CSCE 633 Machine Learning : Assignment 1

February 2024

Question 1:

Given the function $f(x, y) = x^2 + \ln(y) + xy + y^3$, we need to find the gradient of f and evaluate it at the point $(x, y) = (10, -10)$.

The gradient of a scalar-valued function of multiple variables is a vector containing the partial derivatives of the function with respect to each variable.

The partial derivatives of f are given by:

$$\frac{\partial f}{\partial x} = 2x + y$$

$$\frac{\partial f}{\partial y} = \frac{1}{y} + x + 3y^2$$

Now, let's evaluate these partial derivatives at the point $(10, -10)$:

$$\frac{\partial f}{\partial x} = 2(10) + (-10) = 10$$

$$\frac{\partial f}{\partial y} = \frac{1}{-10} + 10 + 3(-10)^2 = -\frac{1}{10} + 10 + 300 = 310 - \frac{1}{10}$$

So, the gradient of f at $(10, -10)$ is $\nabla f(10, -10) = [10, 309.9]$.

Question 2: Given the function $f(x, y, z) = \tanh(x^3y^3) + \sin(z^2)$, we need to find the gradient of f and evaluate it at the point $(x, y, z) = (-1, 0, \frac{\pi}{2})$.

The gradient of a scalar-valued function of multiple variables is a vector containing the partial derivatives of the function with respect to each variable.

The partial derivatives of f are given by:

$$\frac{\partial f}{\partial x} = 3x^2y^3 \cdot \operatorname{sech}^2(x^3y^3)$$

$$\frac{\partial f}{\partial y} = 3x^3y^2 \cdot \operatorname{sech}^2(x^3y^3)$$

$$\frac{\partial f}{\partial z} = 2z \cdot \cos(z^2)$$

Now, let's evaluate these partial derivatives at the point $(x, y, z) = (-1, 0, \frac{\pi}{2})$:

$$\frac{\partial f}{\partial x} = 3(-1)^2(0)^3 \cdot \operatorname{sech}^2((-1)^3(0)^3) = 0$$

$$\frac{\partial f}{\partial y} = 3(-1)^3(0)^2 \cdot \operatorname{sech}^2((-1)^3(0)^3) = 0$$

$$\frac{\partial f}{\partial z} = 2\left(\frac{\pi}{2}\right) \cdot \cos\left(\left(\frac{\pi}{2}\right)^2\right) = \pi \cdot \cos\left(\frac{\pi^2}{4}\right)$$

So, the gradient of f at $(-1, 0, \frac{\pi}{2})$ is $\nabla f(-1, 0, \frac{\pi}{2}) = [0, 0, \pi \cdot \cos\left(\frac{\pi^2}{4}\right)]$.

Question 3: Given matrices:

$$A = \begin{bmatrix} 10 \\ -5 \\ 2 \\ 8 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 3 & 0 & 1 \end{bmatrix}$$

To compute the product of matrices A and B, we perform matrix multiplication as follows:

$$AB = \begin{bmatrix} 10 \\ -5 \\ 2 \\ 8 \end{bmatrix} \begin{bmatrix} 0 & 3 & 0 & 1 \end{bmatrix}$$

Matrix multiplication involves taking the dot product of rows from the first matrix with columns from the second matrix. Let's compute each element of the resulting matrix:

$$AB = \begin{bmatrix} 10(0) & 10(3) & 10(0) & 10(1) \\ -5(0) & -5(3) & -5(0) & -5(1) \\ 2(0) & 2(3) & 2(0) & 2(1) \\ 8(0) & 8(3) & 8(0) & 8(1) \end{bmatrix}$$

Simplifying each term:

$$AB = \begin{bmatrix} 0 & 30 & 0 & 10 \\ 0 & -15 & 0 & -5 \\ 0 & 6 & 0 & 2 \\ 0 & 24 & 0 & 8 \end{bmatrix}$$

Finally:

$$AB = \begin{bmatrix} 0 & 30 & 0 & 10 \\ 0 & -15 & 0 & -5 \\ 0 & 6 & 0 & 2 \\ 0 & 24 & 0 & 8 \end{bmatrix}$$

Question 4: Given matrices:

$$A = \begin{bmatrix} 1 & -1 & 6 & 7 \\ 9 & 0 & 8 & 1 \\ -8 & 1 & 2 & 3 \\ 10 & 4 & 0 & 1 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 6 & 2 & 0 \\ 0 & -1 & 1 \\ -3 & 0 & 4 \\ 3 & 4 & 7 \end{bmatrix}$$

To compute the product of matrices A and B, we perform matrix multiplication as follows:

$$AB = \begin{bmatrix} 1 & -1 & 6 & 7 \\ 9 & 0 & 8 & 1 \\ -8 & 1 & 2 & 3 \\ 10 & 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 2 & 0 \\ 0 & -1 & 1 \\ -3 & 0 & 4 \\ 3 & 4 & 7 \end{bmatrix}$$

Matrix multiplication involves taking the dot product of rows from the first matrix with columns from the second matrix. Let's compute each element of the resulting matrix:

$$AB = \begin{bmatrix} 1(6)+(-1)(0)+6(-3)+7(3) & 1(2)+(-1)(-1)+6(0)+7(4) & 1(0)+(-1)(1)+6(4)+7(7) \\ 9(6)+0(0)+8(-3)+1(3) & 9(2)+0(-1)+8(0)+1(4) & 9(0)+0(1)+8(4)+1(7) \\ -8(6)+1(0)+2(-3)+3(3) & -8(2)+1(-1)+2(0)+3(4) & -8(0)+1(1)+2(4)+3(7) \\ 10(6)+4(0)+0(-3)+1(3) & 10(2)+4(-1)+0(0)+1(4) & 10(0)+4(1)+0(4)+1(7) \end{bmatrix}$$

Simplifying each term:

$$AB = \begin{bmatrix} 9 & 31 & 72 \\ 33 & 22 & 39 \\ -45 & -5 & 30 \\ 63 & 20 & 11 \end{bmatrix}$$