Assignment 13

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Download the latex-tikz codes from

https://github.com/neharani289/MatrixTheory/Assignment13

1 Problem

(hoffman/page198/9) :
Let **A** be an $n \times n$ matrix with characteristics polynomial

$$f = (x - c_1)^{d_1} ....(x - c_k)^{d_k}$$

Show that

$$c_1 d_1 + ... + c_k d_k = trace(A)$$

2 Solution

Given	Let **A** be an $n \times n$ $\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$ and Characteristics polynomial $f = (x - c_1)^{d_1} ....(x - c_k)^{d_k}$
To prove	$c_1 d_1 + ... + c_k d_k = trace(A)$
proof	Characteristics polynomial; $f = (x - c_1)^{d_1} ....(x - c_k)^{d_k}$ here, $c_1, ..., c_k$ are the distinct eigen values. and $d_1, ..., d_k$ denotes the repetition of eigen values

	Therefore, $\boxed{c_1 d_1 + c_2 d_2 + ... + c_k d_k = \sum_i \lambda_i}$ =Sum of all eigen values. Using JCF concept; For every matrix **A** there exist a invertible matrix **P** such that $\mathbf{PAP^{-1}} = \mathbf{J}$ where J has Jordan Canonical form. $\mathbf{J} = \begin{pmatrix} J_1 & & & \\ & J_2 & & \\ & & \ddots & \\ & & & J_p \end{pmatrix}$,each block J_i is a square matrix of the form $\mathbf{J_i} = \begin{pmatrix} \lambda_i & 1 & & \\ & \lambda_i & \ddots & \\ & & \ddots & 1 \\ & & & \lambda_i \end{pmatrix}$ where, J is called Jordan normal form of **A** and $\mathbf{J_i}$ is called a Jordan block of **A** Consider, two $n \times n$ matrices B and C and they are called similar if there exists an invertible $n \times n$ matrix P such that $\mathbf{C} = \mathbf{PBP^{-1}}$ which implies B and C have the same eigen values. $\implies trace(ABC) = trace(BAC) = trace(CAB)$ $\implies trace(A) = trace(P^{-1}JP) = trace(PP^{-1}J) = trace(J)$ $\boxed{trace(J) = \sum_i \lambda_i}$ where, λ_i are eigen values of **A**. $\boxed{\implies c_1 d_1 + c_2 d_2 + ... + c_k d_k = \sum_i \lambda_i = traceA}$ Hence, Proved.

TABLE 1: Solution Summary