Vector Operations

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1. Let $\mathbf{u} = (1, -2, 4)$, $\mathbf{v} = (3, 5, 1)$, and $\mathbf{w} = (2, 1, -3)$.

a. $3\mathbf{u} - 2\mathbf{v}$

The result of $3\mathbf{u} - 2\mathbf{v}$ is:

$$\begin{aligned} 3\mathbf{u} - 2\mathbf{v} &= 3(1, -2, 4) - 2(3, 5, 1) \\ &= (3, -6, 12) - (6, 10, 2) \\ &= (-3, -16, 10) \end{aligned}$$

b. $5\mathbf{u} + 3\mathbf{v} - 4\mathbf{w}$

The result of $5\mathbf{u} + 3\mathbf{v} - 4\mathbf{w}$ is:

$$5\mathbf{u} + 3\mathbf{v} - 4\mathbf{w} = 5(1, -2, 4) + 3(3, 5, 1) - 4(2, 1, -3)$$

$$\begin{aligned} &= (5, -10, 20) + (9, 15, 3) - (8, 4, +12) \\ &= (6, 1, 35) \end{aligned}$$

c. $\mathbf{u} \cdot \mathbf{v}$, $\mathbf{v} \cdot \mathbf{w}$, $\mathbf{u} \cdot \mathbf{w}$

The dot products are:

$$\mathbf{u} \cdot \mathbf{v} = (1, -2, 4) \cdot (3, 5, 1) = (1)(3) + (-2)(5) + (4)(1) = 3 - 10 + 4 = -3$$

$$\begin{aligned} \mathbf{v} \cdot \mathbf{w} &= (3, 5, 1) \cdot (2, 1, -3) = (3)(2) + (5)(1) + (1)(-3) = 6 + 5 - 3 = 8 \\ \mathbf{u} \cdot \mathbf{w} &= (1, -2, 4) \cdot (2, 1, -3) = (1)(2) + (-2)(1) + (4)(-3) = 2 - 2 - 12 = -12 \end{aligned}$$

d. $\|\mathbf{u}\|$, $\|\mathbf{v}\|$, $\|\mathbf{w}\|$

The magnitudes are:

$$\begin{aligned} \|\mathbf{u}\| &= \sqrt{1^2 + (-2)^2 + 4^2} = \sqrt{1 + 4 + 16} = \sqrt{21} \\ \|\mathbf{v}\| &= \sqrt{3^2 + 5^2 + 1^2} = \sqrt{9 + 25 + 1} = \sqrt{35} \\ \|\mathbf{w}\| &= \sqrt{2^2 + 1^2 + (-3)^2} = \sqrt{4 + 1 + 9} = \sqrt{14} \end{aligned}$$

e. If θ is the angle between u and v, find $\cos\theta$

The cosine of the angle θ between $\mathbf{u}$ and $\mathbf{v}$ can be calculated using the dot product and magnitudes:

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{-3}{\sqrt{21}\sqrt{35}}$$

f. If θ is the angle between v and w, find $\cos\theta$

The cosine of the angle θ between $\mathbf{v}$ and $\mathbf{w}$ can be calculated using the dot product and magnitudes:

$$\cos\theta = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|} = \frac{8}{\sqrt{35}\sqrt{14}}$$

g. If θ is the angle between u and w, find $\cos\theta$

The cosine of the angle θ between $\mathbf{u}$ and $\mathbf{w}$ can be calculated using the dot product and magnitudes:

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{w}}{\|\mathbf{u}\|\|\mathbf{w}\|} = \frac{-12}{\sqrt{21}\sqrt{14}}$$

h. Find distance(u, v)

The distance between $\mathbf{u}$ and $\mathbf{v}$ is:

$$\text{distance}(\mathbf{u},\mathbf{v}) = \|\mathbf{u}-\mathbf{v}\|$$
$$= \|(1,-2,4)-(3,5,1)\|$$
$$= \|(-2,-7,3)\|$$
$$= \sqrt{(-2)^2+(-7)^2+3^2}$$
$$= \sqrt{4+49+9}$$
$$= \sqrt{62}$$

i. Find distance(w, v)

