Assignment 17

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Download the latex-tikz codes from

https://github.com/neharani289/MatrixTheory/Assignment17

1 Problem

(ugcjune/2018/28) :

If $\mathbf{A}$ is a 2×2 matrix over $\mathbb{R}$ with $\det(\mathbf{A}+\mathbf{I}) = 1 + \det(\mathbf{A})$, then we can conclude that

1) $\det(\mathbf{A}) = 0$
2) $\mathbf{A} = 0$
3) $tr(\mathbf{A}) = 0$
4) $\mathbf{A}$ is non singular.

2 Solution

Given	Let $\mathbf{A}$ be a 2×2 matrix over $\mathbb{R}$. $$\mathbf{A} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$ $\implies$ Eigen value of $\mathbf{A} = \lambda_1, \lambda_2$ $$(\mathbf{A}+\mathbf{I}) = \begin{pmatrix} \lambda_1 + 1 & 0 \\ 0 & \lambda_2 + 1 \end{pmatrix}$$ $\implies$ Eigen value of $(\mathbf{A}+\mathbf{I}) = \lambda_1 + 1, \lambda_2 + 1$ Since, $$\det(\mathbf{A}+\mathbf{I}) = 1 + \det(\mathbf{A})$$ Trace of any matrix is sum of its eigen values. Determinant of matrix is product of its eigen values $$\implies (\lambda_1 + 1)(\lambda_2 + 1) = 1 + (\lambda_1 \lambda_2)$$

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$$\implies \boxed{\lambda_1 + \lambda_2 = 0}$$

$$\implies \boxed{tr(\mathbf{A}) = 0}$$

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<tr><td>Option 1 : $\det \mathbf{A} = 0$</td><td>

Let,

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}$$

$$\det \mathbf{A} = \begin{vmatrix} 0 & -1 \\ 0 & 0 \end{vmatrix} = 0$$

$$(\mathbf{A} + \mathbf{I}) = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

$$\det(\mathbf{A} + \mathbf{I}) = \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 1$$

$$\implies \det(\mathbf{A} + \mathbf{I}) = 1 + \det(\mathbf{A})$$

Conclusion:
1) $tr(\mathbf{A}) = 0$
2) $\det \mathbf{A} = 0$
3) $\mathbf{A} \neq \mathbf{0}$
4) $\mathbf{A}$ is singular.

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<tr><td>Option 2 : $\mathbf{A} = 0$</td><td>

Let,

$$\mathbf{A} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\det \mathbf{A} = 0$$

$$\det(\mathbf{A} + \mathbf{I}) = 1$$

$$\implies \det(\mathbf{A} + \mathbf{I}) = 1 + \det(\mathbf{A})$$

Conclusion:
1) $tr(\mathbf{A}) = 0$
2) $\det \mathbf{A} = 0$
3) $\mathbf{A} = \mathbf{0}$
4) $\mathbf{A}$ is singular.

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<tr><td>Option 4: $\mathbf{A}$ is non singular</td><td>

Non Singular Matrix: A non-singular matrix is a square one whose determinant is not zero.Thus, a non-singular matrix is also known as a full rank matrix.

Let,

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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	$$\det \mathbf{A} = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = -1$$ $$(\mathbf{A}+\mathbf{I}) = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$$ $$\det(\mathbf{A}+\mathbf{I}) = \begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} = 0$$ $$\implies \det(\mathbf{A}+\mathbf{I}) = 1 + \det(\mathbf{A})$$ Conclusion: 1) $tr(\mathbf{A}) = 0$ 2) $\det \mathbf{A} \neq 0$ 3) $\mathbf{A} \neq \mathbf{0}$ 4) $\mathbf{A}$ is non singular.
Conclusion	In all options, $tr(\mathbf{A}) = 0$ satisfied. Thus, Option 3 is correct.

TABLE 1: Solution Summary