

Assignment 12

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Download the latex-tikz codes from

<https://github.com/neharani289/MatrixTheory/Assignment12>

1 PROBLEM

(hoffman/page189/5) :

Let

$$\mathbf{A} = \begin{pmatrix} 6 & -3 & -2 \\ 4 & -1 & -2 \\ 10 & -5 & -3 \end{pmatrix} \quad (1.0.1)$$

Is A similar over the field R to a diagonal matrix ?

Is A similar over the field C to a diagonal matrix?

2 EXPLANATION

Theorem 2	<p>Let T be the linear operator on a finite dimensional space V and c_1, \dots, c_k be a distinct characteristic values of T and let W_i be the null space of $(T - c_i I)$ then</p> <p>1) T is diagonalizable</p> <p>2) characteristic polynomial for T is $f = (x - c_1)^{d_1} \dots (x - c_k)^{d_k}$ and</p> <p>3) $\dim W_i = d_i, i = 1, \dots, k$</p>
Concept for diagonalization	<p>A linear operator T on a n-dimensional space V is diagonalizable, if and only if T has n distinct characteristic vectors or null spaces corresponding to the characteristic values</p>

TABLE 1: Illustration of theorem.

3 SOLUTION

Given	<p>Let the given matrix be</p> $A = \begin{pmatrix} 6 & -3 & -2 \\ 4 & -1 & -2 \\ 10 & -5 & -3 \end{pmatrix}$
Finding Characteristics polynomial	<p>Characteristics polynomial of the matrix A is $\det(xI - A)$</p> $\det(xI - A) = \begin{vmatrix} (x-6) & 3 & 2 \\ -4 & (x+1) & 2 \\ -10 & 5 & x+3 \end{vmatrix}$ <p>Characteristic Polynomial = $(x-2)(x^2+1) = (x-2)(x-i)(x+i)$</p>
Checking whether A similar over the field R to a diagonal matrix	<p>As the characteristics polynomial is not product of linear factors over R. Therefore from Theorem 2, A is not diagonalizable over R</p>
Checking whether A similar over the field C to a diagonal matrix	<p>The Characteristic Polynomial can be written as a product of linear factors over C i.e</p> $\det(xI - A) = (x-2)(x-i)(x+i)$ <p>To find characteristic values of the operator $\det(xI - A) = 0$ which gives $c_1 = 2, c_2 = i, c_3 = -i$</p> <p>Thus over C matrix A has three distinct characteristic values.</p> <p>There will be atleast one characteristics vector i.e., one dimension with each characteristics value .</p> <p>From Theorem 2; $\sum_i W_i = n = 3$, which is equal to \dim of A.</p> <p>Thus , A is diagonalizable over C.</p>
Conclusion	<p>1) A is not diagonalizable over R.</p> <p>2) A is diagonalizable over C.</p>

TABLE 2: Finding of characteristics values and diagonalization over different field