Assignment 15

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Download the latex-tikz codes from

https://github.com/neharani289/MatrixTheory/Assignment15

1 Problem

(hoffman/page213/3) :

Find a projection $\mathbf{E}$ which projects $\mathbb{R}^2$ onto the subspace spanned by $(1,-1)$ along the subspace spanned by $(1,2)$.

2 Solution

Given	Let $\mathbf{x} \in \mathbb{R}^2$ $$\mathbf{x} = a\begin{pmatrix}1\\-1\end{pmatrix} + b\begin{pmatrix}1\\2\end{pmatrix} \tag{2.0.1}$$ where $\begin{pmatrix}a\\b\end{pmatrix}$ is representation of $\mathbf{x}$ in new basis.
To find	$$\mathbf{E}(\mathbf{x}) = a\begin{pmatrix}1\\-1\end{pmatrix} \tag{2.0.2}$$
Finding a Projection $\mathbf{E}$	As, $\begin{pmatrix}1\\-1\end{pmatrix}, \begin{pmatrix}1\\2\end{pmatrix}$ are linearly independent. Therefore,$\{\begin{pmatrix}1\\-1\end{pmatrix}, \begin{pmatrix}1\\2\end{pmatrix}\}$ is a basis of $\mathbb{R}^2$ As $\mathbf{x} \in \mathbb{R}^2$ $$\implies \mathbf{x} = a\begin{pmatrix}1\\-1\end{pmatrix} + b\begin{pmatrix}1\\2\end{pmatrix} \tag{2.0.3}$$

	$\implies \mathbf{x} = \begin{pmatrix}1 & 1\\ -1 & 2\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix} \qquad (2.0.4)$ $\implies \begin{pmatrix}a\\ b\end{pmatrix} = \begin{pmatrix}1 & 1\\ -1 & 2\end{pmatrix}^{-1}\mathbf{x} \qquad (2.0.5)$ $\implies \begin{pmatrix}a\\ b\end{pmatrix} = \begin{pmatrix}\frac{2}{3} & \frac{-1}{3}\\ \frac{1}{3} & \frac{1}{3}\end{pmatrix}\mathbf{x} \qquad (2.0.6)$ Therefore, $\mathbf{a} = \begin{pmatrix}\frac{2}{3} & \frac{-1}{3}\end{pmatrix}\mathbf{x} \qquad (2.0.7)$ Projection of $\mathbf{x}$ on subspace spanned by $\begin{pmatrix}1\\ -1\end{pmatrix}$; $\mathbf{E}(\mathbf{x}) = \mathbf{a}\begin{pmatrix}1\\ -1\end{pmatrix} \qquad (2.0.8)$ $\mathbf{E}(\mathbf{x}) = \begin{pmatrix}1\\ -1\end{pmatrix}\begin{pmatrix}\frac{2}{3} & \frac{-1}{3}\end{pmatrix}\mathbf{x} \qquad (2.0.9)$ $\mathbf{E}(\mathbf{x}) = \begin{pmatrix}\frac{2}{3} & \frac{-1}{3}\\ \frac{-2}{3} & \frac{1}{3}\end{pmatrix}\mathbf{x} \qquad (2.0.10)$ $\implies \mathbf{E} = \begin{pmatrix}\frac{2}{3} & \frac{-1}{3}\\ \frac{-2}{3} & \frac{1}{3}\end{pmatrix} \qquad (2.0.11)$
Verification	If $n \times n$ matrix $\mathbf{E}$ is projection matrix, then $\mathbf{E}^2 = \mathbf{E}$ $\mathbf{E^2} = \begin{pmatrix}\frac{2}{3} & \frac{-1}{3}\\ \frac{-2}{3} & \frac{1}{3}\end{pmatrix}\begin{pmatrix}\frac{2}{3} & \frac{-1}{3}\\ \frac{-2}{3} & \frac{1}{3}\end{pmatrix} \implies \mathbf{E} = \mathbf{E^2} = \begin{pmatrix}\frac{2}{3} & \frac{-1}{3}\\ \frac{-2}{3} & \frac{1}{3}\end{pmatrix} \qquad (2.0.12)$ Hence, Verified.

TABLE 1: Finding Projection Matrix