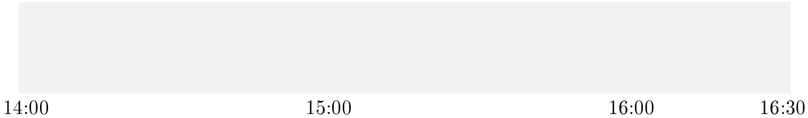
Welcome to

Mathematics I

with Dr Neil Course

Information about this course

- ≈ 12 classes. Friday afternoons 2pm-4:30pm.



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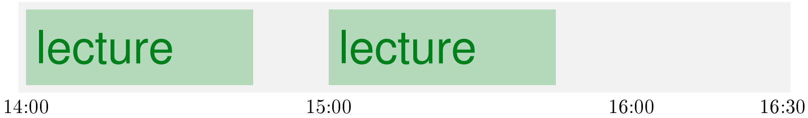
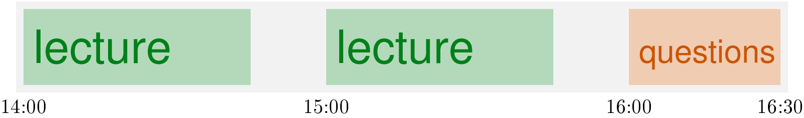
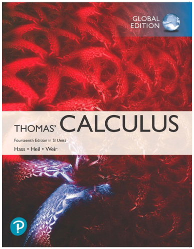


Information about this course

- $\approx$ 12 classes. Friday afternoons 2pm-4:30pm.
- 2 lectures with a break between.
- Then I will answer your questions.



The Book



Joel R. Hass, Christopher E. Heil and Maurice D. Weir,
Thomas' Calculus,
14th Edition in SI Units, Pearson.

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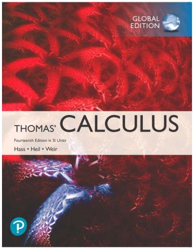

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This is a required purchase.
You need to have this book to be able to do the homework.

Your Mathematics courses:

MATH113 MATH114 MATH215 MATH216

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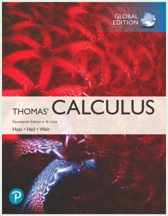
MATH113 MATH114 MATH215 MATH216



Calculus

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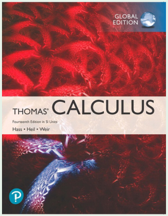
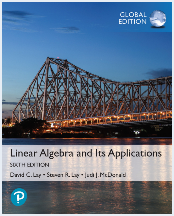
MATH113 MATH114 MATH215 MATH216



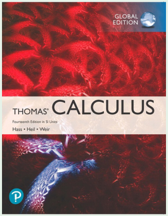
Calculus



Linear Algebra

Your Mathematics courses:

MATH113 MATH114



Calculus

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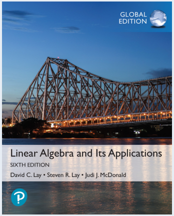
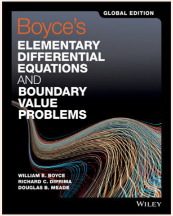


Linear Algebra

MATH216



Differential Equations

1

2

3

4

5

6

7

1. Functions

2

3

4

5

6

7

1. Functions

2. Limits and Continuity

3

4

5

6

7

1. Functions

2. Limits and Continuity

3. Derivatives

4

5

6

7

1. Functions
2. Limits and Continuity
3. Derivatives
4. Applications of Derivatives
5
6
7

1. Functions

2. Limits and Continuity

3. Derivatives

4. Applications of Derivatives

5. Integrals

6

7

1. Functions
2. Limits and Continuity
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5. Integrals
6. Applications of Integrals
7

1. Functions
2. Limits and Continuity
3. Derivatives
4. Applications of Derivatives
5. Integrals
6. Applications of Integrals
7. Trancendental Functions

Topic	Duration
1. Functions	1 week
2. Limits and Continuity	2 weeks
3. Derivatives	4 weeks
4. Applications of Derivatives	
5. Integrals	3 weeks
6. Applications of Integrals	
7. Trancendental Functions	2 weeks

Exams and homework

(This information may change based on the University's decisions)

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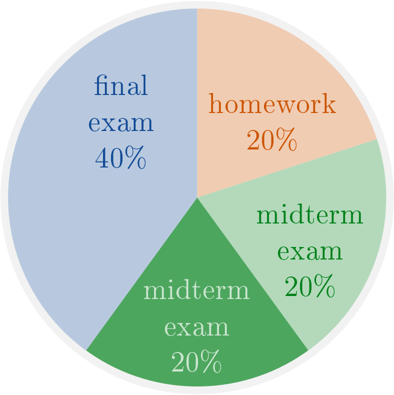

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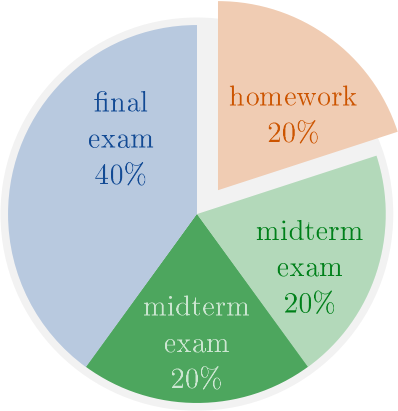

using Pearson MyLab Math

one piece of homework for each lesson

deadline = end of term

more details in O'Learn

Exams and homework

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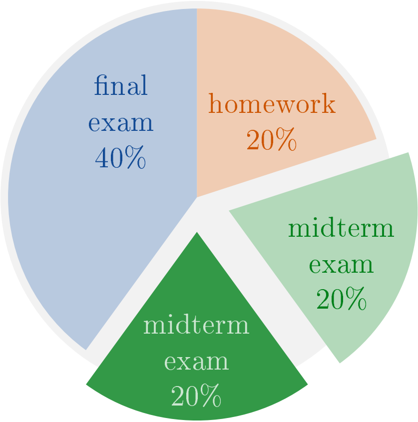

2 midterm exams
(face-to-face)

Exams and homework

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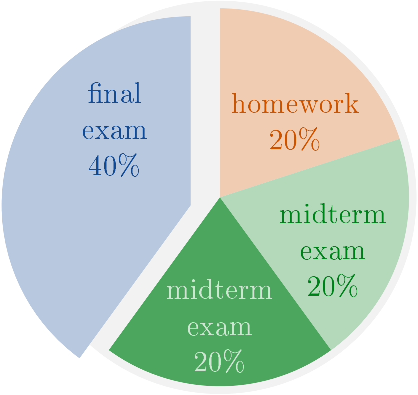
(face-to-face)



Expectations

If this was a classroom course, the expectation is that for each hour of lectures, you would study an extra 1-2 hours outside of class.

classroom course	lectures (5 hours)	other study (5-10 hours)

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For an online course, you are still expected to study a total of 10-15 hours each week.

online course	class (2.5 hours)	other study (7.5-12.5 hours)

This may include:

- Do the online homework on MyLab;

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- Watch online videos;

$\vdots$

slide number

99.9 Section Title

current section

Lecture 1

- Information about this course
- A.1 Real Numbers and the Real Line
- 1.1 Functions and Their Graphs
- 1.2 Combining Functions; Shifting and Scaling Graphs
- 1.3 Trigonometric Functions

The Natural Numbers

The set

$$\mathbb{N} = \{1, 2, 3, 4, 5, 6, \dots\}$$

is called the set of *natural numbers*. These are the first numbers that children learn. For example

$$\begin{aligned}
2 \in \mathbb{N} &\quad \text{means "2 is a natural number"} \\
7 \in \mathbb{N} &\quad \text{means "7 is a natural number"} \\
\tfrac{1}{2} \notin \mathbb{N} &\quad \text{means "}\tfrac{1}{2}\text{ is } \textbf{not} \text{ a natural number"} \\
0 \notin \mathbb{N} &\quad \text{means "0 is } \textbf{not} \text{ a natural number"} \\
-5 \notin \mathbb{N} &\quad \text{means "}-5\text{ is } \textbf{not} \text{ a natural number"}
\end{aligned}$$

In the natural numbers, we can do "+" and "×"

$$2 + 7 = 9 \in \mathbb{N}, \qquad\qquad 2 \times 7 = 14 \in \mathbb{N}.$$

However we can not do "−" because

$$2 - 7 \notin \mathbb{N}.$$

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So we invent new numbers!

The Integers

The set

$$\mathbb{Z} = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$

is called the set of *integers*. We use a $\mathbb{Z}$ for the German word 'zahlen' (numbers).

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$$3+4 \in \mathbb{Z}, \qquad 3-4 \in \mathbb{Z}, \qquad 3 \times 4 \in \mathbb{Z}, \qquad 3 \div 4 \notin \mathbb{Z},$$

$$3+(-5) \in \mathbb{Z}, \quad 3-(-5) \in \mathbb{Z}, \quad 3 \times (-5) \in \mathbb{Z}, \quad 3 \div (-5) \notin \mathbb{Z}.$$

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The Rational Numbers

The set

$$\mathbb{Q} = \{\text{all fractions}\} = \left\{ \frac{a}{b} \;\middle|\; a, b \in \mathbb{Z} \text{ and } b \neq 0 \right\}$$

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$$0 = \frac{0}{1} \in \mathbb{Q} \qquad \frac{100}{13} \in \mathbb{Q}$$

$$1 = \frac{1}{1} \in \mathbb{Q} \qquad \sqrt{2} \notin \mathbb{Q}$$

$$\frac{3}{4} \in \mathbb{Q} \qquad -4 = \frac{8}{-2} \in \mathbb{Q}$$

$$\pi \notin \mathbb{Q} \qquad 0.12345 = \frac{12345}{100000} \in \mathbb{Q}.$$

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In $\mathbb{Q}$ we can do "$+$", "$-$", "$\times$" and "$\div$(by a number $\neq 0$)".

Are we happy now?

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No!

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Why?

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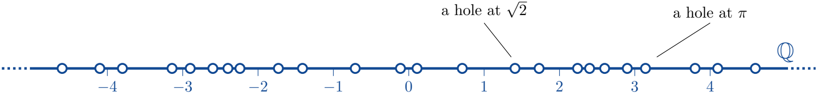
Why?

Because if we draw all the rational numbers in a line, then the line has lots of holes in it. In fact, $\mathbb{Q}$ has ∞ many holes in it.



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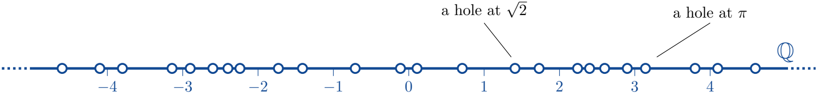

So we invent new numbers!

