Reducing Memory Access Latencies using Data Compression in Sparse, Iterative Linear Solvers

All-College Thesis Defense

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Motivation

- Solving sparse linear systems used in many computations
- Billions of variables or more
- Spend most of the time fetching data from memory

Mathematics of Conjugate Gradient

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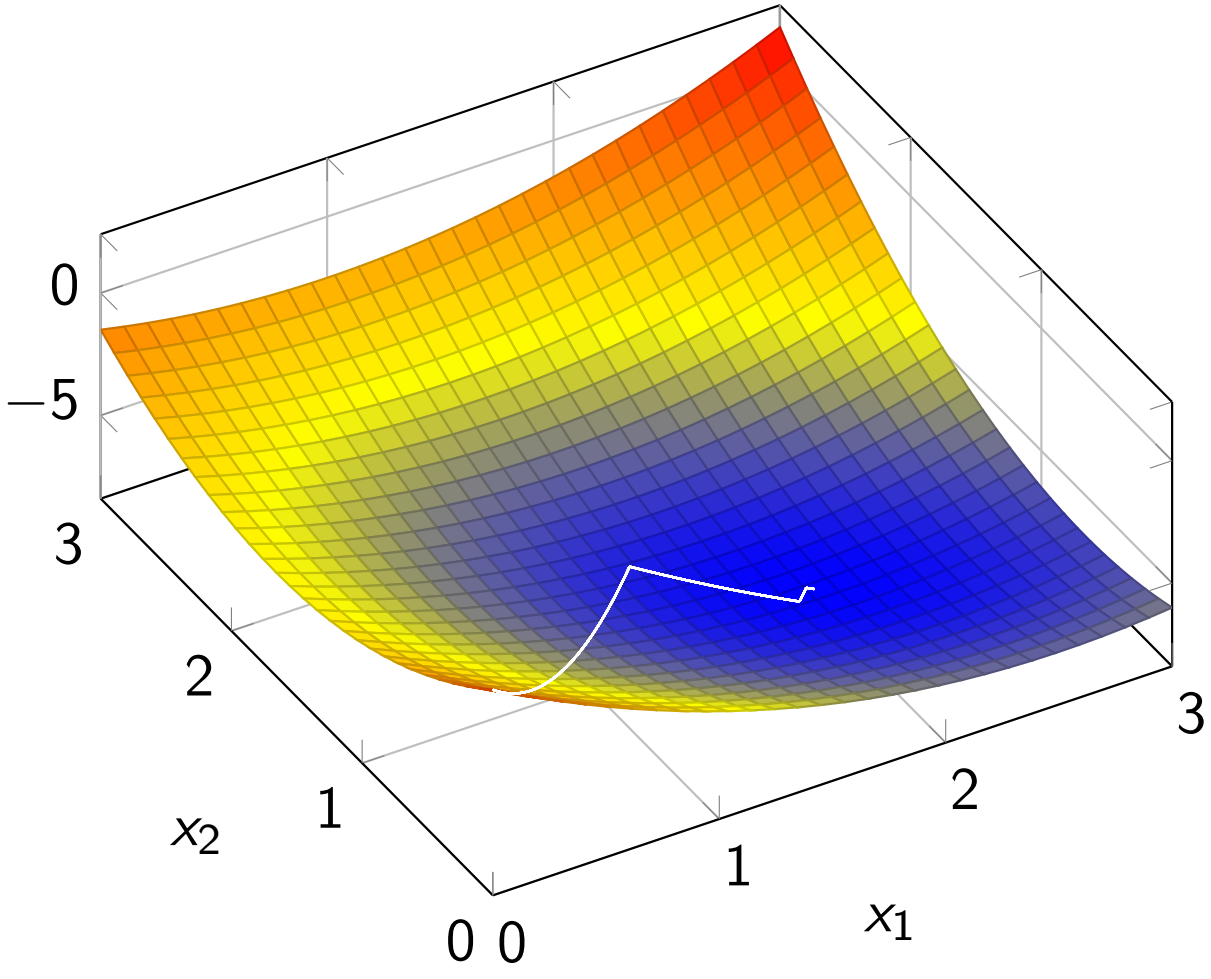
