Assignment 10

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***Abstract*—This document solves a problem based on Linear Transformation .**

Download latex-tikz codes from

https://github.com/neharani289/MatrixTheory/blob/master/Assignment10

1 Problem

Let $\mathbf{T}$ be the linear transformation from $\mathbb{R}^3$ into $\mathbb{R}^2$ defined by

$$\mathbf{T}(x_1, x_2, x_3) = (x_1 + x_2, 2x_3 - x_1) \tag{1.0.1}$$

If $\beta \ = (\alpha_1, \alpha_2, \alpha_3)$ and $\beta' = (\beta_1, \beta_2)$ where

$$\alpha_{\mathbf{1}} = (1, 0, -1), \alpha_{\mathbf{2}} = (1, 1, 1), \alpha_{\mathbf{3}} = (1, 0, 0)$$

$$\beta_{\mathbf{1}} = (0, 1), \beta_{\mathbf{2}} = (1, 0)$$

What is the matrix of $\mathbf{T}$ relative to the pair β, β'

2 Solution

Let

$$\beta = (\alpha_1, \alpha_2, \alpha_3) \tag{2.0.1}$$

$$\implies \beta = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ -1 & 1 & 0 \end{pmatrix} \tag{2.0.2}$$

and

$$\beta' = \{\beta_1, \beta_2\} \tag{2.0.3}$$

$$\implies \beta' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.0.4}$$

T is defined by

$$T(\mathbf{x}) = \mathbf{Ax} \tag{2.0.5}$$

using equation (1.0.1)

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ 2x_3 - x_1 \end{pmatrix} \tag{2.0.6}$$

R.H.S of the equation can be written as a product of 2×3 and 3×1 matrices,

$$= \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \tag{2.0.7}$$

$$\implies \mathbf{A} = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \end{pmatrix} \tag{2.0.8}$$

Now,

$$T(\beta) = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \end{pmatrix} \beta \tag{2.0.9}$$

$$T(\beta) = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ -1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 \\ -3 & 1 & -1 \end{pmatrix} \tag{2.0.10}$$

To find relative matrix we will use row reduce augmented matrix.

$$\left(\begin{array}{ccc|cc} 1 & 2 & 1 & 0 & 1 \\ -3 & 1 & -1 & 1 & 0 \end{array}\right) \tag{2.0.11}$$

$$\left(\begin{array}{ccc|cc} 1 & 2 & 1 & 0 & 1 \\ -3 & 1 & -1 & 1 & 0 \end{array}\right) \xleftrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|cc} -3 & 1 & -1 & 1 & 0 \\ 1 & 2 & 1 & 0 & 1 \end{array}\right) \tag{2.0.12}$$

Hence the matrix of $\mathbf{T}$ in the order basis of $\beta^{'}$

$$\mathbf{B} = \begin{pmatrix} -3 & 1 & -1 \\ 1 & 2 & 1 \end{pmatrix} \tag{2.0.13}$$

Therefore matrix of relative to the pair β , $\beta^{'}$

$$T(\beta) = \mathbf{A}\beta = \mathbf{B}\beta^{'} = \begin{pmatrix} -3 & 1 & -1 \\ 1 & 2 & 1 \end{pmatrix} \beta^{'} \tag{2.0.14}$$