Assignment 15

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Download the latex-tikz codes from

https://github.com/neharani289/MatrixTheory/Assignment15

1 Problem

(hoffman/page213/3) :

Find a projection $\mathbf{E}$ which projects $\mathbb{R}^2$ onto the subspace spanned by $(1, -1)$ along the subspace spanned by $(1, 2)$.

2 Solution

Given	Let $\mathbf{x} \in \mathbb{R}^2$ $$\mathbf{x} = a\begin{pmatrix}1\\-1\end{pmatrix} + b\begin{pmatrix}1\\2\end{pmatrix} \tag{2.0.1}$$ where $\begin{pmatrix}a\\b\end{pmatrix}$ is representation of $\mathbf{x}$ in new basis.
To find	$$\mathbf{E}(\mathbf{x}) = a\begin{pmatrix}1\\-1\end{pmatrix} \tag{2.0.2}$$
Finding a Projection $\mathbf{E}$	As, $\begin{pmatrix}1\\-1\end{pmatrix}, \begin{pmatrix}1\\2\end{pmatrix}$ are linearly independent. Therefore,$\{\begin{pmatrix}1\\-1\end{pmatrix}, \begin{pmatrix}1\\2\end{pmatrix}\}$ is a basis of $\mathbb{R}^2$ As $\mathbf{x} \in \mathbb{R}^2$ $$\implies \mathbf{x} = a\begin{pmatrix}1\\-1\end{pmatrix} + b\begin{pmatrix}1\\2\end{pmatrix} \tag{2.0.3}$$

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$$\implies \mathbf{x} = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.0.4}$$

$$\implies \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}^{-1} \mathbf{x} \tag{2.0.5}$$

$$\implies \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix} \mathbf{x} \tag{2.0.6}$$

Therefore,

$$\mathbf{a} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \end{pmatrix} \mathbf{x} \tag{2.0.7}$$

Projection of $\mathbf{x}$ on subspace spanned by $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$;

$$\mathbf{E}(\mathbf{x}) = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2.0.8}$$

$$\mathbf{E}(\mathbf{x}) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \end{pmatrix} \mathbf{x} \tag{2.0.9}$$

$$\mathbf{E}(\mathbf{x}) = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \mathbf{x} \tag{2.0.10}$$

$$\implies \mathbf{E} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \tag{2.0.11}$$

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<td>Verification</td>
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If $n \times n$ matrix $\mathbf{E}$ is projection matrix, then

$$\mathbf{E}^2 = \mathbf{E}$$

$$\mathbf{E^2} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \implies \mathbf{E} = \mathbf{E^2} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \tag{2.0.12}$$

Hence, Verified.

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TABLE 1: Finding Projection Matrix