Assignment 15

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Download the latex-tikz codes from

https://github.com/neharani289/MatrixTheory/Assignment15

1 Problem

(hoffman/page213/3) :

Find a projection $\mathbf{E}$ which projects $\mathbb{R}^2$ onto the subspace spanned by $(1, -1)$ along the subspace spanned by $(1, 2)$.

2 Solution

Given	Let $\begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$ $\begin{pmatrix} x \\ y \end{pmatrix} = a\begin{pmatrix} 1 \\ -1 \end{pmatrix} + b\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ where $\begin{pmatrix} a \\ b \end{pmatrix}$ is representation of $\begin{pmatrix} x \\ y \end{pmatrix}$ in new basis.
To find	$\mathbf{E}\begin{pmatrix} x \\ y \end{pmatrix} = a\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
Finding a Projection $\mathbf{E}$	We know in standard order basis ; $\begin{pmatrix} x \\ y \end{pmatrix} : \begin{pmatrix} 1 \\ 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} y$ Express $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ in the basis $\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 0 \end{pmatrix} = p\begin{pmatrix} 1 \\ -1 \end{pmatrix} + q\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ where $\begin{pmatrix} p \\ q \end{pmatrix}$ is representation of $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ in the new basis. $\implies \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix}$ similarly;

	$\begin{pmatrix}0\\1\end{pmatrix} = r\begin{pmatrix}1\\-1\end{pmatrix} + s\begin{pmatrix}1\\2\end{pmatrix}$ $\implies \begin{pmatrix}r\\s\end{pmatrix} = \begin{pmatrix}\frac{-1}{3}\\\frac{1}{3}\end{pmatrix}$ Substitute in standard order basis we get; $\begin{pmatrix}x\\y\end{pmatrix} : \left(\frac{2}{3}x - \frac{1}{3}y\right)\begin{pmatrix}1\\-1\end{pmatrix} + \left(\frac{1}{3}x + \frac{1}{3}y\right)\begin{pmatrix}1\\2\end{pmatrix}$ Since, $\mathbf{E}\begin{pmatrix}x\\y\end{pmatrix} = a\begin{pmatrix}1\\-1\end{pmatrix}$ $\implies \begin{pmatrix}x\\y\end{pmatrix} : \begin{pmatrix}\frac{2}{3} & \frac{-1}{3}\\\frac{-2}{3} & \frac{1}{3}\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$ $\implies \boxed{\mathbf{E} = \begin{pmatrix}\frac{2}{3} & \frac{-1}{3}\\\frac{-2}{3} & \frac{1}{3}\end{pmatrix}}$

TABLE 1: Solution Summary