 3z = 0$$
$$x + 2y + z = 0$$

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$3x + y - 2z = 0$

The first and 3rd eqn being the same, we have only two independent equations :-

$-2x + y + 3z = 0$
$x + 2y + z = 0$

let $\boxed{z = 1}$

$-2x + y = -3$
$(x + 2y = -1) \times 2$

$\cancel{-2x} + y = -3$
$\cancel{2x} + 4y = -2$

$5y = -5$

$\boxed{y = -1}$

$x + 2(-1) = -1$
$\boxed{x = 1}$

For $\lambda = 3$, the independent eigen vectors are :

$$X = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

(iii) Putting $\lambda = 6$, we get

$$\begin{bmatrix} 1-6 & 1 & 3 \\ 1 & 5-6 & 1 \\ 3 & 1 & 1-6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$-5x + y + 3z = 0$$
$$x - y + z = 0$$
$$3x + y - 5z = 0$$

The first and third eq^n being the same, so we have only 2 independent equations :

For $\lambda = 3$, the independent eigen vectors are :-

$$X = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

(iii) Putting $\lambda = 6$, we get

$$\begin{bmatrix} 1-6 & 1 & 3 \\ 1 & 5-6 & 1 \\ 3 & 1 & 1-6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$-5x + y + 3z = 0$$
$$x - y + z = 0$$
$$3x + y - 5z = 0$$

The first and third eq^n being the same, so we have only 2 independent equations :

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$$-5x + y + 3z = 0$$
$$x - y + z = 0$$

let $\boxed{z = 1}$

$$-5x + y = -3$$
$$(x - y = -1) \times 5$$

$$-5x + y = -3$$
$$5x - 5y = -5$$

$$-4y = -8$$

$$\boxed{y = 2}$$

$$x - 2 + 1 = 0$$
$$\boxed{x = 1}$$

For $\lambda = 6$, the independent eigen vectors are

$$X = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

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(ii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Sol. The characteristic eq^n of given matrix A is $|A-\lambda I| = 0$.

i.e. $\begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0$

$R_2 \to R_2 + R_3$

$\begin{vmatrix} 6-\lambda & -2 & 2 \\ 0 & 2-\lambda & 2-\lambda \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0$

$(6-\lambda)\left[(2-\lambda)(3-\lambda)-(\lambda-2)\right]+2\left[0-2(2-\lambda)\right]$

$+2\left[0-2(2-\lambda)\right] = 0$

$(6-\lambda)\left[6-2\lambda-3\lambda+\lambda^2-\lambda+2\right]+2\left[-4+2\lambda\right]+$

$2\left[-4+2\lambda\right] = 0$

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$(6-\lambda)[\lambda^2-6\lambda+8]+(-8+4\lambda)-8+4\lambda=0$

$(6-\lambda)[\lambda^2-4\lambda-2\lambda+8]-16+8\lambda=0$

$(6-\lambda)[\lambda(\lambda-4)-2(\lambda-4)]+8(\lambda-2)=0$

$(6-\lambda)(\lambda-4)(\lambda-2)+8(\lambda-2)=0$

$(\lambda-2)[(6-\lambda)(\lambda-4)+8]=0$

$(\lambda-2)[6\lambda-24-\lambda^2+4\lambda+8]=0$

$(\lambda-2)[-\lambda^2+10\lambda-16]=0$

$-(\lambda-2)[\lambda^2-10\lambda+16]=0$

$-(\lambda-2)[\lambda^2-8\lambda-2\lambda+16]=0$

$-(\lambda-2)[\lambda(\lambda-8)-2(\lambda-8)]=0$

$-(\lambda-2)(\lambda-8)(\lambda-2)=0$

$\lambda=2,8,2$

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Thus the eigen value of given matrix A are 2, 8, 2.

Now, if x, y, z be the components of an eigen vector corresponding to the eigen value λ, we have

$$[A - \lambda I] X = \begin{bmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

(i) Putting $\lambda = 2$, we have:

$$\begin{bmatrix} 6-2 & -2 & 2 \\ -2 & 3-2 & -1 \\ +2 & -1 & 3-2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$4x - 2y + 2z = 0$$
$$2x - y + z = 0$$

let $y = k_1$, $z = k_2$

$$2x - k_1 + k_2 = 0$$

$$2x = k_1 - k_2$$

$$x = \frac{k_1 - k_2}{2}$$

$$x = \frac{k_1}{2} - \frac{k_2}{2}$$

So, Independent eigen vectors for $\lambda = 2$ are.

$$X = \begin{bmatrix} 1/2\, k_1 - 1/2\, k_2 \\ k_1 + 0k_2 \\ 0k_1 + k_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2\, k_1 \\ k_1 \\ 0k_1 \end{bmatrix} + \begin{bmatrix} -1/2\, k_2 \\ 0k_2 \\ k_2 \end{bmatrix}$$

$$= 1/2\, \cancel{k_1} \begin{bmatrix} k_1 \\ 2k_1 \\ 0k_1 \end{bmatrix} + \left(-\frac{1}{2}\right) \begin{bmatrix} k_2 \\ 0 \\ -2k_2 \end{bmatrix}$$

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$$= \frac{1}{2} k_1 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \left(\frac{-1}{2}\right) k_2 \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \quad \text{for } \lambda = 2.$$

(ii) Putting $\lambda = 8$, we get

$$\begin{bmatrix} 6-8 & -2 & 2 \\ -2 & 3-8 & -1 \\ 2 & -1 & 3-8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$-2x - 2y + 2z = 0$$

$$-2x - 5y - z = 0$$

$$2x - y - 5z = 0$$

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The first and third eqn being the same, we have:

$$\cancel{\left(-x + (-y) + z = 0\right) \times 2}$$

$$\cancel{-2x - 5y - z = 0}$$

$$-2x - 2y + 2z = 0$$
$$-2x - 5y - z = 0$$
$$+ \quad + \quad +$$

let $\boxed{z = 1}$

$$\cancel{-2x} - 2y = -2$$
$$\cancel{-2x} - 5y = 1$$
$$+ \quad + \quad -$$

$$3y = -3$$

$$\boxed{y = -1}$$

$$-2x - 2(-1) + 2(1) = 0$$
$$-2x + 2 + 2 = 0$$
$$+2x = +4$$
$$\boxed{x = 2}$$

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So, Independent Eigen vector for $\lambda = 8$ are

$$X = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$