The Real Numbers

The set

$$\mathbb{R} = \{\text{all numbers which can be written as a decimal}\}$$

is called the set of *real numbers*. For example

$$\begin{aligned} 0 = 0.0 &\in \mathbb{R} & \frac{100}{13} &= 7.692307\ldots \in \mathbb{R} \\ \frac{23}{99} = 0.232323\ldots &\in \mathbb{R} & \sqrt{2} &= 1.414213\ldots \in \mathbb{R} \\ \frac{3}{4} = 0.75 &\in \mathbb{R} & \frac{123}{999} &= 0.123123\ldots \in \mathbb{R} \\ \pi = 3.141592\ldots &\in \mathbb{R} & \frac{12345}{100000} &= 0.12345 \in \mathbb{R}. \end{aligned}$$

The real numbers are complete – this means that if we draw all the real numbers in a line, then there are no holes in the line.

Are we happy now?

Are we happy now?

Yes! Now we have enough numbers to do Calculus.

Intervals

Definition

A subset of $\mathbb{R}$ is called an *interval* if

1. it contains atleast 2 numbers; and
2. it doesn't have any holes in it.

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Example

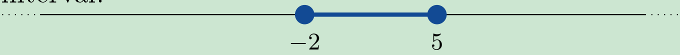
The set $\{x \mid x \text{ is a real number and } x > 6\}$ is an interval.

Because 6 is not in this set, we use $\circ$ at 6.

A.1 Real Numbers and the Real Line

Example

The set of all real numbers x such that $-2 \leq x \leq 5$ is an interval.



Because -2 and 5 are in this set, we use ● at -2 and 5.

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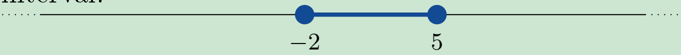

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Example

The set $\{x \mid x \in \mathbb{R} \text{ and } x \neq 0\}$ is not an interval.

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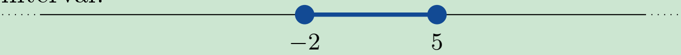

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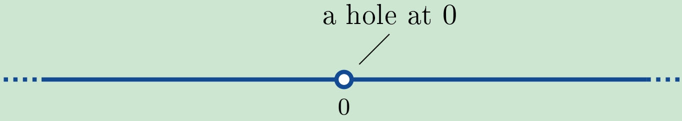


A finite interval is

- *closed* if it contains both its endpoints;
- *half-open* if it contains one of its endpoints;
- *open* if it does not contain its endpoints;

A.1 Real Numbers and the Real Line

Notation	Set	Type	Picture
(a, b)	$\{x \mid a < x < b\}$	open	
$[a, b]$	$\{x \mid a \leq x \leq b\}$	closed	
$[a, b)$	$\{x \mid a \leq x < b\}$	half open	
$(a, b]$	$\{x \mid a < x \leq b\}$	half open	

An infinite interval is

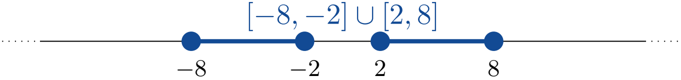
- *closed* if it contains a finite endpoint;
- *open* if it is not closed.

There is one exception to this rule: The whole real line is called both open and closed.

A.1 Real Numbers and the Real Line

Notation	Set	Type	Picture
(a, ∞)	$\{x \mid a < x\}$	open	
$[a, \infty)$	$\{x \mid a \leq x\}$	closed	
$(-\infty, b)$	$\{x \mid x < b\}$	open	
$(-\infty, b]$	$\{x \mid x \leq b\}$	closed	
$(-\infty, \infty)$	$\mathbb{R}$	both open and closed	

We can combine two (or more) intervals with the notation $\cup$.
For example, $[-8,-2] \cup [2,8]$ is called the *union* of $[-8,-2]$ and $[2,8]$ and is shown below.



Functions and Their Graphs

$$y = f(x)$$

"y eşittir f x"

"y is equal to f of x"

dependent variable

$$y = f(x)$$

function

independent variable

"y eşittir f x"

"y is equal to f of x"

Definition

A *function* from a set D to a set Y is a rule that assigns a unique element of Y to each element of D.

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Definition

The set D of all possible values of x is called the *domain* of f.

Definition

The set Y is called the *target* of f.

Definition

The set of all possible values of $f(x)$ is called the *range* of f.

If f is a function with domain D and target Y, we can write

$$f : \underbrace{D}_{\text{domain}} \to \underbrace{Y}_{\text{target}}$$

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$f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$.

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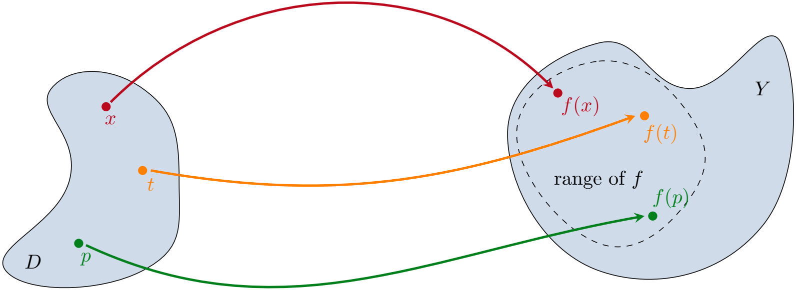
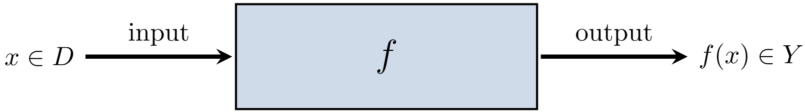
Example

$f : (-\infty, \infty) \to [0, \infty),\ f(x) = x^2.$



x
t
p
D
$f(x)$
$f(t)$
$f(p)$
range of f
Y


$x \in D$
input
f
output
$f(x) \in Y$

Sometimes we want to use the largest possible domain for a function. This is called the *natural domain* of the function.

Sometimes we will want to use a smaller domain.

function	domain (x)	range (y)
$y = x^2$	$(-\infty, \infty)$	
$y = \frac{1}{x}$	$\{x \mid x \in \mathbb{R}, x \neq 0\}$	
$y = \sqrt{x}$	$[0, \infty)$	
$y = \sqrt{4 - x}$		
$y = \sqrt{1 - x^2}$		

function	domain (x)	range (y)
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
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$y = \sqrt{4 - x}$	$(-\infty, 4]$	
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$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$		

function	domain (x)	range (y)
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = \frac{1}{x}$	$\{x \mid x \in \mathbb{R}, x \neq 0\}$	$\{y \mid y \in \mathbb{R}, y \neq 0\}$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	

function	domain (x)	range (y)
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = \frac{1}{x}$	$\{x \mid x \in \mathbb{R}, x \neq 0\}$	$\{y \mid y \in \mathbb{R}, y \neq 0\}$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4-x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1-x^2}$	$[-1, 1]$	$[0, 1]$

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$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = \frac{1}{x}$	$\{x \mid x \in \mathbb{R}, x \neq 0\}$	$\{y \mid y \in \mathbb{R}, y \neq 0\}$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$
$y = x^2$	$[1, 2]$	
$y = x^2$	$[2, \infty)$	
$y = x^2$	$(-\infty, -2]$	

function	domain (x)	range (y)
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = \frac{1}{x}$	$\{x \mid x \in \mathbb{R}, x \neq 0\}$	$\{y \mid y \in \mathbb{R}, y \neq 0\}$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$
$y = x^2$	$[1, 2]$	$[1, 4]$
$y = x^2$	$[2, \infty)$	
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$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
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$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$
$y = x^2$	$[1, 2]$	$[1, 4]$
$y = x^2$	$[2, \infty)$	$[4, \infty)$
$y = x^2$	$(-\infty, -2]$	

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$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$
$y = x^2$	$[1, 2]$	$[1, 4]$
$y = x^2$	$[2, \infty)$	$[4, \infty)$
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$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
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$y = x^2$	$[1, 2]$	$[1, 4]$
$y = x^2$	$[2, \infty)$	$[4, \infty)$
$y = x^2$	$(-\infty, -2]$	$[4, \infty)$
$y = 1 + x^2$	$[1, 3)$	
$y = 1 - \sqrt{x}$	$[0, \infty)$	

function	domain (x)	range (y)
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = \frac{1}{x}$	$\{x \mid x \in \mathbb{R}, x \neq 0\}$	$\{y \mid y \in \mathbb{R}, y \neq 0\}$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$
$y = x^2$	$[1, 2]$	$[1, 4]$
$y = x^2$	$[2, \infty)$	$[4, \infty)$
$y = x^2$	$(-\infty, -2]$	$[4, \infty)$
$y = 1 + x^2$	$[1, 3)$	$[2, 10)$
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$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
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$y = x^2$	$[1, 2]$	$[1, 4]$
$y = x^2$	$[2, \infty)$	$[4, \infty)$
$y = x^2$	$(-\infty, -2]$	$[4, \infty)$
$y = 1 + x^2$	$[1, 3)$	$[2, 10)$
$y = 1 - \sqrt{x}$	$[0, \infty)$	$(-\infty, 1]$

Graphs of Functions

Definition

The *graph* of f is the set containing all the points (x, y) which satisty $y = f(x)$.

Example

Graph the function $y = 1 + x^2$ over the interval $[-2, 2]$.

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STEP 1: Make a table of (x, y) points which satisfy $y = 1 + x^2$.

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STEP 1: Make a table of (x, y) points which satisfy $y = 1 + x^2$.

x	y
-2	5
-1	2
0	1
1	2
$\frac{3}{2}$	$\frac{13}{4} = 3\frac{1}{4}$
2	5

STEP 2: Plot these points.

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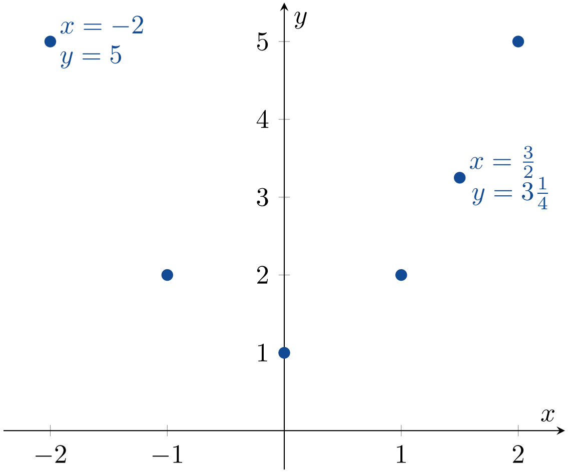
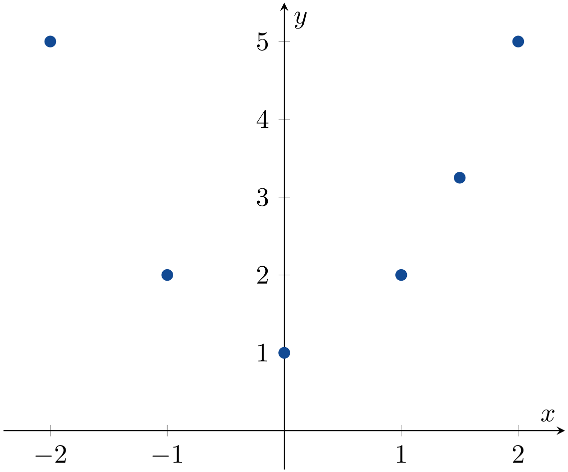
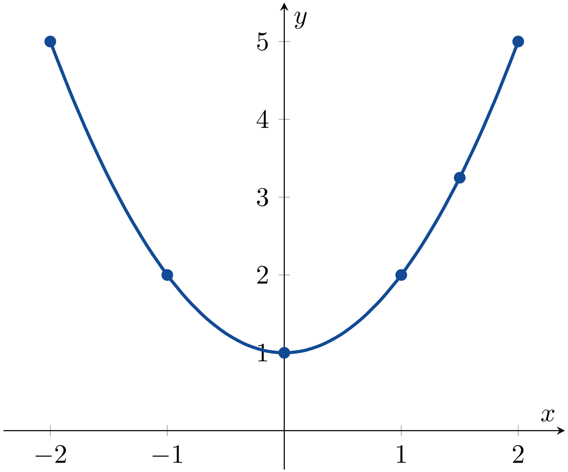


STEP 3: Draw a smooth curve through these points.

