

# Assignment 15

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Download the latex-tikz codes from

<https://github.com/neharani289/MatrixTheory/Assignment15>

## 1 PROBLEM

(hoffman/page213/3) :

Find a projection  $\mathbf{E}$  which projects  $\mathbb{R}^2$  onto the subspace spanned by  $(1, -1)$  along the subspace spanned by  $(1, 2)$ .

## 2 SOLUTION

Given	<p>Let <math>\mathbf{x} \in \mathbb{R}^2</math></p> $\mathbf{x} = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} + b \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.1)$ <p>where <math>\begin{pmatrix} a \\ b \end{pmatrix}</math> is representation of <math>\mathbf{x}</math> in new basis.</p>
To find	$\mathbf{E}(\mathbf{x}) = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.2)$
Finding a Projection $\mathbf{E}$	<p>As, <math>\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}</math> are linearly independent.</p> <p>Therefore, <math>\{\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}\}</math> is a basis of <math>\mathbb{R}^2</math></p> <p>As <math>\mathbf{x} \in \mathbb{R}^2</math></p> $\Rightarrow \mathbf{x} = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} + b \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.3)$

	$\Rightarrow \mathbf{x} = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.0.4)$ $\Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}^{-1} \mathbf{x} \quad (2.0.5)$ $\Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix} \mathbf{x} \quad (2.0.6)$ <p>Therefore,</p> $a = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \end{pmatrix} \mathbf{x} \quad (2.0.7)$ <p>Projection of <math>\mathbf{x}</math> on subspace spanned by <math>\begin{pmatrix} 1 \\ -1 \end{pmatrix}</math>;</p> $\mathbf{E}(\mathbf{x}) = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.8)$ <p>Substituting (2.0.7) in (2.0.8)</p> $\mathbf{E}(\mathbf{x}) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \end{pmatrix} \mathbf{x} \quad (2.0.9)$ $\mathbf{E}(\mathbf{x}) = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \mathbf{x} \quad (2.0.10)$ $\Rightarrow \mathbf{E} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \quad (2.0.11)$
Verification	<p>If <math>n \times n</math> matrix <math>\mathbf{E}</math> is projection matrix, then  <math>\mathbf{E}^2 = \mathbf{E}</math></p> $\mathbf{E}^2 = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \Rightarrow \mathbf{E} = \mathbf{E}^2 = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \quad (2.0.12)$ <p>Hence, Verified.</p>

TABLE 1: Finding Projection Matrix