Assignment 4

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***Abstract*—This document solves a question based on triangle using linear algebra.**

All the codes for the figure in this document can be found at

https://github.com/neharani289/ee14014/tree/master/Assignment4

1 Problem

In $\triangle ABC$, the bisector $\mathbf{AD}$ of $\angle A \perp$ to side $\mathbf{BC}$.Show that $\mathbf{AB} = \mathbf{AC}$ and $\triangle ABC$ is isosceles.

2 Solution

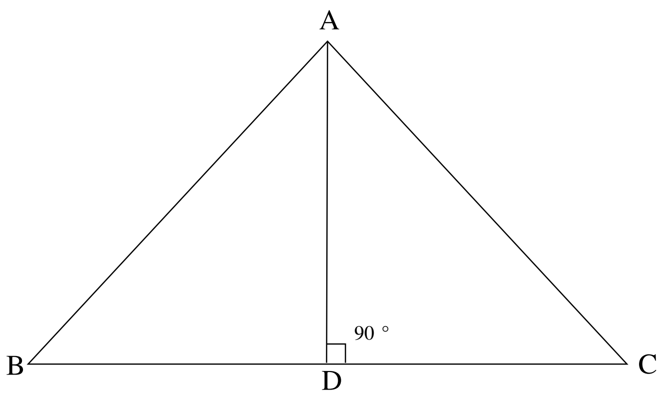

Fig. 1: Isosceles Triangle with $\mathbf{AD} \perp \mathbf{BC}$

Given, line AD is perpendicular to line BC which implies the inner product is zero

$$(\mathbf{B}-\mathbf{D})^T(\mathbf{A}-\mathbf{D}) = (\mathbf{D}-\mathbf{A})^T(\mathbf{B}-\mathbf{D}) \quad = 0 \tag{2.0.1}$$

$$(\mathbf{C}-\mathbf{D})^T(\mathbf{A}-\mathbf{D}) = (\mathbf{D}-\mathbf{A})^T(\mathbf{C}-\mathbf{D}) \quad = 0 \tag{2.0.2}$$

Consider$\triangle BAD$ and $\triangle CAD$;
Taking inner product of sides BA and AD

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{A}-\mathbf{D}) = \|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{D}\|\cos BAD \tag{2.0.3}$$

The angle BAD from the above equation is:

$$\cos BAD = \frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{A}-\mathbf{D})}{\|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{D}\|} \tag{2.0.4}$$

Taking inner product of sides CA and AD

$$(\mathbf{C}-\mathbf{A})^T(\mathbf{A}-\mathbf{D}) = \|\mathbf{C}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{D}\|\cos CAD \tag{2.0.5}$$

The angle CAD from the above equation is:

$$\cos CAD = \frac{(\mathbf{C}-\mathbf{A})^T(\mathbf{A}-\mathbf{D})}{\|\mathbf{C}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{D}\|} \tag{2.0.6}$$

from equation (2.0.4) and (2.0.6)

$$\angle BAD = \angle CAD \tag{2.0.7}$$

Now using pythagorus law;

$$\|\mathbf{B}-\mathbf{A}\|^2 = \|\mathbf{A}-\mathbf{D}\|^2 + \|\mathbf{B}-\mathbf{D}\|^2 \tag{2.0.8}$$

$$\|\mathbf{C}-\mathbf{A}\|^2 = \|\mathbf{A}-\mathbf{D}\|^2 + \|\mathbf{C}-\mathbf{D}\|^2 \tag{2.0.9}$$

using (2.0.7) and (2.0.2)we can conclude ;

$$\|\mathbf{B}-\mathbf{A}\| = \|\mathbf{C}-\mathbf{A}\| \tag{2.0.10}$$

Thus, $\triangle ABC$ is isosceles triangle.
Hence proved.