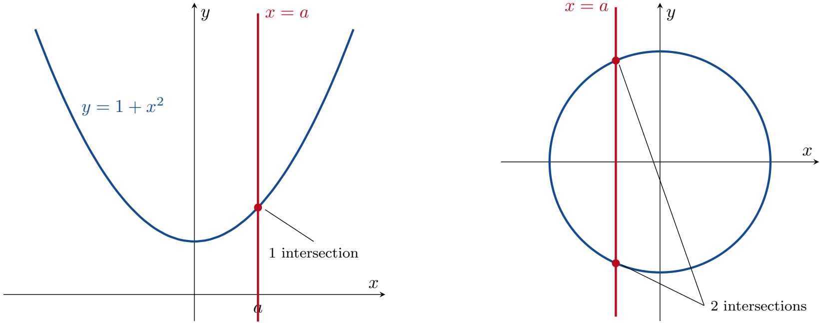


STEP 3: Draw a smooth curve through these points.

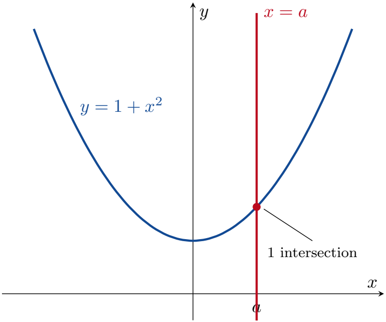


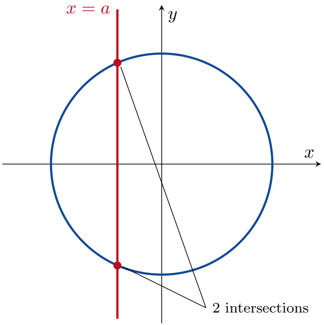
The Vertical Line Test

Not every curve that you draw is a graph of a function.

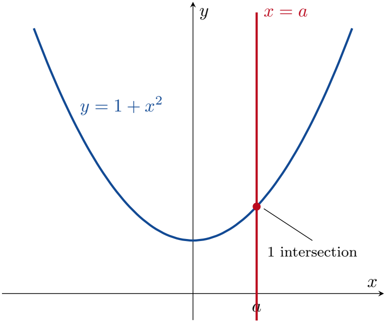


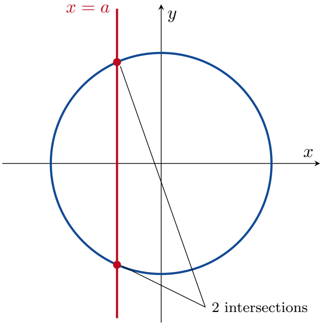
A function can have only one value $f(x)$ for each $x \in D$. This means that a vertical line can intersect the graph of a function at most once.





A circle can not be the graph of a function because some vertical lines intersect the circle at two points.



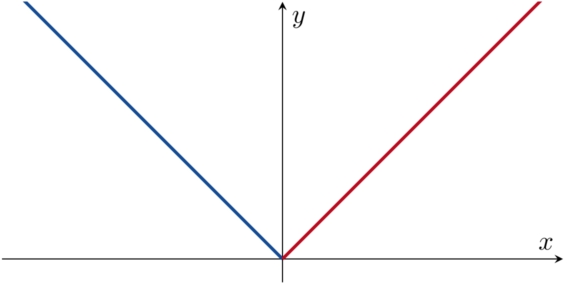


If $a \in D$, then the vertical line $x = a$ will intersect the graph of $f : D \to Y$ only at the point $(a, f(a))$.

Piecewise-Defined Functions

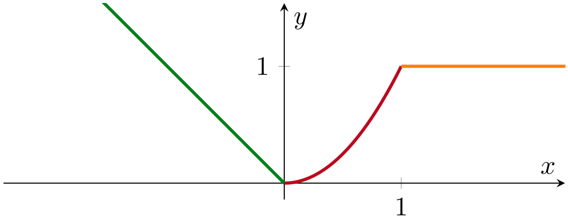
Example

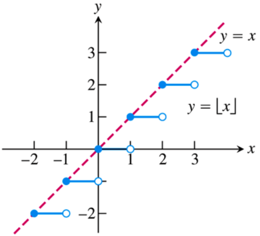
$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$



Example

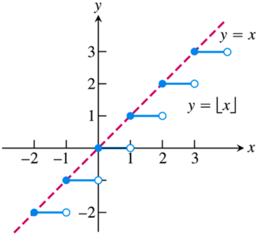
$$f(x) = \begin{cases} -x & x < 0 \\ x^2 & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

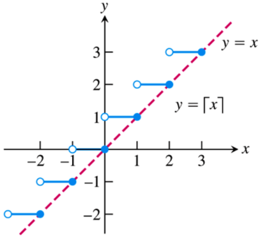




EXAMPLE 5 The function whose value at any number x is the *greatest integer less than or equal to* x is called the **greatest integer function** or the **integer floor function**. It is denoted $\lfloor x \rfloor$. Figure 1.10 shows the graph. Observe that

$$\lfloor 2.4 \rfloor = 2, \quad \lfloor 1.9 \rfloor = 1, \quad \lfloor 0 \rfloor = 0, \quad \lfloor -1.2 \rfloor = -2,$$
$$\lfloor 2 \rfloor = 2, \quad \lfloor 0.2 \rfloor = 0, \quad \lfloor -0.3 \rfloor = -1, \quad \lfloor -2 \rfloor = -2.$$





EXAMPLE 5 The function whose value at any number x is the *greatest integer less than or equal to* x is called the **greatest integer function** or the **integer floor function**. It is denoted $\lfloor x \rfloor$. Figure 1.10 shows the graph. Observe that

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$$\lfloor 2 \rfloor = 2, \quad \lfloor 0.2 \rfloor = 0, \quad \lfloor -0.3 \rfloor = -1, \quad \lfloor -2 \rfloor = -2.$$

EXAMPLE 6 The function whose value at any number x is the *smallest integer greater than or equal to* x is called the **least integer function** or the **integer ceiling function**. It is denoted $\lceil x \rceil$. Figure 1.11 shows the graph. For positive values of x, this function might represent, for example, the cost of parking x hours in a parking lot that charges \$1 for each hour or part of an hour.

Increasing and Decreasing Functions

Definition

Let I be an interval. Let $f : I \to \mathbb{R}$ be a function.

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1. f is called *increasing on* I if

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for all $x_1, x_2 \in I$ which satisfy $x_1 < x_2$;

Increasing and Decreasing Functions

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Let I be an interval. Let $f : I \to \mathbb{R}$ be a function.

1. f is called *increasing on I* if
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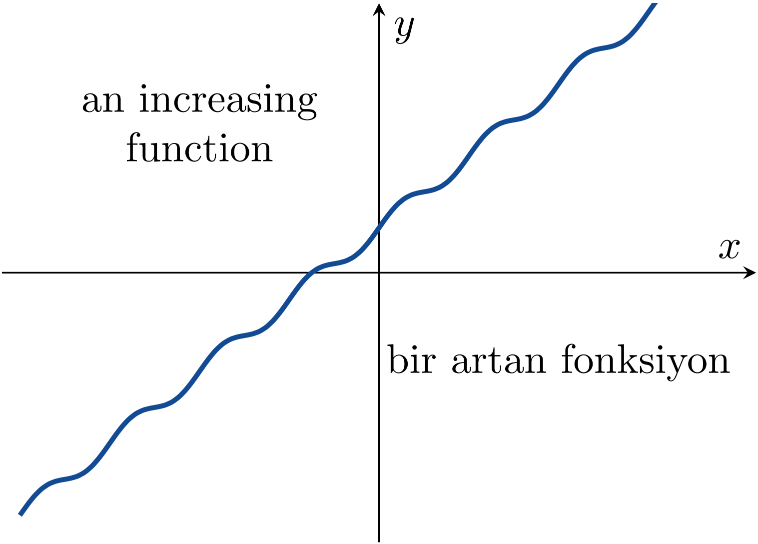
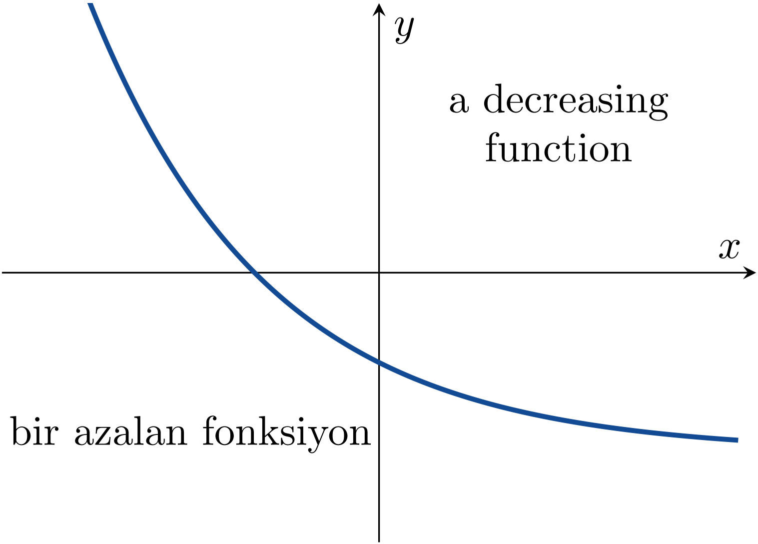
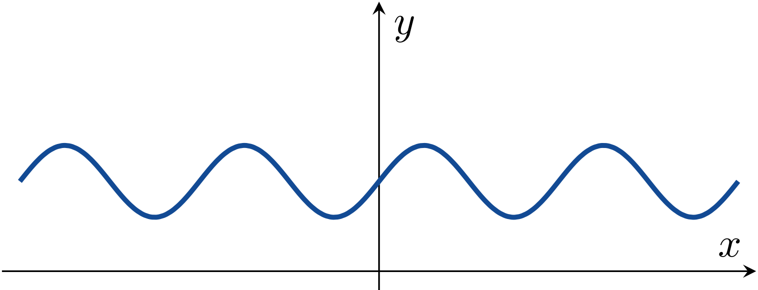
for all $x_1, x_2 \in I$ which satisfy $x_1 < x_2$;
2. f is called *decreasing on I* if
$$f(x_1) > f(x_2)$$
for all $x_1, x_2 \in I$ which satisfy $x_1 < x_2$.


y
an increasing
function
x
bir artan fonksiyon


y
a decreasing function
x
bir azalan fonksiyon



not increasing not decreasing	artmıyor azalmıyor

Even Functions and Odd Functions

Recall that

- $2, 4, 6, 8, 10, \dots$ are even numbers; and
- $1, 3, 5, 7, 9, \dots$ are odd numbers.

