İSTANBUL OKAN ÜNİVERSİTESİ
MÜHENDİSLİK FAKÜLTESİ
MÜHENDİSLİK TEMEL BİLİMLERİ BÖLÜMÜ

2019–20 **MATH216 Mathematics IV – Exercise Sheet 3** N. Course

In the exams, you will typically not be told if an equation is linear, separable, exact, homogeneous, etc – you should be able to determine this yourself. You can use Exercises 15 and 16 to practise.

Exercise 15 (First Order ODEs). Find the general solutions of the following ODEs:

(a) $9yy' + 4x = 0$.

(b) $y' + (x+1)y^3 = 0$.

(c) $\frac{dx}{dt} = 3t(x+1)$.

(d) $y' + \csc y = 0$.

(e) $x' \sin 2t = x \cos 2t$.

(f) $y' = (y-1)\cot x$.

(g) $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right) y = e^{-2x}$.

(h) $(3x^2 + y^2)dx - 2xydy = 0$.

(i) $y' = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$.

(j) $e^{\frac{x}{y}}(y-x)\frac{dy}{dx} + y(1 + e^{\frac{x}{y}}) = 0$.

(k) $(2x+3y)dx + (3x+2y)dy = 0$.

(l) $(x^3 + \frac{y}{x})dx + (y^2 + \ln x)dy = 0$.

(m) $(e^x \sin y + \tan y)dx + (e^x \cos y + x\sec^2 y)dy = 0$.

(n) $ydx + (2x - ye^y)dy = 0$.

(o) $xy' + y = y^{-2}$

(p) $y' = y(xy^3 - 1)$.

(q) $(1+x^2)y' = 2xy(y^3 - 1)$.

Exercise 16 (Initial Value Problems). Solve the following IVPs:

(a) $\begin{cases} y' = x^3 e^{-y} \\ y(2) = 0 \end{cases}$

(b) $\begin{cases} y\frac{dy}{dx} = 4x(y^2+1)^{\frac{1}{2}} \\ y(0) = 1 \end{cases}$

(c) $\begin{cases} y' = y\cot x \\ y(\frac{\pi}{2}) = 2 \end{cases}$

(d) $\begin{cases} y' + 3(y-1) = 2x \\ y(0) = 1 \end{cases}$

(e) $\begin{cases} \frac{dy}{dx} = \frac{10}{(x+y)e^{x+y}} - 1 \\ y(0) = 0 \end{cases}$

(f) $\begin{cases} (4x^2 - 2y^2)y' = 2xy \\ y(3) = -5 \end{cases}$

(g) $\begin{cases} (x-y)dx + (3x+y)dy = 0 \\ y(3) = -2 \end{cases}$

(h) $\begin{cases} \frac{dy}{dx} = \frac{x^3 - xy^2}{x^2 y} \\ y(1) = 1 \end{cases}$

(i) $\begin{cases} (xy+1)ydx + (2y-)dy = 0 \\ y(0) = 3 \end{cases}$.

(j) $\begin{cases} y' - \frac{1}{x}y = y^2 \\ y(1) = 2 \end{cases}$

Exercise 17 (Homogeneous Second Order Linear ODEs with constant coefficients). Solve the following IVPs:

(a) $\begin{cases} y'' - 3y' + 2y = 0 \\ y(0) = 1 \\ y'(0) = 1 \end{cases}$

(b) $\begin{cases} y'' + 4y' + 3y = 0 \\ y(0) = 2 \\ y'(0) = -1 \end{cases}$

(c) $\begin{cases} y'' + 3y' = 0 \\ y(0) = -2 \\ y'(0) = 3 \end{cases}$

(d) $\begin{cases} y'' + 5y' + 3y = 0 \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$

Exercise 18 (Fundamental Sets of Solutions). In each of the following: Verify that y_1 and y_2 are solutions of the given ODE; calculate the Wronskian of y_1 and y_2; and determine if they form a fundamental set of solutions.

(a) $t^2 y'' - 2y = 0; \quad y_1(t) = t^2, \quad y_2(t) = t^{-1}$

(b) $y'' + 4y = 0; \quad y_1(t) = \cos 2t, \quad y_2(t) = \sin 2t$

(c) $y'' - 2y + y = 0; \quad y_1(t) = e^t, \quad y_2(t) = te^t$

(d) $(1 - x\cot x)y'' - xy' + y = 0 \ (0 < x < \pi); \quad y_1(x) = x, \quad y_2(x) = \sin x$