Assignment 15

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Download the latex-tikz codes from

https://github.com/neharani289/MatrixTheory/Assignment15

1 Problem

(hoffman/page213/3) :

Find a projection $\mathbf{E}$ which projects $\mathbb{R}^2$ onto the subspace spanned by $(1, -1)$ along the subspace spanned by $(1, 2)$.

2 Solution

Given	Let $\begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$ $$\begin{pmatrix} x \\ y \end{pmatrix} = a\begin{pmatrix} 1 \\ -1 \end{pmatrix} + b\begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.1)$$ where $\begin{pmatrix} a \\ b \end{pmatrix}$ is representation of $\begin{pmatrix} x \\ y \end{pmatrix}$ in new basis.
To find	$$\mathbf{E}\begin{pmatrix} x \\ y \end{pmatrix} = a\begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.2)$$
Finding a Projection $\mathbf{E}$	We know in standard order basis ; $$\begin{pmatrix} x \\ y \end{pmatrix} : \begin{pmatrix} 1 \\ 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} y \quad (2.0.3)$$ Express $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ in the basis $\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$\begin{pmatrix}1\\0\end{pmatrix} = p\begin{pmatrix}1\\-1\end{pmatrix} + q\begin{pmatrix}1\\2\end{pmatrix} \tag{2.0.4}$$

where $\begin{pmatrix}p\\q\end{pmatrix}$ is representation of $\begin{pmatrix}1\\0\end{pmatrix}$ in the new basis.

$$\implies \begin{pmatrix}p\\q\end{pmatrix} = \begin{pmatrix}\frac{2}{3}\\\frac{1}{3}\end{pmatrix} \tag{2.0.5}$$

similarly;

$$\begin{pmatrix}0\\1\end{pmatrix} = r\begin{pmatrix}1\\-1\end{pmatrix} + s\begin{pmatrix}1\\2\end{pmatrix} \tag{2.0.6}$$

$$\implies \begin{pmatrix}r\\s\end{pmatrix} = \begin{pmatrix}\frac{-1}{3}\\\frac{1}{3}\end{pmatrix} \tag{2.0.7}$$

Substitute (2.0.5) and (2.0.7) in (2.0.3) we get;

$$\begin{pmatrix}x\\y\end{pmatrix} : \begin{pmatrix}\frac{2}{3}\\\frac{1}{3}\end{pmatrix}x + \begin{pmatrix}\frac{-1}{3}\\\frac{1}{3}\end{pmatrix}y = \begin{pmatrix}\frac{2}{3} & \frac{1}{3}\\\frac{-1}{3} & \frac{1}{3}\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} \tag{2.0.8}$$

From (2.0.1) and (2.0.8) ;

$$\begin{pmatrix}x\\y\end{pmatrix} : \begin{pmatrix}\frac{2}{3} & \frac{-1}{3}\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}\frac{1}{3} & \frac{1}{3}\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} \tag{2.0.9}$$

From (2.0.2) and (2.0.9);

$$\mathbf{E}\begin{pmatrix}x\\y\end{pmatrix} = \left(\tfrac{2}{3}x - \tfrac{1}{3}y\right)\begin{pmatrix}1\\-1\end{pmatrix} \tag{2.0.10}$$

$$\implies \mathbf{E}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}\frac{2}{3}\\\frac{-2}{3}\end{pmatrix}x + \begin{pmatrix}\frac{-1}{3}\\\frac{1}{3}\end{pmatrix}y \tag{2.0.11}$$

$$\mathbf{E}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}\frac{2}{3} & \frac{-1}{3}\\\frac{-2}{3} & \frac{1}{3}\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} \tag{2.0.12}$$

	Hence, $$\implies \mathbf{E} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \quad (2.0.13)$$
Verification	If $n \times n$ matrix $\mathbf{E}$ is projecion matrix, then $$\mathbf{E}^2 = \mathbf{E}$$ $$\mathbf{E^2} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \implies \mathbf{E} = \mathbf{E^2} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \quad (2.0.14)$$ Hence, Verified.

TABLE 1: Finding Projection Matrix