Even Functions and Odd Functions

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- $2, 4, 6, 8, 10, \dots$ are even numbers; and
- $1, 3, 5, 7, 9, \dots$ are odd numbers.

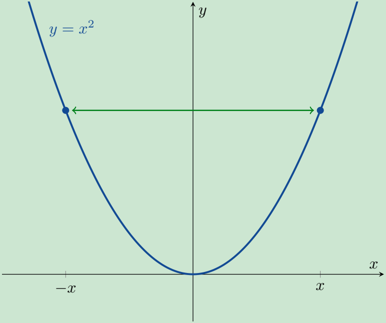
Definition

1. $f : D \to \mathbb{R}$ is an *even function* if $f(-x) = f(x)$ for all $x \in D$;
2. $f : D \to \mathbb{R}$ is an *odd function* if $f(-x) = -f(x)$ for all $x \in D$.

Example

$f(x) = x^2$ is an even function because

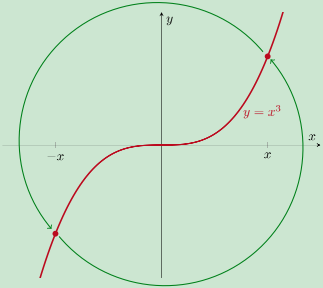
$$f(-x) = (-x)^2 = x^2 = f(x).$$



Example

$f(x) = x^3$ is an odd function because

$$f(-x) = (-x)^3 = -x^3 = -f(x).$$



Example

Is $f(x) = x^2 + 1$ even, odd or neither?

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Is $f(x) = x^2 + 1$ even, odd or neither?

Since

$$f(-x) = (-x)^2 + 1 = x^2 + 1 = f(x),$$

f is an even function.

Example

Is $g(x) = x + 1$ even, odd or neither?

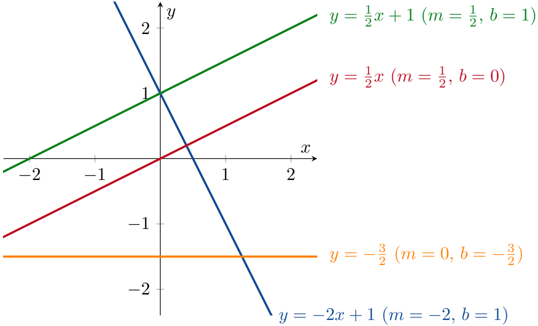
Example

Is $g(x) = x + 1$ even, odd or neither?

Since $g(-2) = -2 + 1 = -1$ and $g(2) = 3$, we have $g(-2) \neq g(2)$ and $g(-2) \neq -g(2)$. Hence g is neither even nor odd.

Linear Functions

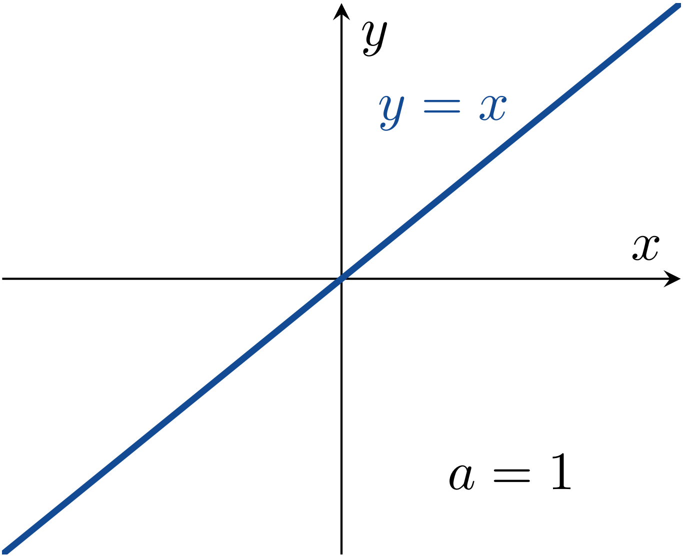
$$f(x) = mx + b$$
$$(m, b \in \mathbb{R})$$



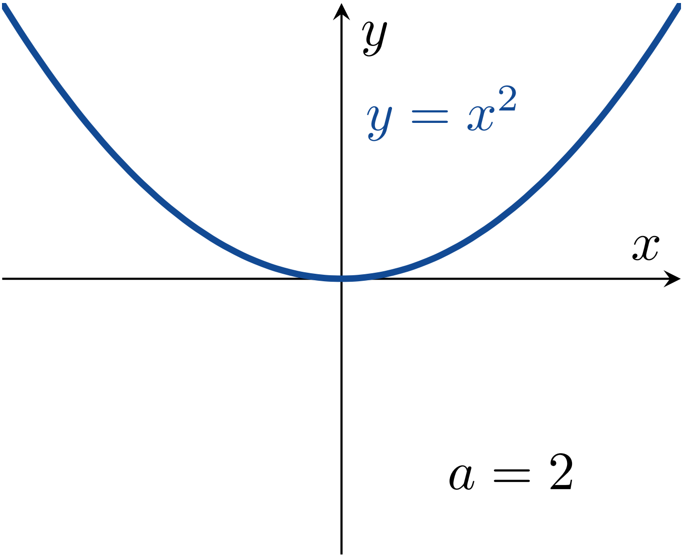
Power Functions

$$f(x) = x^a$$

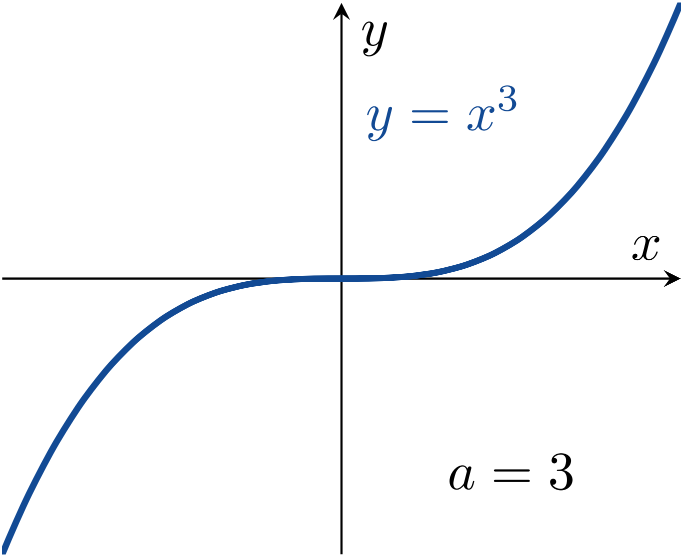
$(a \in \mathbb{R})$
"x to the power of a"



