

# Assignment 17

Neha Rani  
EE20MTECH14014

Download the latex-tikz codes from

<https://github.com/neharani289/MatrixTheory/Assignment17>

## 1 PROBLEM

(ugcjune/2018/28) :

If  $\mathbf{A}$  is a  $2 \times 2$  matrix over  $\mathbb{R}$  with  $\det(\mathbf{A} + \mathbf{I}) = 1 + \det(\mathbf{A})$ , then we can conclude that

- 1)  $\det(\mathbf{A}) = 0$
- 2)  $\mathbf{A} = 0$
- 3)  $\text{tr}(\mathbf{A}) = 0$
- 4)  $\mathbf{A}$  is non singular.

## 2 SOLUTION

Given	<p><math>\mathbf{A}</math> be a <math>2 \times 2</math> matrix over <math>\mathbb{R}</math> with</p> $\det(\mathbf{A} + \mathbf{I}) = 1 + \det(\mathbf{A})$
Explanation	<p>If <math>\mathbf{X}</math> is an eigen vector of matrix <math>\mathbf{A}</math> corresponding to the eigen value <math>\lambda</math> i.e</p> $\mathbf{AX} = \lambda\mathbf{X}$ <p>then, <math>(\mathbf{I} + \mathbf{A})\mathbf{X} = (1 + \lambda)\mathbf{X}</math></p> <p>Thus, <math>\mathbf{X}</math> is an eigen vector of <math>(\mathbf{A} + \mathbf{I})</math> corresponding to the eigen value <math>(1 + \lambda)</math>.</p> <p>Let <math>\lambda_1, \lambda_2</math> be two eigen values of <math>\mathbf{A}</math> and <math>(1 + \lambda_1), (1 + \lambda_2)</math> be the eigen values of <math>(\mathbf{A} + \mathbf{I})</math>.</p> <p><math>\Rightarrow</math> Eigen value of <math>\mathbf{A} = \lambda_1, \lambda_2</math></p> <p><math>\Rightarrow</math> Eigen value of <math>(\mathbf{A} + \mathbf{I}) = \lambda_1 + 1, \lambda_2 + 1</math></p>

	<p>Since,</p> $\det(\mathbf{A} + \mathbf{I}) = 1 + \det(\mathbf{A})$ <p>Trace of any matrix is sum of its eigen values.</p> <p>Determinant of matrix is product of its eigen values</p> $\Rightarrow (\lambda_1 + 1)(\lambda_2 + 1) = 1 + (\lambda_1 \lambda_2)$ $\Rightarrow \boxed{\lambda_1 + \lambda_2 = 0}$ $\Rightarrow \boxed{tr(\mathbf{A}) = 0}$
Option 1 : $\det \mathbf{A} = 0$	<p>Let,</p> $\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}$ $\det \mathbf{A} = \begin{vmatrix} 0 & -1 \\ 0 & 0 \end{vmatrix} = 0$ $(\mathbf{A} + \mathbf{I}) = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ $\det(\mathbf{A} + \mathbf{I}) = \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 1$ $\Rightarrow \det(\mathbf{A} + \mathbf{I}) = 1 + \det(\mathbf{A})$ <p>Conclusion:</p> <ol style="list-style-type: none"> <li>1) <math>tr(\mathbf{A}) = 0</math></li> <li>2) <math>\det \mathbf{A} = 0</math></li> <li>3) <math>\mathbf{A} \neq \mathbf{0}</math></li> <li>4) <math>\mathbf{A}</math> is singular.</li> </ol>
Option 2 : $\mathbf{A} = \mathbf{0}$	<p>Let,</p> $\mathbf{A} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $\det \mathbf{A} = 0$ $\det(\mathbf{A} + \mathbf{I}) = 1$ $\Rightarrow \det(\mathbf{A} + \mathbf{I}) = 1 + \det(\mathbf{A})$

	<p>Conclusion:</p> <ol style="list-style-type: none"> <li>1) <math>tr(\mathbf{A}) = 0</math></li> <li>2) <math>\det \mathbf{A} = 0</math></li> <li>3) <math>\mathbf{A} = \mathbf{0}</math></li> <li>4) <math>\mathbf{A}</math> is singular.</li> </ol>
Option 4: $\mathbf{A}$ is non singular	<p>Non Singular Matrix: A non-singular matrix is a square one whose determinant is not zero. non-singular matrix is also a full rank matrix.</p> <p>Let,</p> $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\det \mathbf{A} = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = -1$ $(\mathbf{A} + \mathbf{I}) = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$ $\det (\mathbf{A} + \mathbf{I}) = \begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} = 0$ $\implies \det (\mathbf{A} + \mathbf{I}) = 1 + \det(\mathbf{A})$ <p>Conclusion:</p> <ol style="list-style-type: none"> <li>1) <math>tr(\mathbf{A}) = 0</math></li> <li>2) <math>\det \mathbf{A} \neq 0</math></li> <li>3) <math>\mathbf{A} \neq \mathbf{0}</math></li> <li>4) <math>\mathbf{A}</math> is non singular.</li> </ol>
Conclusion	<p>In all options, <math>tr(\mathbf{A}) = 0</math> satisfied.</p> <p>Thus, Option 3 is correct.</p>

TABLE 1: Solution Summary