Assignment 12

Neha Rani
EE20MTECH14014

Download the latex-tikz codes from

https://github.com/neharani289/MatrixTheory/Assignment12

1 Problem

(hoffman/page189/5) :
Let

$$\mathbf{A} = \begin{pmatrix} 6 & -3 & -2 \\ 4 & -1 & -2 \\ 10 & -5 & -3 \end{pmatrix} \tag{1.0.1}$$

Is A similar over the field R to a diagonal matrix ?
Is A similar over the field C to a diagonal matrix?

2 Solution

Theorem 2	Let T be the linear operator on a finite dimensional space V and c_1,..,c_k be a distinct characteristic values of T and let W_i be the null space of $(T - c_i I)$ then 1)T is diagonalizable 2) characteristic polynomial for T is $f = (x - c_1)^{d_1} \ldots.(x - c_k)^{d_k}$ and 3) $dimW_i = d_i$, $i = 1, \ldots k$
Condition for diagonalization	A linear operator T on a n-dimensional space V is diagonalizable , if and only if T has n distinct characteristic vectors or null spaces corresponding to the characteristic values

Given	Let the given matrix be $\mathbf{A}=\begin{pmatrix} 6 & -3 & -2 \\ 4 & -1 & -2 \\ 10 & -5 & -3 \end{pmatrix}$
Finding Characteristics polynomial	Characteristics polynomial of the matrix A is $det(xI - A)$ $\det(xI - A)= \begin{vmatrix} (x-6) & 3 & 2 \\ -4 & (x+1) & 2 \\ -10 & 5 & x+3 \end{vmatrix}$ Characteristic Polynomial $= (x-2)(x^2+1) = (x-2)(x-i)(x+i)$
Checking whether A similar over the field R to a diagonal matrix	As the characteristics polynomial is not product of linear factors over R . Therefore from Theorem 2, A is not diagonalizable over R
Checking whether A similar over the field C to a diagonal matrix	The Characteristic Polynomial can be wriiten as a product of linear factors over C i.e $\det(xI - A) = (x-2)(x-i)(x+i)$ To find characteristic values of the operator $\det(xI - A) = 0$ which gives $c_1 = 2, c_2 = i, c_3 = -i$ Thus over C matrix A has three distinct characteristic values. There will be atleast one characteristics vector i.e., one dimension with each characteristics value . From Theorem 2; $\sum_i W_i = n = 3$, which is equal to dim of A. Thus , A is diagonalizable over C.
Conclusion	1) A is not diagonalizable over R. 2) A is diagonalizable over C.

TABLE 0: Illustration of Theorem and finding of characteristics values over different field.