

Deep Inelastic Scattering (DIS)



**Ingo Schienbein
UGA/LPSC Grenoble**



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Disclaimer

This lecture has profited a lot from the following resources:

- The text book by **Halzen&Martin**, Quarks and Leptons
- The text book by **Ellis, Stirling & Webber**, QCD and Collider Physics
- The lecture on DIS at the CTEQ school in 2012 by **F. Olness**
- The lecture on DIS given by **F. Gelis** Saclay in 2006

Lecture I

- I. Kinematics of Deep Inelastic Scattering
2. Cross sections for inclusive DIS (photon exchange)
3. OPE
4. Longitudinal and Transverse Structure functions
5. CC and NC DIS
6. Bjorken scaling

Lecture II

7. The Parton Model
8. Which partons?
9. Structure functions in the parton model
10. The pQCD formalism
11. NLO corrections to DIS
12. Parton evolution

Not covered

- DIS with massive quarks
- Target mass corrections
- QCD studies with neutrinos
- Polarised DIS
- DIS off spin-1 targets

VII. The Parton Model

Naive parton model

- The historical ('naive') parton model describes the nucleon as a collection made of **point-like constituents** called '**partons**'.

Today the Feynman partons are understood to be identical with the quarks postulated by **Gell-Mann**.

- At high momentum ('infinite momentum frame') the **partons are free**.

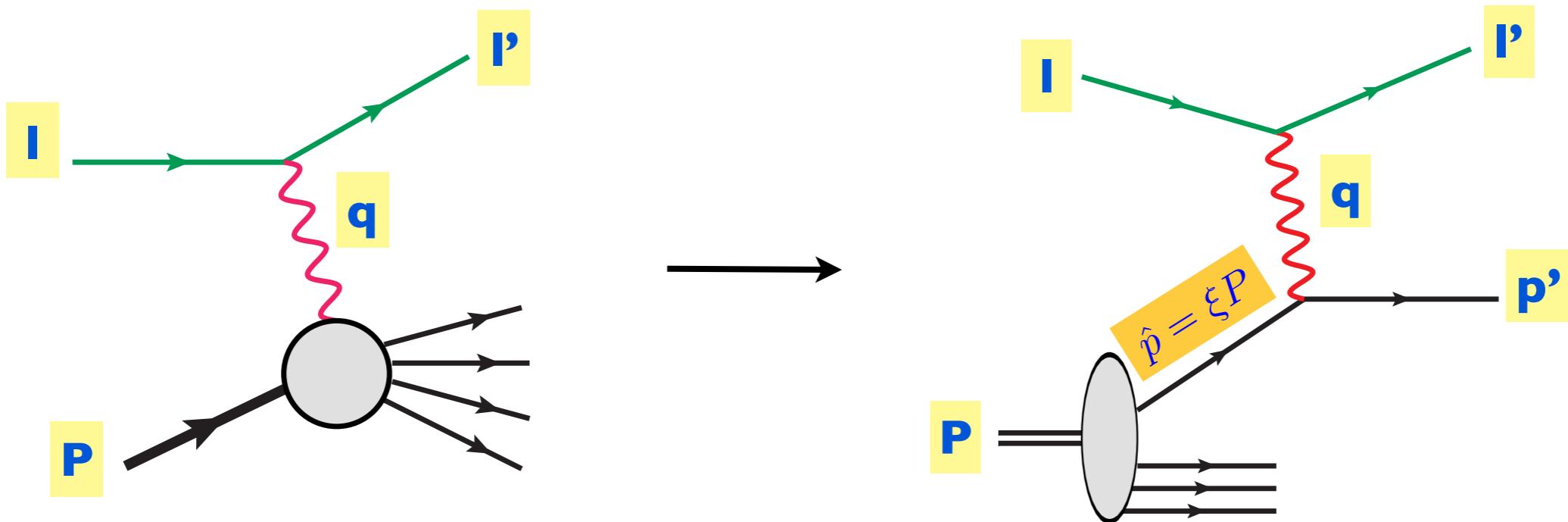
Therefore, the interaction of one parton with the electron does not affect the other partons. This leads to scaling in $x = Q^2 / (2 M v)$ as we will see below.

- In an IMF, the proton is moving very fast, $\mathbf{P} \sim (\mathbf{E}_p, 0, 0, \mathbf{E}_p)$ with $\mathbf{E}_p \gg \mathbf{M}$.

The quark-parton is moving parallel with the proton, carrying a fraction ξ of its momentum:

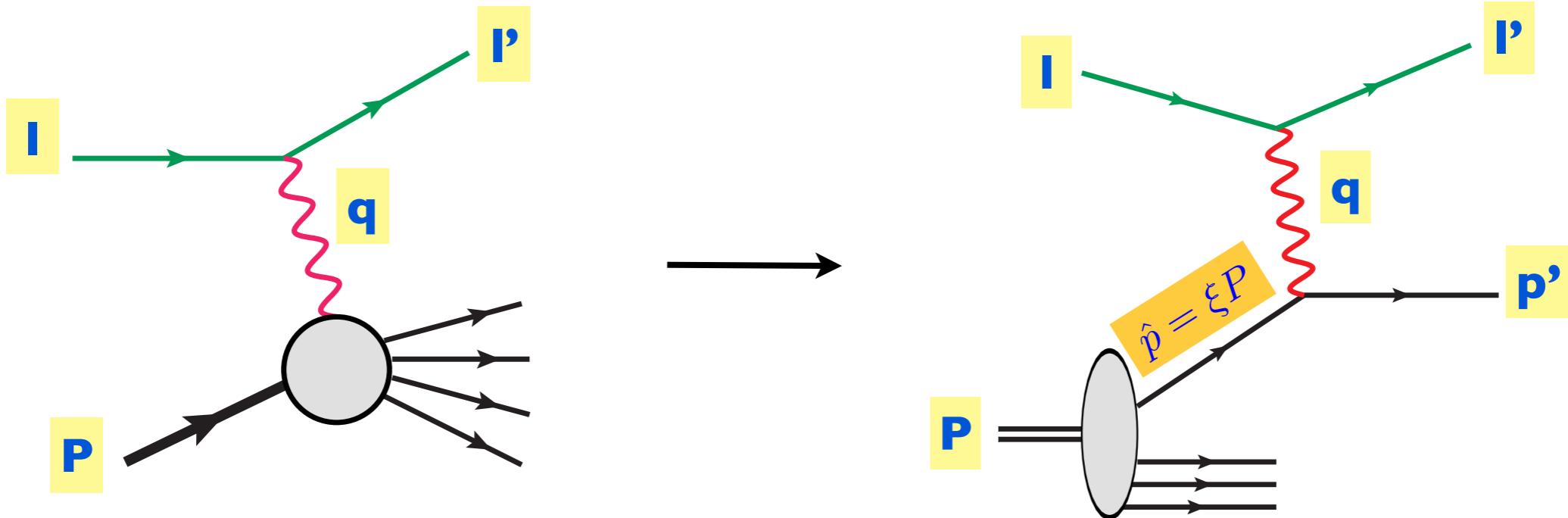
$$\hat{\mathbf{p}} = \xi \mathbf{P}$$

Naive parton model



$$d\sigma(e(l) + N(P) \rightarrow e(l') + X(p_X)) = \sum_i \int_0^1 d\xi f_i(\xi) d\hat{\sigma}(e(l) + i(\xi P) \rightarrow e(l') + i(p'))$$

Naive parton model

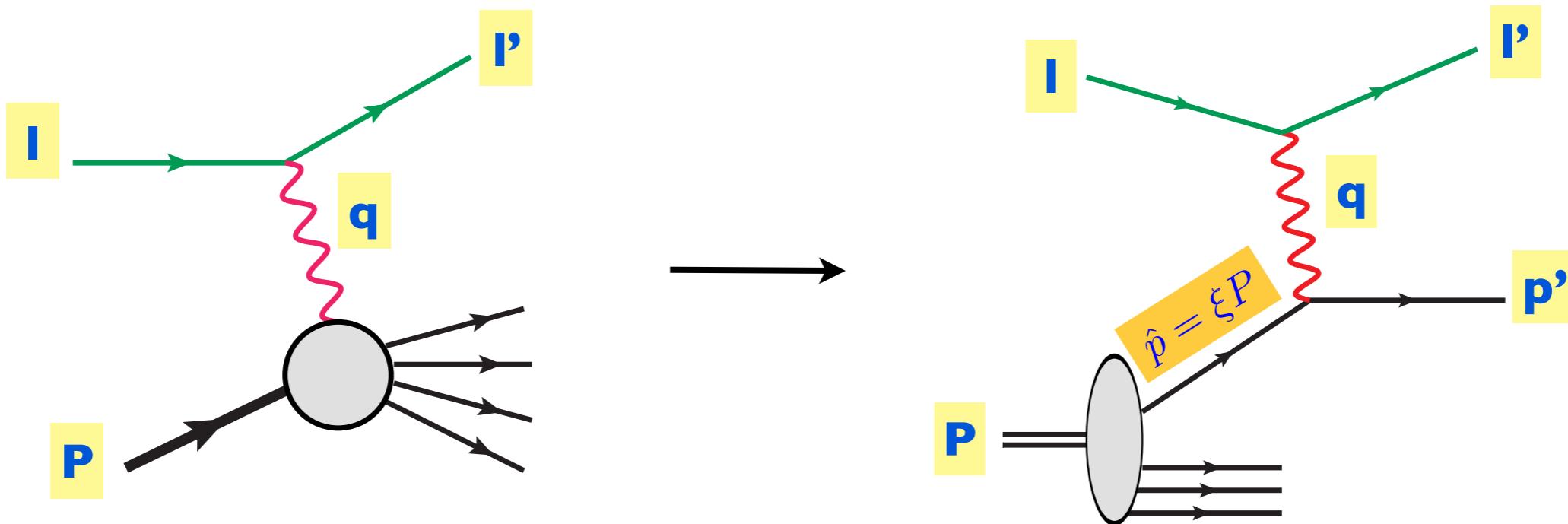


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Elastic scattering off
point-like parton

We know how to
calculate this!

Naive parton model



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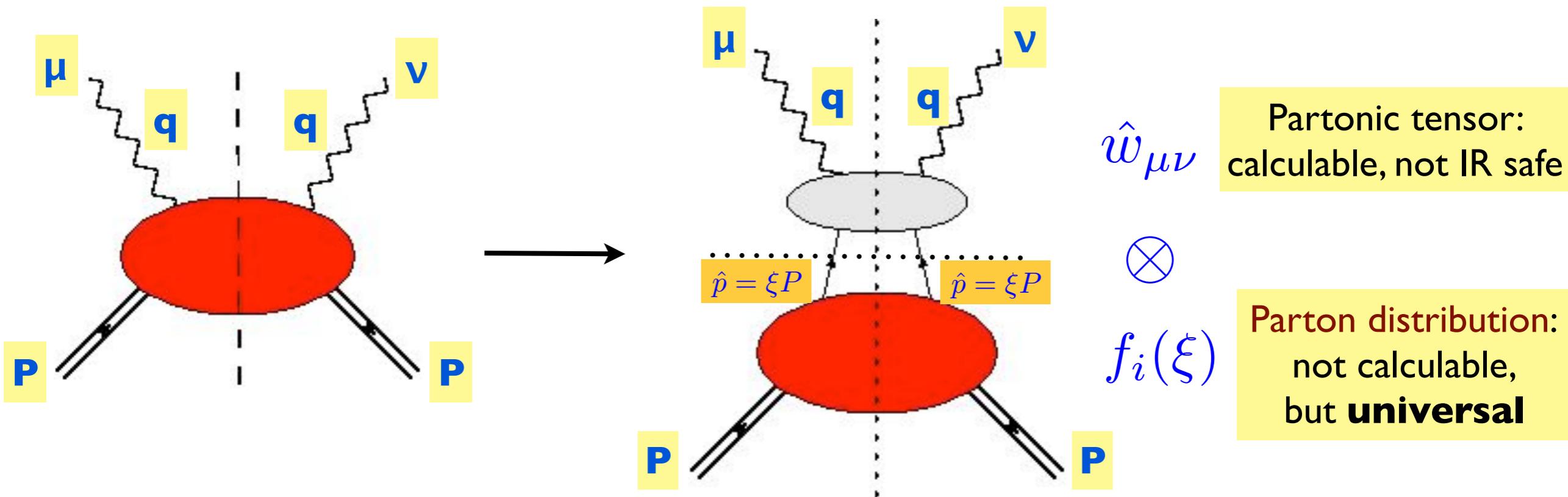
Incoherent sum
over all possible
partonic processes

Parton density:
 $f_i(\xi)d\xi$ = number of partons of
type ' i ', carrying a momentum
fraction of the proton
momentum in $[\xi, \xi+d\xi]$

Elastic scattering off
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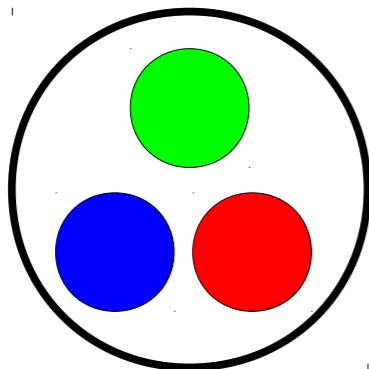


$$W^{\mu\nu}(P, q) = \sum_i \int_x^1 \frac{d\xi}{\xi} f_i(\xi) \hat{w}_i^{\mu\nu}(\xi P, q)$$

VIII. Which partons?

Which Partons?

- It is tempting to identify the partons with the quarks of Gell-Mann's quark model
- The proton consists of three **valence quarks** (uud) which carry the quantum numbers of the proton (electric charge, baryon number): **u=u_v, d=d_v**



$$u(x, Q) = 2 \delta(x - \frac{1}{3})$$

$$d(x, Q) = 1 \delta(x - \frac{1}{3})$$

Perfect Scaling PDFs
Q independent

Quark Number Sum Rule

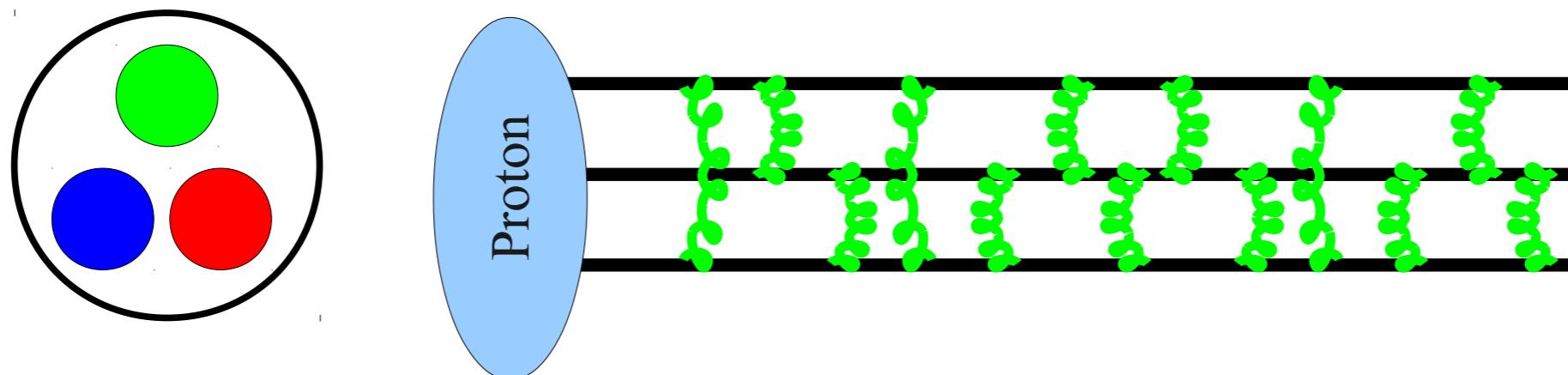
$$\langle q \rangle = \int_0^1 dx q(x) \quad \langle u \rangle = 2 \quad \langle d \rangle = 1 \quad \langle s \rangle = 0$$

Quark Momentum Sum Rule

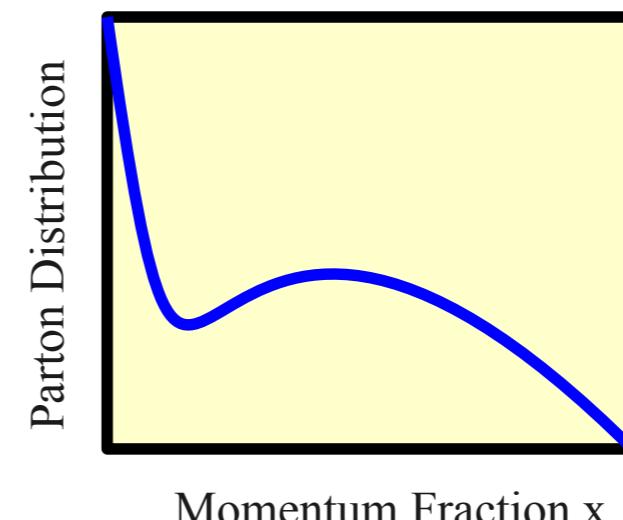
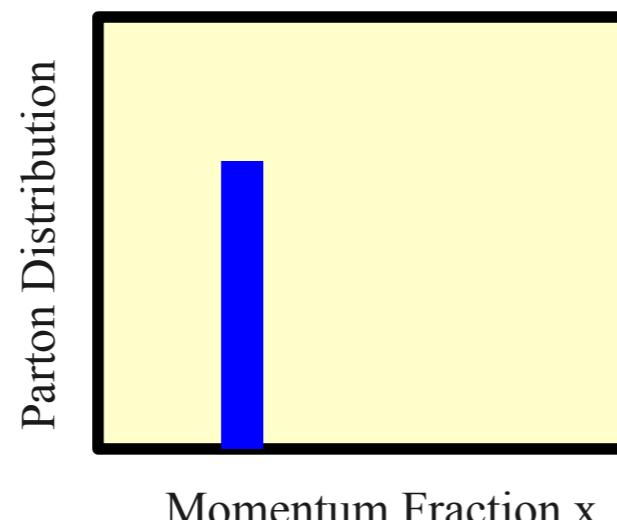
$$\langle x q \rangle = \int_0^1 dx x q(x) \quad \langle x u \rangle = \frac{2}{3} \quad \langle x d \rangle = \frac{1}{3}$$

Which Partons?

