Assignment 11

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Download the latex-tikz codes from

https://github.com/neharani289/MatrixTheory/Assignment11

1 Problem

(UGC-june2017,71) :
Let **V** be the vector space of polynomials of degree at most 3 in a varible x with coefficients in $\mathbb{R}$. Let **T**=d/dx be the linear transformation of **V** to itself given by differentiation.

Which of the following are correct?

1) **T** is invertible
2) 0 is an eigenvalue of **T**
3) There is a basis with respect to which the matrix of **T** is nilpotent.
4) The matrix of **T** with respect to the basis $\left(1, 1+x, 1+x+x^2, 1+x+x^2+x^3\right)$ is diagonal.

2 Solution

Checking whether matrix **T** is nilpotent	Let **V**= $\mathrm{P}_3(x)$ **T**:**V**$\rightarrow$ **V** **T**(P(x)) = P'(x) Standard basis of P(x)=$\left(1, x, x^2, x^3\right)$ differentiating wrt x to find matrix ; $\mathrm{T}(1)=0=\mathrm{a}_1 x + b_1 x + c_1 x^2 + d_1 x^3$ $\mathrm{T}(x)=1=\mathrm{a}_2 + b_2 x + c_2 x^2 + d_2 x^3$ $\mathrm{T}(\mathrm{x}^2) = 2x = a_3 + b_3 x + c_3 x^2 + d_3 x^3$ $\mathrm{T}(\mathrm{x}^3) = 3x^2 = a_4 + b_4 x + c_4 x^2 + d_4 x^3$ Representing **T** in matrix form ; $\mathrm{T}=\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ from the above matrix **T** we can say it is nilpotent matrix.

Checking for eigen value of **T**	T=$\begin{pmatrix} 0-\lambda & 1 & 0 & 0 \\ 0 & 0-\lambda & 2 & 0 \\ 0 & 0 & 0-\lambda & 3 \\ 0 & 0 & 0 & 0-\lambda \end{pmatrix}$ $\implies \lambda = 0$
Checking whether **T** is invertible	Since $\det T = 0$. Therefore **T** is not invertible
Matrix **T** is diagonal matrix	Let basis of P(x) be B=$\left(1, 1+x, 1+x+x^2, 1+x+x^2+x^3\right)$ Differentiating wrt x ; T(1)=0=$a_1 x + b_1(1+x) + c_1(1+x+x^2) + d_1(1+x+x^2+x^3)$ T(1+x)=1=$a_2 + b_2(1+x) + c_2(1+x+x^2) + d_2(1+x+x^2x^3)$ T(1+x+x^2) $= 1+2x = a_3 + b_3(1+x) + c_3(1+x+x^2) + d_3(1+x+x^2+x^3)$ T(1+x+x^2+x^3) $= 1+2x+3x^2 = a_4 + b_4(1+x) + c_4(1+x+x^2)$ $+d_4(1+x+x^2+x^3)$ T=$\begin{pmatrix} 0 & 1 & -1 & -1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ above matrix is not a diagonal matrix
Conclusion	Thus we can conclude Option 2) and 3) are correct.