Assignment 14

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Download the latex-tikz codes from

https://github.com/neharani289/MatrixTheory/Assignment14

1 Problem

(hoffman/page208/1b) :
Find an invertible matrix $\mathbf{P}$ such that $\mathbf{P^{-1}AP}$ and $\mathbf{P^{-1}BP}$ are both diagonal where $\mathbf{A}$ and $\mathbf{B}$ are real matrices.

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{1.0.1}$$

$$\mathbf{B} = \begin{pmatrix} 1 & a \\ a & 1 \end{pmatrix} \tag{1.0.2}$$

2 Explanation

Characteristic Polynomial	For an $n \times n$ matrix $\mathbf{A}$, characteristic polynomial is defined by, $p(x) = \lvert x\mathbf{I} - \mathbf{A} \rvert$
Theorem	Acc to theorem 8, if a 2×2 matrix has two characteristics values then the $\mathbf{P}$ that diagonalize $\mathbf{A}$ will necessarily also diagonalize any $\mathbf{B}$ that commutes with $\mathbf{A}$.

TABLE 1: Definitions and theorem used

3 Solution

Characteristic polynomial	$\begin{aligned} p(x) &= \lvert x\mathbf{I} - \mathbf{A} \rvert \\ &= \begin{vmatrix} x-1 & 1 \\ 1 & x-1 \end{vmatrix} \\ &= (x-1)(x-1) - 1 \\ &= x^2 - 2x \\ &= x(x-2) \end{aligned}$
Characteristic values	$p(x) = 0$ $\implies x(x-2) = 0$ $\implies c_1 = 0, c_2 = 2$
Basis for characteristic values	Basis for Characteristics value $c_1 = 0$ will be obtained by solving homogenous equation $(\mathbf{A} - c_1\mathbf{I})\,x = 0$ $\implies \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} x = 0$ After solving Basis for characteristics value c_1 is $\mathbf{B_1} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ Similarly for c_2 we get; $\mathbf{B_2} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $c_1 : \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $c_2 : \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$
Invertible matrix	Now, Invertible matrix $\mathbf{P}$ is given by So, $\mathbf{P} = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$ $\mathbf{P^{-1}} = \begin{pmatrix} \frac{-1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ $\implies \mathbf{P^{-1}AP} = \begin{pmatrix} \frac{-1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$ $\implies \mathbf{P^{-1}BP} = \begin{pmatrix} \frac{-1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & a \\ a & 1 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1-a & 0 \\ 0 & 1-a \end{pmatrix}$

TABLE 2: Solution Summary