- The valence quarks are bound together by gluons which leads to a **smearing of the delta distributions** and the gluon can fluctuate into a '**sea' of quark-anti-quark pairs: $S(x)=\bar{u}(x)=\bar{d}(x)=s(x)=\bar{s}(x)$**
- **$u(x)=\bar{u}(x)+\bar{u}(x)$, $d(x)=\bar{d}(x)+\bar{d}(x)$, $\bar{u}(x)$, $\bar{d}(x)$, $s(x)$, ...**



Gluons allow partons to exchange momentum fraction



Which Partons?

- Problem, **experimentally** it is found that the **momentum fractions** of the quark partons do not add up to 1!

$$\sum_i \int_0^1 dx \ x[q_i(x) + \bar{q}_i(x)] \sim 0.5$$

- **Gluons carry ~half of the momentum but don't couple to photons**
- We have to include a gluon distribution **G(x)**

Which Partons?

In summary, the following picture emerges:

- The proton consists of three **valence quarks** (uud) which carry the quantum numbers of the proton (electric charge, baryon number): **u_v, d_v**
- There is a **sea of light quark-antiquark pairs**: **u ubar, d dbar, s sbar, ...**

When probed at a scale **Q**, the sea contains all quark flavours with mass **m_q<<Q**

- **u(x) = u_v(x) + ubar(x), d(x) = d_v(x) + dbar(x)**
- There is gluon distribution **G(x)** which carries about 50% of the proton momentum

Quark number sum rule: $\int_0^1 dx u_v(x) = 2, \quad \int_0^1 dx d_v(x) = 1, \quad \int_0^1 dx (s - \bar{s})(x) = 0$

Momentum sum rule: $\int_0^1 dx x \left\{ \sum_i [q_i(x) + \bar{q}_i(x)] + G(x) \right\} = 1$

IX. Structure functions in the parton model

A Parton Model Calculation

- As a little exercise, let's calculate the contribution of a parton of type i, carrying the fraction ξ of the nucleon momentum, to the partonic tensor:

$$4\pi \hat{w}_i^{\mu\nu} = \int \frac{d^4 p'}{(2\pi)^4} 2\pi \delta(p'^2) (2\pi)^4 \delta^{(4)}(\xi P + q - p') \left\langle \langle \xi P | J^{\mu\dagger}(0) | p' \rangle \langle p' | J^\nu(0) | \xi P \rangle \right\rangle_{\text{spin}}$$

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standard current for point-like fermion as in QED

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\end{aligned}$$

As expected for elastic scattering, the result is $\sim \delta(1 - \hat{x})$ with $\hat{x} = \frac{Q^2}{2\hat{p} \cdot q} = \frac{x}{\xi}$.

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F_1 and F_2 in the Parton Model

- A parton of type i , carrying the fraction ξ of the nucleon momentum, gives the following contribution to the hadronic tensor:

$$4\pi \hat{w}_i^{\mu\nu} = (2\pi)\xi\delta(\xi - x)e_i^2 \left[-\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + \frac{2\xi}{P \cdot q} \left(P^\mu - q^\mu \frac{P \cdot q}{q^2}\right) \left(P^\nu - q^\nu \frac{P \cdot q}{q^2}\right) \right]$$

- If there are $f_i(\xi)d\xi$ partons of type i with a momentum fraction between ξ and $\xi + d\xi$, we have

$$W^{\mu\nu} = \sum_i \int_x^1 \frac{d\xi}{\xi} f_i(\xi) \hat{w}_i^{\mu\nu}$$

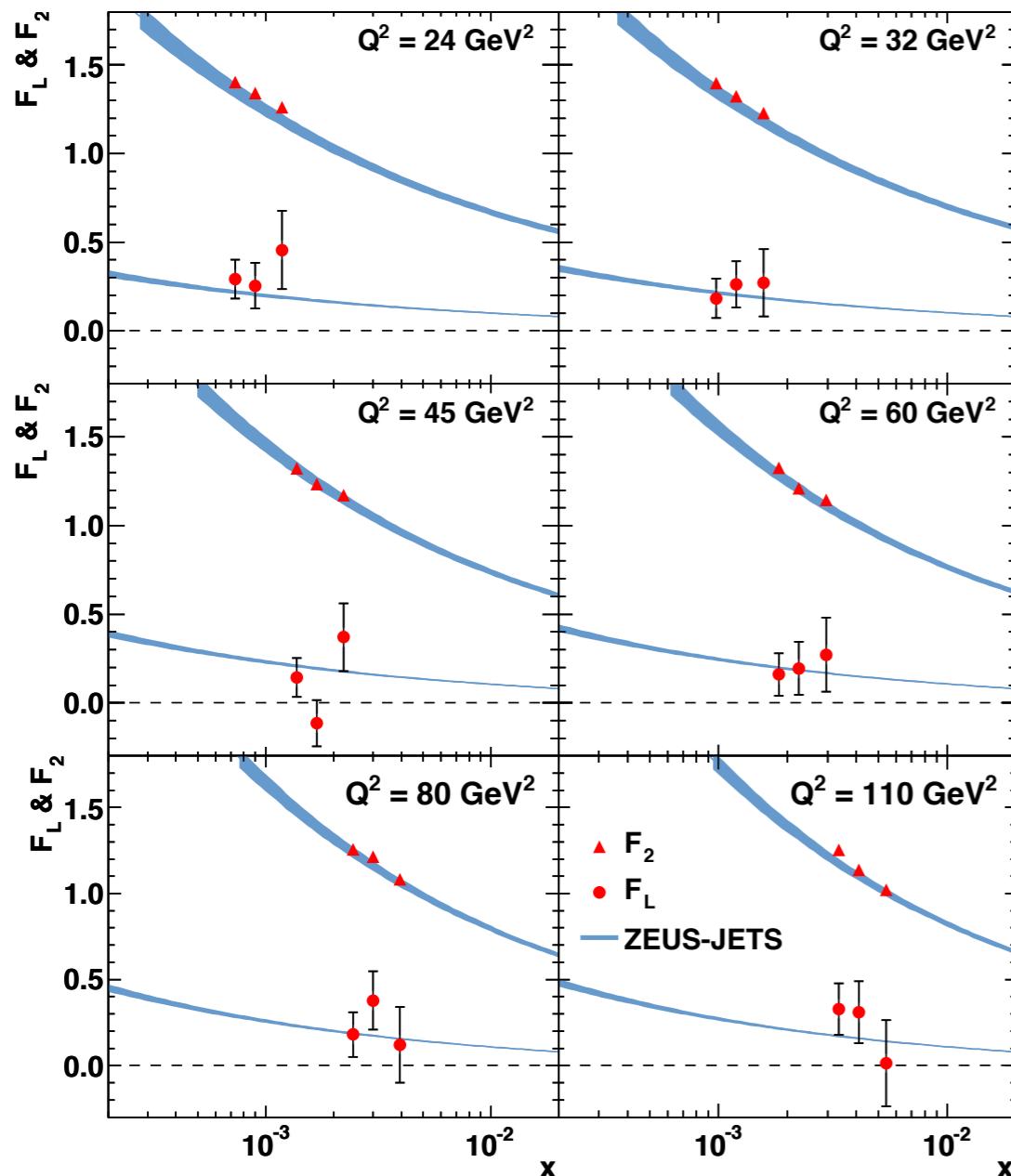
- One obtains the following structure functions:

$$F_1(x) = \frac{1}{2} \sum_i e_i^2 f_i(x) \quad , \quad F_2(x) = 2x F_1(x)$$

F_2 and F_L in the Parton Model

- This model provides an explicit realization of **Bjorken scaling**
- The **Callan-Gross relation** $F_L = F_2 - F_T = 0$ implies $F_2 = 2x F_L = F_T$
 - The observation of this property provides further support of the fact that the **partons are spin-1/2 fermions**
 - If the partons were spin-0 particles, we would have $F_T = 0$ and hence $F_2 = F_L$
- Caveats and puzzles (to be addressed later):
 - The naive parton model assumes that the partons are **free**. How can this be true in a strongly bound state?
 - One would like to have a field theoretic description (QCD) of what is going on, including the effects of interactions and quantum fluctuations.

F_L is small but not zero



- **Callan-Gross relation: $F_L=0$**
- **F_L is non-zero if:**
 - quarks are massive
 - Z-boson exchange is included
 - QCD corrections are included

We note that F_L is quite smaller than F_2 .

Homework Problem

Leading order parton model expressions for structure functions in **e+N→e+X** (γ -exchange) and **$\nu_\mu + N \rightarrow \mu^- + X$** DIS, assuming a diagonal **CKM-matrix, $m_c=0$**

$$\begin{aligned}
F_2^{ep} &= \frac{4}{9}x [u + \bar{u} + c + \bar{c}] \\
&\quad + \frac{1}{9}x [d + \bar{d} + s + \bar{s}] \\
F_2^{en} &= \frac{4}{9}x [d + \bar{d} + c + \bar{c}] \\
&\quad + \frac{1}{9}x [u + \bar{u} + s + \bar{s}] \\
F_2^{\nu p} &= 2x [d + s + \bar{u} + \bar{c}] \\
F_2^{\nu n} &= 2x [u + s + \bar{d} + \bar{c}] \\
F_2^{\bar{\nu} p} &= 2x [u + c + \bar{d} + \bar{s}] \\
F_2^{\bar{\nu} n} &= 2x [d + c + \bar{u} + \bar{s}] \\
F_3^{\nu p} &= 2 [d + s - \bar{u} - \bar{c}] \\
F_3^{\nu n} &= 2 [u + s - \bar{d} - \bar{c}] \\
F_3^{\bar{\nu} p} &= 2 [u + c - \bar{d} - \bar{s}] \\
F_3^{\bar{\nu} n} &= 2 [d + c - \bar{u} - \bar{s}]
\end{aligned}$$

- Verify this list!
Check for isospin symmetry,
How are the structure functions F_3 in neutrino and anti-neutrino DIS related?

- These different observables are used to dis-entangle the flavor structure of the PDFs

Homework Problem

Verify the following sum rules in the parton model:

Adler
(1966)

$$\int_0^1 \frac{dx}{2x} [F_2^{\nu n} - F_2^{\nu p}] = 1$$

Bjorken
(1967)

$$\int_0^1 \frac{dx}{2x} [F_2^{\bar{\nu} p} - F_2^{\nu p}] = 1$$

Gross Llewellyn-
Smith
(1969)

$$\int_0^1 dx [F_3^{\nu p} + F_3^{\bar{\nu} p}] = 6$$

Gottfried
(1967) if $\bar{u} = \bar{d}$

$$\int_0^1 \frac{dx}{x} [F_2^{ep} - F_2^{en}] = \frac{1}{3}$$

Experimentally: @ $Q^2=4$ GeV²
I_G=0.235 +/- 0.026
 Conclusion? [cf. hep-ph/0311091]

Homework
(19???)

$$\frac{5}{18} F_2^{\nu N} - F_2^{eN} = ?$$

$$F_2^{lN} := (F_2^{lp} + F_2^{ln})/2 \quad (l = \nu, e)$$

Homework Problem

- Calculate the structure functions $\mathbf{F}_1(\mathbf{x})$ and $\mathbf{F}_2(\mathbf{x})$ for a massless spin-0 parton ‘i’ of electric charge \mathbf{e}_i and parton distribution $\mathbf{q}_i(\mathbf{x})$.

The scalar-scalar-photon vertex reads:

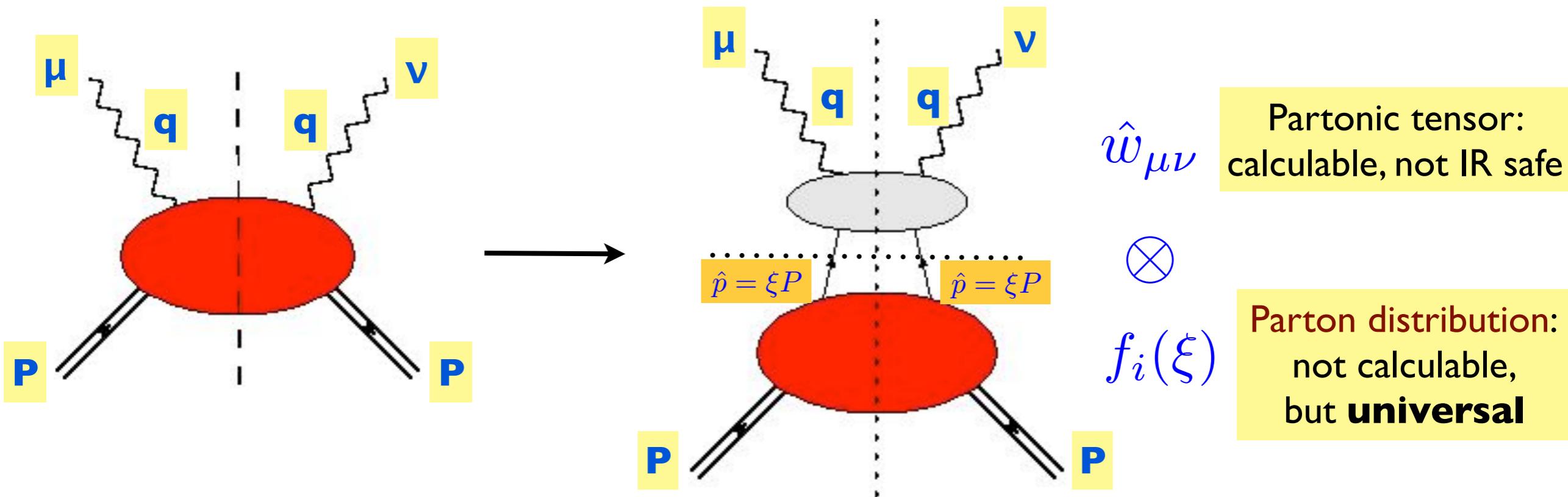
$$-iee_q(p' + p)^\mu \delta_{ij}$$

The result for the partonic tensor is:

$$\hat{w}_i^{\mu\nu} = \frac{2}{Q^2} e_i^2 x^3 p_\perp^\mu p_\perp^\nu \delta(\xi - x)$$

How do the results change if the incoming parton remains massless but the outgoing parton has a mass m?

