# Assignment 4

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***Abstract*—This document solves a question based on triangle using linear algebra.**

All the codes for the figure in this document can be found at

https://github.com/neharani289/ee14014/tree/master/Assignment4

## 1 Problem

In $\triangle ABC$, the bisector $\mathbf{AD}$ of $\angle A \perp$ to side $\mathbf{BC}$.Show that $\mathbf{AB} = \mathbf{AC}$ and $\triangle ABC$ is isosceles.

## 2 Solution

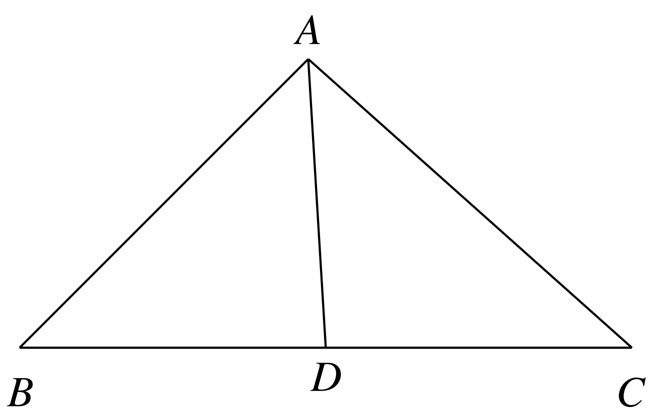


Fig. 1: Isosceles Triangle with $\mathbf{AD} \perp \mathbf{BC}$

Given, line AD is perpendicular to line BC which implies the inner product is zero.

$$(B - D)^T (A - D) = (D - A)^T (B - D) \quad = 0 \tag{2.0.1}$$

In $\triangle BAD$ and $\triangle CAD$ ;
Taking inner product of sides BA and AD

$$(\mathbf{B} - \mathbf{A})^T (\mathbf{A} - \mathbf{D}) = \|\mathbf{B} - \mathbf{A}\| \|\mathbf{A} - \mathbf{D}\| \cos BAD \tag{2.0.2}$$

The angle BAD from the above equation is:

$$\cos BAD = \frac{(\mathbf{B} - \mathbf{A})^T (\mathbf{A} - \mathbf{D})}{\|\mathbf{B} - \mathbf{A}\| \|\mathbf{A} - \mathbf{D}\|} \tag{2.0.3}$$

Taking inner product of sides BA and AD

$$(\mathbf{C} - \mathbf{A})^T (\mathbf{A} - \mathbf{D}) = \|\mathbf{C} - \mathbf{A}\| \|\mathbf{A} - \mathbf{D}\| \cos CAD \tag{2.0.4}$$

The angle CAD from the above equation is:

$$\cos CAD = \frac{(\mathbf{C} - \mathbf{A})^T (\mathbf{A} - \mathbf{D})}{\|\mathbf{C} - \mathbf{A}\| \|\mathbf{A} - \mathbf{D}\|} \tag{2.0.5}$$

from equation (2.0.3) and (2.0.5)

$$\angle BAD = \angle CAD \tag{2.0.6}$$

Similarly;

$$(\mathbf{A} - \mathbf{D})^T (\mathbf{D} - \mathbf{B}) = \|\mathbf{A} - \mathbf{D}\| \|\mathbf{D} - \mathbf{B}\| \cos ADB \tag{2.0.7}$$

$$(\mathbf{A} - \mathbf{D})^T (\mathbf{D} - \mathbf{C}) = \|\mathbf{A} - \mathbf{D}\| \|\mathbf{D} - \mathbf{C}\| \cos ADC \tag{2.0.8}$$

from equation (2.0.7) and (2.0.8) we can say;

$$\angle ADB = \angle ADC = 90° \tag{2.0.9}$$

and

$$\|\mathbf{A} - \mathbf{D}\| = \|\mathbf{A} - \mathbf{D}\| \tag{2.0.10}$$

By, ASA property $\triangle BAD \cong \triangle CAD$ , hence triangles are congruent.
Now by using CSCT property;

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{A} - \mathbf{C}\| \tag{2.0.11}$$

By converse of isosceles triangle theorem $\triangle ABC$ is isosceles.
Hence proved.