Assignment 13

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Download the latex-tikz codes from

https://github.com/neharani289/MatrixTheory/Assignment13

1 Problem

(hoffman/page198/9) :
Let $\mathbf{A}$ be an $n \times n$ matrix with characteristics polynomial

$$f = (x - c_1)^{d_1}....(x - c_k)^{d_k}$$

Show that

$$c_1 d_1 + ... + c_k d_k = tr(A)$$

2 Solution

Given	Let $\mathbf{A}$ be an $n \times n$ $\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$ and Characteristics polynomial $f = (x - c_1)^{d_1}....(x - c_k)^{d_k}$
To prove	$c_1 d_1 + ... + c_k d_k = tr(A)$
proof	Characteristics polynomial; $f = (x - c_1)^{d_1}....(x - c_k)^{d_k}$ here, $c_1, ..., c_k$ are the distinct eigen values. and $d_1, ..., d_k$ denotes the repetition of eigen values

	Therefore, $\boxed{c_1d_1 + c_2d_2 + ... + c_kd_k = \sum_i \lambda_i}$ =Sum of all eigen values. As we know , Trace of a matrix is the sum of its eigen values. $\implies tr(A) = \sum_i \lambda_i$ therefore, $\boxed{\implies c_1d_1 + c_2d_2 + ... + c_kd_k = \sum_i \lambda_i = tr(A)}$ Hence, Proved.

TABLE 1: Solution Summary