Reducing Memory Access Latencies using Data Compression in Sparse, Iterative Linear Solvers

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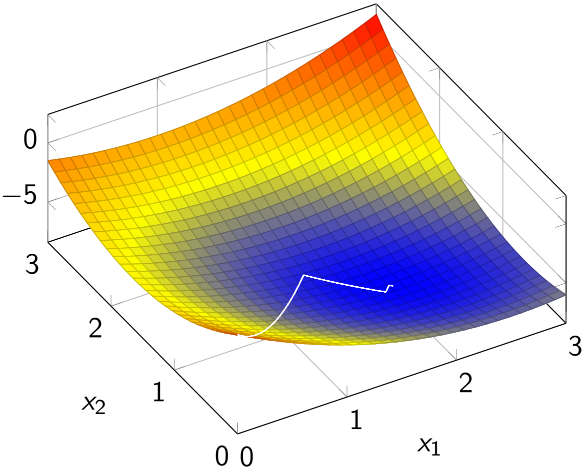
Motivation

- Sparse linear systems used by many scientific computations
- Problems can be large, with millions of variables
- Arithmetic is faster than fetching data from memory

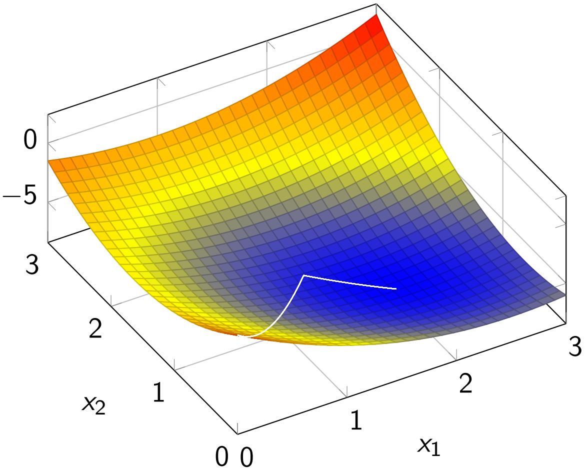
Mathematics of Conjugate Gradient

- Solving $\mathbf{A}\vec{x} = \vec{b}$
- Minimizing $f(\vec{x}) = \frac{1}{2}\vec{x}^T\mathbf{A}\vec{x} - \vec{b} \cdot \vec{x}$
- Note that $\nabla f(\vec{x}) = \mathbf{A}\vec{x} - \vec{b}$ when $\mathbf{A}$ is symmetric

Mathematics of Conjugate Gradient



Mathematics of Conjugate Gradient



Solver Description

- Approximating the steady state heat equation in 3 dimensions
  - $0 = \frac{\partial}{\partial x} u(\vec{x}) + \frac{\partial}{\partial y} u(\vec{x}) + \frac{\partial}{\partial z} u(\vec{x})$
- Preconditioned Conjugate Gradient was used
  - Preconditioned with a 3-level multigrid preconditioner using Symmetric Gauss-Seidel step smoother
- Matrix store in CSR format
  - Stores the column index and value for each nonzero entry
- 3 compressible data structures
  - Vector Values
  - Matrix Indices
  - Matrix Values

Main Data Access Pattern

```
for row in rows do
    for nonzero entry in row do
        LOAD entry's value
        LOAD entry's column index
        LOAD vector value for column index
    end for
    WRITE vector value for row
end for
```

- need random vector reads
- need vector writes
- need both forward and backward iteration of matrix rows

Compression Methods

- Mixed Floating Point Precision
- SZ Compression
- Elias Gamma and Delta Codings
- ZFP Compression
- Huffman Coding
- Op Code Compression

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Mixed Floating Point Precision

- Single Precision takes half the storage space
- But drops from 15-17 significant digits to 6-9 digits
- Certain vectors can be lower precision without slowing convergence
  - $\vec{b}, \vec{x}, \mathbf{A}\vec{d}$

Squeeze “SZ” Compression

- Tries to predict each value from the previous few
- Enforces a minimum accuracy
- Available prediction functions are chosen based on the type of data
- Compression rate is highly dependent on local patterns in the data