So far: Naive Parton Model



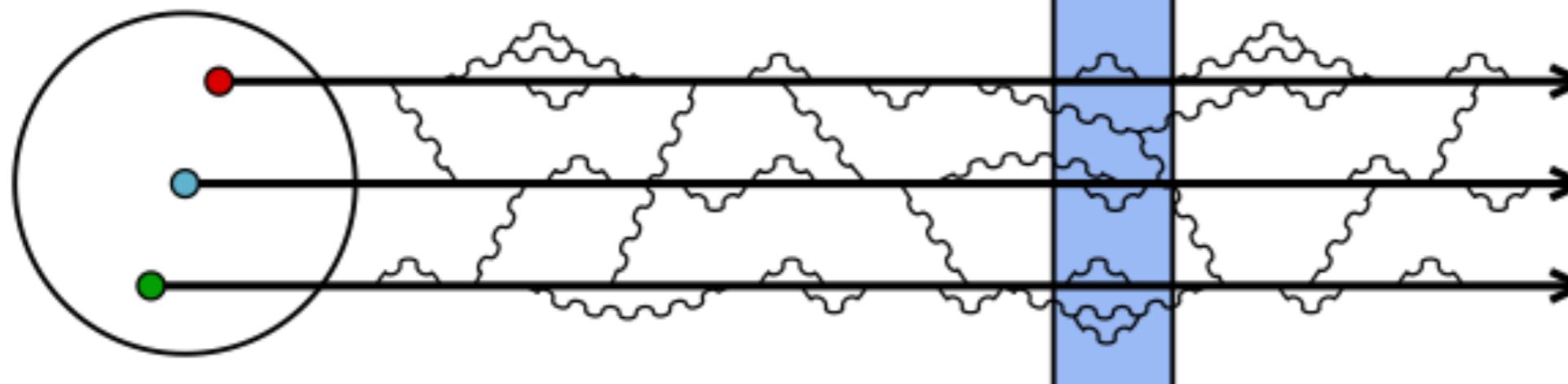
$$W^{\mu\nu}(P, q) = \sum_i \int_x^1 \frac{d\xi}{\xi} f_i(\xi) \hat{w}_i^{\mu\nu}(\xi P, q)$$

F_2 and F_L in the Parton Model

- This model provides an explicit realization of **Bjorken scaling**
- The **Callan-Gross relation** $F_L = F_2 - F_T = 0$ implies $F_2 = 2x F_L = F_T$
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- Caveats and puzzles (to be addressed later):
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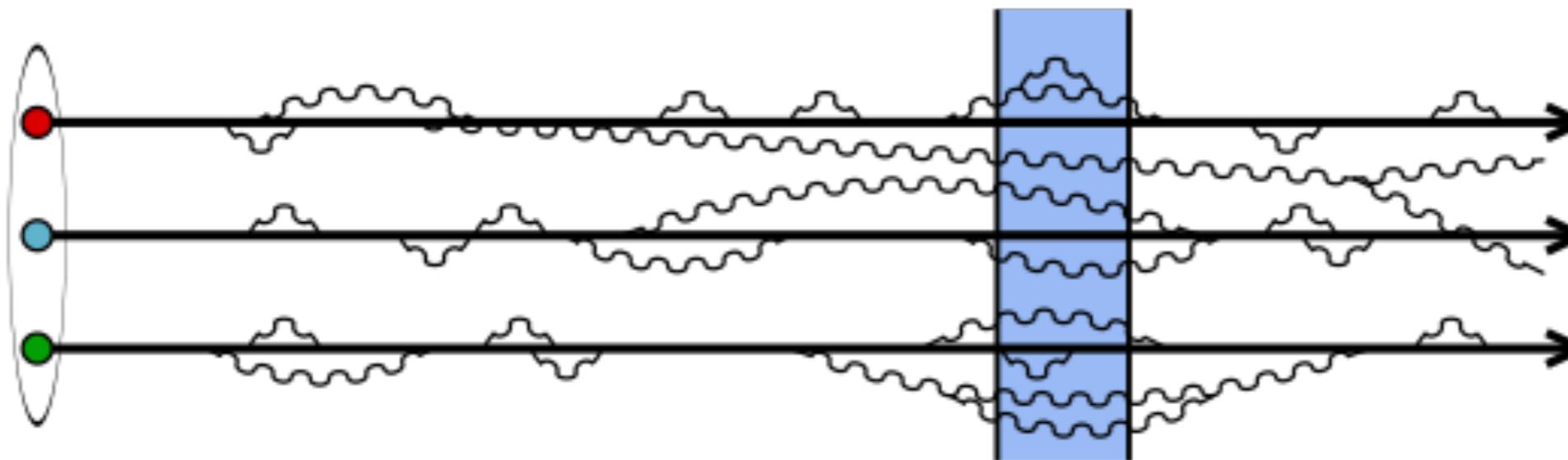
Field theory point of view

Lecture by
F. Gelis 2006



- A **nucleon at rest** is a very complicated object
- Contains **fluctuations at all space-time scales** smaller than its own size
- Only the fluctuations that are longer lived than the external probe participate in the interaction process
- The only role of **short-lived fluctuations** is to renormalize the masses and couplings
- Interactions are very complicated if the constituents of the nucleon have a non-trivial dynamics over time-scales comparable to those of the probe

Field theory point of view



- **Dilation** of all internal time-scales for a **high-energy nucleon**
- Interactions among **constituents** now take place over time-scales that are longer than the characteristic time-scale of the probe
 - **the constituents behave as if they were free**, the probability of having interactions between the constituents **during the time-scale of the probe** is suppressed
- Many fluctuations live long enough to be seen by the probe. The nucleon appears to be **denser at higher energy** (it contains more gluons)
- Proofs from first principles of **QCD** show that the parts involving long and short time-scales can be **separated into independent factors** (Factorization theorems)

X.The pQCD formalism

Quantum Chromodynamics (QCD)

QCD: A QFT for the strong interactions



- Statement: Hadronic matter is made of spin-1/2 quarks [$\leftrightarrow SU(3)_{\text{fl}}$]
- Baryons like $\Delta^{++} = |u^\uparrow u^\uparrow u^\uparrow\rangle$ forbidden by Pauli exclusion/Fermi-Dirac stat.
Need additional colour degree of freedom!
- Local $SU(3)$ -color gauge symmetry:

$$\mathcal{L}_{\text{QCD}} = \sum_{q=u,d,s,c,b,t} \bar{q}(i\cancel{\partial} - m_q)q - g\bar{q}\cancel{G}q - \frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu} + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{ghost}}$$

- Fundamental d.o.f.: quark and gluon fields
- Free parameters:
 - gauge coupling: g
 - quark masses: $m_u, m_d, m_s, m_c, m_b, m_t$

Quantum Chromodynamics (QCD)

Properties:

- **Confinement and Hadronization:**

- Free quarks and gluons have not been observed:
 - A) They are **confined** in color-neutral hadrons of size $\sim 1 \text{ fm}$.
 - B) They **hadronize** into the observed hadrons.
- Hadronic energy scale: a few hundred MeV [$1 \text{ fm} \leftrightarrow 200 \text{ MeV}$]
- Strong coupling large at long distances ($\gtrsim 1 \text{ fm}$): '**IR-slavery**'
- Hadrons and hadron masses enter the game



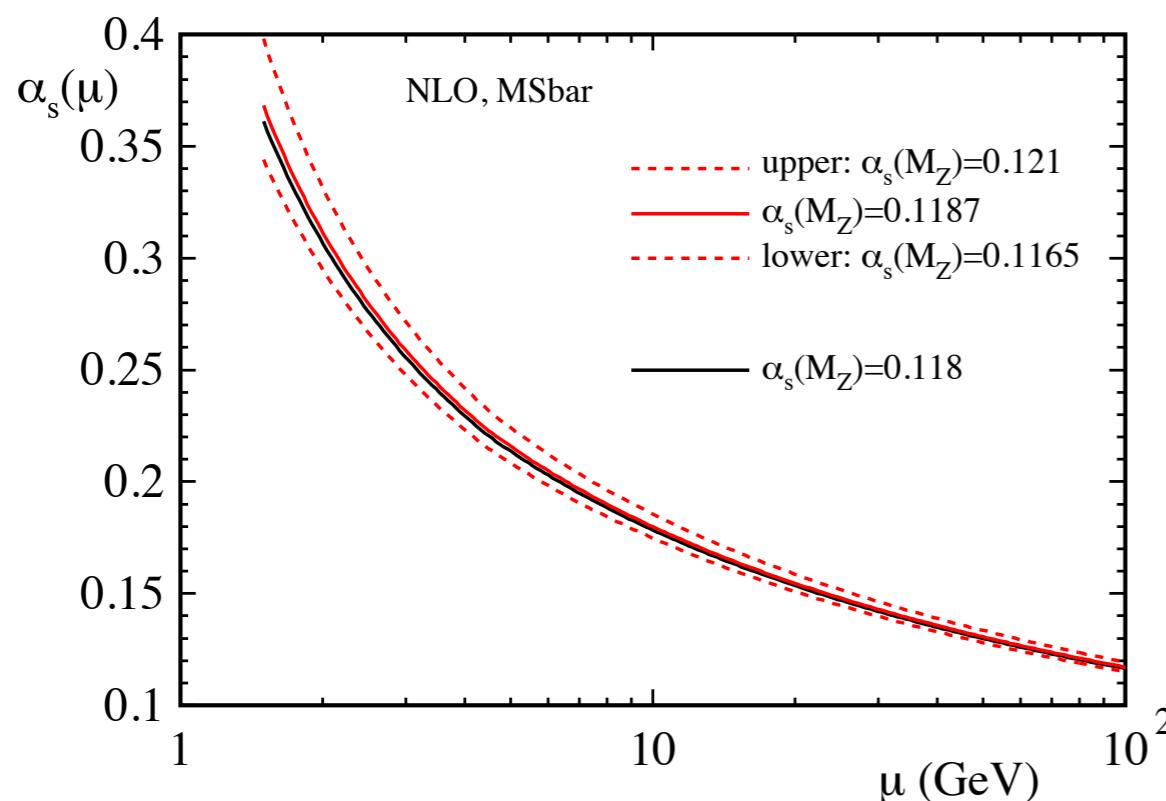
- **Asymptotic freedom:**

- Strong coupling small at short distances: **perturbation theory**
- Quarks and gluons behave as free particles at asymptotically large energies

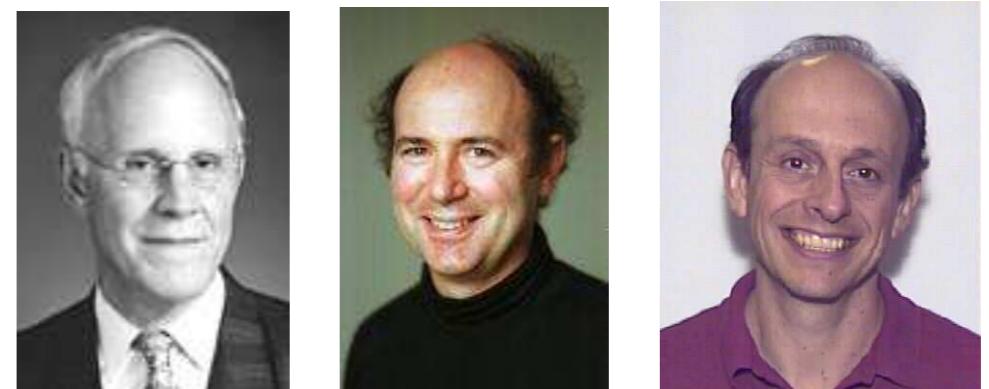
Asymptotic Freedom

Renormalization of UV-divergences:
Running coupling constant $a_s := \alpha_s/(4\pi)$

$$a_s(\mu) = \frac{1}{\beta_0 \ln(\mu^2/\Lambda^2)}$$



- Gross, Wilczek ('73); Politzer ('73)



Non-abelian gauge theories:
negative beta-functions

$$\frac{da_s}{d \ln \mu^2} = -\beta_0 a_s^2 + \dots$$

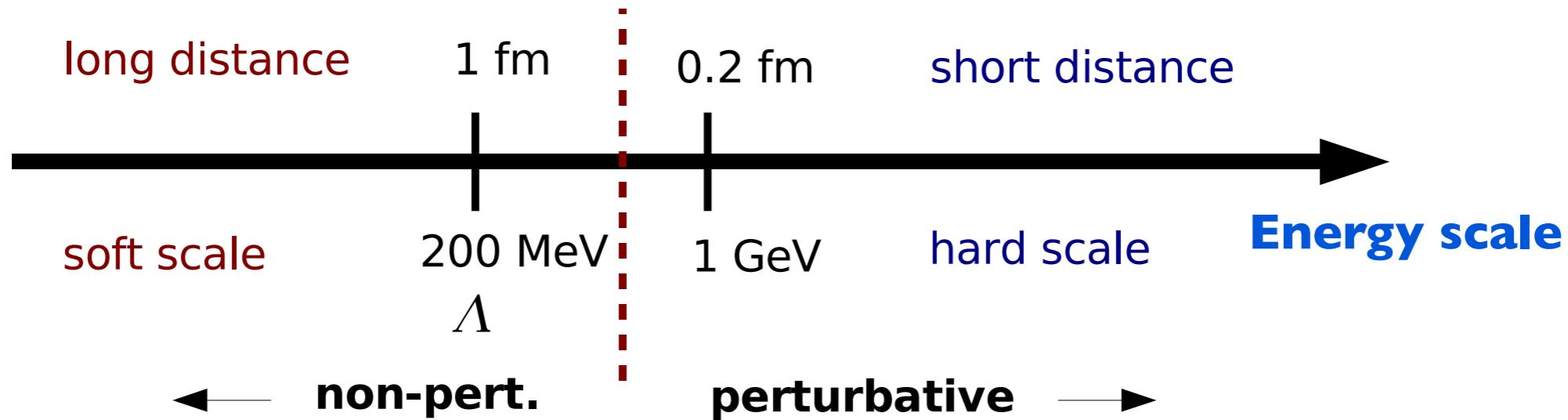
where $\beta_0 = \frac{11}{3} C_A - \frac{2}{3} n_f$

⇒ asympt. freedom: $a_s \searrow$ for $\mu \nearrow$

- Nobel Prize 2004

The perturbative QCD formalism

Two key ingredients:



Asympt. freedom \rightarrow pQCD possible if **all** scales hard

Factorisation \rightarrow

⁴⁶

Possible to **separate** hard and soft scales
soft part : **universal**
hard part : **perturbative**

The perturbative QCD formalism

QCD factorization theorems:

$$d\sigma = \text{PDF} \otimes d\hat{\sigma} + \text{remainder}$$

- PDF:
 - Proton composed of partons = quarks, gluons
 - Structure of proton described by parton distribution functions (PDF)
 - Factorization theorems provide field theoretic definition of PDFs
 - PDFs **universal** → **PREDICTIVE POWER**
- Hard part $d\hat{\sigma}$:
 - depends on the process
 - calculable order by order in **perturbation theory**
 - Factorization theorems prescribe how to calculate $d\hat{\sigma}$:
“ $d\hat{\sigma}$ = partonic cross section - mass factorization”
- Statement about error: remainder suppressed by hard scale