The distance between $\mathbf{w}$ and $\mathbf{v}$ is:

$$\text{distance}(\mathbf{w},\mathbf{v}) = \|\mathbf{w}-\mathbf{v}\|$$
$$= \|(2,1,-3)-(3,5,1)\|$$
$$= \|(-1,-4,-4)\|$$
$$= \sqrt{(-1)^2+(-4)^2+(-4)^2}$$
$$= \sqrt{1+16+16}$$
$$= \sqrt{33}$$

j. Find distance(u, w)

The distance between $\mathbf{u}$ and $\mathbf{w}$ is:

$$\text{distance}(\mathbf{u},\mathbf{w}) = \|\mathbf{u}-\mathbf{w}\|$$
$$= \|(1,-2,4)-(2,1,-3)\|$$
$$= \|(-1,-3,7)\|$$
$$= \sqrt{(-1)^2+(-3)^2+7^2}$$
$$= \sqrt{1+9+49}$$
$$= \sqrt{59}$$

k. Find projection$(\mathbf{u}, \mathbf{v})$

The projection of $\mathbf{v}$ onto $\mathbf{u}$ is:

$$\text{projection}(\mathbf{u}, \mathbf{v}) = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v}$$

$$= \frac{-3}{35}(3, 5, 1)$$

$$= (-\frac{9}{35}, -\frac{15}{35}, -\frac{3}{35})$$

l. Find projection$(\mathbf{w}, \mathbf{v})$

The projection of $\mathbf{v}$ onto $\mathbf{w}$ is:

$$\text{projection}(\mathbf{w}, \mathbf{v}) = \frac{\mathbf{w} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v}$$

$$= \frac{8}{35}(3, 5, 1)$$

$$= (\frac{24}{35}, \frac{40}{35}, \frac{8}{35})$$

m. Find projection$(\mathbf{u}, \mathbf{w})$

The projection of $\mathbf{w}$ onto $\mathbf{u}$ is:

$$\text{projection}(\mathbf{u}, \mathbf{w}) = \frac{\mathbf{u} \cdot \mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w}$$

$$= \frac{-12}{14}(2, 1, -3)$$

$$= (-\frac{12}{7}, -\frac{6}{7}, \frac{18}{7})$$

2. Let $\mathbf{u} = (1, 3, -4)$, $\mathbf{v} = (2, 1, 5)$, and $\mathbf{w} = (3, -2, 6)$.

a. $3\mathbf{u} - 2\mathbf{v}$

The result of $3\mathbf{u} - 2\mathbf{v}$ is:

$$3\mathbf{u} - 2\mathbf{v} = 3\begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} - 2\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 9 \\ -12 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ 10 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 7 \\ -22 \end{pmatrix}$$

b. $5\mathbf{u} + 3\mathbf{v} - 4\mathbf{w}$

The result of $5\mathbf{u} + 3\mathbf{v} - 4\mathbf{w}$ is:

$$5\mathbf{u} + 3\mathbf{v} - 4\mathbf{w} = 5\begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} + 3\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} - 4\begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ 15 \\ -20 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \\ 15 \end{pmatrix} - \begin{pmatrix} 12 \\ +8 \\ 24 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 26 \\ -29 \end{pmatrix}$$

c. $\mathbf{u} \cdot \mathbf{v}$, $\mathbf{v} \cdot \mathbf{w}$, $\mathbf{u} \cdot \mathbf{w}$

The dot products are:

$$\mathbf{u} \cdot \mathbf{v} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} = (1)(2) + (3)(1) + (-4)(5) = 2 + 3 - 20 = -15$$

$$\mathbf{v} \cdot \mathbf{w} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} = (2)(3) + (1)(-2) + (5)(6) = 6 - 2 + 30 = 34$$

$$\mathbf{u} \cdot \mathbf{w} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} = (1)(3) + (3)(-2) + (-4)(6) = 3 - 6 - 24 = -27$$

d. $\|\mathbf{u}\|$, $\|\mathbf{v}\|$, $\|\mathbf{w}\|$

The magnitudes are:

$$\|\mathbf{u}\| = \sqrt{1^2 + 3^2 + (-4)^2} = \sqrt{1 + 9 + 16} = \sqrt{26}$$

$$\|\mathbf{v}\| = \sqrt{2^2 + 1^2 + 5^2} = \sqrt{4 + 1 + 25} = \sqrt{30}$$

$$\|\mathbf{w}\| = \sqrt{3^2 + (-2)^2 + 6^2} = \sqrt{9 + 4 + 36} = \sqrt{49} = 7$$

e. If θ is the angle between u and v, find $\cos\theta$

The cosine of the angle θ between **u** and **v** can be calculated using the dot product and magnitudes:

