

Assignment 15

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Download the latex-tikz codes from

<https://github.com/neharani289/MatrixTheory/Assignment15>

1 PROBLEM

(hoffman/page213/3) :

Find a projection \mathbf{E} which projects \mathbb{R}^2 onto the subspace spanned by $(1, -1)$ along the subspace spanned by $(1, 2)$.

2 SOLUTION

Given	<p>Let $\begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$</p> $\begin{pmatrix} x \\ y \end{pmatrix} = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} + b \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.1)$ <p>where $\begin{pmatrix} a \\ b \end{pmatrix}$ is representation of $\begin{pmatrix} x \\ y \end{pmatrix}$ in new basis.</p>
To find	$\mathbf{E} \begin{pmatrix} x \\ y \end{pmatrix} = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.2)$
Finding a Projection \mathbf{E}	<p>As, $\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ are linearly independent.</p> <p>Therefore, $\{\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}\}$ is a basis of \mathbb{R}^2</p> <p>As $\begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$</p>

	$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} + b \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.3)$ $\Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \frac{2x-y}{3} \\ \frac{x+y}{3} \end{pmatrix} \quad (2.0.4)$ <p>Projection of $\begin{pmatrix} x \\ y \end{pmatrix}$ on subspace spanned by $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$;</p> $\mathbf{E} \begin{pmatrix} x \\ y \end{pmatrix} = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.5)$ $\Rightarrow \mathbf{E} \begin{pmatrix} x \\ y \end{pmatrix} = \left(\frac{2x-y}{3} \right) \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.6)$ $\Rightarrow \mathbf{E} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (2.0.7)$ $\Rightarrow \mathbf{E} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \quad (2.0.8)$
Verification	<p>If $n \times n$ matrix \mathbf{E} is projection matrix, then</p> $\mathbf{E}^2 = \mathbf{E}$ $\mathbf{E}^2 = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \Rightarrow \mathbf{E} = \mathbf{E}^2 = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \quad (2.0.9)$ <p>Hence, Verified.</p>

TABLE 1: Finding Projection Matrix