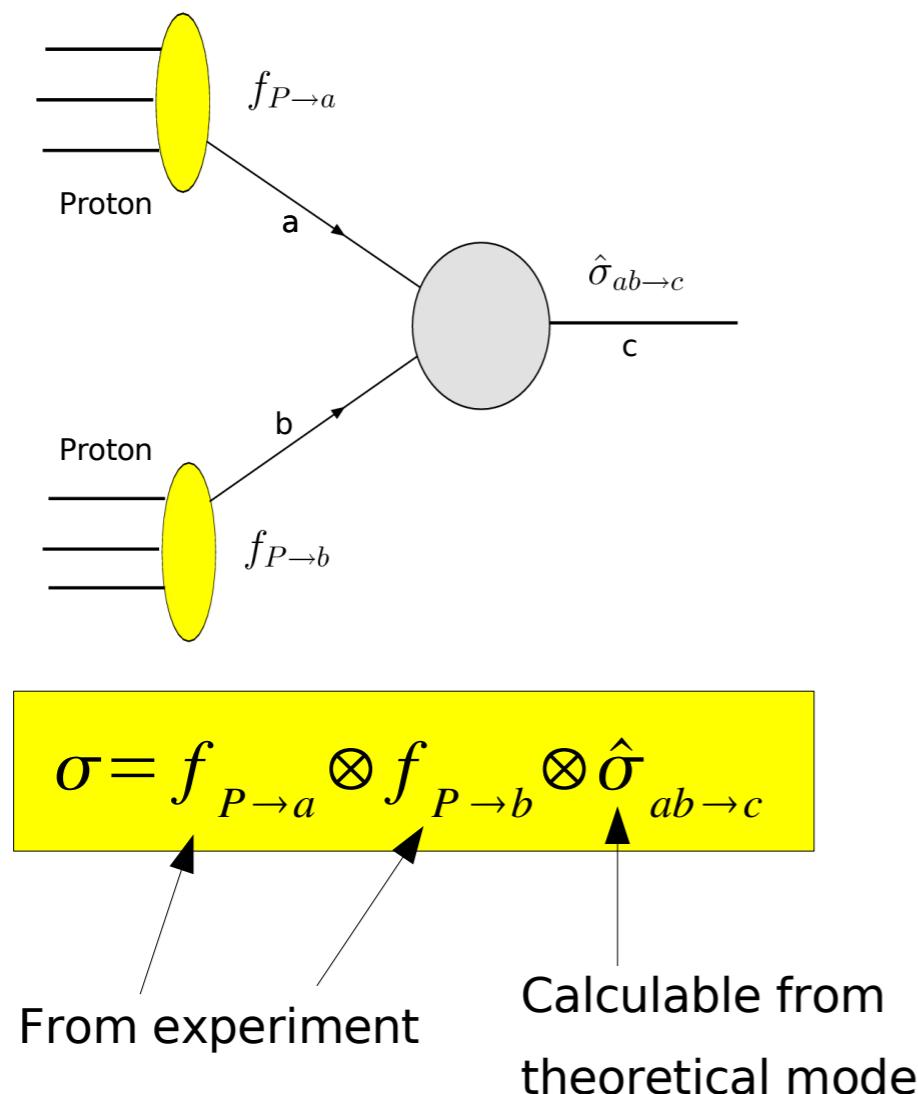
Original **factorization proofs** considered **massless** partons

Factorization theorems

Inclusive DIS:

$$W^{\mu\nu} = \sum_i \int_x^1 \frac{d\xi}{\xi} f_i(\xi, \mu_F^2, \mu_R^2) \hat{w}_i^{\mu\nu}(\xi, Q^2, \mu_F^2/Q^2, \mu_R^2/Q^2, \alpha_s(\mu_R^2)) + \mathcal{O}(Q^2/\Lambda^2)$$

In addition to inclusive DIS, there are factorization formulas for other processes



Parton Distribution Functions (PDFs)

$$f_{P \rightarrow a, b}(x, \mu^2)$$

- ★ Universal
- ★ Describe the structure of hadrons
- ★ The key to calculations involving hadrons in the initial state!!!

The hard part $\hat{\sigma}_{ab \rightarrow c}(\mu^2)$

- ★ Free of short distance scales
- ★ Calculable in perturbation theory
- ★ Depends on the process

Predictive Power

Universality: same PDFs/FFs enter different processes:

- DIS:

$$F_2^A(x, Q^2) = \sum_i [f_i^A \otimes C_{2,i}] (x, Q^2)$$

- DY:

$$\sigma_{A+B \rightarrow \ell^+ + \ell^- + X} = \sum_{i,j} f_i^A \otimes f_j^B \otimes \hat{\sigma}^{i+j \rightarrow \ell^+ + \ell^- + X}$$

- A+B -> H + X:

$$\sigma_{A+B \rightarrow H + X} = \sum_{i,j,k} f_i^A \otimes f_j^B \otimes \hat{\sigma}^{i+j \rightarrow k + X} \otimes D_k^H$$

- Predictions for unexplored kinematic regions
and for your favorite new physics process

How to compute the hard cross section?

LO:

$$d\hat{\sigma}^{(0)} = d\sigma^{(0)}$$

Remove long distance part
(soft scales)

NLO:

$$d\hat{\sigma}^{(1)} = d\sigma^{(1)} - \text{collinear subtractions}$$

Mass factorization
Factorization scheme dependent

Renormalized partonic cross section

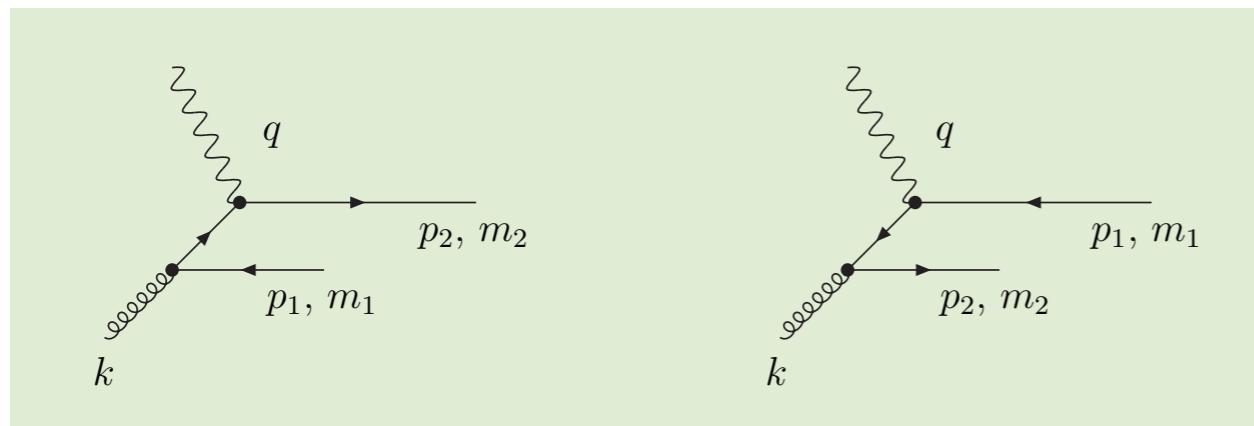
UV div.

(IR div. cancel between real and virtual diagrams)

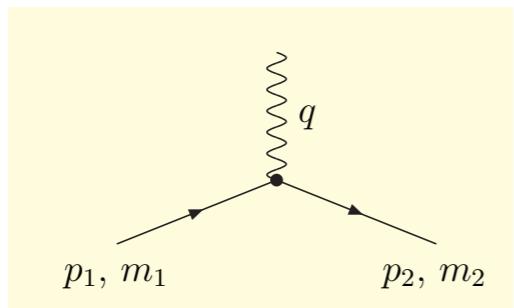
- The good news is at LO the **hard cross section is just the partonic cross section**
- At higher orders, **divergences** appear which have to be **regularized** and then be treated with procedures called “**renormalization**” and “**mass factorization**”

XI. NLO corrections to DIS

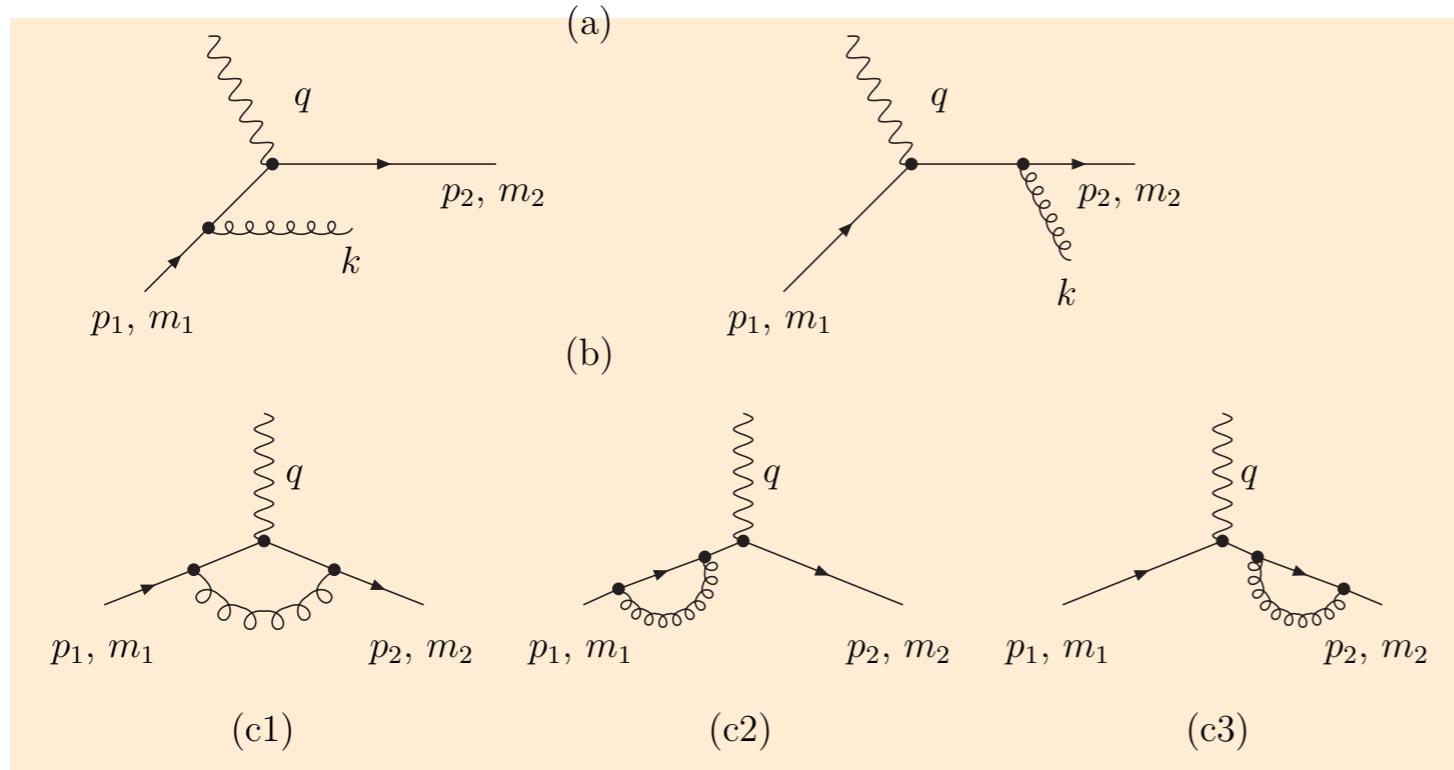
NLO corrections to DIS



Boson gluon fusion



LO: quark initiated



NLO: quark initiated

(b) NLO real (R)
(c) NLO virtual (V)

The anatomy of divergences

- There are **3 types of divergences** which appear (quite generally at higher orders) and which **need to be regularized** in order to have mathematically well defined objects
 - **UV** divergences (due to high energy modes in loop diagrams)
 - **Soft** divergences (due to the emission of soft/low energy gluons)
 - **Collinear** divergences (due to collinear parton branchings)
- Soft (S) and Collinear (C) divergences involve **propagators which are close to their mass shell**/which propagate long distances. They are therefore both **IR** divergences.
- The **Collinear** divergences are also called **Mass singularities** because they appear for massless particles. For massive particles the “singularities” are regularized/finite and give just very large contributions to the scattering amplitude for scales much larger than the mass

The anatomy of divergences

- There are **3 types of divergences** which appear (quite generally at higher orders) and which **need to be regularized** in order to have mathematically well defined objects
- **UV** divergences (due to high energy modes in loop diagrams)
 - Treated by the **renormalization procedure**
 - All parameters of the Lagrangian are **redefined in a renormalization scheme**, acquire a **renormalization scale dependence** (running couplings) and need to be ‘measured’ at one scale
 - We can obtain the running parameter at another scale with differential equations called **renormalization group equations** (RGEs)
- **Soft** divergences (due to the emission of soft/low energy gluons)
- **Collinear** divergences (due to collinear parton branchings)

The anatomy of divergences

- There are **3 types of divergences** which appear (quite generally at higher orders) and which **need to be regularized** in order to have mathematically well defined objects
 - **UV** divergences? **Renormalization!**
 - **Soft** divergences (due to the emission of soft/low energy gluons)
 - For ‘reasonable observables’ (IR safe), the soft divergences cancel in the sum of real and virtual contributions: **Reals + Virtuals = finite** (KLN theorem)
 - **Collinear** divergences (due to collinear parton branchings)

The anatomy of divergences

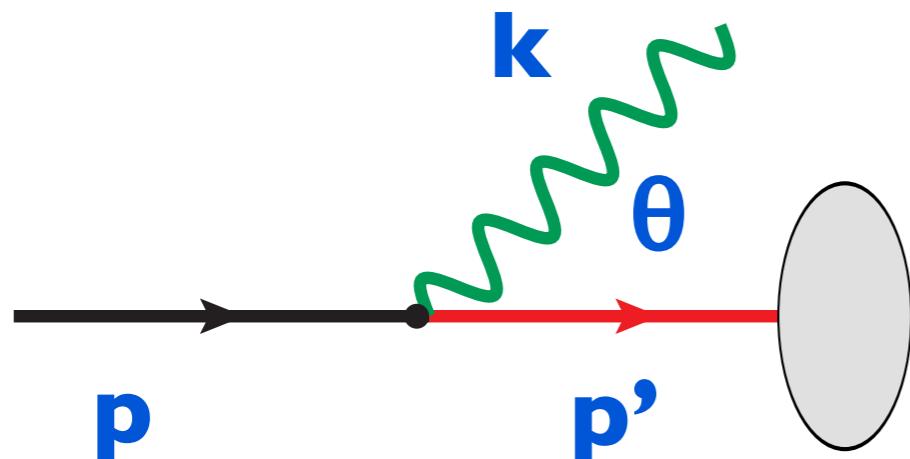
- There are **3 types of divergences** which appear (quite generally at higher orders) and which **need to be regularized** in order to have mathematically well defined objects
- **UV** divergences? **Renormalization!**
- **Soft** divergences? **Cancel!**
- **Collinear** divergences (due to collinear parton branchings)
 - Treated by the **mass factorization procedure**
(multiplicative mass factorization is very similar to multiplicative renormalization)
 - PDFs and Fragmentation functions are **redefined in a factorization scheme**,
acquire a **factorization scale dependence** and we need to ‘measure’ them at one scale
 - We can obtain the evolution from one scale to another with renormalization group equations known as **DGLAP** equations

The anatomy of divergences

- There are **3 types of divergences** which appear (quite generally at higher orders) and which **need to be regularized** in order to have mathematically well defined objects
 - **UV** divergences? **Renormalization!**
 - **Soft** divergences? **Cancel!**
 - **Collinear** divergences? **Mass factorization!**

The collinear divergences will be absorbed into a redefinition of the PDFs (or FFs).
The redefinition depends on the factorization scheme.

Where do IR divergences come from?



Neglecting the electron mass:

$$\mathbf{p} = (E, 0, 0, E)$$

$$\mathbf{k} = (\omega, \omega \sin \theta, 0, \omega \cos \theta)$$

$$\mathbf{p}' = \mathbf{p} - \mathbf{k}$$

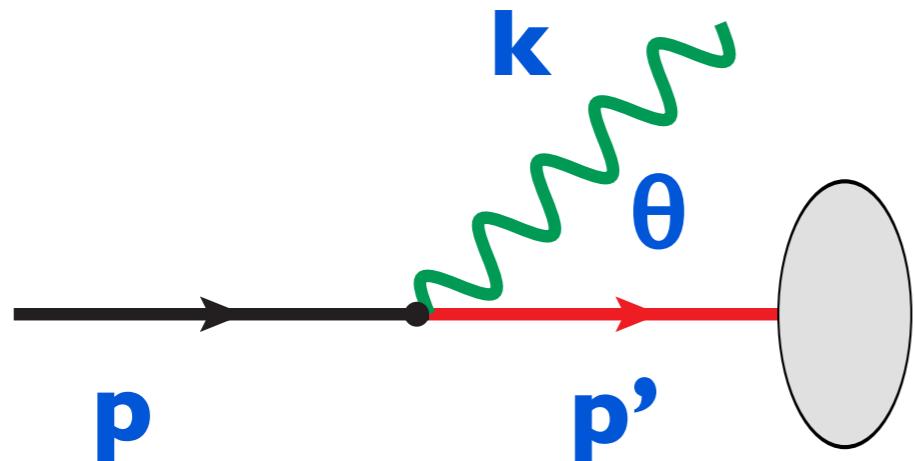
The red propagator:

$$\frac{i}{\not{p}'} = \frac{i\not{p}''}{\not{p}'^2} \sim \frac{1}{\not{p}'^2} = \frac{1}{\mathbf{p}^2 - 2\mathbf{p} \cdot \mathbf{k} + \mathbf{k}^2} = \frac{-1}{2\mathbf{p} \cdot \mathbf{k}} = \frac{-1}{E\omega(1 - \cos \theta)}$$

We see that the propagator is **divergent** for

- a) $\omega \rightarrow 0$ (soft photon emission)
- b) $\theta \rightarrow 0$ (collinear emission of photon)

Where do IR divergences come from?