- If $\mathbf{A}$ is symmetric, then $\nabla f(\vec{x}) = \mathbf{A}\vec{x} - \vec{b}$
- $-\nabla f(\vec{x})$ is in direction of maximal decrease

Mathematics of Conjugate Gradient



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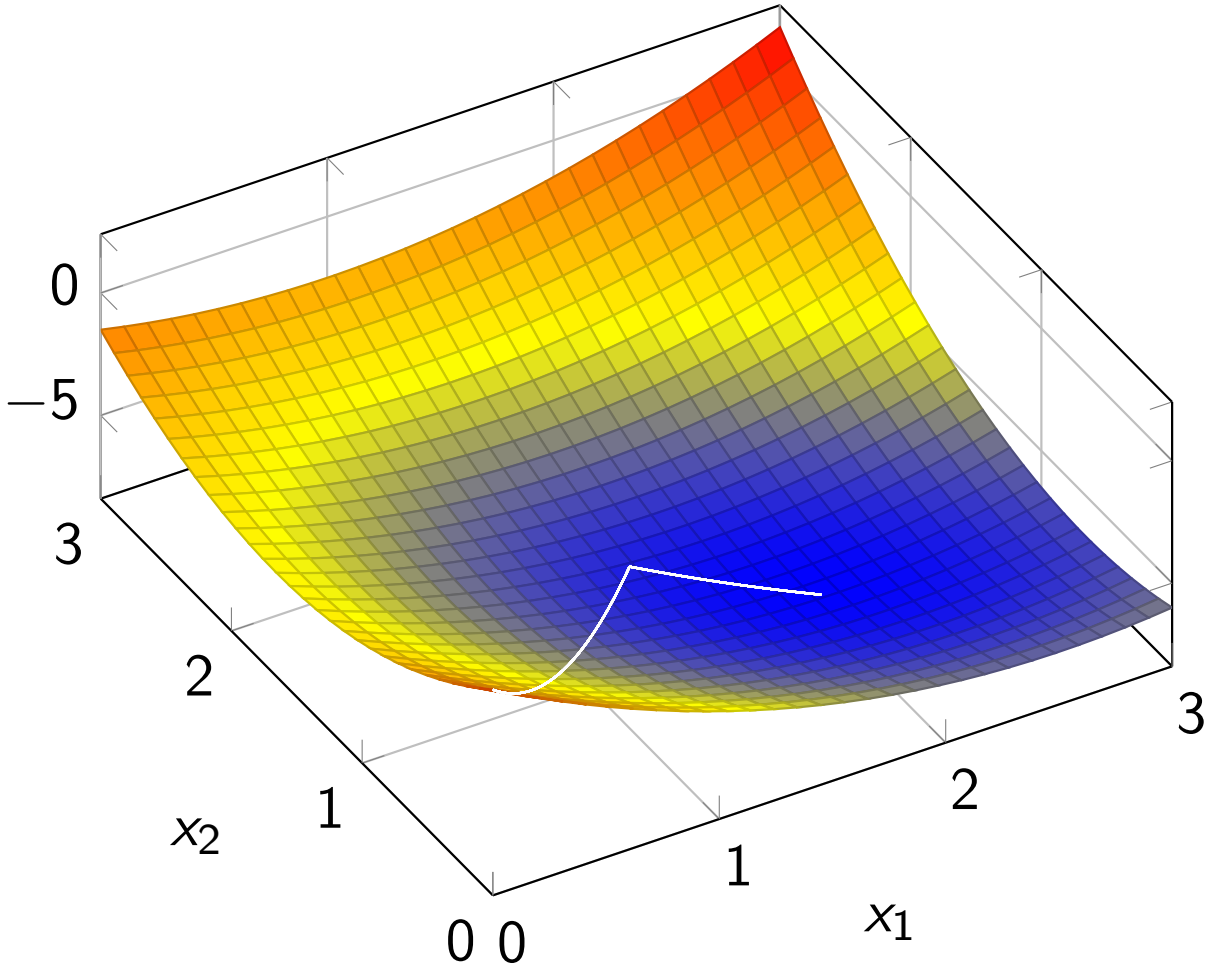

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- 3 compressible data structures
  - Vector Values
  - Matrix Indices
  - Matrix Values

Main Data Access Pattern

```
for row in rows do
  for nonzero entry in row do
    LOAD entry's value
    LOAD entry's column index
