Assignment 15

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Download the latex-tikz codes from

https://github.com/neharani289/MatrixTheory/Assignment15

1 Problem

(hoffman/page213/3) :

Find a projection $\mathbf{E}$ which projects $\mathbb{R}^2$ onto the subspace spanned by $(1, -1)$ along the subspace spanned by $(1, 2)$.

2 Solution

Given	Let $\mathbf{x} \in \mathbb{R}^2$ $$\mathbf{x} = a\begin{pmatrix} 1 \\ -1 \end{pmatrix} + b\begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.1)$$ where $\begin{pmatrix} a \\ b \end{pmatrix}$ is representation of $\mathbf{x}$ in new basis.
To find	$$\mathbf{E}(\mathbf{x}) = a\begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.2)$$
Finding a Projection $\mathbf{E}$	As, $\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ are linearly independent. Therefore,$\{\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}\}$ is a basis of $\mathbb{R}^2$ As $\mathbf{x} \in \mathbb{R}^2$ $$\implies \mathbf{x} = a\begin{pmatrix} 1 \\ -1 \end{pmatrix} + b\begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.3)$$

	$\implies \mathbf{x} = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}$ (2.0.4) $\implies \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}^{-1} \mathbf{x}$ (2.0.5) $\implies \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix}\mathbf{x}$ (2.0.6) Therefore, $\mathbf{a} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \end{pmatrix}\mathbf{x}$ (2.0.7) Projection of $\mathbf{x}$ on subspace spanned by $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$; $\mathbf{E}(\mathbf{x}) = \mathbf{a}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (2.0.8) $\mathbf{E}(\mathbf{x}) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}\begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \end{pmatrix}\mathbf{x}$ (2.0.9) $\mathbf{E}(\mathbf{x}) = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix}\mathbf{x}$ (2.0.10) $\implies \mathbf{E} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix}$ (2.0.11)
Verification	If $n \times n$ matrix $\mathbf{E}$ is projection matrix, then $\mathbf{E}^2 = \mathbf{E}$ $\mathbf{E^2} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix}\begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \implies \mathbf{E} = \mathbf{E^2} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix}$ (2.0.12) Hence, Verified.

TABLE 1: Finding Projection Matrix