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{-15}{\sqrt{26}\sqrt{30}}$$

f. If θ is the angle between v and w, find $\cos\theta$

The cosine of the angle θ between **v** and **w** can be calculated using the dot product and magnitudes:

$$\cos\theta = \frac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|} = \frac{34}{\sqrt{30} \cdot 7}$$

g. If θ is the angle between u and w, find $\cos\theta$

The cosine of the angle θ between **u** and **w** can be calculated using the dot product and magnitudes:

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{w}}{\|\mathbf{u}\|\|\mathbf{w}\|} = \frac{-27}{\sqrt{26} \cdot 7}$$

h. Find distance$(\mathbf{u}, \mathbf{v})$

The distance between **u** and **v** is:

$$\text{distance}(\mathbf{u}, \mathbf{v}) = \|\mathbf{u} - \mathbf{v}\|$$

$$= \left\| \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \right\|$$

$$
\begin{aligned}
&= \left\| \begin{pmatrix} -1 \\ 2 \\ -9 \end{pmatrix} \right\| \\
&= \sqrt{(-1)^2 + 2^2 + (-9)^2} \\
&= \sqrt{1 + 4 + 81} \\
&= \sqrt{86}
\end{aligned}
$$

i. Find distance$(\mathbf{w}, \mathbf{v})$

The distance between **w** and **v** is:

$$
\begin{aligned}
\text{distance}(\mathbf{w}, \mathbf{v}) &= \|\mathbf{w} - \mathbf{v}\| \\
&= \left\| \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \right\| \\
&= \left\| \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \right\| \\
&= \sqrt{1^2 + (-3)^2 + 1^2} \\
&= \sqrt{1 + 9 + 1} \\
&= \sqrt{11}
\end{aligned}
$$

j. Find distance$(\mathbf{u}, \mathbf{w})$

The distance between **u** and **w** is:

$$
\begin{aligned}
\text{distance}(\mathbf{u}, \mathbf{w}) &= \|\mathbf{u} - \mathbf{w}\| \\
&= \left\| \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} \right\| \\
&= \left\| \begin{pmatrix} -2 \\ 5 \\ -10 \end{pmatrix} \right\| \\
&= \sqrt{(-2)^2 + 5^2 + (-10)^2} \\
&= \sqrt{4 + 25 + 100} \\
&= \sqrt{129}
\end{aligned}
$$

k. Find projection$(\mathbf{u}, \mathbf{v})$

The projection of **v** onto **u** is:

$$
\begin{aligned}
\text{projection}(\mathbf{u}, \mathbf{v}) &= \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \mathbf{v} \\
&= \frac{-15}{30} \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \\
&= -\frac{1}{2} \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \\
&= \begin{pmatrix} -\frac{1}{7} \\ \frac{2}{7} \\ -\frac{4}{7} \end{pmatrix}
\end{aligned}
$$

l. Find projection($\mathbf{w}, \mathbf{v}$)

The projection of $\mathbf{v}$ onto $\mathbf{w}$ is:

$$\text{projection}(\mathbf{w}, \mathbf{v}) = \frac{\mathbf{w} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v}$$
$$= \frac{34}{30}\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$
$$= \frac{17}{15}\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$
$$= \begin{pmatrix} \frac{34}{15} \\ \frac{17}{15} \\ \frac{85}{15} \end{pmatrix}$$

m. Find projection($\mathbf{u}, \mathbf{w}$)

The projection of $\mathbf{w}$ onto $\mathbf{u}$ is:

$$\text{projection}(\mathbf{u}, \mathbf{w}) = \frac{\mathbf{u} \cdot \mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w}$$
$$= \frac{-27}{49}\begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$$
$$= -\frac{27}{49}\begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$$
$$= \begin{pmatrix} -\frac{81}{49} \\ \frac{54}{49} \\ -\frac{162}{49} \end{pmatrix}$$