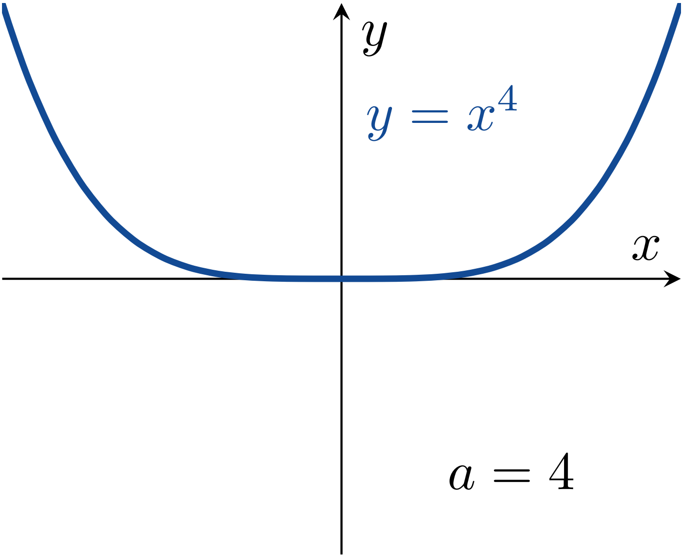
y
$y = x$
x
$a = 1$



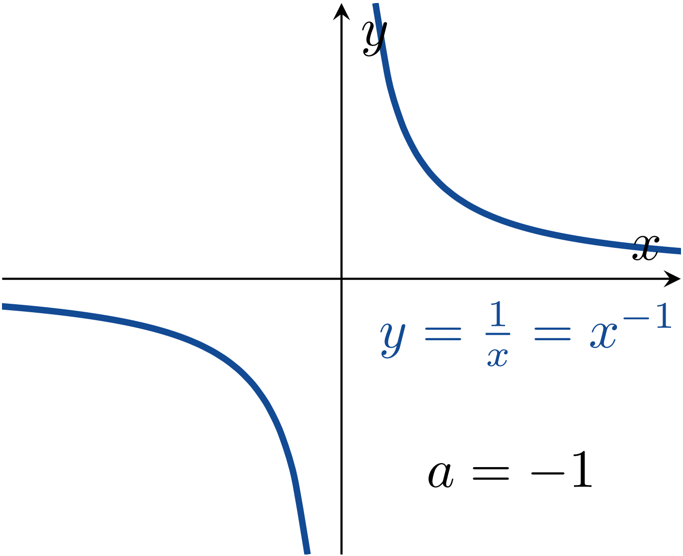
$a = 2$



$a = 3$

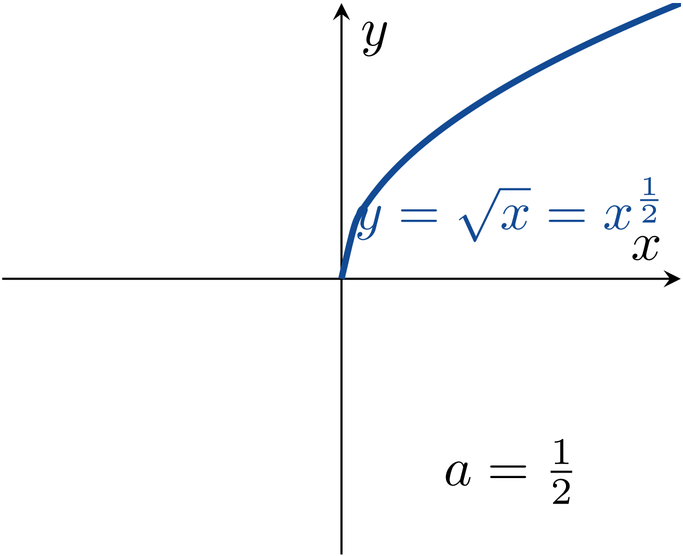


$$a = 4$$

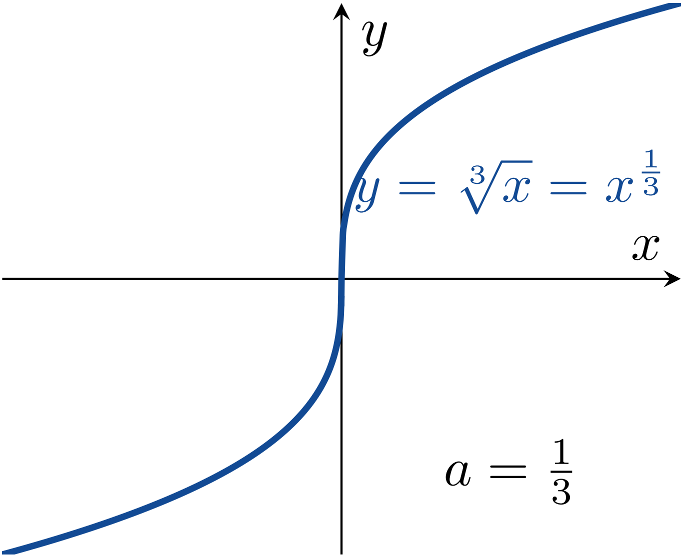


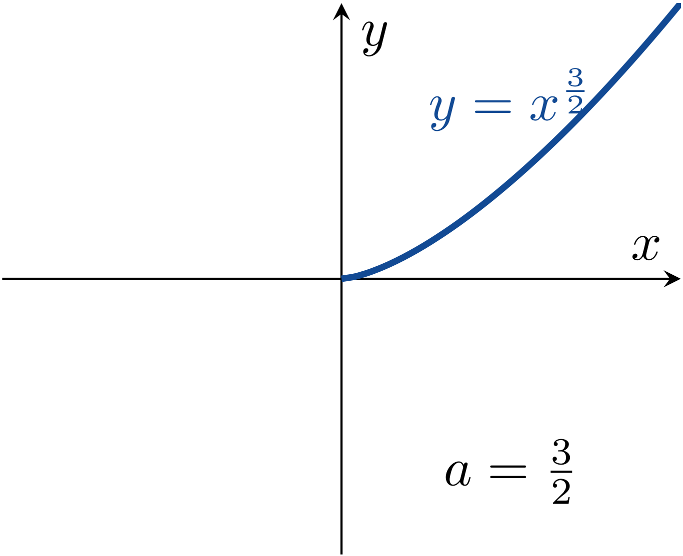
$$y = \frac{1}{x} = x^{-1}$$

$$a = -1$$

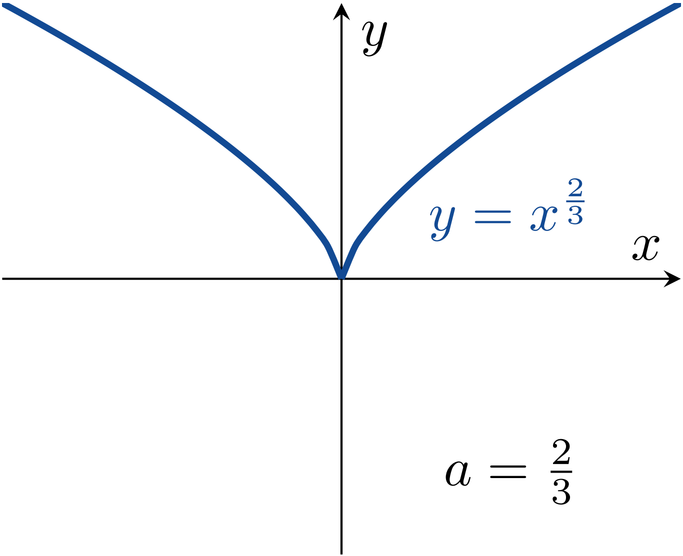


$$a = \frac{1}{2}$$


y
$y = \sqrt[3]{x} = x^{\frac{1}{3}}$
x
$a = \frac{1}{3}$



$$a = \frac{3}{2}$$



$$a = \frac{2}{3}$$

Polynomials

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

$$(n \in \mathbb{N} \cup \{0\},\ a_j \in \mathbb{R}).$$

Polynomials

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

$$(n \in \mathbb{N} \cup \{0\},\ a_j \in \mathbb{R}).$$

The (natural) domain of a polynomial is always $(-\infty, \infty)$. If $n > 0$ and $a_n \neq 0$, then n is called the *degree* of $p(x)$.

Rational Functions

$$\underbrace{f(x)}_{\text{rational function}} = \frac{p(x)}{q(x)}$$

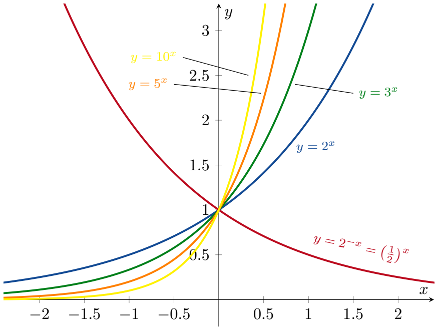
polynomial

Example

$$f(x) = \frac{2x^3 - 3}{7x + 4}$$

Exponential Functions

$$f(x) = a^x$$
$$(a \in \mathbb{R},\ a > 0,\ a \neq 1)$$



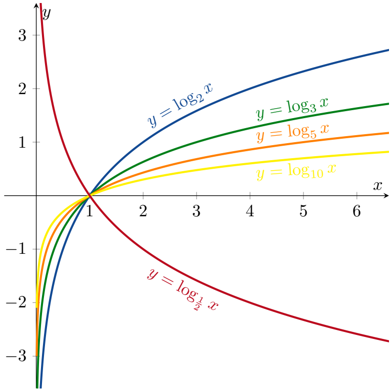
The (natural) domain of an exponential function is $(-\infty, \infty)$.

Logarithmic Functions

$$y = \log_a x \iff x = a^y$$

$$(a \in \mathbb{R},\ a > 0,\ a \neq 1)$$

"log base a of x"



Break

We will continue at 2pm

Combining Functions; Shifting and Scaling Graphs

12

Sums, Differences, Products and Quotients

Consider $f : D(f) \to \mathbb{R}$ and $g : D(g) \to \mathbb{R}$.

Sums, Differences, Products and Quotients

Consider $f : D(f) \to \mathbb{R}$ and $g : D(g) \to \mathbb{R}$. We can define 4 new functions:

$$(f+g)(x) = f(x) + g(x)$$
$$(f-g)(x) = f(x) - g(x)$$
$$(fg)(x) = f(x)g(x)$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

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The domain of $(f+g)$, $(f-g)$ and (fg) is $D(f) \cap D(g)$.

Sums, Differences, Products and Quotients

Consider $f : D(f) \to \mathbb{R}$ and $g : D(g) \to \mathbb{R}$. We can define 4 new functions:

$$\begin{aligned}
(f+g)(x) &= f(x)+g(x)\\
(f-g)(x) &= f(x)-g(x)\\
(fg)(x) &= f(x)g(x)\\
\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}
\end{aligned}$$

The domain of $(f+g)$, $(f-g)$ and (fg) is $D(f) \cap D(g)$.
The domain of $\left(\frac{f}{g}\right)$ is $D(f) \cap \{x \in D(g) \mid g(x) \neq 0\}$.

Example

Consider $f : [0, \infty) \to \mathbb{R}$, $f(x) = \sqrt{x}$ and $g : (-\infty, 1] \to \mathbb{R}$, $g(x) = \sqrt{1-x}$.

Example

Consider $f : [0, \infty) \to \mathbb{R}$, $f(x) = \sqrt{x}$ and $g : (-\infty, 1] \to \mathbb{R}$, $g(x) = \sqrt{1 - x}$. Find $(f + g)$, $(f - g)$, (fg), $\left(\frac{f}{g}\right)$ and $\left(\frac{g}{f}\right)$ and find the domains of these 5 functions.

$$D(f) = [0, \infty),\ f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1],\ g(x) = \sqrt{1-x}$$

Function	**Formula**	**Domain**
$f+g$		
$f-g$		
fg		
$\frac{f}{g}$		
$\frac{g}{f}$		

$D(f) = [0, \infty),\ f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1],\ g(x) = \sqrt{1-x}$

Function	Formula	Domain
$f+g$	$\sqrt{x} + \sqrt{1-x}$	
$f-g$		
fg		
$\frac{f}{g}$		
$\frac{g}{f}$		

$D(f) = [0, \infty),\ f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1],\ g(x) = \sqrt{1-x}$

Function	Formula	Domain
$f+g$	$\sqrt{x} + \sqrt{1-x}$	$[0,1]$
$f-g$		
fg		
$\frac{f}{g}$		
$\frac{g}{f}$		

$D(f) = [0, \infty),\ f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1],\ g(x) = \sqrt{1-x}$

Function	**Formula**	**Domain**
$f+g$	$\sqrt{x} + \sqrt{1-x}$	$[0,1]$
$f-g$	$\sqrt{x} - \sqrt{1-x}$	$[0,1]$
fg	$\sqrt{x}\sqrt{1-x} = \sqrt{x-x^2}$	$[0,1]$
$\frac{f}{g}$		
$\frac{g}{f}$		

1.2 Combining Functions; Shifting and Scaling Graphs

$$D(f) = [0, \infty),\ f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1],\ g(x) = \sqrt{1-x}$$

Function	**Formula**	**Domain**
$f+g$	$\sqrt{x} + \sqrt{1-x}$	$[0, 1]$
$f-g$	$\sqrt{x} - \sqrt{1-x}$	$[0, 1]$
fg	$\sqrt{x}\sqrt{1-x} = \sqrt{x - x^2}$	$[0, 1]$
$\frac{f}{g}$	$\sqrt{\frac{x}{1-x}}$	
$\frac{g}{f}$	$\sqrt{\frac{1-x}{x}}$	

$D(f) = [0, \infty),\ f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1],\ g(x) = \sqrt{1-x}$

Function	**Formula**	**Domain**
$f+g$	$\sqrt{x}+\sqrt{1-x}$	$[0,1]$
$f-g$	$\sqrt{x}-\sqrt{1-x}$	$[0,1]$
fg	$\sqrt{x}\sqrt{1-x}=\sqrt{x-x^2}$	$[0,1]$
$\frac{f}{g}$	$\sqrt{\frac{x}{1-x}}$	$[0,1)$
$\frac{g}{f}$	$\sqrt{\frac{1-x}{x}}$	

$D(f) = [0, \infty),\ f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1],\ g(x) = \sqrt{1-x}$

Function	Formula	Domain
$f+g$	$\sqrt{x} + \sqrt{1-x}$	$[0,1]$
$f-g$	$\sqrt{x} - \sqrt{1-x}$	$[0,1]$
fg	$\sqrt{x}\sqrt{1-x} = \sqrt{x-x^2}$	$[0,1]$
$\frac{f}{g}$	$\sqrt{\frac{x}{1-x}}$	$[0,1)$
$\frac{g}{f}$	$\sqrt{\frac{1-x}{x}}$	$(0,1]$

Composite Functions

$$f \circ g(x) = f(g(x))$$

is the *composite* of f and g.

