

# Assignment 10

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**Abstract—This document solves a problem based on Linear Transformation .**

Download latex-tikz codes from

<https://github.com/neharani289/MatrixTheory/blob/master/Assignment10>

## 1 PROBLEM

Let  $\mathbf{T}$  be the linear transformation from  $\mathbb{R}^3$  into  $\mathbb{R}^2$  defined by

$$\mathbf{T}(x_1, x_2, x_3) = (x_1 + x_2, 2x_3 - x_1). \quad (1.0.1)$$

If  $\mathbb{B} = (\alpha_1, \alpha_2, \alpha_3)$  and  $\mathbb{B}' = (\beta_1, \beta_2)$  where

$$\alpha_1 = (1, 0, -1), \alpha_2 = (1, 1, 1), \alpha_3 = (1, 0, 0)$$

$$\beta_1 = (0, 1), \beta_2 = (1, 0)$$

What is the matrix of  $\mathbf{T}$  relative to the pair  $\mathbb{B}, \mathbb{B}'$

## 2 SOLUTION

Let,

$$\mathbb{B} = (\alpha_1, \alpha_2, \alpha_3) \quad (2.0.1)$$

$$\mathbb{B} = (1, 0, -1), (1, 1, 1), (1, 0, 0) \quad (2.0.2)$$

and

$$\mathbb{B}' = (\beta_1, \beta_2)$$

$$\mathbb{B}' = (0, 1), (1, 0) \quad (2.0.3)$$

We know that,

$$[T\alpha]_{\mathbb{B}'} = \mathbf{A}[\alpha]_{\mathbb{B}} \quad (2.0.4)$$

where  $\mathbf{A}$  is called the matrix of  $\mathbf{T}$  relative to ordered basis  $\mathbb{B}, \mathbb{B}'$  and  $\alpha$  is any vector in the space formed using basis vectors  $\mathbb{B}$ .

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ 2x_3 - x_1 \end{pmatrix} \quad (2.0.5)$$

Now, to find the matrix of  $\mathbf{T}$  relative to the pair  $\mathbb{B}'$

$$\begin{aligned} T(\alpha_1) &= T(1, 0, -1) = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \\ T(\alpha_2) &= T(1, 1, 1) = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \\ T(\alpha_3) &= T(1, 0, 0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{aligned} \quad (2.0.6)$$

thus, the transformation matrix is,

$$\begin{pmatrix} 1 & 2 & 1 \\ -3 & 1 & 0 \end{pmatrix} \quad (2.0.7)$$

$$[T\alpha]_{\mathbb{B}'} = \mathbf{A}[\alpha]_{\mathbb{B}} \quad (2.0.8)$$

Now, since the transformation has to be found relative to the pair  $\mathbb{B}, \mathbb{B}'$  row reduce the augmented matrix,

$$\left( \begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 2 & 1 \\ 1 & 0 & -3 & -3 & 1 & 0 \end{array} \right) \xrightarrow{R_1 \leftrightarrow R_2} \left( \begin{array}{ccc|ccc} 1 & 0 & -3 & -3 & 1 & 0 \\ 0 & 1 & 1 & 1 & 2 & 1 \end{array} \right) \quad (2.0.9)$$

Thus the matrix of  $\mathbf{T}$  wrt to  $\mathbb{B}, \mathbb{B}'$  is

$$\begin{pmatrix} -3 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix} \quad (2.0.10)$$