

Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with $\mu = 45$ minutes and $\sigma = 8$ minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

- A. 0.3875
- B. 0.2676
- C. 0.5
- D. 0.6987

Answer: $P(X > 50) = 1 - P(X \leq 50)$.

$$Z = (X - 45)/8.0 = (50 - 45)/8.0$$

Thus the question can be answered by using the normal table to find

$$P(X \leq 50) = P(Z \leq (50 - 45)/8.0) = P(Z \leq 0.625) = 73.4\%$$

Probability that the service manager will not meet his demand will be $= 100 - 73.4 = 26.6\%$ or 0.2676

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $\mu = 38$ and Standard deviation $\sigma = 6$. For each statement below, please specify True/False. If false, briefly explain why.

- A. More employees at the processing center are older than 44 than between 38 and 44.

Answer: False. Since the μ is given 38.

- B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer: True

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between $2X_1$ and $X_1 + X_2$? Discuss both their distributions and parameters.

Answer: As both are independent normal random variables, $X_1 + X_2$ is normal with $N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$. And $2X_1$ will just scale the normal distribution by 2 times.

4. Let $X \sim N(100, 20^2)$. Find two values, a and b , symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

Answer: 48.5, 151.5

- A. 90.5, 105.9
B. 80.2, 119.8
C. 22, 78
D. 48.5, 151.5
E. 90.1, 109.9
5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $\text{Profit}_1 \sim N(5, 3^2)$ and $\text{Profit}_2 \sim N(7, 4^2)$ respectively. Both the profits are in \$ Million. Answer the following questions about the total profit of the company in Rupees. Assume that \$1 = Rs. 45

- A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Answer:

$$\text{Mean} = 5 + 7 = 12$$

$$\text{Mean Profit} = 12 * 45 = 540 \text{ Million}$$

$$\text{SD} = 3^2 + 4^2 = 25 = 25 * 45 = 315 \text{ Million}$$

$$\text{stats.norm.interval}(0.95, 540, 315)$$

Range is (-77.38865513011706, 1157.388655130117) Millions

- B. Specify the 5th percentile of profit (in Rupees) for the company

Answer:

$$\text{5th percentile} = -1.64$$

$$X = \mu + Z\sigma = 540 + (-1.64) * 315$$

$$\text{np.round}(X, 2)$$

$$= 23.4 \text{ Million}$$

- C. Which of the two divisions has a larger probability of making a loss in a given year?

Answer: Division 2 with distribution $N(7, 4^2)$