We read $f \circ g$ as "f composed with g".

Composite Functions

$$f \circ g(x) = f(g(x))$$

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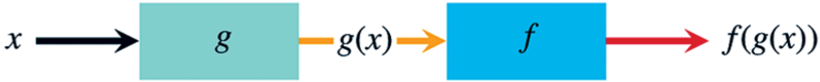
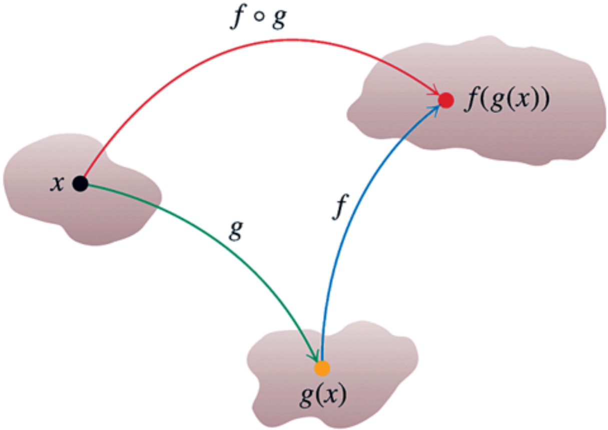
We read $f \circ g$ as "f composed with g".

The domain of $f \circ g$ consists of all numbers x in the domain of g for which the number $g(x)$ lies in the domain of f.


x
g
g(x)
f
f(g(x))


$f \circ g$
$f(g(x))$
x
f
g
$g(x)$

Example

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$, and find the domains of these 4 functions.

Note that the domain of f is $[0, \infty)$ and the domain of g is $(-\infty, \infty)$.

Function	Formula	Domain
$f \circ g$		
$g \circ f$		
$f \circ f$		
$g \circ g$		

Example

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$, and find the domains of these 4 functions.

Note that the domain of f is $[0, \infty)$ and the domain of g is $(-\infty, \infty)$.

Function	**Formula**	**Domain**
$f \circ g$	$f(g(x)) = \sqrt{x+1}$	
$g \circ f$		
$f \circ f$		
$g \circ g$		

Example

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$, and find the domains of these 4 functions.

Note that the domain of f is $[0, \infty)$ and the domain of g is $(-\infty, \infty)$.

Function	**Formula**	**Domain**
$f \circ g$	$f(g(x)) = \sqrt{x+1}$	$[-1, \infty)$
$g \circ f$		
$f \circ f$		
$g \circ g$		

Example

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$, and find the domains of these 4 functions.

Note that the domain of f is $[0, \infty)$ and the domain of g is $(-\infty, \infty)$.

Function	**Formula**	**Domain**
$f \circ g$	$f(g(x)) = \sqrt{x+1}$	$[-1, \infty)$
$g \circ f$	$g(f(x)) = \sqrt{x} + 1$	$[0, \infty)$
$f \circ f$		
$g \circ g$		

Example

If $f(x)=\sqrt{x}$ and $g(x)=x+1$, find $f\circ g$, $g\circ f$, $f\circ f$ and $g\circ g$, and find the domains of these 4 functions.

Note that the domain of f is $[0,\infty)$ and the domain of g is $(-\infty,\infty)$.

Function	Formula	Domain
$f\circ g$	$f(g(x))=\sqrt{x+1}$	$[-1,\infty)$
$g\circ f$	$g(f(x))=\sqrt{x}+1$	$[0,\infty)$
$f\circ f$	$f(f(x))=\sqrt{\sqrt{x}}=x^{\frac{1}{4}}$	$[0,\infty)$
$g\circ g$		

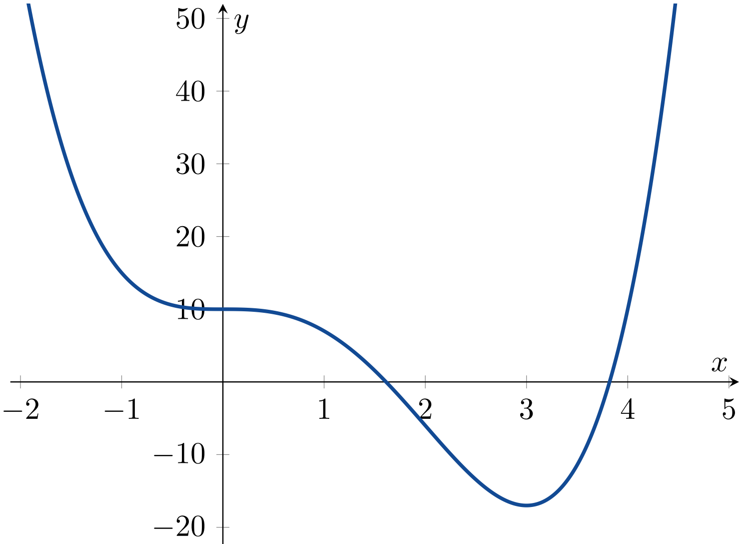
Example

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$, and find the domains of these 4 functions.

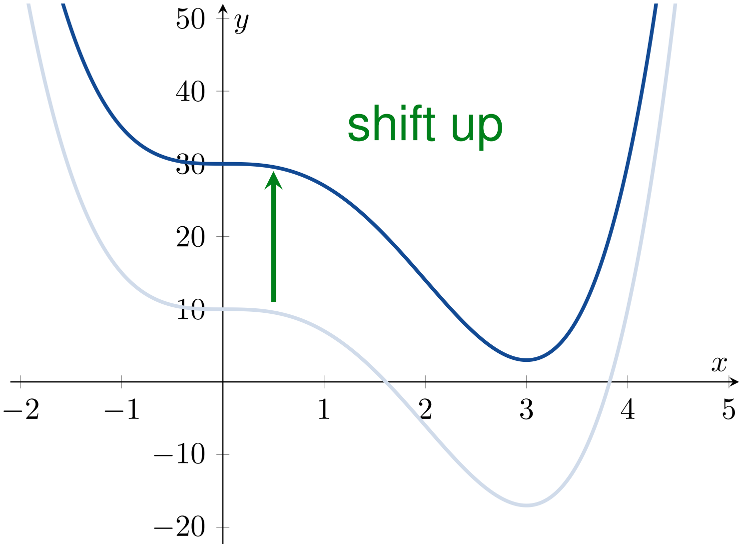
Note that the domain of f is $[0, \infty)$ and the domain of g is $(-\infty, \infty)$.

Function	Formula	Domain
$f \circ g$	$f(g(x)) = \sqrt{x+1}$	$[-1, \infty)$
$g \circ f$	$g(f(x)) = \sqrt{x} + 1$	$[0, \infty)$
$f \circ f$	$f(f(x)) = \sqrt{\sqrt{x}} = x^{\frac{1}{4}}$	$[0, \infty)$
$g \circ g$	$g(g(x)) = (x+1) + 1 = x + 2$	$(-\infty, \infty)$

