Assignment 15

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Download the latex-tikz codes from

https://github.com/neharani289/MatrixTheory/Assignment15

1 Problem

(hoffman/page213/3) :

Find a projection $\mathbf{E}$ which projects $\mathbb{R}^2$ onto the subspace spanned by $(1, -1)$ along the subspace spanned by $(1, 2)$.

2 Solution

Given	Let $\begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$ $$\begin{pmatrix} x \\ y \end{pmatrix} = a\begin{pmatrix} 1 \\ -1 \end{pmatrix} + b\begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{2.0.1}$$ where $\begin{pmatrix} a \\ b \end{pmatrix}$ is representation of $\begin{pmatrix} x \\ y \end{pmatrix}$ in new basis.
To find	$$\mathbf{E}\begin{pmatrix} x \\ y \end{pmatrix} = a\begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2.0.2}$$
Finding a Projection $\mathbf{E}$	As, $\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ are linearly independent. Therefore,$\{\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}\}$ is a basis of $\mathbb{R}^2$ As $\begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$ $$\implies \begin{pmatrix} x \\ y \end{pmatrix} = a\begin{pmatrix} 1 \\ -1 \end{pmatrix} + b\begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{2.0.3}$$

	$$\implies \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \frac{2x-y}{3} \\ \frac{x+y}{3} \end{pmatrix} \quad (2.0.4)$$ Projection of $\begin{pmatrix} x \\ y \end{pmatrix}$ on subspace spanned by $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$; $$\mathbf{E}\begin{pmatrix} x \\ y \end{pmatrix} = a\begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.5)$$ $$\implies \mathbf{E}\begin{pmatrix} x \\ y \end{pmatrix} = \left(\frac{2x-y}{3}\right)\begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.6)$$ $$\implies \mathbf{E}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \quad (2.0.7)$$ $$\implies \mathbf{E} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \quad (2.0.8)$$
Verification	If $n \times n$ matrix $\mathbf{E}$ is projection matrix, then $$\mathbf{E}^2 = \mathbf{E}$$ $$\mathbf{E}^2 = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix}\begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \implies \mathbf{E} = \mathbf{E}^2 = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \quad (2.0.9)$$ Hence, Verified.

TABLE 1: Finding Projection Matrix