Including the electron mass:

$$p = (E, 0, 0, |\vec{p}|) = (E, 0, 0, E\beta)$$

$$\text{with } \beta = \sqrt{1 - m^2/E^2}$$

$$\mathbf{k} = (\omega, \omega \sin \theta, 0, \omega \cos \theta)$$

$$\mathbf{p}' = \mathbf{p} - \mathbf{k}$$

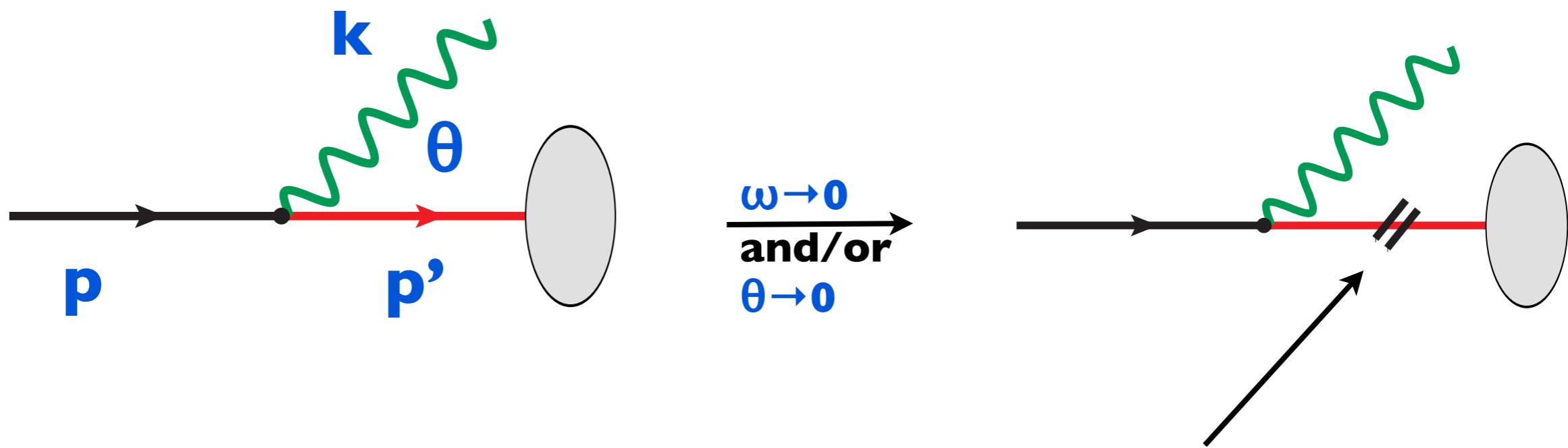
The red propagator:

$$\frac{i}{\not{p}'' - m} = \frac{i(\not{p}'' + m)}{\not{p}'^2 - m^2} \sim \frac{1}{\not{p}'^2 - m^2} = \frac{1}{\not{p}^2 - 2\mathbf{p} \cdot \mathbf{k} + \mathbf{k}^2 - m^2} = \frac{-1}{2\mathbf{p} \cdot \mathbf{k}} = \frac{-1}{E\omega(1 - \beta \cos \theta)}$$

We see that the propagator is **divergent/large** for

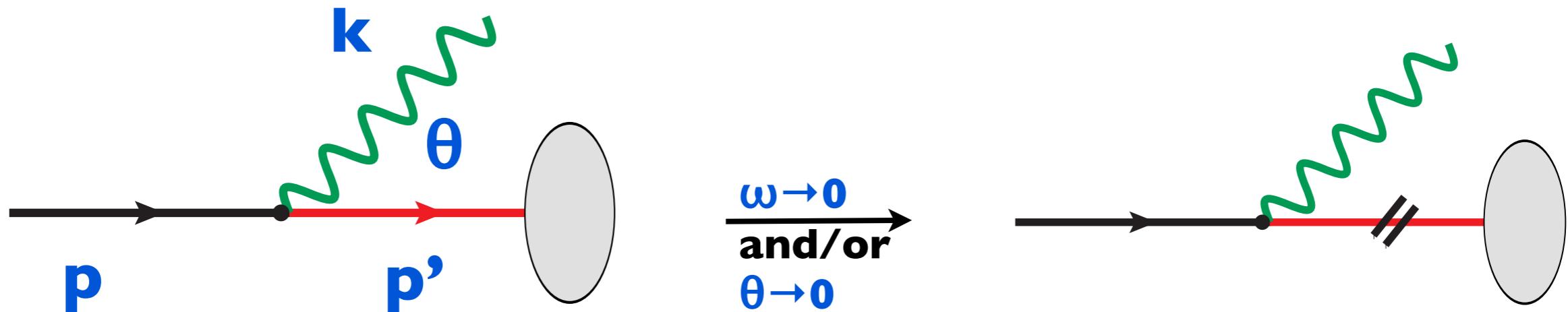
- a) $\omega \rightarrow 0$ (soft divergence)
- b) $\theta \rightarrow 0$ (collinear ‘divergence’ regulated by the mass)

Where do IR divergences come from?



The cut indicates that propagator
is on-shell, travels a long distance
before interaction with the blob

Where do IR divergences come from?

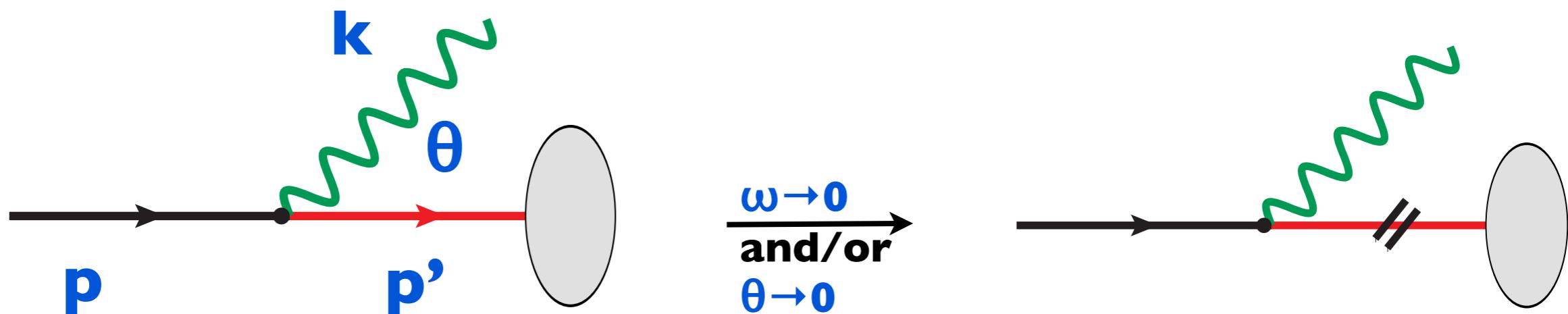


Discussion:

- In the soft/collinear regions the cross section factorizes into two independent pieces which are convoluted (the phase space factorizes as well):
 - a) the probability for emitting a soft/collinear photon
 - b) the cross section for the interaction of the on-shell red line with the blob
- The soft part cancels once virtual corrections are included'
- The dominant leading logarithmic contributions to the amplitude come from the collinear regions. The collinear emission probability can be interpreted as finding an electron with a momentum fraction z inside a parent electron:

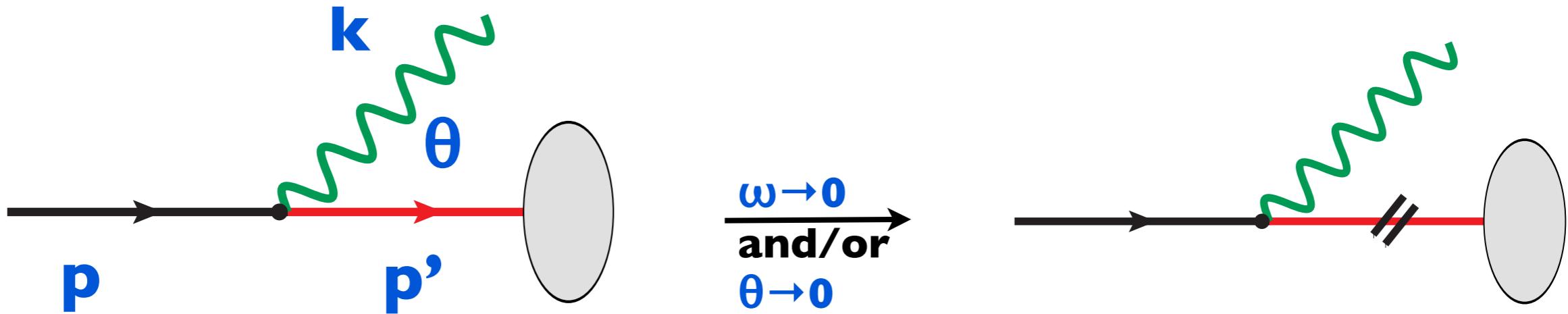
$$\Gamma_{ee}(z, \mu^2) = \frac{\alpha}{2\pi} P_{ee}^{(0)}(z) \ln \mu^2/m^2$$

Not yet ready with the discussion



- The collinear logarithms [$\alpha_P \ln(\mu^2/m^2)$] can in principle be kept in fixed order perturbation theory since the fine structure constant is small.
- Conversely, they can be **resummed to all orders** by introducing **parton distributions inside an electron** (with an electron parton and a photon parton inside the electron).
- The collinear terms can then be absorbed into the PDFs which can be evolved with (inhomogeneous) DGLAP equations and convoluted with ‘partonic processes’
- This “QED structure functions method” has been widely used at LEP and HERA to calculate the QED radiative corrections in the leading log approximation (which reproduces the exact result to 1% ... 2% accuracy)

...more to say...



- There is a fundamental difference between QED and QCD!
- The results for QED are reliable in fixed order perturbation theory or we can subtract the long distance terms and introduce QED structure functions as a technique to resum large collinear logs to all orders using the renormalization group
- On the other hand, a quark propagating a long distance will not be free/hadronize!

The results with an almost on-shell free quark propagator are certainly not reliable and we are FORCED to subtract these long distance pieces from the cross section (and to replace them by experimentally determined PDFs)

Mass factorization

Let's calculate a cross section/structure function in the QCD improved parton model at NLO!

It is given by a convolution of the partonic cross section (already renormalized) with PDFs:

$$\sigma = \sum_i \tilde{f}_i \otimes \tilde{\sigma}_i$$

↑
independent of $\mu=\mu_R$ ↑ depends on $\mu=\mu_R$

The partonic cross section still contains collinear singularities (poles in $1/\epsilon$ in dim. reg.).

The collinear pieces factorize into universal factorization constants and hard scattering cross sections. This is the crucial mass factorization relation.

$$\tilde{\sigma}_i = \sum_l \Gamma_{li} \otimes \hat{\sigma}_l$$

↑
factorization constants
containing the divergence ↑
finite (IR-safe) hard
scattering cross sections

Mass factorization

The factorization constants have to be determined once and for all. They can then be used in all kinds of processes to do the mass factorization (analogue: renormalization constants)

Using dim. reg. with
d=4+ ϵ dimensions

$$\Gamma_{ij}(z, \mu_F^2, \mu^2) = \delta_{ij} \delta(1-z) + \Gamma_{ij}^{(1)}(z, \mu_F^2, \mu^2)$$

$$\Gamma_{ij}^{(1)} = \frac{\alpha_s(\mu^2)}{2\pi} \left[P_{ij}(z) \frac{2}{\epsilon} + F_{ij} \right]$$

$\mu = \mu_R$ is the scale of dim. reg.
 μ_F is the factorization scale

$$F_{ij}^{\overline{\text{MS}}} = P_{ij}(z) \left(\gamma_E - \ln(4\pi) + \ln \frac{\mu_F^2}{\mu^2} \right)$$

We can now express the cross section as convolution of renormalized PDFs with hard scattering cross sections:

$$\sigma = \sum_i \sum_l \tilde{f}_i \otimes \Gamma_{li} \otimes \hat{\sigma}_i = \sum_l f_l \otimes \hat{\sigma}_l \quad \text{with} \quad f_l = \sum_i \Gamma_{li} \otimes \tilde{f}_i$$

The desired end result:
65

renormalized PDFs

Construction of the hard scattering cross sections

$$\tilde{\sigma}_i = \sum_l \Gamma_{li} \otimes \hat{\sigma}_l$$

The mass factorization relation can be inverted
in a **perturbative approach**

$$\Gamma_{ij} = \delta_{ij} + \Gamma_{ij}^{(1)} + \dots$$

$$\tilde{\sigma} = \tilde{\sigma}^{(0)} + \tilde{\sigma}^{(1)} + \dots$$

$$\hat{\sigma} = \hat{\sigma}^{(0)} + \hat{\sigma}^{(1)} + \dots$$

Entering the perturbative expressions into the mass factorization formula and comparing the lhs with the rhs we find iteratively (exercise!):

$$\hat{\sigma}_i^{(0)} = \tilde{\sigma}_i^{(0)}$$

$$\hat{\sigma}_i^{(1)} = \tilde{\sigma}_i^{(1)} - \Gamma_{li}^{(1)} \otimes \tilde{\sigma}_l^{(0)} = \tilde{\sigma}_i^{(1)} - \Gamma_{li}^{(1)} \otimes \hat{\sigma}_l^{(0)}$$

Wilson Coefficients

In DIS the hard scattering cross sections are called Wilson coefficients:

$$x^{-1}F_2 = q \otimes C_{2,q} + g \otimes C_{2,g}$$

$$x^{-1}F_L = q \otimes C_{L,q} + g \otimes C_{L,g}$$

$$F_1 = q \otimes C_{1,q} + g \otimes C_{1,g}$$

The Wilson coefficients have a perturbative expansion in the strong coupling constant:

$$C_{i,q}(z, Q^2/\mu^2) = C_{i,q}^{(0)}(z) + C_{i,q}^{(1)}(z, Q^2/\mu^2) + \dots$$

$$C_{i,g}(z, Q^2/\mu^2) = C_{i,g}^{(0)}(z) + C_{i,g}^{(1)}(z, Q^2/\mu^2) + \dots$$

We recover the leading order parton model results:

$$C_{2,q}^{(0)}(z, Q^2, Q^2/\mu^2) = e_q^2 \delta(1-z), C_{2,g}^{(0)} = 0, C_{L,q}^{(0)} = 0, C_{L,g}^{(0)} = 0$$

$$F_2(x, Q^2) = x \sum_q e_q^2 (q + \bar{q})(x, Q^2) = 2x F_1(x, Q^2), F_L(x, Q^2) = 0$$

Higher order Wilson coefficient functions

Reference	Boson	SFs	Order	Coefficients	Scheme	Comments	
BBDM'78 [18]	NC,CC \pm	F_2, F_L, F_3	α_S^1	C_2	$\overline{\text{MS}}$	$C_{3,+}^{(1)}(x) = C_{3,-}^{(1)}(x)$	NLO
AEM'78 [70]	NC,CC \pm	F_2	α_S^1	C_2	$\overline{\text{MS}}$	—	
FP'82 [62]	NC,CC \pm	F_2	α_S^1	C_2	$\overline{\text{MS}}$	—	
GMMPS'91 [71]	NC,CC $+$	F_L	α_S^2	$C_{L,q}(x), C_{L,g}(x)$	$\overline{\text{MS}}$	$C_{L,g}$ corrected in [72]	NNLO
NZ'91 [73]	NC,CC \pm	F_2	α_S^2	$C_{2,q}(x)$	$\overline{\text{MS}}$	first calc.	
ZN'91 [72]	NC,CC $+$	F_2, F_L	α_S^2	$C_{2,g}(x), C_{L,g}(x)$	$\overline{\text{MS}}$	first calc.	
ZN'92 [74]	NC,CC $+$	F_2	α_S^2	C_2	$\overline{\text{MS}}$	—	
NV'00 [75]	NC,CC $+$	F_2, F_L	α_S^2	C_2^{NS}	$\overline{\text{MS}}$	x -space param.	
NV'00 [76]	NC,CC $+$	F_2	α_S^2	C_2^S	$\overline{\text{MS}}$	x -space param.	
ZN'92 [77]	NC,CC $+$	F_3	α_S^2	$C_{3,-}^{(2)}(x)$	$\overline{\text{MS}}$	first calc.	
MV'00 [78]	NC,CC $+$	F_2, F_L, F_3	α_S^2	—	$\overline{\text{MS}}$	all N , confirms [72, 73, 77]	
MRV'08 [79]	CC $-$	F_2, F_L, F_3	α_S^2	$\delta C_{2,L,3}^{(2)}(x)$	$\overline{\text{MS}}$	x -space param., $\delta C_L^{(2)}$ new	
MVV'09 [80]	CC $+$	F_3	α_S^2	$C_{3,-}^{(2)}(x)$	$\overline{\text{MS}}$	x -space param.	
VVM'05 [81]	NC,CC $+$	F_2, F_L	α_S^3	C_2, C_L	$\overline{\text{MS}}$	x -space calc. and param.	N ³ LO
MVV'02 [82]	NC,CC $+$	F_2	α_S^3	C_2^{NS}	$\overline{\text{MS}}$	x -space param.	
MVV'05 [83]	NC,CC $+$	F_L	α_S^3	C_L^{NS}	$\overline{\text{MS}}$	x -space param.	
MR'07 [84]	CC $-$	F_2, F_L, F_3	α_S^3	—	$\overline{\text{MS}}$	N -space, fixed $N \leq 10$	
MRV'08 [79]	CC $-$	F_2, F_L, F_3	α_S^3	$\delta C_{2,L,3}^{(2)}(N)$	$\overline{\text{MS}}$	N -space, first 5 moments	
MVV'09 [80]	CC $+$	F_3	α_S^3	—	$\overline{\text{MS}}$	x -space calc.	

