Assignment 11

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Download the latex-tikz codes from

https://github.com/neharani289/MatrixTheory/Assignment11

1 Problem

(UGC-june2017,71) :
Let **V** be the vector space of polynomials of degree at most 3 in a varible x with coefficients in $\mathbb{R}$. Let **T**=d/dx be the linear transformation of **V** to itself given by differentiation.

Which of the following are correct?

1) **T** is invertible
2) 0 is an eigenvalue of **T**
3) There is a basis with respect to which the matrix of **T** is nilpotent.
4) The matrix of **T** with respect to the basis $\left(1, 1+x, 1+x+x^2, 1+x+x^2+x^3\right)$ is diagonal.

2 Solution

Checking whether matrix **T** is nilpotent	Let **V**= $P_3(x)$ **T**:**V**$\rightarrow$ **V** **T**(P(x)) = P'(x) Standard basis of P(x)=$\left(1, x, x^2, x^3\right)$ for finding P'(x) , differentiating the standard basis; $T(1)=0=a_1x+b_1x+c_1x^2+d_1x^3$ $T(x)=1=a_2+b_2x+c_2x^2+d_2x^3$ $T(x^2)=2x=a_3+b_3x+c_3x^2+d_3x^3$ $T(x^3)=3x^2=a_4+b_4x+c_4x^2+d_4x^3$ Representing **T** in matrix form ; $T=\begin{pmatrix}0&1&0&0\\0&0&2&0\\0&0&0&3\\0&0&0&0\end{pmatrix}$ from the above matrix **T** we can say it is nilpotent matrix.

Checking for eigen value of **T**	T=$\begin{pmatrix} 0-\lambda & 1 & 0 & 0 \\ 0 & 0-\lambda & 2 & 0 \\ 0 & 0 & 0-\lambda & 3 \\ 0 & 0 & 0 & 0-\lambda \end{pmatrix}$ $\implies \lambda = 0$
Checking whether **T** is invertible	Since $\det T = 0$. Therefore **T** is not invertible
Matrix **T** is diagonal matrix	Let basis of P(x) be B'=$\left(1, 1+x, 1+x+x^2, 1+x+x^2+x^3\right)$ Differentiating wrt x ; T(1)=0=$a_1 x + b_1(1+x) + c_1(1+x+x^2) + d_1(1+x+x^2+x^3)$ T(1+x)=1=$a_2 + b_2(1+x) + c_2(1+x+x^2) + d_2(1+x+x^2x^3)$ T(1+x+x^2) $= 1+2x = a_3 + b_3(1+x) + c_3(1+x+x^2) + d_3(1+x+x^2+x^3)$ T(1+x+x^2+x^3) $= 1+2x+3x^2 = a_4 + b_4(1+x) + c_4(1+x+x^2)$ $+d_4(1+x+x^2+x^3)$ T=$\begin{pmatrix} 0 & 1 & -1 & -1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ above matrix is not a diagonal matrix
Conclusion	Thus we can conclude Option 2) and 3) are correct.