Assignment 6

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***Abstract*—This document explains QR decomposition of a 2x2 Matrix.**

Download python codes from

https://github.com/neharani289/MatrixTheory/tree/master/Assignment6/codes

Download latex-tikz codes from

https://github.com/neharani289/MatrixTheory/tree/master/Assignment6

1 Problem

Find QR decomposition for matrix

$$\mathbf{V} = \begin{pmatrix} 12 & \frac{7}{2} \\ \frac{7}{2} & -10 \end{pmatrix} \tag{1.0.1}$$

2 Solution

Let $\mathbf{x}$ and $\mathbf{y}$ be the column vectors of the given matrix.

$$\mathbf{x} = \begin{pmatrix} 12 \\ \frac{7}{2} \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{y} = \begin{pmatrix} \frac{7}{2} \\ -10 \end{pmatrix} \tag{2.0.2}$$

The column vectors can be expressed as follows,

$$\mathbf{x} = k_1\mathbf{u}_1 \tag{2.0.3}$$

$$\mathbf{y} = r_1\mathbf{u}_1 + k_2\mathbf{u}_2 \tag{2.0.4}$$

Here,

$$k_1 = \|\mathbf{x}\| \tag{2.0.5}$$

$$\mathbf{u}_1 = \frac{\mathbf{x}}{k_1} \tag{2.0.6}$$

$$r_1 = \frac{\mathbf{u}_1^T\mathbf{y}}{\|\mathbf{u}_1\|^2} \tag{2.0.7}$$

$$\mathbf{u}_2 = \frac{\mathbf{y} - r_1\mathbf{u}_1}{\|\mathbf{y} - r_1\mathbf{u}_1\|} \tag{2.0.8}$$

$$k_2 = \mathbf{u}_2^T\mathbf{y} \tag{2.0.9}$$

The (2.0.3) and (2.0.4) can be written as,

$$\begin{pmatrix} \mathbf{x} & \mathbf{y} \end{pmatrix} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix} \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{2.0.10}$$

$$\begin{pmatrix} \mathbf{x} & \mathbf{y} \end{pmatrix} = \mathbf{QR} \tag{2.0.11}$$

Now, $\mathbf{R}$ is an upper triangular matrix and also,

$$\mathbf{Q}^T\mathbf{Q} = \mathbf{I} \tag{2.0.12}$$

Now using equations (2.0.5) to (2.0.9) we get,

$$k_1 = \sqrt{\left(\frac{7}{2}\right)^2 + 12^2} = \frac{25}{2} \tag{2.0.13}$$

$$\mathbf{u}_1 = \begin{pmatrix} \frac{24}{25} \\ \frac{7}{25} \end{pmatrix} \tag{2.0.14}$$

$$r_1 = \begin{pmatrix} \frac{24}{25} & \frac{7}{25} \end{pmatrix} \begin{pmatrix} \frac{7}{2} \\ -10 \end{pmatrix} = \frac{14}{25} \tag{2.0.15}$$

$$\mathbf{u}_2 = \begin{pmatrix} \frac{7}{25} \\ -\frac{24}{25} \end{pmatrix} \tag{2.0.16}$$

$$k_2 = \begin{pmatrix} \frac{7}{25} & -\frac{24}{25} \end{pmatrix} \begin{pmatrix} \frac{7}{2} \\ -10 \end{pmatrix} = \frac{529}{50} \tag{2.0.17}$$

Thus putting the values from (2.0.13) to (2.0.17) in (2.0.10) we obtain QR decomposition,

$$\begin{pmatrix} 12 & \frac{7}{2} \\ \frac{7}{2} & -10 \end{pmatrix} = \begin{pmatrix} \frac{24}{25} & \frac{7}{25} \\ \frac{7}{25} & -\frac{24}{25} \end{pmatrix} \begin{pmatrix} \frac{25}{2} & \frac{14}{25} \\ 0 & \frac{529}{50} \end{pmatrix} \tag{2.0.18}$$

which can also be written as,

$$\begin{pmatrix} 12 & \frac{7}{2} \\ \frac{7}{2} & -10 \end{pmatrix} = \begin{pmatrix} -\frac{24}{25} & -\frac{7}{25} \\ -\frac{7}{25} & \frac{24}{25} \end{pmatrix} \begin{pmatrix} -\frac{25}{2} & -\frac{14}{25} \\ 0 & -\frac{529}{50} \end{pmatrix} \tag{2.0.19}$$