

Assignment 15

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Download the latex-tikz codes from

<https://github.com/neharani289/MatrixTheory/Assignment15>

1 PROBLEM

(hoffman/page213/3) :

Find a projection \mathbf{E} which projects \mathbb{R}^2 onto the subspace spanned by $(1, -1)$ along the subspace spanned by $(1, 2)$.

2 SOLUTION

Given	<p>Let $\mathbf{x} \in \mathbb{R}^2$</p> $\mathbf{x} = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} + b \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.1)$ <p>where $\begin{pmatrix} a \\ b \end{pmatrix}$ is representation of \mathbf{x} in new basis.</p>
To find	$\mathbf{E}(\mathbf{x}) = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.2)$
Finding a Projection \mathbf{E}	<p>As, $\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ are linearly independent.</p> <p>Therefore, $\left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$ is a basis of \mathbb{R}^2</p> <p>As $\mathbf{x} \in \mathbb{R}^2$</p> $\Rightarrow \mathbf{x} = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} + b \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.3)$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.0.4)$$

$$\Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}^{-1} \mathbf{x} \quad (2.0.5)$$

$$\Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \mathbf{x} \quad (2.0.6)$$

Therefore,

$$\mathbf{a} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \end{pmatrix} \mathbf{x} \quad (2.0.7)$$

Projection of \mathbf{x} on subspace spanned by $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$;

$$\mathbf{E}(\mathbf{x}) = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.8)$$

$$\mathbf{E}(\mathbf{x}) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \end{pmatrix} \mathbf{x} \quad (2.0.9)$$

$$\mathbf{E}(\mathbf{x}) = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \mathbf{x} \quad (2.0.10)$$

$$\Rightarrow \mathbf{E} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \quad (2.0.11)$$

Verification

If $n \times n$ matrix \mathbf{E} is projection matrix, then

$$\mathbf{E}^2 = \mathbf{E}$$

$$\mathbf{E}^2 = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \Rightarrow \mathbf{E} = \mathbf{E}^2 = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \quad (2.0.12)$$

Hence, Verified.

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TABLE 1: Finding Projection Matrix