3. Let u = (2,-5,4,6,-3) and v = (5,-2,1,-7,-4) w = (2,1,-3) find:

a. $4\mathbf{u} - 3\mathbf{v}$

The result of $4\mathbf{u} - 3\mathbf{v}$ is:

$$4\mathbf{u} - 3\mathbf{v} = (8, -20, 16, 24, -12) - (15, -6, 3, -21, -12) = (-7, -14, 13, 45, 0).$$

b. $5\mathbf{u} + 2\mathbf{v} - 2\mathbf{w}$

The result of $5\mathbf{u} + 2\mathbf{v} - 2\mathbf{w}$ is:

$$5\mathbf{u} + 2\mathbf{v} - 2\mathbf{w} = (10, -25, 20, 30, -15) + (10, -4, 2, -14, -8) - (4, 2, -6, 0, 0) = (16, -31, 28, 16, -23).$$

c. $\mathbf{u} \cdot \mathbf{v}$, $\mathbf{v} \cdot \mathbf{w}$, $\mathbf{u} \cdot \mathbf{w}$

The dot products are:

$$\mathbf{u} \cdot \mathbf{v} = (2)(5) + (-5)(-2) + (4)(1) + (6)(-7) + (-3)(-4) = 10 + 10 + 4 - 42 + 12 = -6$$
$$\mathbf{v} \cdot \mathbf{w} = (5)(2) + (-2)(1) + (1)(-3) + (-7)(0) + (-4)(0) = 10 - 2 - 3 + 0 + 0 = 5$$
$$\mathbf{u} \cdot \mathbf{w} = (2)(2) + (-5)(1) + (4)(-3) + (6)(0) + (-3)(0) = 4 - 5 - 12 + 0 + 0 = -13$$

d. $\|\mathbf{u}\|$, $\|\mathbf{v}\|$, $\|\mathbf{w}\|$

The magnitudes are:

$$\|\mathbf{u}\| = \sqrt{2^2 + (-5)^2 + 4^2 + 6^2 + (-3)^2} = \sqrt{4 + 25 + 16 + 36 + 9} = \sqrt{90}$$

$$\|\mathbf{v}\| = \sqrt{5^2 + (-2)^2 + 1^2 + (-7)^2 + (-4)^2} = \sqrt{25 + 4 + 1 + 49 + 16} = \sqrt{95}$$

$$\|\mathbf{w}\| = \sqrt{2^2 + 1^2 + (-3)^2 + 0^2 + 0^2} = \sqrt{4 + 1 + 9 + 0 + 0} = \sqrt{14}$$

e. If θ is the angle between u and v, find $\cos\theta$

The cosine of the angle θ between $\mathbf{u}$ and $\mathbf{v}$ can be calculated using the dot product and magnitudes:

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{-6}{\sqrt{90}\sqrt{95}}$$

f. If θ is the angle between v and w, find $\cos\theta$

The cosine of the angle θ between $\mathbf{v}$ and $\mathbf{w}$ can be calculated using the dot product and magnitudes:

$$\cos\theta = \frac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|} = \frac{5}{\sqrt{95}\sqrt{14}}$$

g. If θ is the angle between u and w, find $\cos\theta$

The cosine of the angle θ between $\mathbf{u}$ and $\mathbf{w}$ can be calculated using the dot product and magnitudes:

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{w}}{\|\mathbf{u}\|\|\mathbf{w}\|} = \frac{-13}{\sqrt{90}\sqrt{14}}$$

h. Find distance$(\mathbf{u}, \mathbf{v})$

The distance between $\mathbf{u}$ and $\mathbf{v}$ is:

$$\begin{aligned}
\text{distance}(\mathbf{u}, \mathbf{v}) = \|\mathbf{u} - \mathbf{v}\| &= \|(2, -5, 4, 6, -3) - (5, -2, 1, -7, -4)\| \\
&= \|(-3, -3, 3, 13, 1)\| \\
&= \sqrt{(-3)^2 + (-3)^2 + 3^2 + 13^2 + 1^2} \\
&= \sqrt{9 + 9 + 9 + 169 + 1} \\
&= \sqrt{197}
\end{aligned}$$