Table 2.1: Massless higher order Wilson coefficient functions in the literature. 'NC' corresponds to neutral current DIS with γ and Z exchange while 'CC \pm ' stands for charged current DIS with $W^+ \pm W^-$ exchange.

XII. Parton evolution

Scale dependence

$$W^{\mu\nu} = \sum_i \int_0^1 \frac{d\xi}{\xi} f_i(\xi, \mu_F^2, \mu_R^2) \hat{w}_i^{\mu\nu}(\xi, Q^2, \mu_F^2/Q^2, \mu_R^2/Q^2, \alpha_s(\mu_R^2)) + \mathcal{O}(Q^2/\Lambda^2)$$

- The PDFs depend on the **factorization scale μ_F** and the **renormalization scale μ_R** .
- Beyond LO the PDFs depend on the **factorization/renormalization scheme**.
- In **DIS** usually the scale choice $\mu_F=\mu_R=Q$ is made
- The **hard part** is also scheme dependent (beyond LO) and one has to use the same scheme as for the PDFs and alphas.
- **Physical observables do not depend on the scales and schemes** up to missing higher orders in the perturbation series.

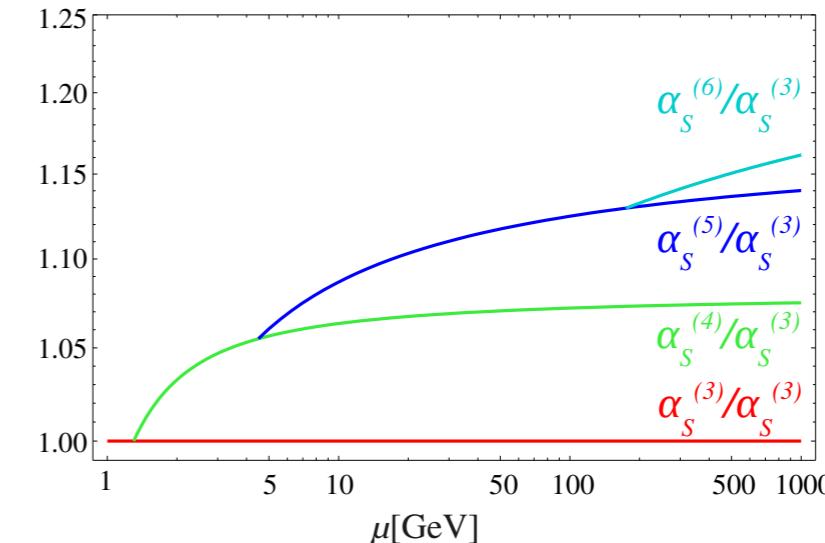
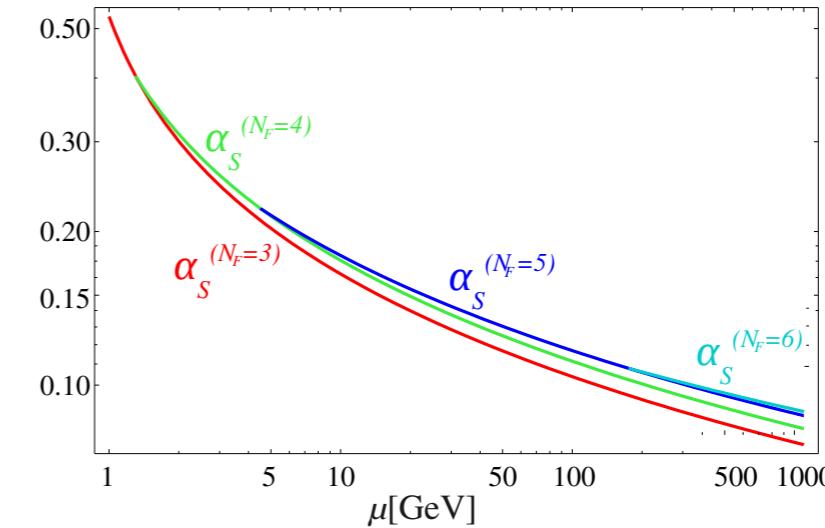
Scale dependence of a_s

The scale dependence of the strong coupling constant is also governed by a renormalization group equation:

$$\frac{\partial a_s}{\partial \ln \mu^2} = \beta[a_s(\mu^2)] = -(\beta_0 a_s^2 + \beta_1 a_s^3 + \beta_2 a_s^4 + \dots)$$

$$a_s := \frac{\alpha_s}{4\pi} \quad , \quad \beta_0 = 11 - \frac{2}{3}n_f \quad , \quad \beta_1 = 102 - \frac{38}{3}n_f \quad \dots$$

The QCD beta-function is negative at small a_s which leads to asymptotic freedom:



Scale dependence of PDFs

We had introduced renormalized dressed PDFs $\mathbf{f}_l(\mathbf{x}, \mu_F^2)$:

$$f_l(x, \mu_F^2) = \tilde{f}_i(y, \mu^2) \otimes \Gamma_{li}(z, \mu_F^2, \mu^2)$$

The dependence of $\mathbf{f}_l(\mathbf{x}, \mu_F^2)$ is necessitated by the fact that the physical cross section has to be independent of μ_F

depends on μ (see above)

$$\begin{aligned}\Gamma_{ij}(z, \mu_F^2, \mu^2) &= \delta_{ij} \delta(1-z) + \Gamma_{ij}^{(1)}(z, \mu_F^2, \mu^2) \\ \Gamma_{ij}^{(1)} &= \frac{\alpha_s(\mu^2)}{2\pi} \left[P_{ij}(z) \frac{2}{\epsilon} + F_{ij} \right] \\ F_{ij}^{\overline{\text{MS}}} &= P_{ij}(z) \left(\gamma_E - \ln(4\pi) + \ln \frac{\mu_F^2}{\mu^2} \right)\end{aligned}$$

We can then calculate the logarithmic derivative w.r.t. μ_F :

$$\begin{aligned}\frac{\partial f_i(x, \mu_F^2)}{\partial \ln \mu_F^2} &= \frac{\alpha_s(\mu^2)}{2\pi} P_{ij}(z) \otimes \tilde{f}_j(y, \mu^2) \\ &\stackrel{\mathcal{O}(\alpha_s)}{=} \frac{\alpha_s(\mu_F^2)}{2\pi} P_{ij}(z) \otimes f_j(y, \mu_F^2)\end{aligned}$$

Scale dependence

The scale dependence of the PDFs is governed by a coupled system of **integro-differential equations**, the **DGLAP** (Dokshitzer, Gribov, Lipatov, Altarelli, Parisi) **evolution equations**:

$$\frac{\partial f_i}{\partial \ln \mu^2} = P_{ij}(x, \mu^2) \otimes f_j(x, \mu^2)$$

Here, a summation over all parton species '**j**' is understood. **Pij** are the **QCD splitting functions** which are known perturbatively up to NNLO (3-loop) and \otimes stands for the **Mellin convolution** of two functions/distributions with support in [0,1]:

$$\begin{aligned} (f \otimes g)(x) &= \int_0^1 dy \int_0^1 dz \delta(x - yz) f(y) g(z) \\ &= \int_0^1 \frac{dy}{y} \theta(0 \leq x/y \leq 1) f(y) g(x/y) \\ &= \int_x^1 \frac{dy}{y} f(y) g(x/y) = \int_x^1 \frac{dy}{y} g(y) f(x/y) \end{aligned}$$

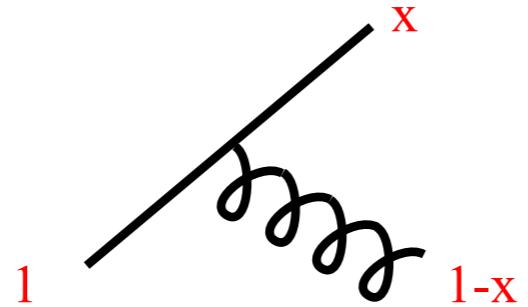
The leading order Splitting Functions

Group theory SU(3): $T_F = 1/2$, $C_F = 4/3$, $C_A = 3$

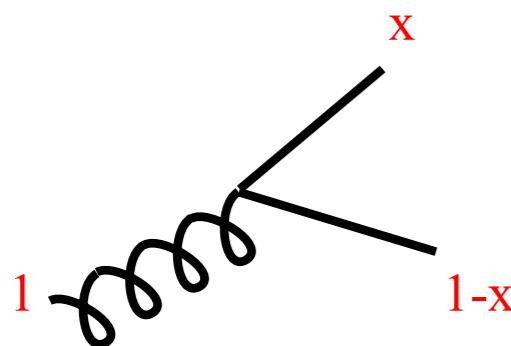
$$P_{ij} = P_{ij}^{(1)} + \frac{\alpha_s}{2\pi} P_{ij}^{(2)} + \dots$$

Read backwards

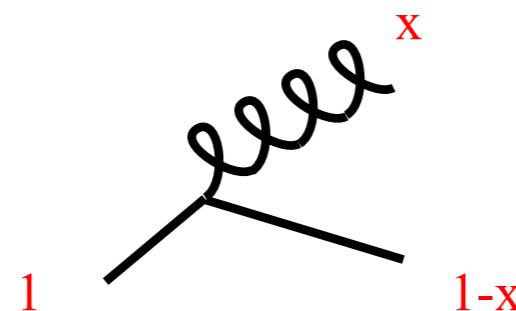
Note singularities



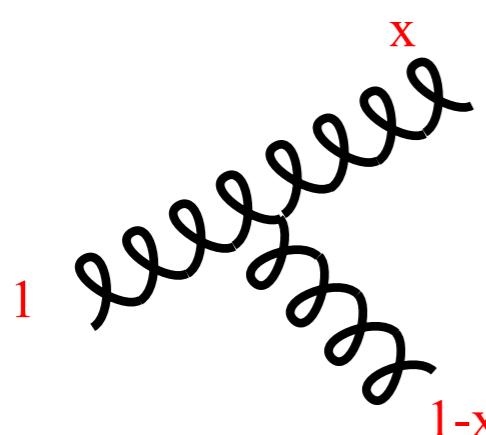
$$P_{qq}^{(1)}(x) = C_F \left[\frac{1+x^2}{1-x} \right]_+$$



$$P_{qg}^{(1)}(x) = T_F [(1-x)^2 + x^2]$$



$$P_{gq}^{(1)}(x) = C_F \left[\frac{(1-x)^2 + 1}{x} \right]$$



$$\begin{aligned} P_{gg}^{(1)}(x) = & 2C_F \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] \\ & + \left[\frac{11}{6}C_A - \frac{2}{3}T_F N_F \right] \delta(1-x) \end{aligned}$$

Homework

Definition of the Plus prescription:

Definition using a test function $f(x)$

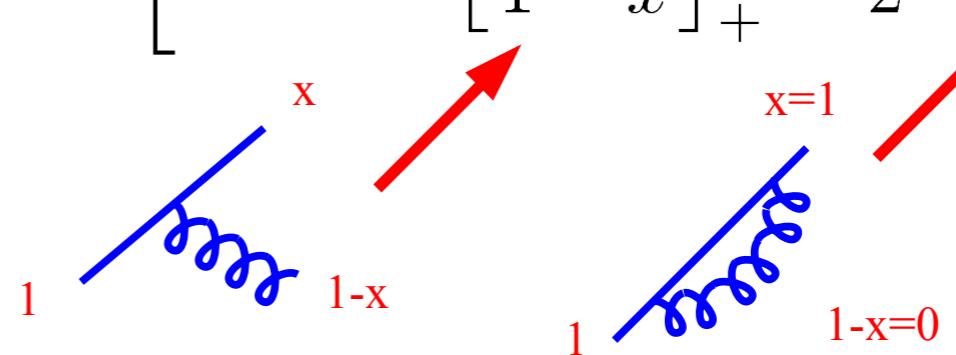
$$\int_0^1 dx \frac{f(x)}{(1-x)_+} = \int_0^1 dx \frac{f(x) - f(1)}{(1-x)}$$

Compute:

$$\int_a^1 dx \frac{f(x)}{(1-x)_+} = ???$$

Verify:

$$P_{qq}^{(1)}(x) = C_F \left[\frac{1+x^2}{1-x} \right]_+ \equiv C_F \left[(1+x^2) \left[\frac{1}{1-x} \right]_+ + \frac{3}{2} \delta(1-x) \right]$$



Homework

Appendix E in R. D. Field, Applications of Perturbative QCD

Definition as a limiting procedure:

$$[F(x)]_+ := \lim_{\beta \rightarrow 0} \left(F(x) \Theta(1 - x - \beta) - \delta(1 - x - \beta) \int_0^{1-\beta} F(y) dy \right)$$

Hard Virtual:

↓ ↓

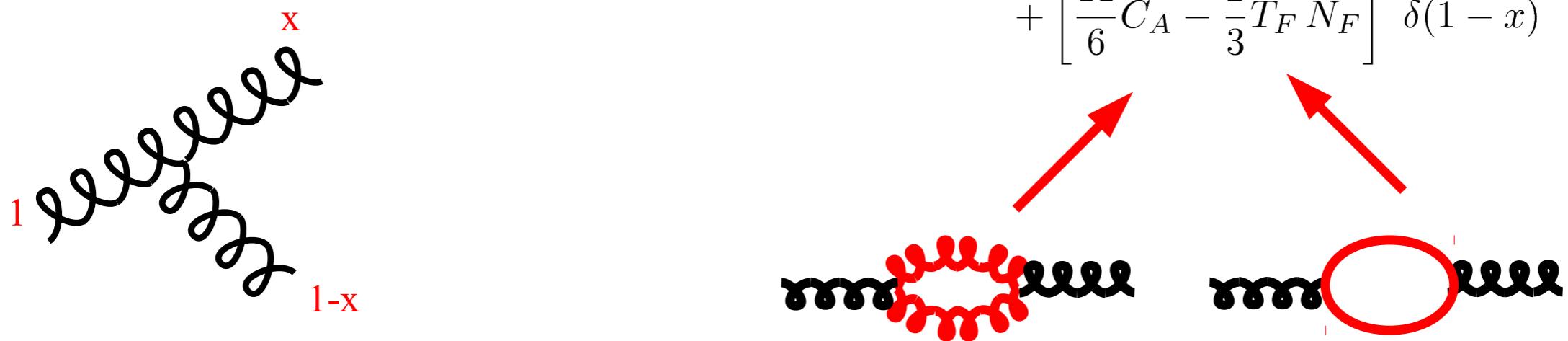
Verify:

$$\frac{1}{(1-x)_+} = \lim_{\beta \rightarrow 0} \left(\frac{1}{1-x} \Theta(1 - x - \beta) + \ln(\beta) \delta(1 - x - \beta) \right)$$

