

Reducing Memory Access Latencies using Data Compression in Sparse, Iterative Linear Solvers

All-College Thesis Defense

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Motivation

- Sparse systems of linear equations used in many computations
- Iterative solvers are used
- Spend most of the time fetching data

Mathematics of Conjugate Gradient

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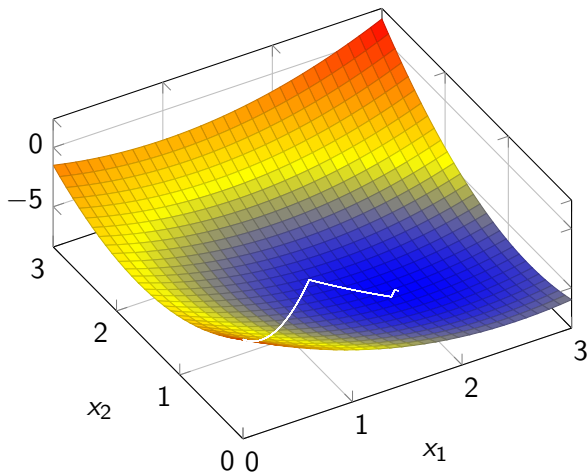
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- If \mathbf{A} is symmetric, then $\nabla f(\vec{x}) = \mathbf{A}\vec{x} - \vec{b}$

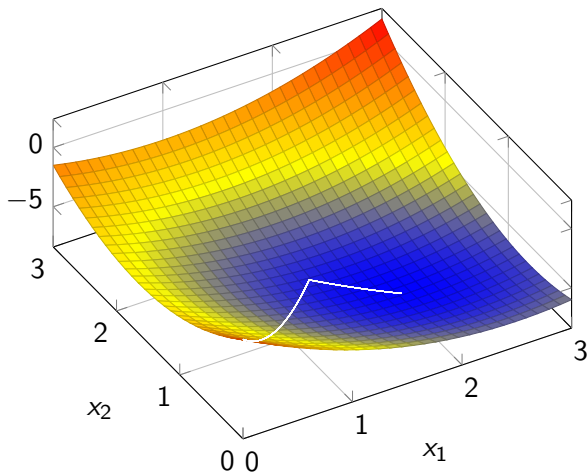
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- $-\nabla f(\vec{x})$ is in direction of maximal decrease

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 - Stores the column index and value for each nonzero entry
- 3 compressible data structures
 - Vector Values
 - Matrix Indices
 - Matrix Values

Compression Methods

- Mixed Floating Point Precision
- SZ Compression
- Elias Gamma and Delta Codings
- ZFP Compression
- Huffman Coding
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Mixed Floating Point Precision

- Two versions of floating point numbers
- Trade off between storage and precision
- Certain vectors can be lower precision without slowing convergence

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- Available prediction functions are based on the data
- Compression rate is highly dependent on local patterns in the data

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 - Very effective for small integers
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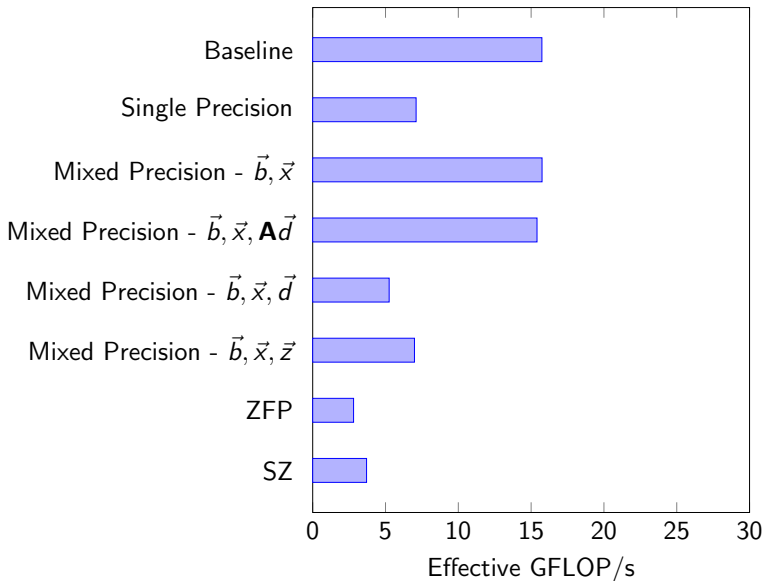
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- For each value, stores the size then the data
 - Very effective for small integers
 - Storing the difference from the previous index reduces the size of values
- Elias Delta Coding is similar, but uses Gamma coding for the length
- Compression rate is only dependent on the magnitude of the values

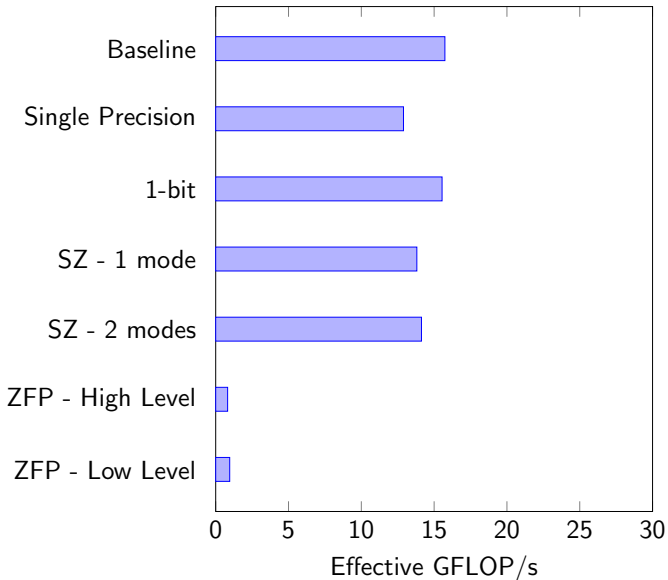
Timing Results

- 60 processes with 96^3 rows each
 - 53,084,160 total rows
- A 20-core, 2.2GHz, Intel Broadwell head node
- Plus five 8-core, 1.7GHz Intel Broadwell nodes

