Assignment 10

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***Abstract*—This document solves a problem based on Linear Transformation .**

Download latex-tikz codes from

https://github.com/neharani289/MatrixTheory/blob/master/Assignment10

1 Problem

Let $\mathbf{T}$ be the linear transformation from $\mathbb{R}^3$ into $\mathbb{R}^2$ defined by

$$\mathbf{T}(x_1, x_2, x_3) = (x_1 + x_2, 2x_3 - x_1). \tag{1.0.1}$$

If $\mathbb{B} = (\alpha_1, \alpha_2, \alpha_3)$ and $\mathbb{B}' = (\beta_1, \beta_2)$ where

$$\alpha_{\mathbf{1}} = (1, 0, -1), \alpha_{\mathbf{2}} = (1, 1, 1), \alpha_{\mathbf{3}} = (1, 0, 0)$$

$$\beta_{\mathbf{1}} = (0, 1), \beta_{\mathbf{2}} = (1, 0)$$

What is the matrix of $\mathbf{T}$ relative to the pair $\mathbb{B}$,$\mathbb{B}'$

2 Solution

Let,

$$\mathbb{B} = (\alpha_1, \alpha_2, \alpha_3) \tag{2.0.1}$$
$$\mathbb{B} = (1, 0, -1), (1, 1, 1), (1, 0, 0) \tag{2.0.2}$$

and

$$\mathbb{B}' = (\beta_1, \beta_2)$$
$$\mathbb{B}' = (0, 1), (1, 0) \tag{2.0.3}$$

We know that,

$$[T\alpha]_{\mathbb{B}'} = \mathbf{A}[\alpha]_{\mathbb{B}} \tag{2.0.4}$$

where $\mathbf{A}$ is called the matrix of T relative to ordered basis $\mathbb{B}$, $\mathbb{B}^{'}$ and α is any vector in the space formed using basis vectors $\mathbb{B}$.

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ 2x_3 - x_1 \end{pmatrix} \tag{2.0.5}$$

Now, to find the matrix of T relative to the pair B' B

$$T(\alpha_1) = T(1, 0, -1) = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$
$$T(\alpha_2) = T(1, 1, 1) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$
$$T(\alpha_3) = T(1, 0, 0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.6}$$

thus, the transformation matrix is,

$$\begin{pmatrix} 1 & 2 & 1 \\ -3 & 1 & 0 \end{pmatrix} \tag{2.0.7}$$

Now, since the transformation has to be found relative to the pair $\mathbb{B}$, $\mathbb{B}'$ row reduce the augmented matrix,

$$\left(\begin{array}{cc|ccc} 0 & 1 & 1 & 2 & 1 \\ 1 & 0 & -3 & 1 & 0 \end{array}\right) \xleftrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{cc|ccc} 1 & 0 & -3 & 1 & 0 \\ 0 & 1 & 1 & 2 & 1 \end{array}\right) \tag{2.0.8}$$

Thus the matrix of $\mathbf{T}$ wrt to $\mathbb{B}$, $\mathbb{B}'$ is

$$\begin{pmatrix} -3 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix} \tag{2.0.9}$$