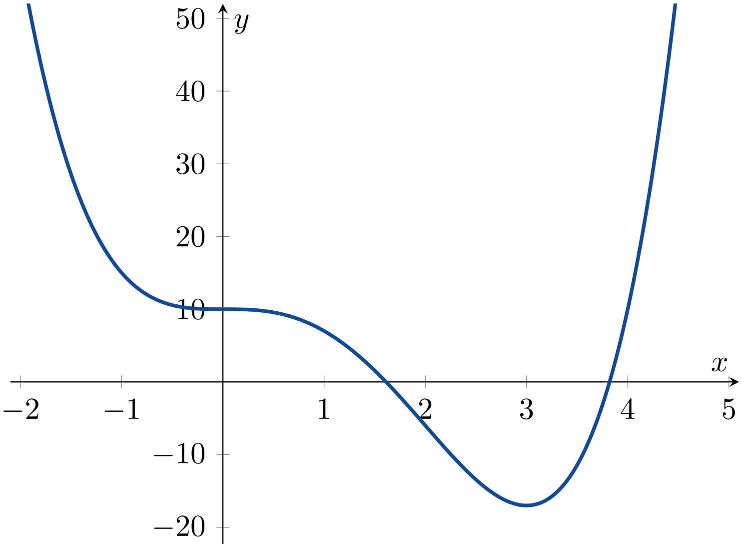
Shifting



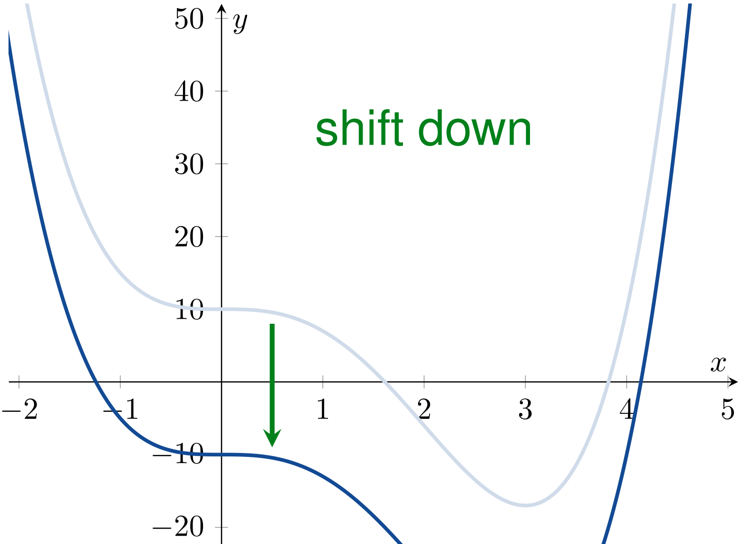
Shifting



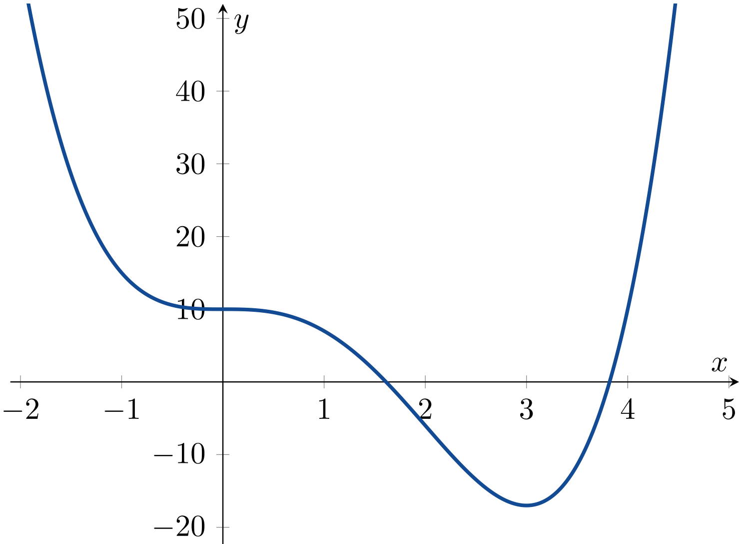
Shifting



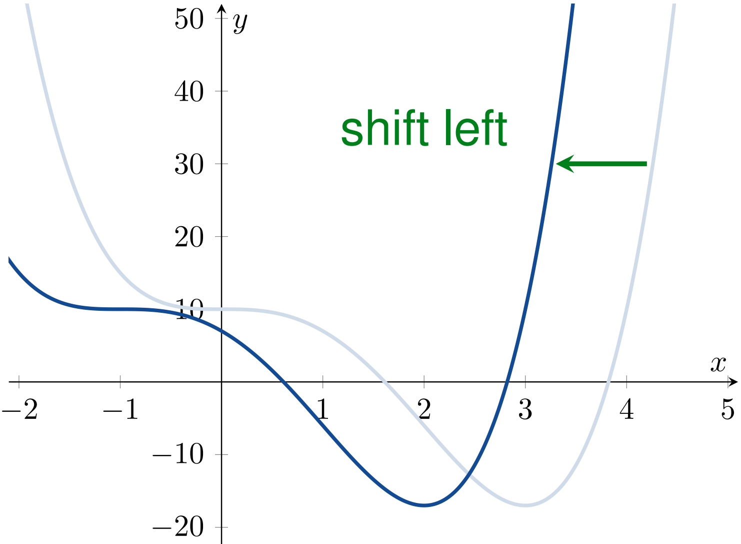
Shifting



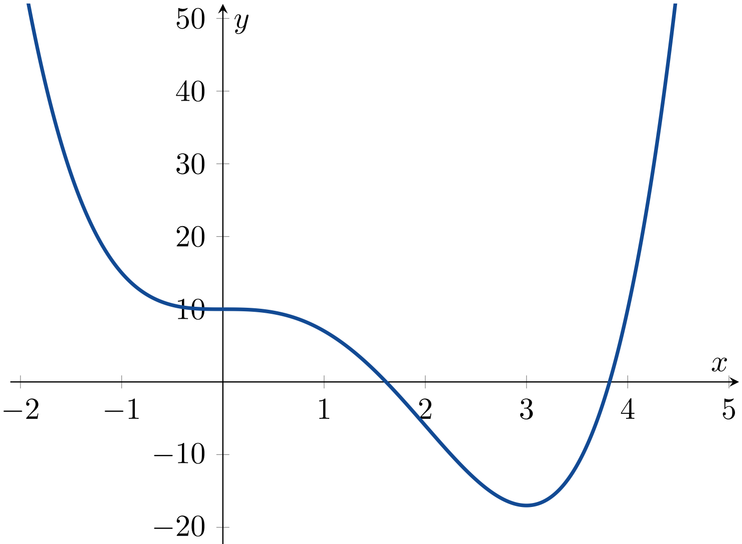
Shifting



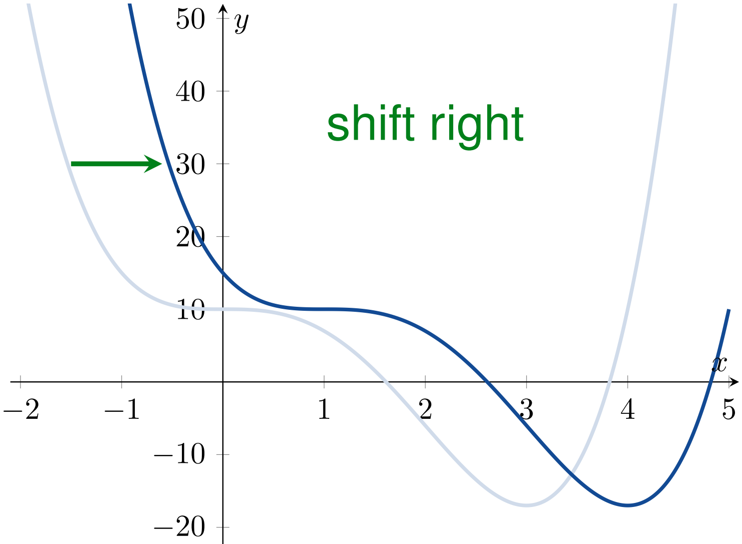
Shifting



Shifting



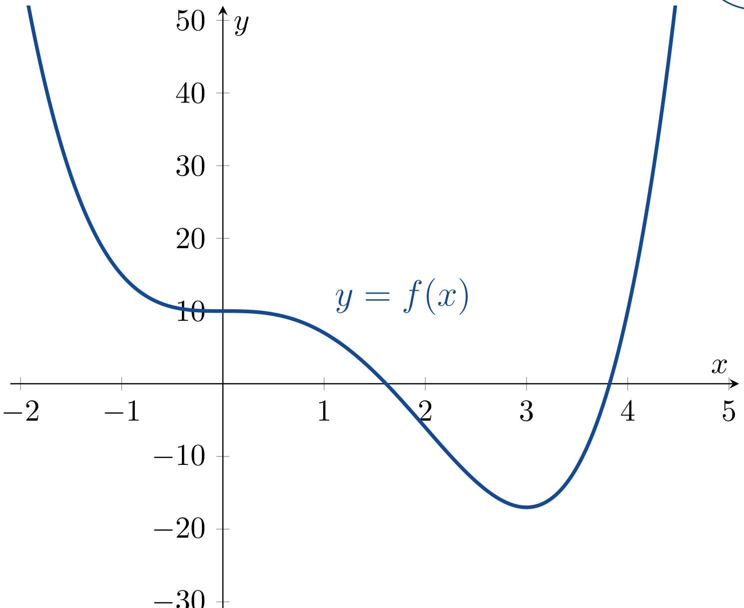
Shifting



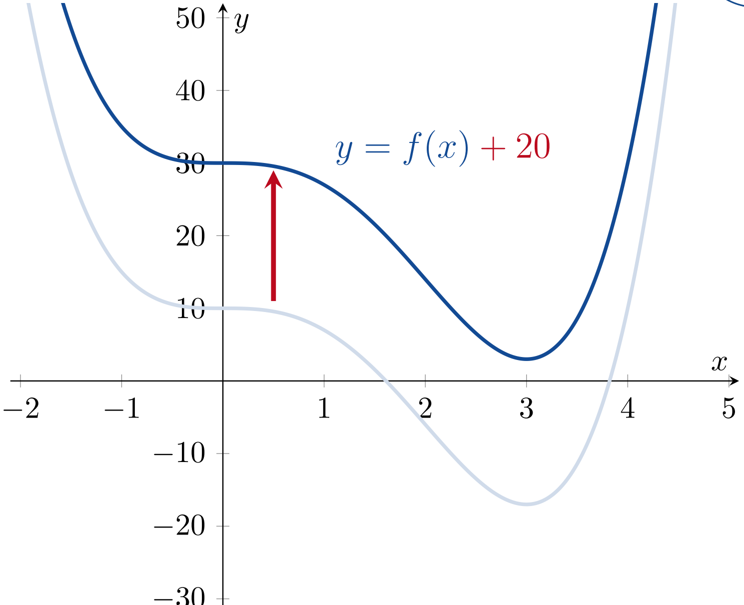
$y = f(x) + k$ shifts the graph up k units.
(or down $|k|$ units if $k < 0$)

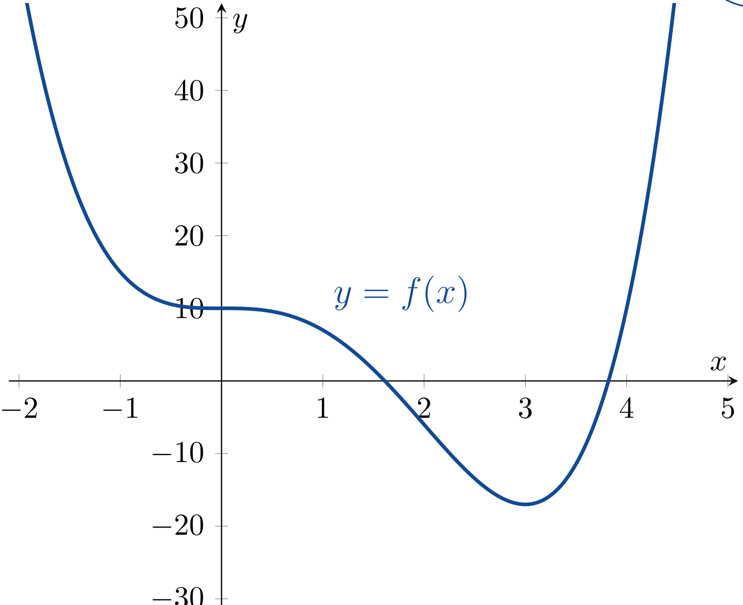
$y = f(x) + k$ shifts the graph up k units.
(or down $|k|$ units if $k < 0$)

$y = f(x + k)$ shifts the graph left k units.
(or right $|k|$ units if $k < 0$)