    LOAD vector value for column index
  end for
  WRITE vector value for row
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- need random vector reads
- need vector writes
- need both forward and backward iteration of matrix rows

Compression Methods

- Mixed Floating Point Precision
- SZ Compression
- Elias Gamma and Delta Codings
- ZFP Compression
- Huffman Coding
- Op Code Compression

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Mixed Floating Point Precision

- Single Precision takes half the storage space
- But drops from 15-17 significant digits to 6-9 digits
- Certain vectors can be lower precision without slowing convergence
  - $\vec{b}, \vec{x}, \mathbf{A}\vec{d}$

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- Available prediction functions are chosen based on the type of data
- Compression rate is highly dependent on local patterns in the data

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- Positive integers
- For each value, stores the number of bits needed then the data
  - Very effective for small integers
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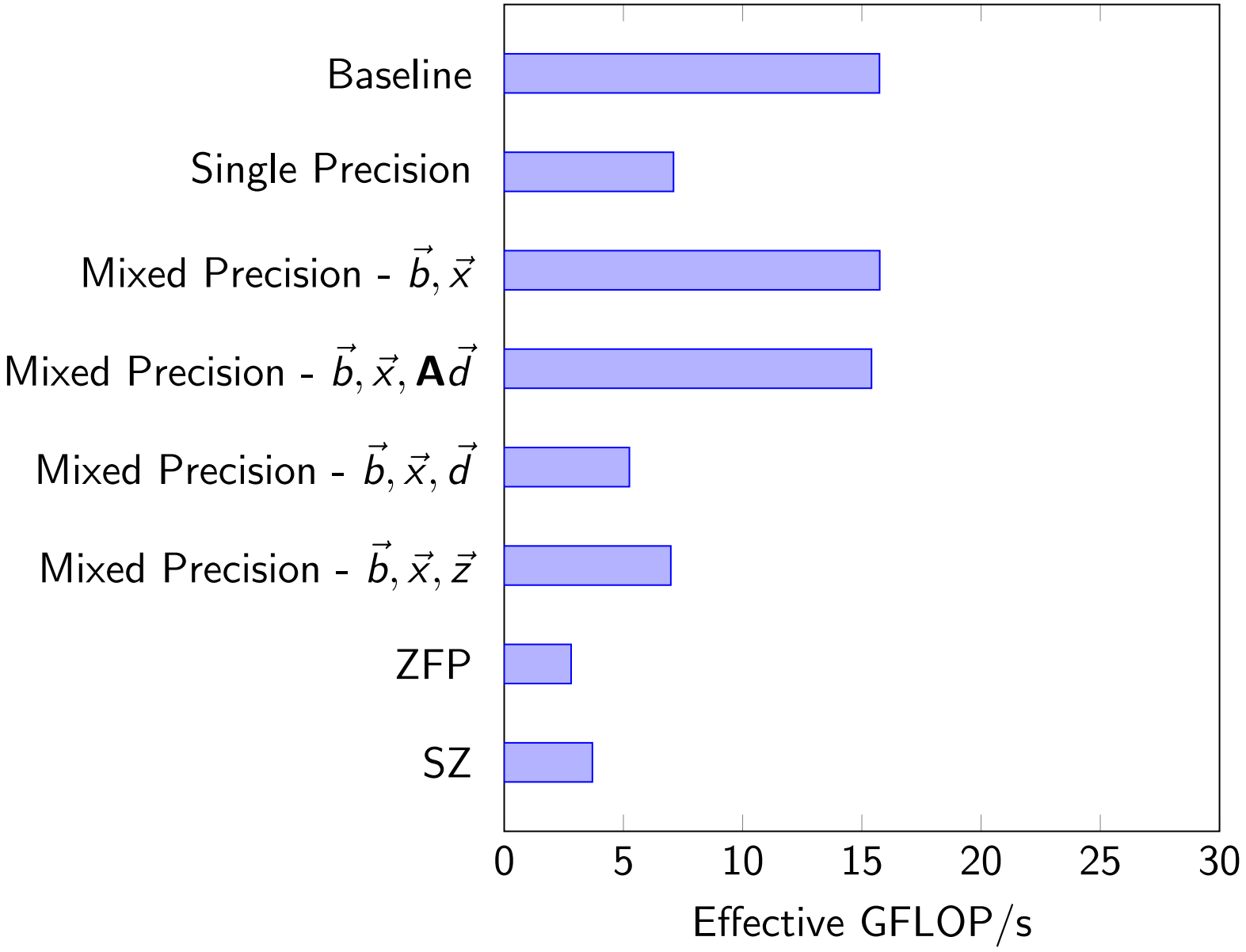
  - Storing the difference from the previous index reduces the size of values
- Elias Delta Coding is similar, but uses Gamma coding for the length
- Compression rate is only dependent on the magnitude of the values

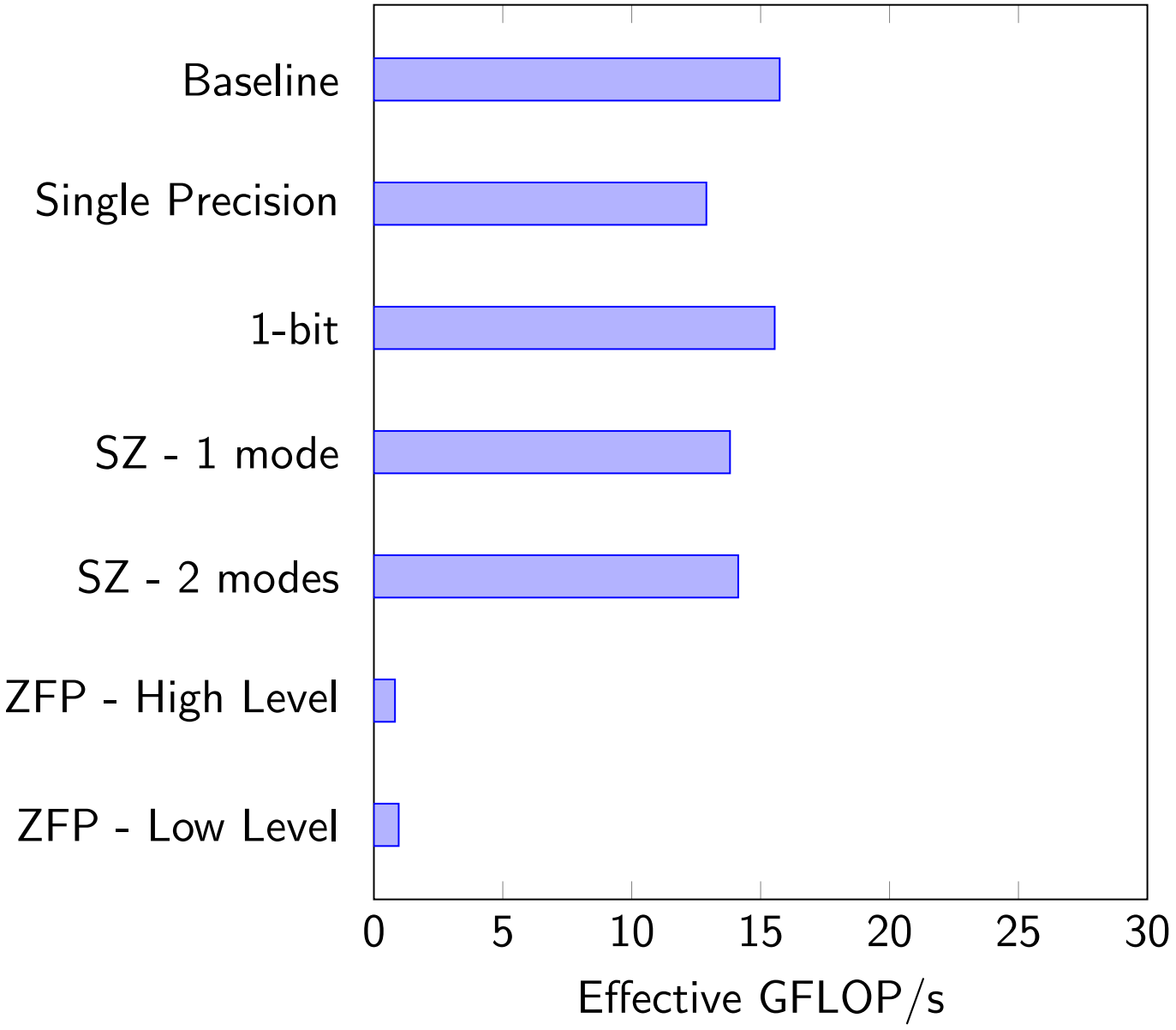
Timing Results

- 60 processes with 96^3 rows each