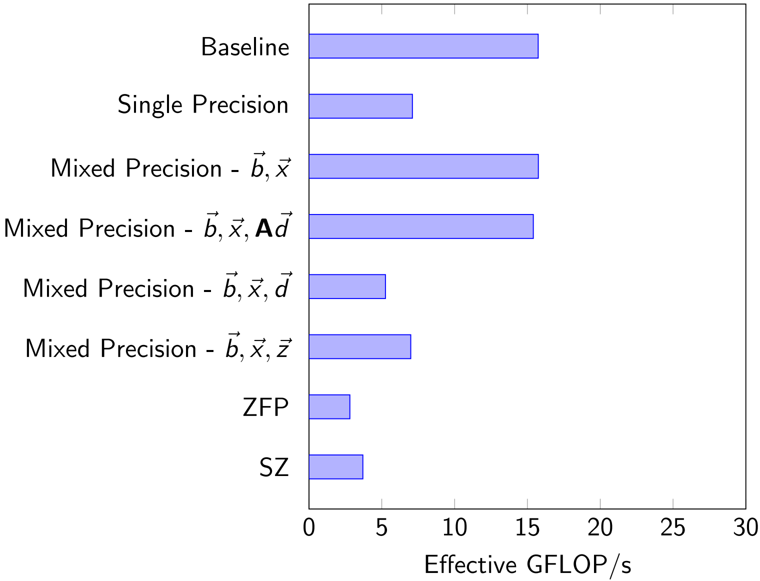
Elias Gamma Coding

- Positive integers
- For each value, stores the number of bits needed then the data
  - Very effective for small integers
  - Storing the difference from the previous index reduces the size of values
- Elias Delta Coding is similar, but uses Gamma coding for the length
- Compression rate is only dependent on the magnitude of the values

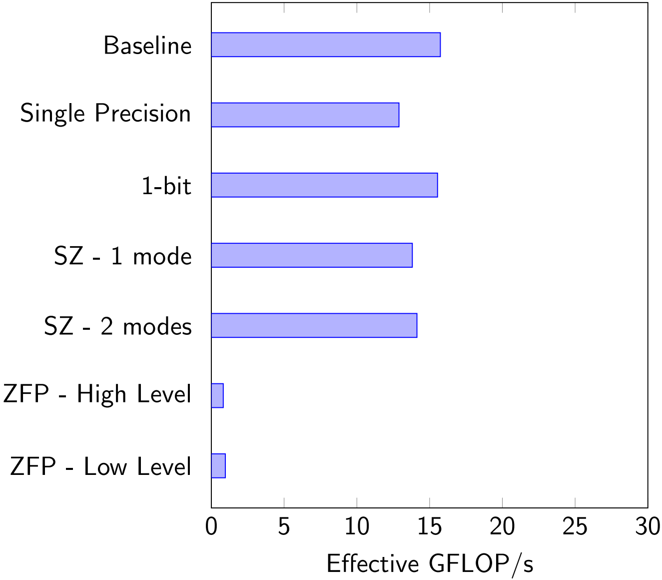
Timing Results

- 60 processes with 96^3 rows each
  - 53,084,160 total rows
- A 20-core, 2.2GHz, Intel Broadwell head node
- Plus five 8-core, 1.7GHz Intel Broadwell nodes
- MPI communication

