

Assignment 15

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Download the latex-tikz codes from

<https://github.com/neharani289/MatrixTheory/Assignment15>

1 PROBLEM

(hoffman/page213/3) :

Find a projection \mathbf{E} which projects \mathbb{R}^2 onto the subspace spanned by $(1, -1)$ along the subspace spanned by $(1, 2)$.

2 SOLUTION

Given	<p>Let $\begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$</p> $\begin{pmatrix} x \\ y \end{pmatrix} = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} + b \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.1)$ <p>where $\begin{pmatrix} a \\ b \end{pmatrix}$ is representation of $\begin{pmatrix} x \\ y \end{pmatrix}$ in new basis.</p>
To find	$\mathbf{E} \begin{pmatrix} x \\ y \end{pmatrix} = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.2)$
Finding a Projection \mathbf{E}	<p>We know in standard order basis ;</p> $\begin{pmatrix} x \\ y \end{pmatrix} : \begin{pmatrix} 1 \\ 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} y \quad (2.0.3)$ <p>Express $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ in the basis $\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}$</p>

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = p \begin{pmatrix} 1 \\ -1 \end{pmatrix} + q \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.4)$$

where $\begin{pmatrix} p \\ q \end{pmatrix}$ is representation of $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ in the new basis.

$$\Rightarrow \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} \quad (2.0.5)$$

similarly;

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = r \begin{pmatrix} 1 \\ -1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.6)$$

$$\Rightarrow \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} \frac{-1}{3} \\ \frac{1}{3} \end{pmatrix} \quad (2.0.7)$$

Substitute (2.0.5) and (2.0.7) in (2.0.3) we get;

$$\begin{pmatrix} x \\ y \end{pmatrix} : \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} x + \begin{pmatrix} \frac{-1}{3} \\ \frac{1}{3} \end{pmatrix} y = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (2.0.8)$$

From (2.0.1) and (2.0.8) ;

$$\begin{pmatrix} x \\ y \end{pmatrix} : \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (2.0.9)$$

From (2.0.2) and (2.0.9);

$$\mathbf{E} \begin{pmatrix} x \\ y \end{pmatrix} = \left(\frac{2}{3}x - \frac{1}{3}y \right) \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.10)$$

$$\Rightarrow \mathbf{E} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{2}{3} \\ \frac{-2}{3} \end{pmatrix} x + \begin{pmatrix} \frac{-1}{3} \\ \frac{1}{3} \end{pmatrix} y \quad (2.0.11)$$

$$\mathbf{E} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (2.0.12)$$

	<p>Hence,</p> $\Rightarrow \mathbf{E} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \quad (2.0.13)$
Verification	<p>If $n \times n$ matrix \mathbf{E} is projection matrix, then</p> $\mathbf{E}^2 = \mathbf{E}$ $\mathbf{E}^2 = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \Rightarrow \mathbf{E} = \mathbf{E}^2 = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \quad (2.0.14)$ <p>Hence, Verified.</p>

TABLE 1: Finding Projection Matrix