

# Assignment 11

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Download the latex-tikz codes from

<https://github.com/neharani289/MatrixTheory/Assignment11>

## 1 PROBLEM

(UGC-june2017,71) :

Let  $V$  be the vector space of polynomials of degree at most 3 in a variable  $x$  with coefficients in  $\mathbb{R}$ . Let  $T=d/dx$  be the linear transformation of  $V$  to itself given by differentiation.

Which of the following are correct?

- 1)  $T$  is invertible
- 2) 0 is an eigenvalue of  $T$
- 3) There is a basis with respect to which the matrix of  $T$  is nilpotent.
- 4) The matrix of  $T$  with respect to the basis  $\{1, 1+x, 1+x+x^2, 1+x+x^2+x^3\}$  is diagonal.

## 2 DEFINITION AND RESULT USED

Nilpotent Matrix	1. If all the eigen values of matrix is zero then it is said to nilpotent matrix 2. Determinant and trace of nilpotent matrix are always zero.
Invertible Matrix	A matrix is said to be invertible matrix if its determinant is non zero.
Diagonal matrix	diagonal matrix is a matrix in which the entries outside the main diagonal are all zero.

## 3 SOLUTION

Given	$T : P_3 \rightarrow P_3$  $T : V \rightarrow V$ be the linear operator given by differentiation wrt $x$ $T(P(x)) \rightarrow P'(x)$  $A$ be the matrix of $T$ wrt some basis for $V$ Assume basis for $V$ be $\{1, x, x^2, x^3\}$
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<p>Checking whether matrix of <math>T</math> is nilpotent</p>	<p><math>T : V \rightarrow V</math>  <math>TP(x) = P'(x)</math>          Differentiating wrt <math>x</math> to find matrix <math>A</math>;  <math>T(1) = 0 = a_1x + b_1x + c_1x^2 + d_1x^3</math>  <math>T(x) = 1 = a_2 + b_2x + c_2x^2 + d_2x^3</math>  <math>T(x^2) = 2x = a_3 + b_3x + c_3x^2 + d_3x^3</math>  <math>T(x^3) = 3x^2 = a_4 + b_4x + c_4x^2 + d_4x^3</math>          Representing <math>A</math> in matrix form ;  <math display="block">A = \begin{pmatrix} 0 &amp; 1 &amp; 0 &amp; 0 \\ 0 &amp; 0 &amp; 2 &amp; 0 \\ 0 &amp; 0 &amp; 0 &amp; 3 \\ 0 &amp; 0 &amp; 0 &amp; 0 \end{pmatrix}</math>          from the above matrix of <math>T</math> we can say it is nilpotent matrix.</p>
<p>Checking eigen value of matrix <math>T</math></p>	$A = \begin{pmatrix} 0 - \lambda & 1 & 0 & 0 \\ 0 & 0 - \lambda & 2 & 0 \\ 0 & 0 & 0 - \lambda & 3 \\ 0 & 0 & 0 & 0 - \lambda \end{pmatrix}$ $\Rightarrow \lambda = 0$
<p>Checking whether matrix of <math>T</math> is invertible</p>	<p>Since <math>\det T = 0</math>.          Therefore <math>T</math> is not invertible</p>
<p>Checking whether Matrix of <math>T</math> is diagonal matrix</p>	<p>Let basis be <math>B' = \{1, 1 + x, 1 + x + x^2, 1 + x + x^2 + x^3\}</math>          Differentiating wrt <math>x</math> ;  <math>T(1) = 0 = a_1x + b_1(1 + x) + c_1(1 + x + x^2) + d_1(1 + x + x^2 + x^3)</math>  <math>T(1 + x) = 1 = a_2 + b_2(1 + x) + c_2(1 + x + x^2) + d_2(1 + x + x^2 + x^3)</math>  <math>T(1 + x + x^2) = 1 + 2x = a_3 + b_3(1 + x) + c_3(1 + x + x^2) + d_3(1 + x + x^2 + x^3)</math>  <math>T(1 + x + x^2 + x^3) = 1 + 2x + 3x^2 = a_4 + b_4(1 + x) + c_4(1 + x + x^2) + d_4(1 + x + x^2 + x^3)</math>  <math display="block">B = \begin{pmatrix} 0 &amp; 1 &amp; -1 &amp; -1 \\ 0 &amp; 0 &amp; 2 &amp; -1 \\ 0 &amp; 0 &amp; 0 &amp; 3 \\ 0 &amp; 0 &amp; 0 &amp; 0 \end{pmatrix}</math>          above matrix is not a diagonal matrix</p>
<p>Conclusion</p>	<p>Thus we can conclude          Option 2) and 3) are correct.</p>