Homework

Observe

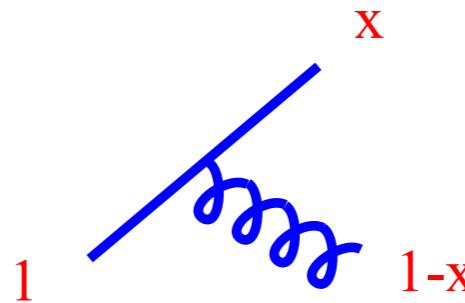
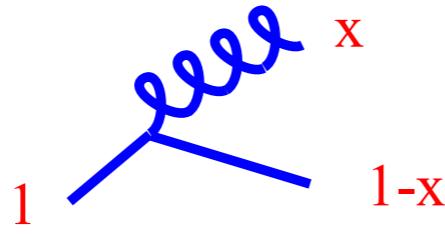
$$P_{gg}^{(1)}(x) = 2C_F \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \left[\frac{11}{6}C_A - \frac{2}{3}T_F N_F \right] \delta(1-x)$$



Homework: Symmetries and Limits

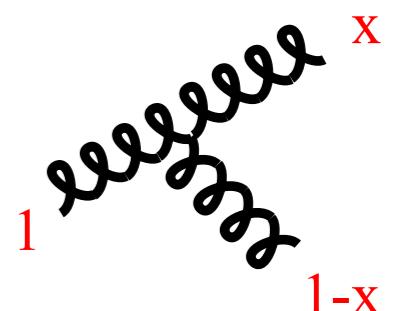
Verify the following relation among the regular parts (from the real graphs)

For the regular part
show:

$$P_{gq}^{(1)}(x) = P_{qq}^{(1)}(1-x)$$


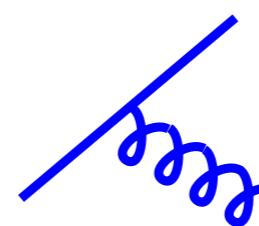
For the regular part
show:

$$P_{gg}^{(1)}(x) = P_{gg}^{(1)}(1-x)$$

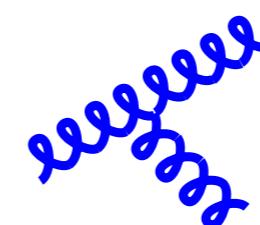


Verify, in the soft limit:

$$P_{qq}^{(1)}(x) \xrightarrow{x \rightarrow 1} 2C_F \frac{1}{(1-x)_+}$$



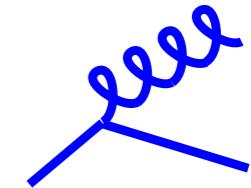
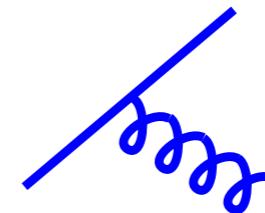
$$P_{gg}^{(1)}(x) \xrightarrow{x \rightarrow 1} 2C_F \frac{1}{(1-x)_+}$$



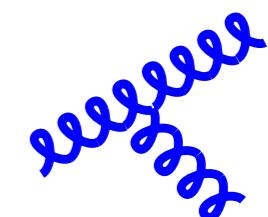
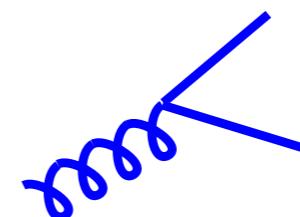
Homework: Conservation rules

Verify conservation of momentum fraction

$$\int_0^1 dx x [P_{qq}(x) + P_{gq}(x)] = 0$$



$$\int_0^1 dx x [P_{qg}(x) + P_{gg}(x)] = 0$$



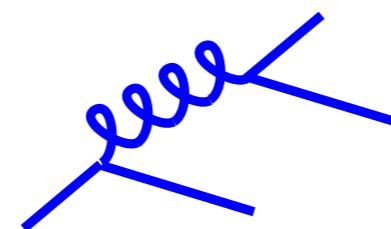
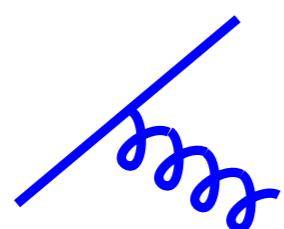
Verify conservation of fermion number

$$\int_0^1 dx [P_{qq}(x) - P_{q\bar{q}}(x)] = 0$$

Homework: Using the real to guess the virtual

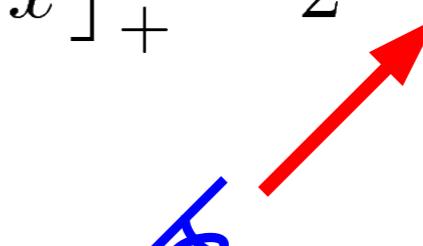
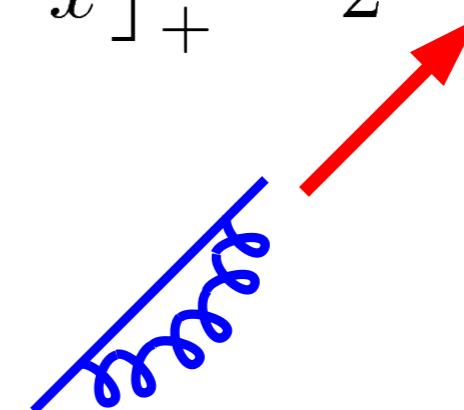
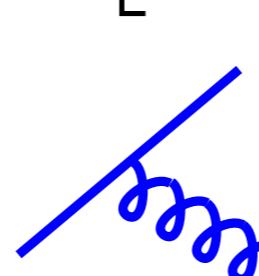
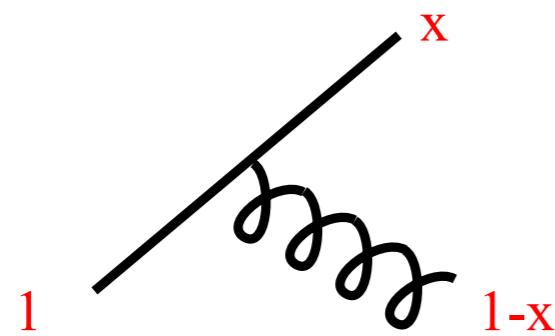
Use conservation of fermion number to compute
the delta function term in $P(q \leftarrow q)$

$$\int_0^1 dx [P_{qq}(x) - P_{q\bar{q}}(x)] = 0$$



*This term only
starts at NNLO*

$$P_{qq}^{(1)}(x) = C_F \left[\frac{1+x^2}{1-x} \right]_+ \equiv C_F \left[(1+x^2) \left[\frac{1}{1-x} \right]_+ + \frac{3}{2} \delta(1-x) \right]$$

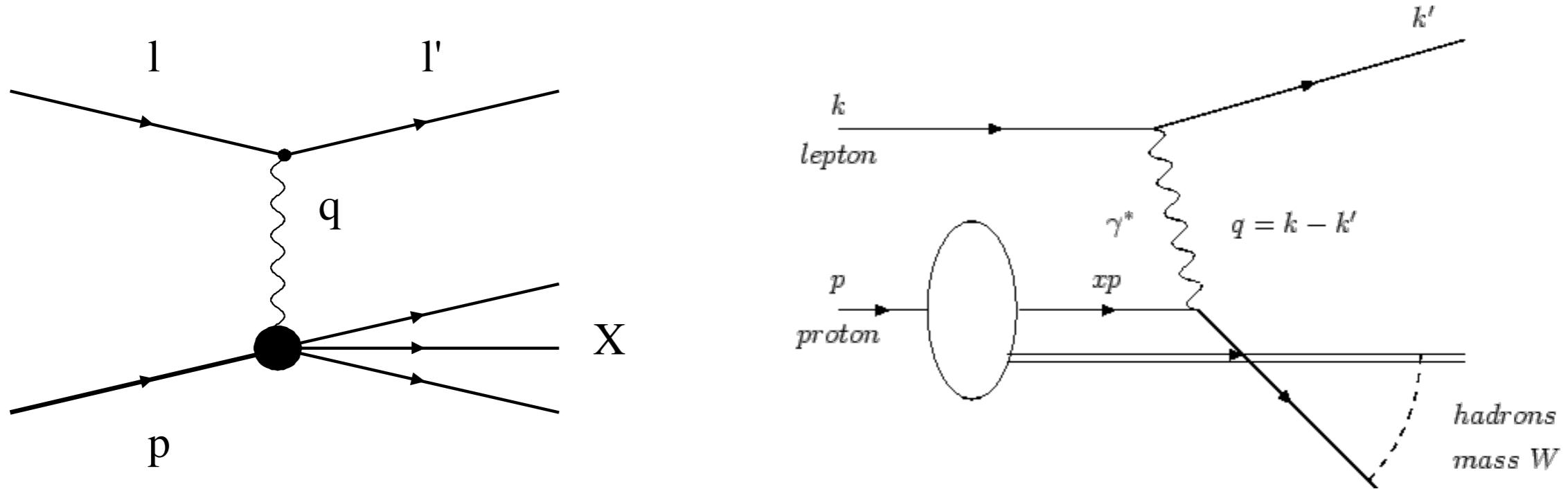


Powerful tool: Since we know real and virtual must balance, we can use to our advantage!!!

The End of Lecture 2

XIII. Quark masses in the parton model

Massless Parton Model



- In factorization ansatz: relate 4-momenta of partons to 4-momenta of hadrons

$$d\sigma[P] = \int_0^1 dx f(x) d\hat{\sigma}[\hat{p} = xP] \quad \text{with} \quad P^2 = \hat{p}^2 = 0$$

- In dynamics: massless parton propagators
- In kinematics: massless partons in phase space
- At higher orders: calculate in n dimensions, renormalization and mass factorization in $\overline{\text{MS}}$
- However, not fully massless (would be unphysical!)

Example: No contribution from $\gamma^* c \rightarrow c$ to DIS structure functions at scale $Q < m_c$ (overestimation).

LO parton kinematics

Proton momentum:

$$P_\mu = (P_0, P_x, P_y, P_z)$$

Light cone coordinates:

$$P_\mu = (P^+, P^-, \vec{P}_T)$$

$$P^\pm = (P_0 \pm P_z)/\sqrt{2}$$

Useful if strongly ordered:

$$P_\mu \simeq (P^+, M^2/(2P^+), \vec{0}_T)$$

$$P^+ \gg P^-, \vec{P}_T$$

Parton momentum:

$$\hat{p}_\mu \simeq (\xi P^+, m^2/(2\xi P^+), \vec{0}_T)$$

LO parton kinematics

Momentum conservation:

$$\delta^{(4)}(q + \hat{p}_i - \hat{p}_j) \rightarrow \delta(\xi - \chi)$$

$$M^2 = m^2 = 0 \Rightarrow \chi = x$$

$$M^2 \neq 0, m^2 = 0 \Rightarrow \chi = xR_M$$

$$M^2 \neq 0, m^2 \neq 0 \Rightarrow \chi = xR_M R_{ij}$$

Nachtmann variable

Crucial observation:
target mass and quark
mass effects factorize!

where

$$\begin{aligned} R_M &= \frac{2}{1 + \sqrt{1 + (4x^2 M^2 / Q^2)}} = \frac{2}{1 + r} , \\ R_{ij} &= \frac{Q^2 - m_i^2 + m_j^2 + \Delta(-Q^2, m_i^2, m_j^2)}{2Q^2} , \end{aligned}$$

$$\Delta(a, b, c) = \sqrt{a^2 + b^2 + c^2 - 2(ab + bc + ca)} ,$$

ACOT scheme

Theoretical basis: Factorization theorem with massive quarks

J. Collins '98

$$d\sigma[P] = \sum_i \int_0^1 d\xi f_i(\xi, \mu_F^2) d\hat{\sigma}(\gamma^* i \rightarrow cX)[\mu_R^2, \mu_F^2/Q^2, m^2/Q^2]|_{\hat{p}^+ = \xi P^+} + \mathcal{O}(\Lambda^2/Q^2)$$

Sum over all possible
subprocesses

Mass term contained in the
hard scattering coefficient

massive hard scattering
cross section; IR safe

massive partonic cross section;
not yet IR safe

collinear subtraction
term, mass fact. with
massive regulator

$$d\hat{\sigma}[m] = d\tilde{\sigma}[m] - d\sigma^{\text{sub}}$$

XIV.Target Mass Corrections

Operator product expansion

$$\begin{aligned}
 & \int d^4x \ e^{iq \cdot x} \langle N | T(J^\mu(x) J^\nu(0)) | N \rangle \\
 = & \sum_k \left(-g^{\mu\nu} q^{\mu_1} q^{\mu_2} + g^{\mu\mu_1} q^\nu q^{\mu_2} + q^\mu q^{\mu_1} g^{\nu\mu_2} + g^{\mu\mu_1} g^{\nu\mu_2} Q^2 \right) \\
 & \times q^{\mu_3} \cdots q^{\mu_{2k}} \frac{2^{2k}}{Q^{4k}} A_{2k} \Pi_{\mu_1 \cdots \mu_{2k}} \\
 & \quad \text{local operators} \\
 & \quad \swarrow \qquad \qquad \qquad \downarrow \\
 & \langle N | \mathcal{O}_{\mu_1 \cdots \mu_{2k}} | N \rangle
 \end{aligned}$$

$$\Pi_{\mu_1 \cdots \mu_{2k}} = p_{\mu_1} \cdots p_{\mu_{2k}} - (g_{\mu_i \mu_j} \text{ terms})$$

$$= \sum_{j=0}^k (-1)^j \frac{(2k-j)!}{2^j (2k)^j} g \cdots g \ p \cdots p$$

Georgi, Politzer (1976)

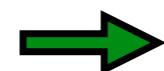
traceless, symmetric
rank- $2k$ tensor

■ **n -th Cornwall-Norton moment of F_2 structure function**

$$M_2^n(Q^2) = \int dx \ x^{n-2} \ F_2(x, Q^2)$$

$$= \sum_{j=0}^{\infty} \left(\frac{M^2}{Q^2} \right)^j \frac{(n+j)!}{j!(n-2)!} \frac{A_{n+2j}}{(n+2j)(n+2j-1)}$$

“quark distribution function”



$$A_n = \int_0^1 dy \ y^n \ F(y)$$

$F(y) \equiv \frac{F_2(y)}{y^2}$

- take inverse Mellin transform (+ tedious manipulations)
- target mass corrected structure function

$$\begin{aligned}
 F_2^{\text{GP}}(x, Q^2) = & \frac{x^2}{r^3} F(\xi) + 6 \frac{M^2}{Q^2} \frac{x^3}{r^4} \int_{\xi}^1 d\xi' F(\xi') \\
 & + 12 \frac{M^4}{Q^4} \frac{x^4}{r^5} \int_{\xi}^1 d\xi' \int_{\xi'}^1 d\xi'' F(\xi'')
 \end{aligned}$$

Nachtmann variable

$$\eta = \xi = \frac{2x}{1+r} \quad r = \sqrt{1 + 4x^2 M^2 / Q^2}$$

... similarly for other structure functions F_1, F_L

TMC: Master formula

$$\begin{aligned} F_1^{\text{TMC}}(x, Q^2) &= \frac{x}{\eta r} F_1^{(0)}(\eta, Q^2) + \frac{M^2 x^2}{Q^2 r^2} h_2(\eta, Q^2) + \frac{2M^4 x^3}{Q^4 r^3} g_2(\eta, Q^2) , \\ F_2^{\text{TMC}}(x, Q^2) &= \frac{x^2}{\eta^2 r^3} F_2^{(0)}(\eta, Q^2) + \frac{6M^2 x^3}{Q^2 r^4} h_2(\eta, Q^2) + \frac{12M^4 x^4}{Q^4 r^5} g_2(\eta, Q^2) , \\ F_3^{\text{TMC}}(x, Q^2) &= \frac{x}{\eta r^2} F_3^{(0)}(\eta, Q^2) + \frac{2M^2 x^2}{Q^2 r^3} h_3(\eta, Q^2) + 0 , \end{aligned}$$