  - 53,084,160 total rows
- A 20-core, 2.2GHz, Intel Broadwell head node
- Plus five 8-core, 1.7GHz Intel Broadwell nodes
- MPI communication

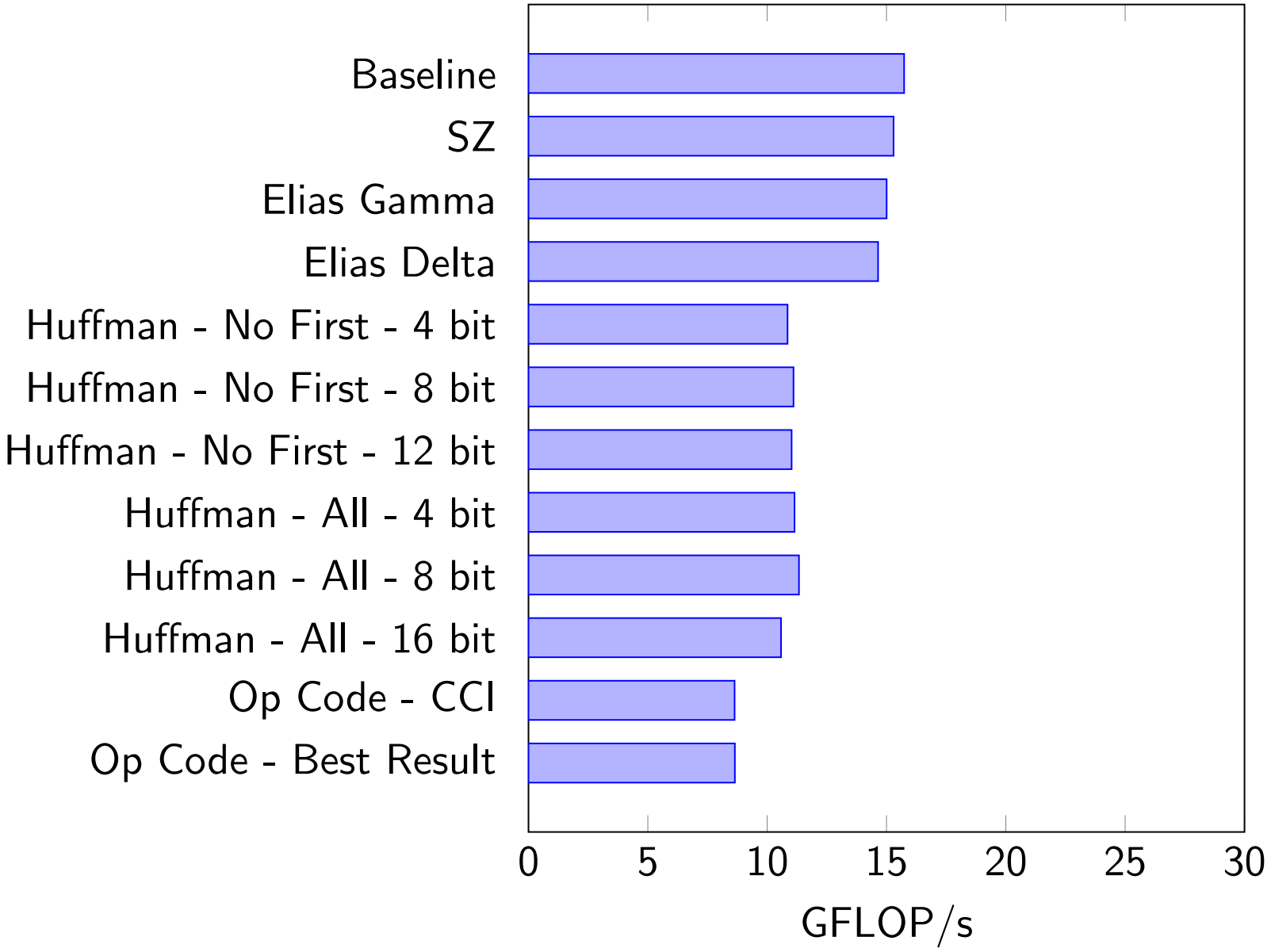
Vector Compression



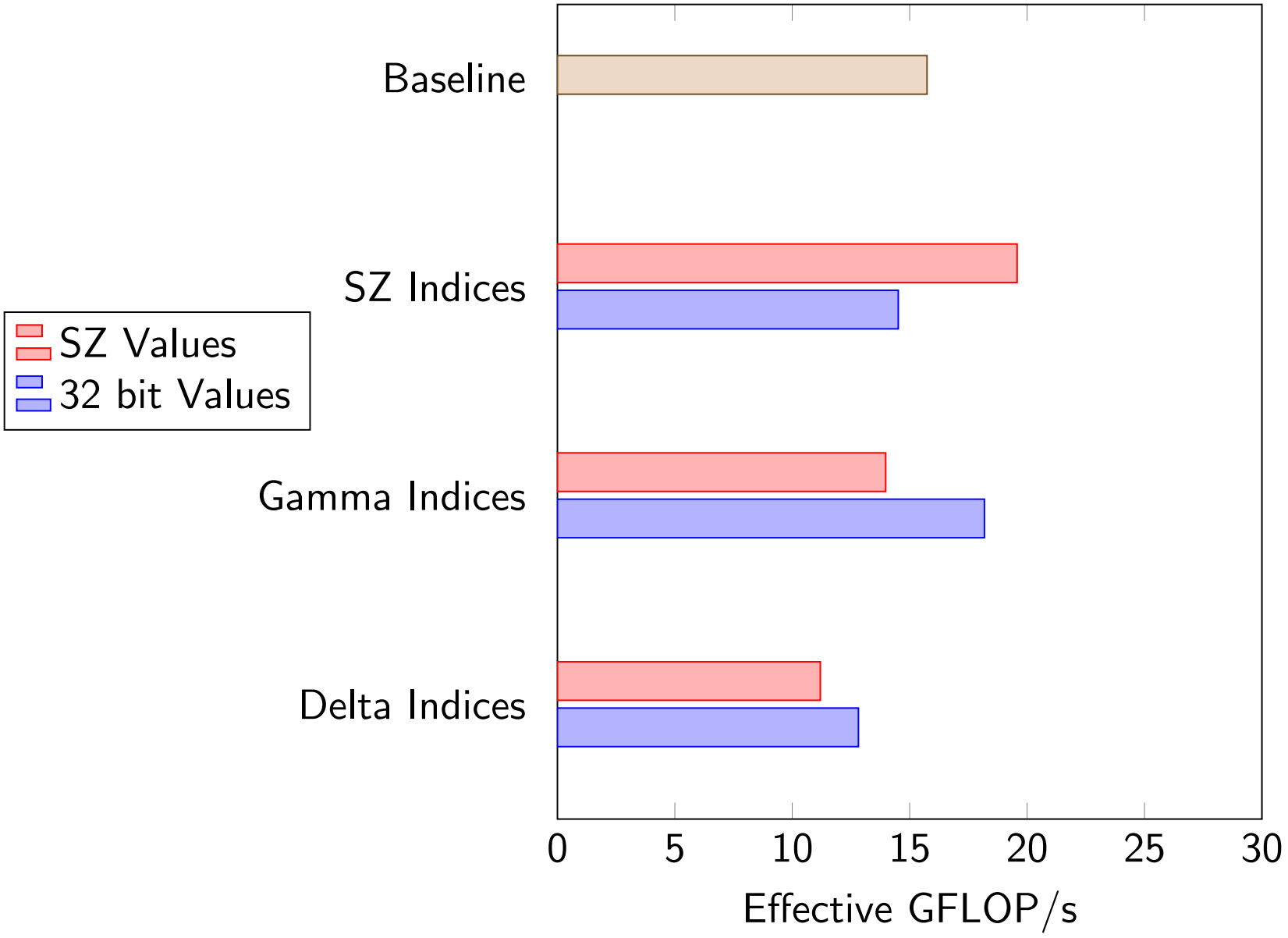
Matrix Value Compression



Matrix Index Compression



Matrix Value and Index Compression



Vector and Matrix Compression

Conclusion

- Iterative linear solvers are memory access bound
- Compressing key data structures provided an 84% increase in performance