

Reducing Memory Latency Using Data Compression

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Compressed Sparse Row Format

Problem Setup

Dense matrix

10	21	0	0
0	17	0	6
0	0	8	28
0	0	0	11

(a)

<i>i</i>	<i>j</i>	Value
0	0	10
0	1	21
1	1	17
1	3	6
2	2	8
2	3	28
3	3	11

(b)

<i>i</i> index	<i>j</i>	Value
0	0	10
2	1	21
4	1	17
6	3	6
	2	8
	3	28
	3	11

(c)

Image Credit: Buono, et al, Optimizing Sparse Linear Algebra for Large-Scale Graph Analytics

Conjugate Gradient

Problem Setup

- 1: **while** $distance(\mathbf{A}\vec{x}, \vec{b}) \geq tolerance$ **do**
- 2: Adjust \vec{x} to reduce $\mathbf{A}\vec{x} - \vec{b}$
- 3: **end while**

High level pseudocode of the Conjugate Gradient method to solve $\mathbf{A}\vec{x} = \vec{b}$ for \vec{x} given a matrix \mathbf{A} and a vector \vec{b} [1]

Main Loops

Problem Setup

```
1: for  $i$  from 0 to  $numRows(\mathbf{A})$  do
2:    $sum \leftarrow 0$ 
3:    $vals \leftarrow rowValues(\mathbf{A}, i)$ 
4:    $inds \leftarrow rowIndices(\mathbf{A}, i)$ 
5:   for  $j$  from 0 to  $nnzInRow(\mathbf{A}, i)$  do
6:      $sum \leftarrow sum + vals[j] \cdot \vec{x}[inds[j]]$ 
7:   end for
8:    $\vec{y}[i] \leftarrow sum$ 
9: end for
```

Pseudocode of the sparse matrix-vector product

$\vec{y} \leftarrow \mathbf{A}\vec{x}$, given a sparse matrix \mathbf{A} , and vectors \vec{y}, \vec{x} [2]

Single Precision Floating Point Numbers

Compression Methods

- Same access restrictions as the double precision version
- Can easily be applied to only certain vectors
 - For example, by using C++'s template system
- Fixed compression ratio of 1:2
 - Worse when mixing precisions

Squeeze (SZ) Compression

Compression Methods

- Utilizes local patterns in the data
- Stores each value as the mode used to (de)compress the data [3]
- The modes used depended on the type of data
- Compression ratio highly dependent on how well the data matches the patterns

Elias Gamma Coding

Compression Methods

- Compresses integers greater than zero by storing the number of bits needed [4]
- To compress a positive integer x :
 - Let $N = \text{ceil}(\log_2(x)) = \text{numBits}(x) - 1$
 - Store N 0's, then the $N + 1$ bits of x
- Compression ratio depends on the size of the differences

Further Compression Methods

Compression Methods

- Other, individual compression methods were tried too
 - Including huffman coding and ZFP compression
- Compression methods can be combined
 - Only combined compressions performed well in practice

References

[1] Y. Saad.

Iterative Methods for Sparse Linear Systems

[2] J. Dongarra, M. Heroux, and P. Luszczek.

HPCG Benchmark: A New Metric for Ranking High Performance Computing Systems

[3] S. Di and F. Cappello.

Fast Error-Bounded Lossy HPC Data Compression with SZ

[4] P. Elias.

Universal Codeword Sets and Representations of the Integers