- Modular, easy to use!
- Resums leading twist TMC to all orders in $(M^2/Q^2)^n$
- Input: standard structure functions in the parton model with $M=0$
 - any order in α_s
 - can include quark masses

TMC important at
large x and small Q^2

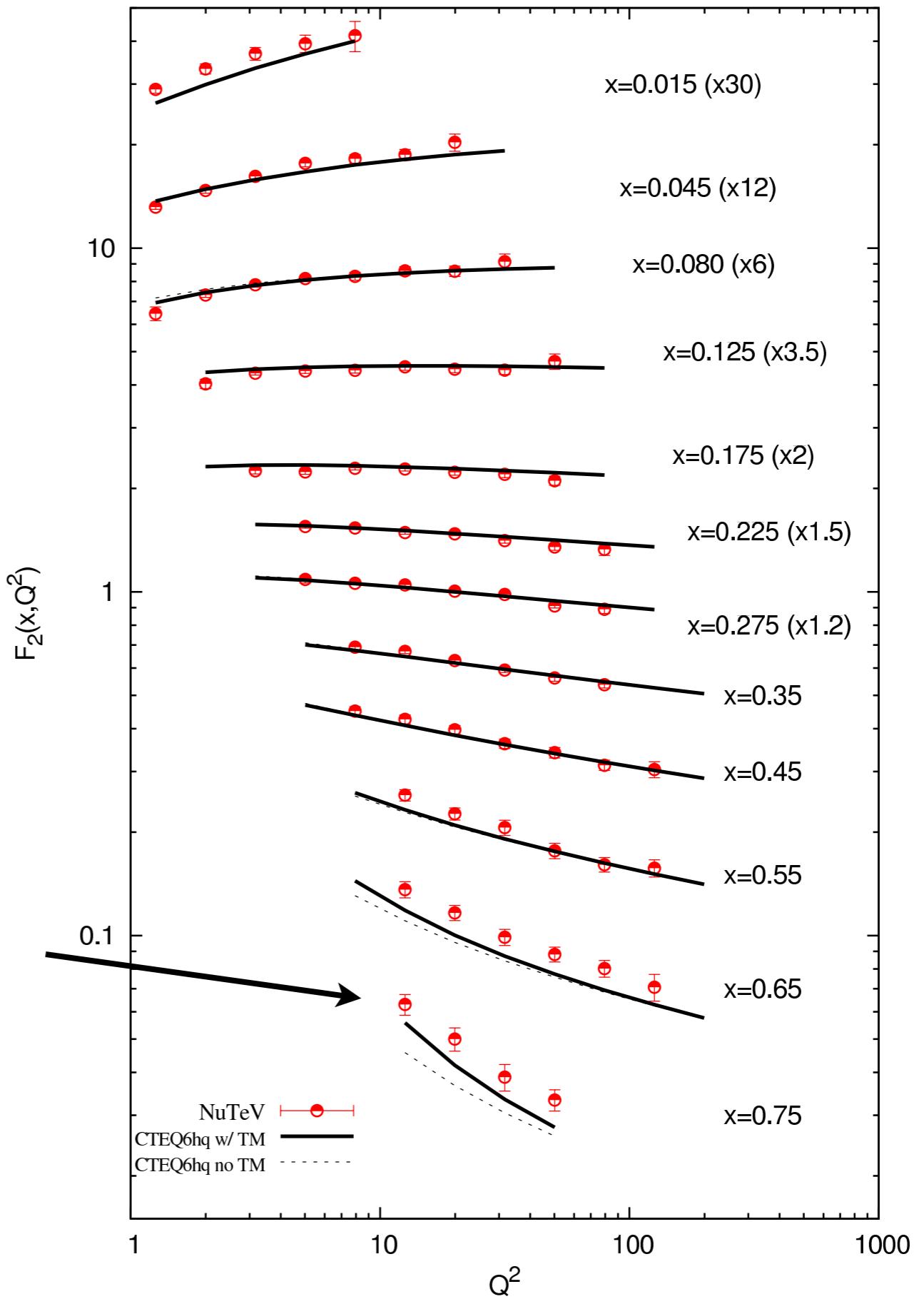


Figure 9. Comparison of the F_2 structure function, with and without target mass corrections, and NuTeV data [64]. The base PDF set is CTEQ6HQ [7].

XVI. QCD studies with neutrinos

Flavor separation of PDFs

NC charged lepton DIS: 2 structure functions (γ -exchange)

$$F_2^\gamma(x) \sim \frac{1}{9}[4(u + \bar{u} + c + \bar{c}) + d + \bar{d} + s + \bar{s}](x)$$

$$F_2^\gamma(x) = 2xF_1^\gamma(x)$$

CC Neutrino DIS: 6 additional structure functions $F_{1,2,3}^{W+}, F_{1,2,3}^{W-}$

$$F_2^{W^+} \sim [d + s + \bar{u} + \bar{c}]$$

$$F_3^{W^+} \sim 2[d + s - \bar{u} - \bar{c}]$$

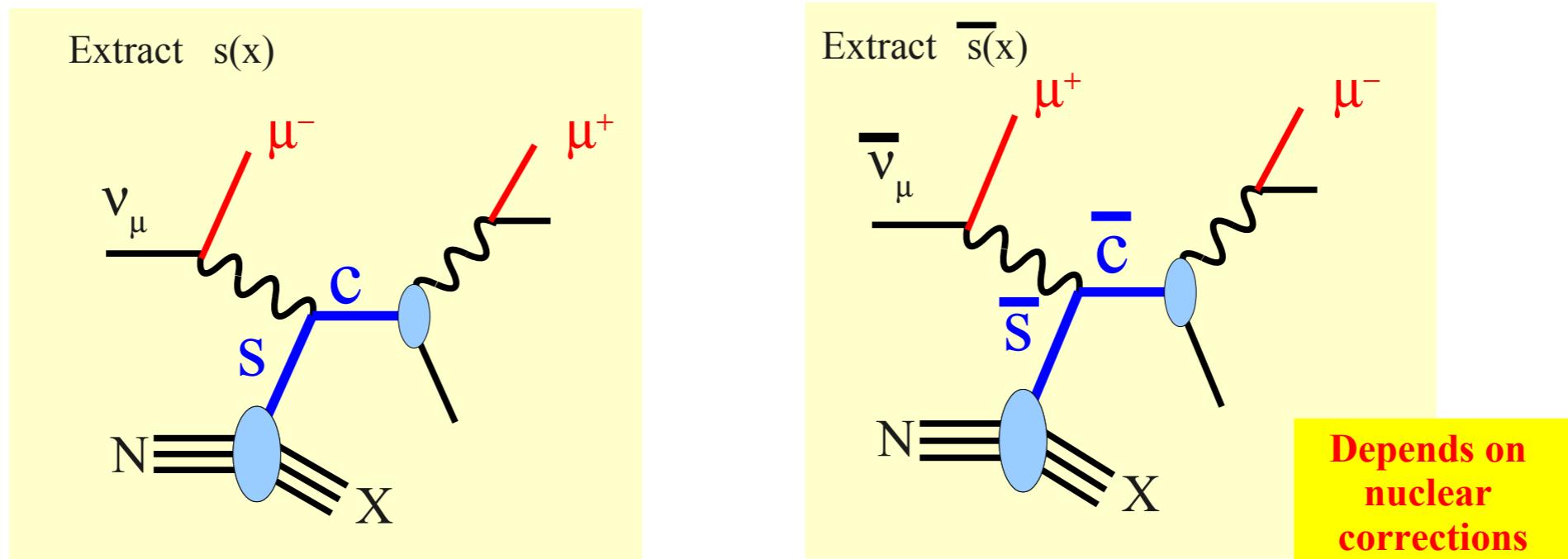
$$F_2^{W^-} \sim [\bar{d} + \bar{s} + u + c]$$

$$F_3^{W^-} \sim 2[u + c - \bar{d} - \bar{s}]$$

Useful/needed to disentangle different quark parton flavors
in a global analysis of proton or nuclear PDFs

Dimuon production and the strange PDF

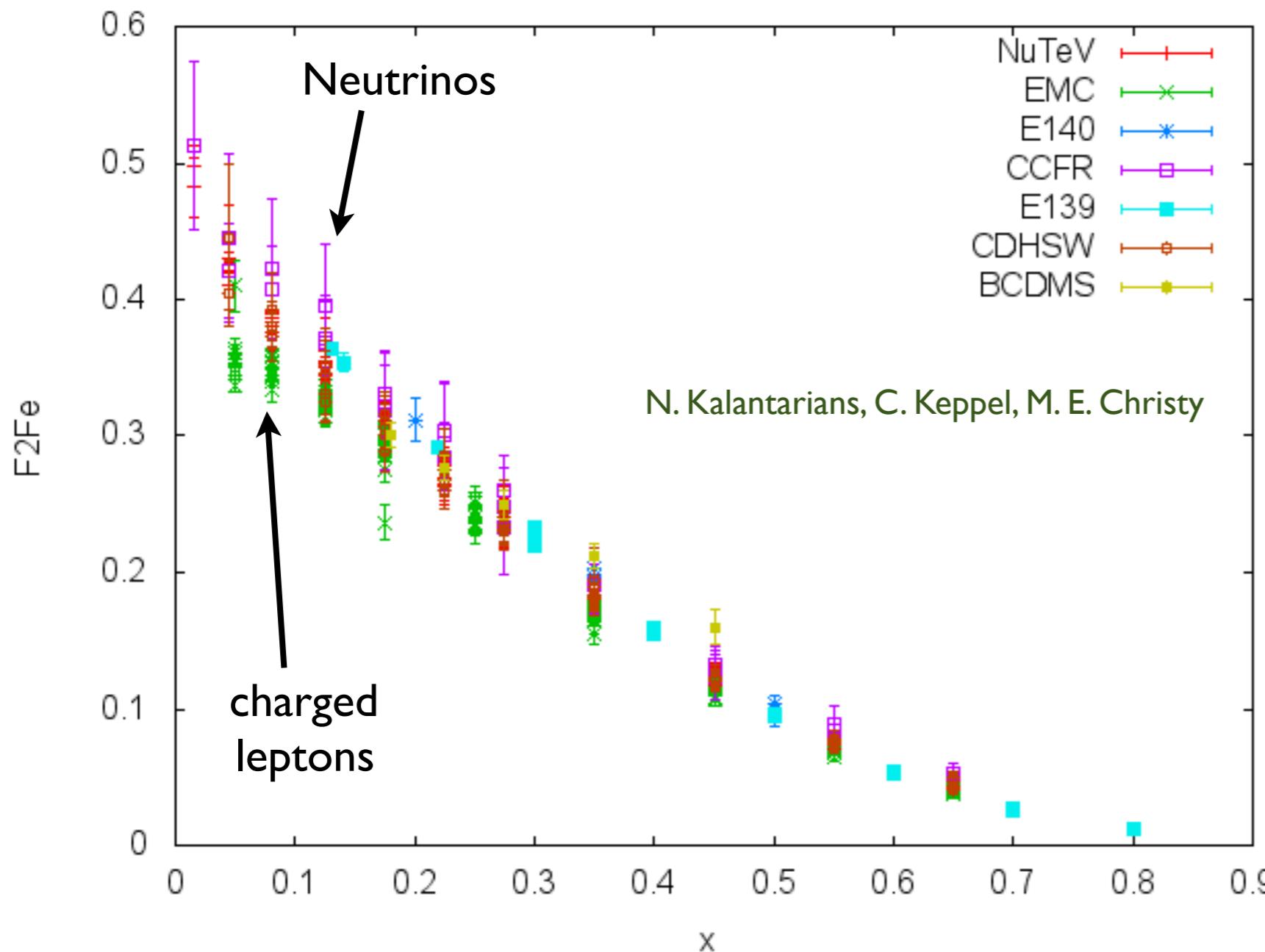
Opposite sign dimuon production in neutrino DIS: $\nu N \rightarrow \mu^+ \mu^- X$



- High-statistics data from CCFR and NuTeV: **Main source** of information!
- $x \sim [0.01, 0.4]$
- νFe DIS: need **nuclear corrections!** Problem: Final State Interactions (FSI)
- CHORUS (νPb): compatible with NuTeV, could be included
- NOMAD (νFe): data not yet published, in principle very interesting

World data on $18/5 F_2^{NC}$ and F_2^{CC} on iron

$$\Delta F_2 = \frac{5}{18} F_2^{CC} - F_2^{NC} \simeq \frac{x}{6} [s(x) + \bar{s}(x)]$$



Data available at Durham database;

Data brought to the same $Q^2=8 \text{ GeV}^2$

Info on nuclear corrections in $\nu\text{-Fe DIS}$ vs l-Fe DIS :
Advantage: no deuterium

Info on strange PDF in iron:

Advantage: inclusive, no FSI

Disadvantage: difference of two large numbers

xF_3 and Isospin Violation

- xF_3 uniquely determined by neutrino-DIS

$$\begin{aligned}\frac{1}{2}F_3^{\nu A}(x) &= d_A + s_A - \bar{u}_A - \bar{c}_A + \dots, \\ \frac{1}{2}F_3^{\bar{\nu} A}(x) &= u_A + c_A - \bar{d}_A - \bar{s}_A + \dots\end{aligned}$$

- The sum is sensitive to the valence quarks

→ Nonsinglet QCD evolution, determination of $\alpha_s(Q)$

- The difference can be used to constrain **isospin violation**

$$\begin{aligned}\Delta xF_3 &= xF_3^{\nu A} - xF_3^{\bar{\nu} A} = 2xs_A^+ - 2xc_A^+ + x\delta I^A + \mathcal{O}(\alpha_S) \\ \delta I^A &= (d_{p/A} - u_{n/A}) + (d_{n/A} - u_{p/A}) + (\bar{d}_{p/A} - \bar{u}_{n/A})(\bar{d}_{n/A} - \bar{u}_{p/A})\end{aligned}$$

Hadronic Precision Observables

$$R^\nu = \frac{\sigma_{\text{NC}}^\nu}{\sigma_{\text{CC}}^\nu} \simeq g_L^2 + r g_R^2$$

$$R^{\bar{\nu}} = \frac{\sigma_{\text{NC}}^{\bar{\nu}}}{\sigma_{\text{CC}}^{\bar{\nu}}} \simeq g_L^2 + r g_R^2$$

$$r = \frac{\sigma_{\text{CC}}^{\bar{\nu}}}{\sigma_{\text{CC}}^\nu}$$

g_L and g_R are effective L and R
νq couplings

$$g_L^2 = \rho^2 \left(\frac{1}{2} - s_w^2 + \frac{5}{9} s_w^{98} \right)$$

$$g_R^2 = \rho^2 \left(\frac{5}{9} s_w^4 \right)$$

Paschos-Wolfenstein (PW):

$$\begin{aligned} R^- &= \frac{\sigma_{\text{NC}}^\nu - \sigma_{\text{NC}}^{\bar{\nu}}}{\sigma_{\text{CC}}^\nu - \sigma_{\text{CC}}^{\bar{\nu}}} \\ &\simeq g_L^2 - g_R^2 = \rho^2 \left(\frac{1}{2} - s_w^2 \right) \end{aligned}$$

QCD for PW-style analysis

$$R^- = \frac{\sigma_{\text{NC}}^\nu - \sigma_{\text{NC}}^{\bar{\nu}}}{\sigma_{\text{CC}}^\nu - \sigma_{\text{CC}}^{\bar{\nu}}} \simeq \frac{1}{2} - s_w^2 + \delta R_A^- + \delta R_{QCD}^- + \delta R_{EW}^-$$

non-isoscalarity
of the target QCD effects higher order
ew effects

$$\delta R_{QCD}^- = \delta R_s^- + \delta R_I^- + \delta R_{NLO}^-$$

due to strangeness
asymmetry:⁹⁹
 $s^- \equiv s - \bar{s} \neq 0$ due to isospin
violation:
 $u^p(x) \neq d^n(x)$ higher order
QCD effects

see, e.g., hep-ph/0405221