Assignment 4

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***Abstract*—This document solves a question based on triangle using linear algebra.**

All the codes for the figure in this document can be found at

https://github.com/neharani289/ee14014/tree/master/Assignment4

1 Problem

In $\triangle ABC$, the bisector $\mathbf{AD}$ of $\angle A \perp$ to side $\mathbf{BC}$.Show that $\mathbf{AB} = \mathbf{AC}$ and $\triangle ABC$ is isosceles.

2 Solution

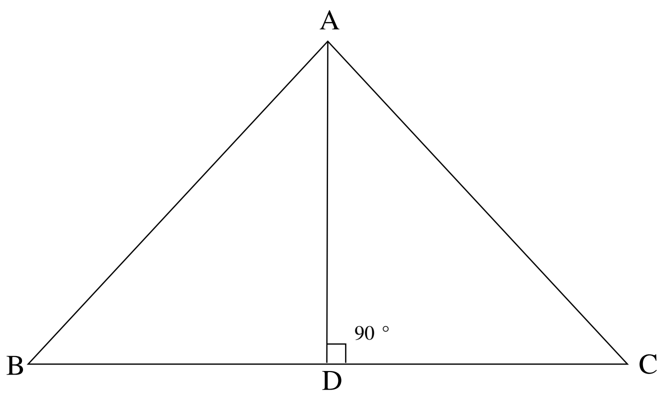

Fig. 1: Isosceles Triangle with $\mathbf{AD} \perp \mathbf{BC}$

In $\triangle BAD$ and $\triangle CAD$ applying cosine law we get;

$$\|\mathbf{B}-\mathbf{D}\|^2 = \|\mathbf{A}-\mathbf{B}\|^2 + \|\mathbf{A}-\mathbf{D}\|^2 \, \|\mathbf{A}-\mathbf{B}\| \, \|\mathbf{A}-\mathbf{D}\| \cos BAD \tag{2.0.1}$$

similarly;

$$\|\mathbf{D}-\mathbf{C}\|^2 = \|\mathbf{A}-\mathbf{C}\|^2 + \|\mathbf{A}-\mathbf{D}\|^2 + \|\mathbf{A}-\mathbf{C}\| \, \|\mathbf{A}-\mathbf{D}\| \cos CAD \tag{2.0.2}$$

Now using pythagorus law;

$$\|\mathbf{A}-\mathbf{B}\|^2 = \|\mathbf{A}-\mathbf{D}\|^2 + \|\mathbf{B}-\mathbf{D}\|^2 \tag{2.0.3}$$

$$\|\mathbf{A}-\mathbf{C}\|^2 = \|\mathbf{A}-\mathbf{D}\|^2 + \|\mathbf{D}-\mathbf{C}\|^2 \tag{2.0.4}$$

From (2.0.1) and (2.0.3)

$$2\,\|\mathbf{A}-\mathbf{D}\|^2 + \|\mathbf{A}-\mathbf{B}\| \, \|\mathbf{A}-\mathbf{D}\| \cos BAD = 0 \tag{2.0.5}$$

From (2.0.2) and (2.0.4)

$$2\,\|\mathbf{A}-\mathbf{D}\|^2 + \|\mathbf{A}-\mathbf{C}\| \, \|\mathbf{A}-\mathbf{D}\| \cos CAD = 0 \tag{2.0.6}$$

equating (2.0.5) and (2.0.6) gives;

$$\cos BAD = \cos CAD \tag{2.0.7}$$

$$\|\mathbf{A}-\mathbf{B}\| = \|\mathbf{A}-\mathbf{C}\| \tag{2.0.8}$$

Thus, $\triangle ABC$ is isosceles triangle.
Hence proved.