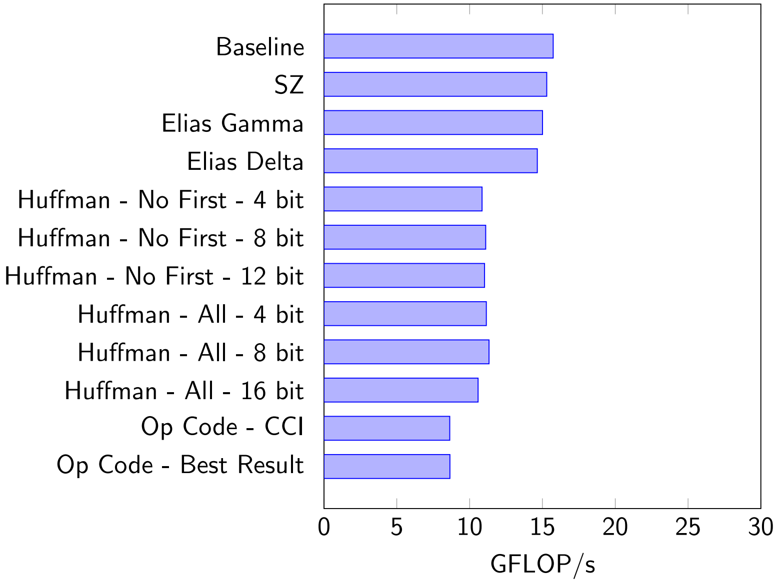
Vector Compression



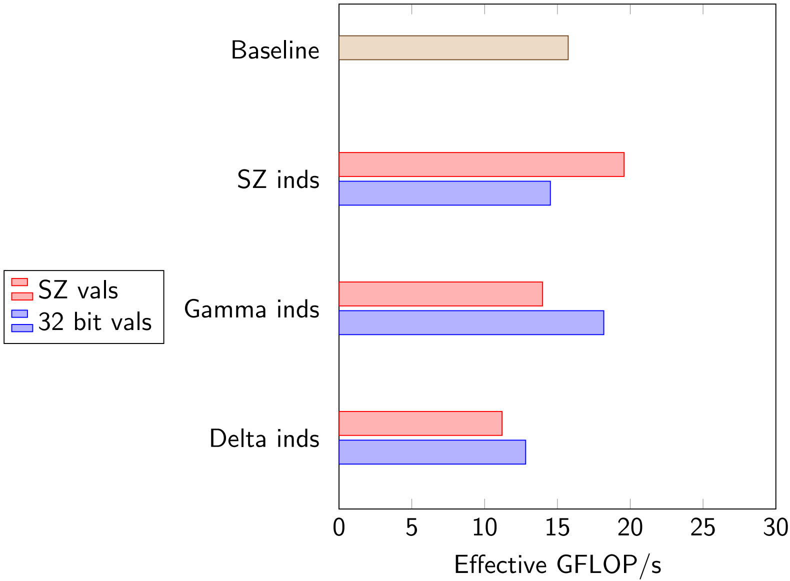
Matrix Value Compression



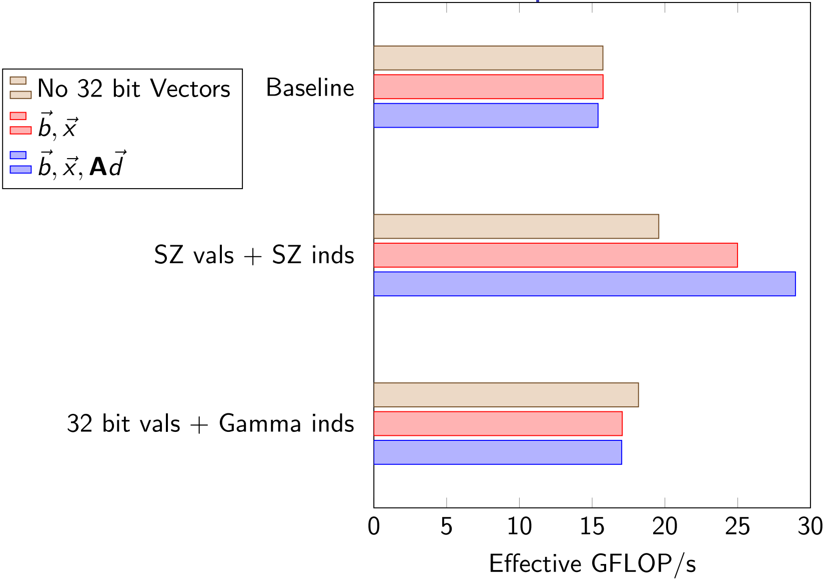
Matrix Index Compression



Matrix Value and Index Compression



Vector and Matrix Compression



Conclusion

- Iterative linear solvers are memory access bound
- Compressing key data structures provided an 84% increase in performance