Assignment 17

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Download the latex-tikz codes from

https://github.com/neharani289/MatrixTheory/Assignment17

1 Problem

(ugcjune/2018/28) :
If $\mathbf{A}$ is a 2×2 matrix over $\mathbb{R}$ with $\det(\mathbf{A}+\mathbf{I}) = 1 + \det(\mathbf{A})$, then we can conclude that

1) $\det(\mathbf{A}) = 0$
2) $\mathbf{A} = 0$
3) $tr(\mathbf{A}) = 0$
4) $\mathbf{A}$ is non singular.

2 Solution

Given	$\mathbf{A}$ be a 2×2 matrix over $\mathbb{R}$ with $$\det(\mathbf{A}+\mathbf{I}) = 1 + \det(\mathbf{A})$$
Explanation	If $\mathbf{X}$ is an eigen vector of matrix $\mathbf{A}$ corresponding to the eigen value λ i.e $$\mathbf{A}\mathbf{X} = \lambda\mathbf{X}$$ then, $(\mathbf{I}+\mathbf{A})\mathbf{X} = (1+\lambda)\mathbf{X}$ Thus, $\mathbf{X}$ is an eigen vector of $(\mathbf{A}+\mathbf{I})$ corresponding to the eigen value $(1+\lambda)$. Let λ_1, λ_2 be two eigen values of $\mathbf{A}$ and $(1+\lambda_1), (1+\lambda_2)$ be the eigen values of $(\mathbf{A}+\mathbf{I})$. $\implies$ Eigen value of $\mathbf{A} = \lambda_1, \lambda_2$ $\implies$ Eigen value of $(\mathbf{A}+\mathbf{I}) = \lambda_1 + 1, \lambda_2 + 1$ Since,

	$\det(\mathbf{A}+\mathbf{I}) = 1 + \det(\mathbf{A})$ Trace of any matrix is sum of its eigen values. Determinant of matrix is product of its eigen values $\implies (\lambda_1+1)(\lambda_2+1) = 1 + (\lambda_1\lambda_2)$ $\implies \boxed{\lambda_1+\lambda_2 = 0}$ $\implies \boxed{tr(\mathbf{A}) = 0}$
Statement 1 : $\det\mathbf{A} = 0$	False
	Let, $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ Here, $\det\mathbf{A} = -1$ and $\det(\mathbf{A}+\mathbf{I}) = 0$ Thus,$1 + \det(\mathbf{A}) = \det(\mathbf{A}+\mathbf{I})$ In this case, $\det\mathbf{A} \neq 0$ but satisfy the given condition i.e $1 + \det(\mathbf{A}) = \det(\mathbf{A}+\mathbf{I})$
Statement 2 : $\mathbf{A} = \mathbf{0}$	False
	Let , $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ Here, $\det\mathbf{A} = 0$ and $\det(\mathbf{A}+\mathbf{I}) = 1$ Thus,$1 + \det(\mathbf{A}) = \det(\mathbf{A}+\mathbf{I})$ In this case, $\mathbf{A} \neq \mathbf{0}$ But , satisfy the given condition i.e $1 + \det(\mathbf{A}) = \det(\mathbf{A}+\mathbf{I})$
Statement 3: $tr(\mathbf{A}) = 0$	True
	The given statement is true for all possible matrices. If $tr\mathbf{A} \neq 0$ then the given condition i.e $1+\det(\mathbf{A}) = \det(\mathbf{A}+\mathbf{I})$ doesn't satisy. Let , $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

	Here, $\det \mathbf{A} = 0$, $\det(\mathbf{A} + \mathbf{I}) = 2, tr\mathbf{A} \neq 0$ Thus, $1 + \det(\mathbf{A}) \neq \det(\mathbf{A} + \mathbf{I})$
Statement4:**A** is non singular	False
	Non Singular Matrix: A non-singular matrix is a square one whose determinant is not zero.non-singular matrix is also a full rank matrix. Let, $\mathbf{A} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Here, $\det \mathbf{A} = 0$ and $\det(\mathbf{A} + \mathbf{I}) = 1$ Thus,$1 + \det(\mathbf{A}) = \det(\mathbf{A} + \mathbf{I})$ In this case, **A** is Singular, But satisfy the given condition i.e $1 + \det(\mathbf{A}) = \det(\mathbf{A} + \mathbf{I})$
Conclusion	Thus, we can conclude Statement 3 is true for all possible matrices which satisfy the given condition i.e $1 + \det(\mathbf{A}) = \det(\mathbf{A} + \mathbf{I})$

TABLE 1: Solution Summary