i. Find distance$(\mathbf{w}, \mathbf{v})$

The distance between $\mathbf{w}$ and $\mathbf{v}$ is:

$$\begin{aligned}
\text{distance}(\mathbf{w}, \mathbf{v}) = \|\mathbf{w} - \mathbf{v}\| &= \|(2, 1, -3, 0, 0) - (5, -2, 1, -7, -4)\| \\
&= \|(-3, 3, -4, 7, 4)\| \\
&= \sqrt{(-3)^2 + 3^2 + (-4)^2 + 7^2 + 4^2} \\
&= \sqrt{9 + 9 + 16 + 49 + 16} \\
&= \sqrt{99}
\end{aligned}$$

j. Find distance$(\mathbf{u}, \mathbf{w})$

The distance between **u** and **w** is:

$$\begin{aligned}
\text{distance}(\mathbf{u}, \mathbf{w}) = \|\mathbf{u} - \mathbf{w}\| &= \|(2, 1, -3, 0, 0) - (5, -2, 1, -7, -4)\| \\
&= \|(0, -6, 7, 6, -3)\| \\
&= \sqrt{0^2 + (-6)^2 + 7^2 + 6^2 + (-3)^2} \\
&= \sqrt{0 + 36 + 49 + 36 + 9} \\
&= \sqrt{130}
\end{aligned}$$

k. Find projection$(\mathbf{u}, \mathbf{v})$

The projection of **v** onto **u** is:

$$\begin{aligned}
\text{projection}(\mathbf{u}, \mathbf{v}) &= \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} \\
&= \frac{-6}{\|\mathbf{v}\|^2}(5, -2, 1, -7, -4) \\
&= \frac{-6}{\left(\sqrt{95}\right)^2}(5, -2, 1, -7, -4) \\
&= \frac{-6}{95}(5, -2, 1, -7, -4) \\
&= \left(-\frac{30}{95}, \frac{12}{95}, -\frac{6}{95}, \frac{42}{95}, \frac{24}{95}\right).
\end{aligned}$$

l. Find projection$(\mathbf{w}, \mathbf{v})$

The projection of **v** onto **w** is:

$$\begin{aligned}
\text{projection}(\mathbf{w}, \mathbf{v}) &= \frac{\mathbf{w} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} \\
&= \frac{5}{\|\mathbf{v}\|^2}(5, -2, 1, -7, -4) \\
&= \frac{5}{\left(\sqrt{95}\right)^2}(5, -2, 1, -7, -4) \\
&= \frac{5}{95}(5, -2, 1, -7, -4) \\
&= \left(\frac{25}{95}, -\frac{10}{95}, -\frac{5}{95}, -\frac{35}{95}, -\frac{20}{95}\right).
\end{aligned}$$

m. Find projection$(\mathbf{u}, \mathbf{w})$

The projection of **w** onto **u** is:

$$\begin{aligned}
\text{projection}(\mathbf{u}, \mathbf{w}) &= \frac{\mathbf{u} \cdot \mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w} \\
&= \frac{-13}{\|\mathbf{w}\|^2}(2, 1, -3, 0, 0) \\
&= \frac{-13}{\left(\sqrt{14}\right)^2}(2, 1, -3, 0, 0) \\
&= \frac{-13}{14}(2, 1, -3, 0, 0) \\
&= \left(-\frac{26}{95}, -\frac{13}{95}, \frac{26}{95}, 0, 0\right).
\end{aligned}$$

4. Normalize each vector:

a. Normalized vector u:

$$\|\mathbf{u}\| = \sqrt{5^2 + (-7)^2} = \sqrt{74}$$
$$\hat{\mathbf{u}} = \frac{\mathbf{u}}{\|\mathbf{u}\|} = \left(\frac{5}{\sqrt{74}}, -\frac{7}{\sqrt{74}}\right).$$

b. Normalized vector v:

$$\|\mathbf{v}\| = \sqrt{1^2 + 2^2 + (-2)^2 + 4^2} = 5$$
$$\hat{\mathbf{v}} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \left(\frac{1}{5}, \frac{2}{5}, -\frac{2}{5}, \frac{4}{5}\right).$$

c. Normalized vector w:

