Assignment 11

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Download the latex-tikz codes from

https://github.com/neharani289/MatrixTheory/Assignment11

1 Problem

(UGC-june2017,71) :
Let V be the vector space of polynomials of degree at most 3 in a variable x with coefficients in $\mathbb{R}$. Let T=d/dx be the linear transformation of V to itself given by differentiation.

Which of the following are correct?

1) T is invertible
2) 0 is an eigenvalue of T
3) There is a basis with respect to which the matrix of T is nilpotent.
4) The matrix of T with respect to the basis $\{1, 1 + x, 1 + x + x^2, 1 + x + x^2 + x^3\}$ is diagonal.

2 Definition and Result used

Nilpotent Matrix	1. If all the eigen values of matrix is zero then it is said to nilpotent matrix 2. Determinant and trace of nilpotent matrix are always zero.
Invertible Matrix	A matrix is said to be invertible matrix if its determinant is non zero.
Diagonal matrix	diagonal matrix is a matrix in which the entries outside the main diagonal are all zero.

3 Solution

Given	$T : P_3 \rightarrow P_3$ $T : V \rightarrow V$ be the linear operator given by differentiation wrt x $T(P(x)) \rightarrow P'(x)$ A be the matrix of T wrt some basis for V Assume basis for V be $\{1, x, x^2, x^3\}$

Checking whether matrix of T is nilpotent	$T : V \to V$ $TP(x) = P'(x)$ Differentiating wrt x to find matrix A; $T(1) = 0 = a_1x + b_1x + c_1x^2 + d_1x^3$ $T(x) = 1 = a_2 + b_2x + c_2x^2 + d_2x^3$ $T(x^2) = 2x = a_3 + b_3x + c_3x^2 + d_3x^3$ $T(x^3) = 3x^2 = a_4 + b_4x + c_4x^2 + d_4x^3$ Representing A in matrix form ; $A=\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ from the above matrix of T we can say it is nilpotent matrix.
Checking eigen value of matrix T	$A = \begin{pmatrix} 0-\lambda & 1 & 0 & 0 \\ 0 & 0-\lambda & 2 & 0 \\ 0 & 0 & 0-\lambda & 3 \\ 0 & 0 & 0 & 0-\lambda \end{pmatrix}$ $\implies \lambda = 0$
Checking whether matrix of T is invertible	Since $\det A = 0$. Therefore matrix of T is not invertible
Checking whether Matrix of T is diagonal matrix	Let basis be $B' = \{1, 1+x, 1+x+x^2, 1+x+x^2+x^3\}$ Differentiating wrt x ; $T(1) = 0 = a_1x + b_1(1+x) + c_1(1+x+x^2) + d_1(1+x+x^2+x^3)$ $T(1+x) = 1 = a_2 + b_2(1+x) + c_2(1+x+x^2) + d_2(1+x+x^2x^3)$ $T(1+x+x^2) = 1+2x = a_3 + b_3(1+x) + c_3(1+x+x^2)$ $+d_3(1+x+x^2+x^3)$ $T(1+x+x^2+x^3) = 1+2x+3x^2 = a_4 + b_4(1+x) + c_4(1+x+x^2)$ $+d_4(1+x+x^2+x^3)$ $B = \begin{pmatrix} 0 & 1 & -1 & -1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ above matrix is not a diagonal matrix
Conclusion	Thus we can conclude Option 2) and 3) are correct.