

# Assignment 4

Neha Rani

**Abstract**—This document solves a question based on triangle using linear algebra.

All the codes for the figure in this document can be found at

<https://github.com/neharani289/ee14014/tree/master/Assignment4>

## 1 PROBLEM

In  $\triangle ABC$ , the bisector  $AD$  of  $\angle A \perp$  to side  $BC$ . Show that  $AB = AC$  and  $\triangle ABC$  is isosceles.

## 2 SOLUTION

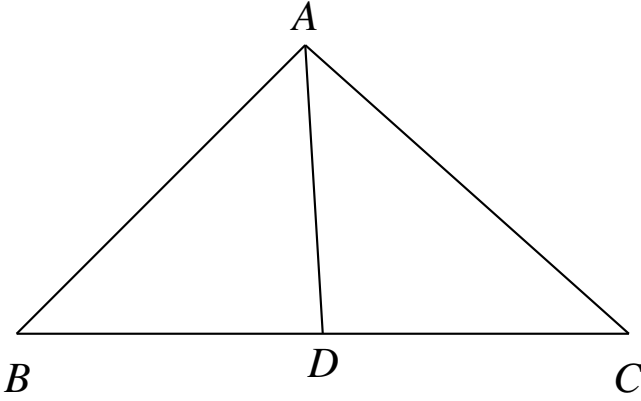


Fig. 1: Isosceles Triangle with  $AD \perp BC$

Given, line  $AD$  is perpendicular to line  $BC$  which implies the inner product is zero.

$$(B - D)^T(A - D) = (D - A)^T(B - D) = 0 \quad (2.0.1)$$

and

$$\|B - D\| = \|D - C\| \quad (2.0.2)$$

In  $\triangle BAD$  and  $\triangle CAD$  ;

Taking inner product of sides  $BA$  and  $AD$

$$(B - A)^T(A - D) = \|B - A\| \|A - D\| \cos BAD \quad (2.0.3)$$

The angle  $BAD$  from the above equation is:

$$\cos BAD = \frac{(B - A)^T(A - D)}{\|B - A\| \|A - D\|} \quad (2.0.4)$$

Taking inner product of sides  $BA$  and  $AD$

$$(C - A)^T(A - D) = \|C - A\| \|A - D\| \cos CAD \quad (2.0.5)$$

The angle  $CAD$  from the above equation is:

$$\cos CAD = \frac{(C - A)^T(A - D)}{\|C - A\| \|A - D\|} \quad (2.0.6)$$

from equation (2.0.4) and (2.0.6)

$$\angle BAD = \angle CAD \quad (2.0.7)$$

Similarly;

$$(A - D)^T(D - B) = \|A - D\| \|D - B\| \cos ADB \quad (2.0.8)$$

$$(A - D)^T(D - C) = \|A - D\| \|D - C\| \cos ADC \quad (2.0.9)$$

from equation (2.0.8) and (2.0.9) we can say;

$$\angle ADB = \angle ADC = 90^\circ \quad (2.0.10)$$

By, ASA property  $\triangle BAD \cong \triangle CAD$  , hence triangles are congruent.

Now by using CSCT property;

$$\|A - B\| = \|A - C\| \quad (2.0.11)$$

By converse of isosceles triangle theorem  $\triangle ABC$  is isosceles.

Hence proved.