$$\|\mathbf{w}\| = \left[\frac{1}{2}, -\frac{1}{3}, \frac{3}{4}\right]\left(\frac{1}{2} \cdot \frac{1}{2}\right) + \left(-\frac{1}{3} \cdot -\frac{1}{3}\right) + \left(\frac{3}{4} \cdot \frac{3}{4}\right)$$
$$\|\mathbf{w}\| = \frac{1}{4} + \frac{1}{9} + \frac{9}{16}$$
$$\|\mathbf{w}\| = \frac{29}{16}$$
$$\|\mathbf{w}\| = \sqrt{w \cdot w} = \sqrt{\frac{29}{16}}$$

$$\hat{\mathbf{w}} = \frac{\mathbf{w}}{\|\mathbf{w}\|} = \left(\frac{1}{\sqrt{\frac{29}{16}}}, -\frac{1}{\sqrt{\frac{29}{16}}}, \frac{3}{\sqrt{\frac{29}{16}}}\right).$$

5. Let u = (1,2,-2), v=(3,-12,4) and a scalar k = -3

a. The norm (magnitude) of vector u, denoted as $\|$u$\|$, is given by:

$$\|\mathbf{u}\| = \sqrt{1^2 + 2^2 + (-2)^2} = \sqrt{1+4+4} = \sqrt{9} = 3$$

So, $\|\mathbf{u}\| = 3$.

b. The norm (magnitude) of vector v, denoted as $\|$v$\|$, is given by:

$$\|\mathbf{v}\| = \sqrt{3^2 + (-12)^2 + 4^2} = \sqrt{9 + 144 + 16} = \sqrt{169} = 13$$

So, $\|\mathbf{v}\| = 13$.

c. The norm (magnitude) of the vector sum u + v, denoted as $\|$u + v$\|$, is given by:

$$\|\mathbf{u}+\mathbf{v}\| = \|(1+3, 2+(-12), -2+4)\| = \|(4, -10, 2)\| = \sqrt{4^2 + (-10)^2 + 2^2} = \sqrt{16+100+4} = \sqrt{120} = 2\sqrt{30}$$

So, $\|\mathbf{u} + \mathbf{v}\| = 2\sqrt{30}$.

d. The norm (magnitude) of the scalar multiple ku, denoted as $\|k\mathbf{u}\|$, where $k = -3$, is given by:

$$\|k\mathbf{u}\| = \|(-3)\cdot(1,2,-2)\| = \|(-3,-6,6)\| = \sqrt{(-3)^2+(-6)^2+6^2} = \sqrt{9+36+36} = \sqrt{81} = 9$$

So, $\|k\mathbf{u}\| = 9$.

6. Find k so that u and v are orthogonal.

a. u = (3, k, -2) and v = (6,-4,-3)

To check for orthogonality, calculate the dot product:

$$\mathbf{u}\cdot\mathbf{v} = 3\cdot 6 + k\cdot(-4) + (-2)\cdot(-3) = 18 - 4k + 6 = 24 - 4k$$

For the vectors to be orthogonal, u $\cdot$ v must be equal to 0:

$$24 - 4k = 0$$

Solving for k:

$$4k = 24 \implies k = 6$$

b. u = (5,k,-4,2) and v = (1,-3,2,2k)

To check for orthogonality, calculate the dot product:

$$\mathbf{u}\cdot\mathbf{v} = 5\cdot 1 + k\cdot(-3) + (-4)\cdot 2 + 2\cdot 2k = 5 - 3k - 8 + 4k = -3k - 3$$

For the vectors to be orthogonal, u $\cdot$ v must be equal to 0:

$$-3k = 3$$

Solving for k:

$$k = -1$$

c. u = (1,7,k+2,-2) and v = (3,k,-3,k)

To check for orthogonality, calculate the dot product:

$$\mathbf{u}\cdot\mathbf{v} = 1\cdot 3 + 7\cdot k + (k+2)\cdot(-3) + (-2)\cdot k = 3 + 7k - 3k - 6 - 2k$$

For the vectors to be orthogonal, u $\cdot$ v must be equal to 0:

$$3 + 7k - 3k - 6 - 2k = 0$$

Solving for k:

$$-2k = 3$$
$$k = 3/(-2)$$
$$k = -\frac{3}{2}$$