Assignment 9

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Download all latex-tikz codes from

https://github.com/neharani289/MatrixTheory/blob/master/Assignment9

1 Problem

Let $\mathbf{A} = \begin{pmatrix} 2 & 0 & i \\ 1 & -3 & -i \\ i & 1 & 1 \end{pmatrix}$, find a row-reduced echelon matrix $\mathbf{R}$ which is row-equivalent to $\mathbf{A}$ and an invertible 3x3 matrix $\mathbf{P}$ such that $\mathbf{R} = \mathbf{P}\ \mathbf{A}$.

2 Solution

Given,

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & i \\ 1 & -3 & -i \\ i & 1 & 1 \end{pmatrix} \tag{2.0.1}$$

Row reduce $\mathbf{A}$ by applying the elementary row operations and equivalently at each operations find the elementary matrix $\mathbf{E}$

$$[\mathbf{A}\ \mathbf{I}] = \left(\begin{array}{ccc|ccc} 2 & 0 & i & 1 & 0 & 0 \\ 1 & -3 & -i & 0 & 1 & 0 \\ i & 1 & 1 & 0 & 0 & 1 \end{array}\right) \tag{2.0.2}$$

$$\xleftrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & -3 & -i & 0 & 1 & 0 \\ 2 & 0 & i & 1 & 0 & 0 \\ i & 1 & 1 & 0 & 0 & 1 \end{array}\right) \tag{2.0.3}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \left(\begin{array}{ccc|ccc} 1 & -3 & -i & 0 & 1 & 0 \\ 0 & 6 & 3i & 1 & -2 & 0 \\ i & 1 & 1 & 0 & 0 & 1 \end{array}\right) \tag{2.0.4}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - iR_1} \left(\begin{array}{ccc|ccc} 1 & -3 & -i & 0 & 1 & 0 \\ 0 & 6 & 3i & 1 & -2 & 0 \\ 0 & 1+3i & 0 & 0 & -i & 1 \end{array}\right) \tag{2.0.5}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{6}} \left(\begin{array}{ccc|ccc} 1 & -3 & -i & 0 & 1 & 0 \\ 0 & 1 & \frac{i}{2} & \frac{1}{6} & -\frac{1}{3} & 0 \\ 0 & 1+3i & 0 & 0 & -i & 1 \end{array}\right) \tag{2.0.6}$$

$$\xleftrightarrow{R_1 \leftarrow\ R_1 + 3R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{i}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{i}{2} & \frac{1}{6} & -\frac{1}{3} & 0 \\ 0 & 1+3i & 0 & 0 & -i & 1 \end{array}\right) \tag{2.0.7}$$

$$\xleftrightarrow{R_3 \leftarrow\ R_3/(3-i)/2} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{i}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{i}{2} & \frac{1}{6} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & -\frac{(i)}{3} & \frac{3+i}{15} & \frac{3+i}{5} \end{array}\right) \tag{2.0.8}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{i}{2}R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{3} & \frac{1-3i}{30} & \frac{1-3i}{10} \\ 0 & 1 & \frac{i}{2} & \frac{1}{6} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & -\frac{(i)}{3} & \frac{3+i}{15} & \frac{3+i}{5} \end{array}\right) \tag{2.0.9}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - \frac{i}{2}R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{3} & \frac{1-3i}{30} & \frac{1-3i}{10} \\ 0 & 1 & 0 & 0 & -\frac{3+i}{10} & \frac{1-3i}{10} \\ 0 & 0 & 1 & -\frac{i}{3} & \frac{3+i}{15} & \frac{3+i}{5} \end{array}\right) \tag{2.0.10}$$

$$= [\mathbf{I}\ \mathbf{E}]$$

Hence, the row reduced matrix that is row equivalent to $\mathbf{A}$ is

$$\mathbf{R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \mathbf{I} \tag{2.0.11}$$

Using Gauss-Jordan Elimination, if there exists an elimentary matrix $\mathbf{E}$ such that $\mathbf{E}[\mathbf{A}\ \mathbf{I}] = [\mathbf{I}\ \mathbf{E}]$ then $\mathbf{E}$ is the inverse of A i.e $\mathbf{E} = \mathbf{A^{-1}}$

$$\mathbf{E} = \mathbf{A^{-1}} = \begin{pmatrix} \frac{1}{3} & \frac{1-3i}{30} & \frac{1-3i}{10} \\ 0 & -\frac{3+i}{10} & \frac{1-3i}{10} \\ -\frac{i}{3} & \frac{3+i}{15} & \frac{3+i}{5} \end{pmatrix} \tag{2.0.12}$$

Since,

$$\mathbf{R} = \mathbf{PA} \implies \mathbf{P} = \mathbf{A}^{-1}\mathbf{R} \tag{2.0.13}$$

$$\mathbf{P} = \begin{pmatrix} \frac{1}{3} & \frac{1-3i}{30} & \frac{1-3i}{10} \\ 0 & -\frac{3+i}{10} & \frac{1-3i}{10} \\ -\frac{i}{3} & \frac{3+i}{15} & \frac{3+i}{5} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.14}$$

Thus,

$$\mathbf{P} = \begin{pmatrix} \frac{1}{3} & \frac{1-3i}{30} & \frac{1-3i}{10} \\ 0 & -\frac{3+i}{10} & \frac{1-3i}{10} \\ -\frac{i}{3} & \frac{3+i}{15} & \frac{3+i}{5} \end{pmatrix}$$

$$\mathbf{R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$