

# Assignment 15

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Download the latex-tikz codes from

<https://github.com/neharani289/MatrixTheory/Assignment15>

## 1 PROBLEM

(hoffman/page213/3) :

Find a projection  $\mathbf{E}$  which projects  $\mathbb{R}^2$  onto the subspace spanned by  $(1, -1)$  along the subspace spanned by  $(1, 2)$ .

## 2 SOLUTION

Given	<p>Let <math>\begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2</math></p> $\begin{pmatrix} x \\ y \end{pmatrix} = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} + b \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ <p>where <math>\begin{pmatrix} a \\ b \end{pmatrix}</math> is representation of <math>\begin{pmatrix} x \\ y \end{pmatrix}</math> in new basis.</p>
To find	$\mathbf{E} \begin{pmatrix} x \\ y \end{pmatrix} = a \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
Finding a Projection $\mathbf{E}$	<p>We know in standard order basis ;</p> $\begin{pmatrix} x \\ y \end{pmatrix} : \begin{pmatrix} 1 \\ 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} y$ <p>Express <math>\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}</math> in the basis <math>\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}</math></p> $\begin{pmatrix} 1 \\ 0 \end{pmatrix} = p \begin{pmatrix} 1 \\ -1 \end{pmatrix} + q \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ <p>where <math>\begin{pmatrix} p \\ q \end{pmatrix}</math> is representation of <math>\begin{pmatrix} 1 \\ 0 \end{pmatrix}</math> in the new basis.</p> $\Rightarrow \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix}$ <p>similarly;</p>

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = r \begin{pmatrix} 1 \\ -1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} \frac{-1}{3} \\ \frac{1}{3} \end{pmatrix}$$

Substitute in standard order basis we get;

$$\begin{pmatrix} x \\ y \end{pmatrix} : \left( \frac{2}{3}x - \frac{1}{3}y \right) \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \left( \frac{1}{3}x + \frac{1}{3}y \right) \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Since,  $\mathbf{E} \begin{pmatrix} x \\ y \end{pmatrix} = a \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$$\Rightarrow \mathbf{E} \begin{pmatrix} x \\ y \end{pmatrix} = \left( \frac{2}{3}x - \frac{1}{3}y \right) \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\Rightarrow \mathbf{E} = \begin{pmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix}$$

TABLE 1: Solution Summary