Assignment 4

Neha Rani

Abstract—**This document solves a question based on triangle using linear algebra.**

All the codes for the figure in this document can be found at

https://github.com/neharani289/ee14014/tree/master/Assignment4

1 Problem

In $\triangle ABC$, the bisector $\mathbf{AD}$ of $\angle A \perp$ to side $\mathbf{BC}$.Show that $\mathbf{AB} = \mathbf{AC}$ and $\triangle ABC$ is isosceles.

2 Solution

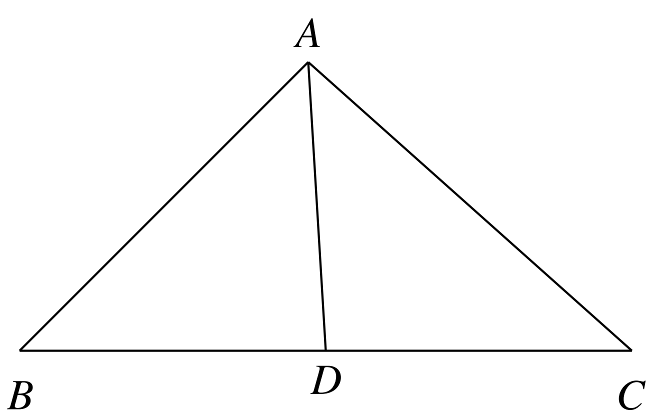

Fig. 1: Isosceles Triangle with $\mathbf{AD} \perp \mathbf{BC}$

Given, line AD is perpendicular to line BC which implies the inner product is zero.

$$(B-D)^T(A-D) = (D-A)^T(B-D) \quad = 0 \tag{2.0.1}$$

and

$$\|\mathbf{B}-\mathbf{D}\| = \|\mathbf{D}-\mathbf{C}\| \tag{2.0.2}$$

In $\triangle BAD$ and $\triangle CAD$;
Taking inner product of sides BA and AD

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{A}-\mathbf{D}) = \|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{D}\|\cos BAD \tag{2.0.3}$$

The angle BAD from the above equation is:

$$\cos BAD = \frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{A}-\mathbf{D})}{\|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{D}\|} \tag{2.0.4}$$

Taking inner product of sides BA and AD

$$(\mathbf{C}-\mathbf{A})^T(\mathbf{A}-\mathbf{D}) = \|\mathbf{C}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{D}\|\cos CAD \tag{2.0.5}$$

The angle CAD from the above equation is:

$$\cos CAD = \frac{(\mathbf{C}-\mathbf{A})^T(\mathbf{A}-\mathbf{D})}{\|\mathbf{C}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{D}\|} \tag{2.0.6}$$

from equation (2.0.4) and (2.0.6)

$$\angle BAD = \angle CAD \tag{2.0.7}$$

Similarly;

$$(\mathbf{A}-\mathbf{D})^T(\mathbf{D}-\mathbf{B}) = \|\mathbf{A}-\mathbf{D}\|\,\|\mathbf{D}-\mathbf{B}\|\cos ADB \tag{2.0.8}$$

$$(\mathbf{A}-\mathbf{D})^T(\mathbf{D}-\mathbf{c}) = \|\mathbf{A}-\mathbf{D}\|\,\|\mathbf{D}-\mathbf{C}\|\cos ADC \tag{2.0.9}$$

from equation (2.0.8) and (2.0.9) we can say;

$$\angle ADB = \angle ADC = 90^\circ \tag{2.0.10}$$

By, ASA property $\triangle BAD \cong \triangle CAD$, hence triangles are congruent.
Now by using CSCT property;

$$\|\mathbf{A}-\mathbf{B}\| = \|\mathbf{A}-\mathbf{C}\| \tag{2.0.11}$$

By converse of isosceles triangle theorem $\triangle ABC$ is isosceles.
Hence proved.