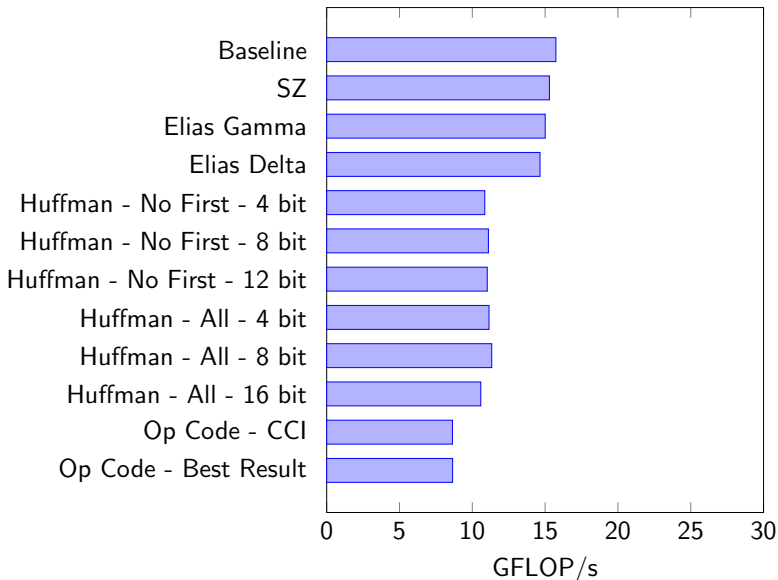
Vector Compression



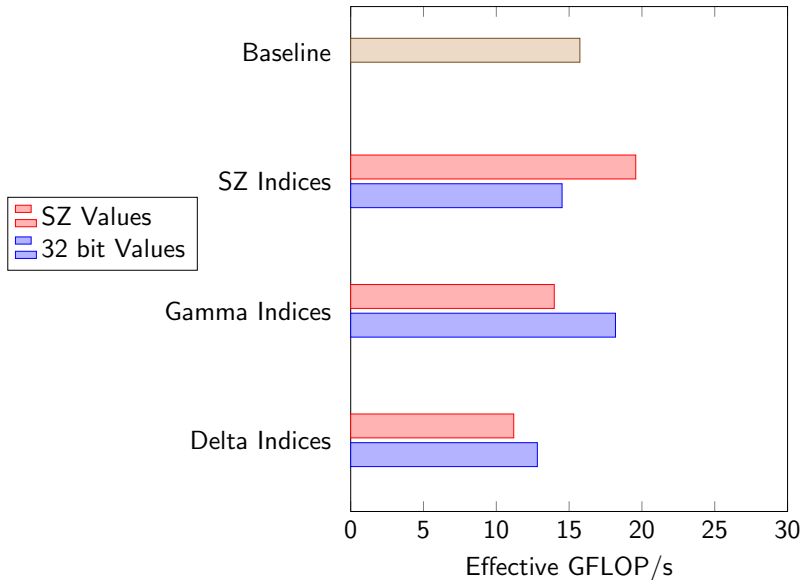
Matrix Value Compression



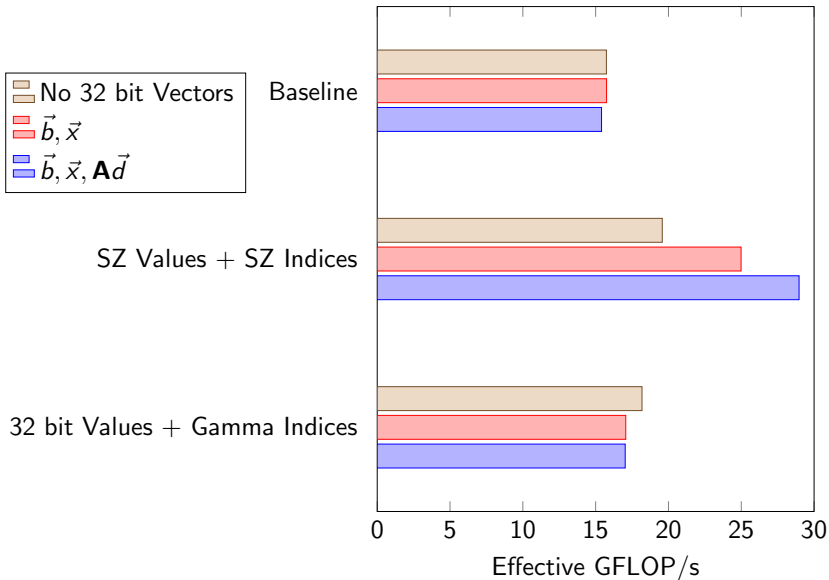
Matrix Index Compression



Matrix Value and Index Compression



Vector and Matrix Compression



Conclusion

- Iterative, sparse linear solvers are memory access bound
- Compressing key data structures provided an 84% increase in performance

Sources

`Github.com/Collegeville/HPCG-ZFP`