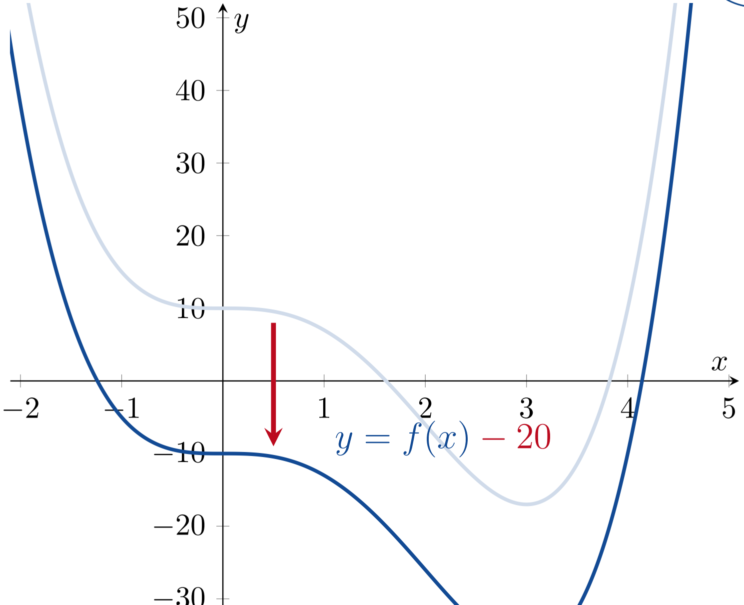


y
x
50
40
30
20
10
−10
−20
−30
−2
−1
1
2
3
4
5
$y = f(x)$

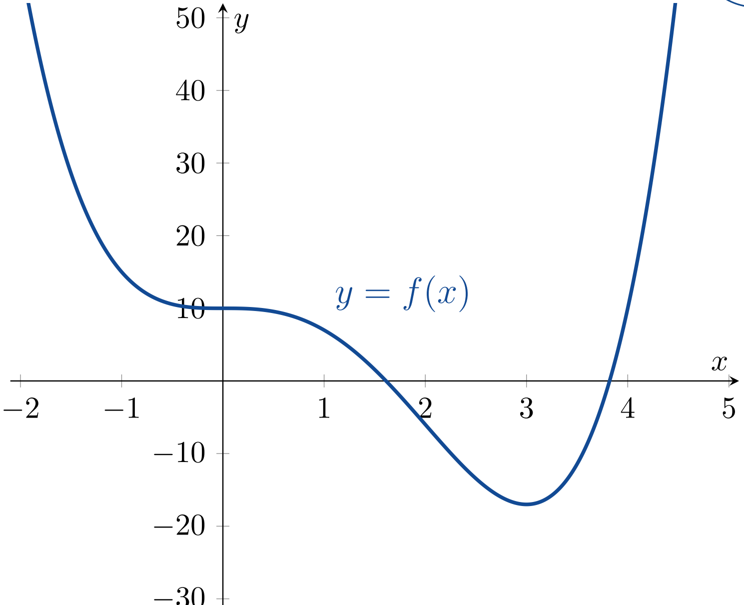

50
40
30
20
10
−10
−20
−30
−2
−1
1
2
3
4
5
x
y
$y = f(x) + 20$

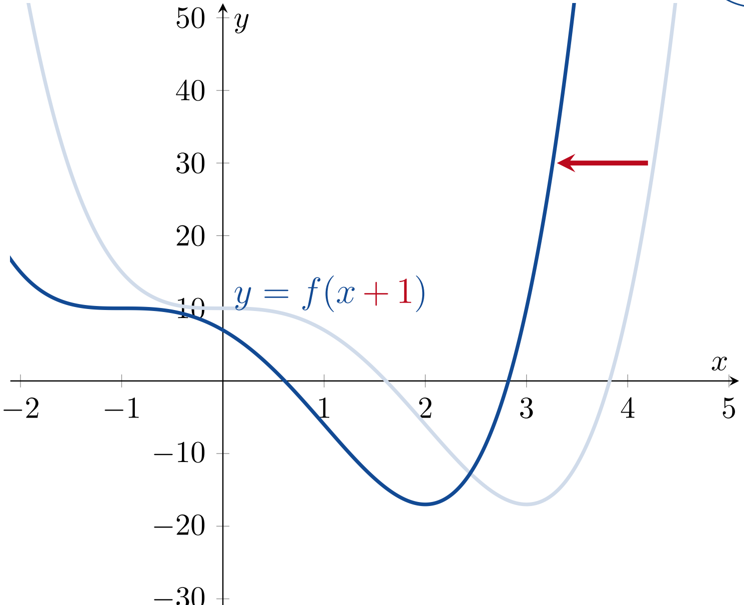


y
x
50
40
30
20
10
−10
−20
−30
−2
−1
1
2
3
4
5
$y = f(x)$

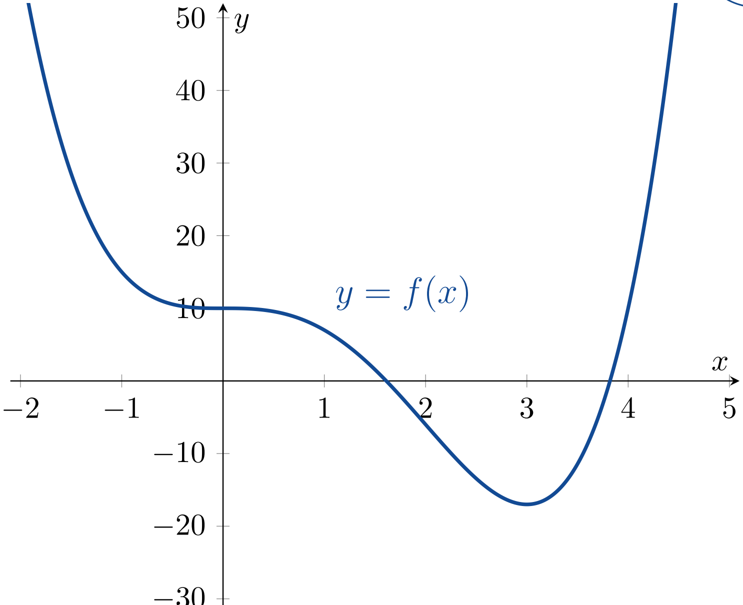


y
x
50
40
30
20
10
-10
-20
-30
-2
-1
1
2
3
4
5
$y = f(x) - 20$


50
40
30
20
10
−10
−20
−30
y
x
−2
−1
1
2
3
4
5
$y = f(x)$

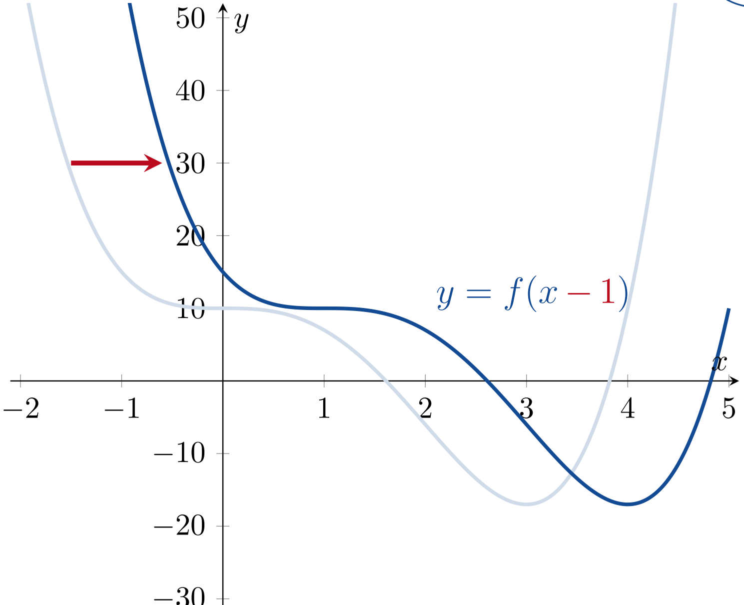


y
50
40
30
20
10
$y = f(x+1)$
x
−2
−1
1
2
3
4
5
−10
−20
−30



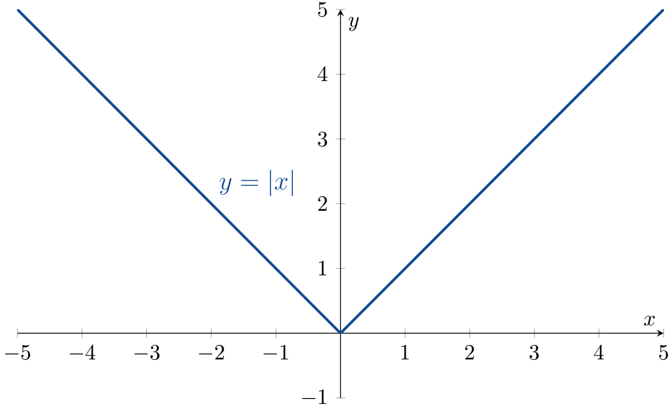
y
50
40
30
20
10
$y = f(x)$
x
−2
−1
1
2
3
4
5
−10
−20
−30

1.2 Combining Functions; Shifting and Scaling Graphs



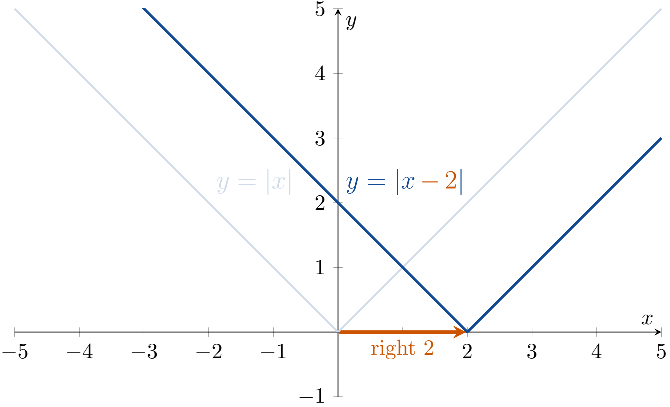
Example

Shift the function $y = |x|$ by 2 units to the right and 1 unit up.



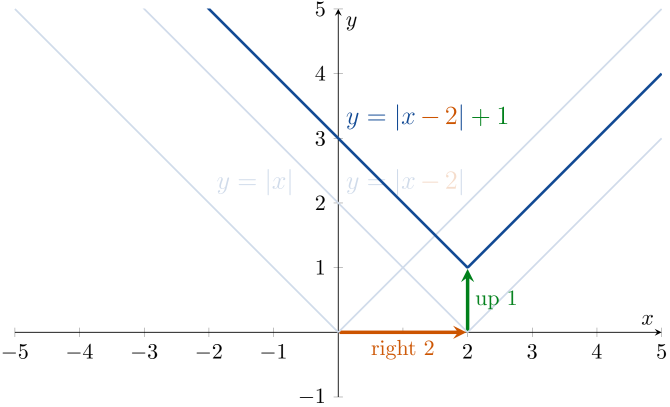
Example

Shift the function $y = |x|$ by 2 units to the right and 1 unit up.



Example

Shift the function $y = |x|$ by 2 units to the right and 1 unit up.



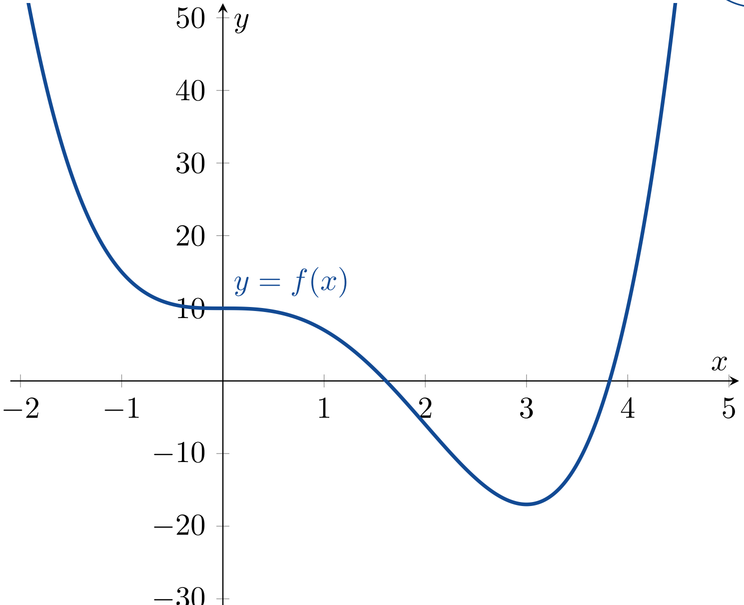
Scaling and Reflecting

For $c > 1$, the graph is scaled:

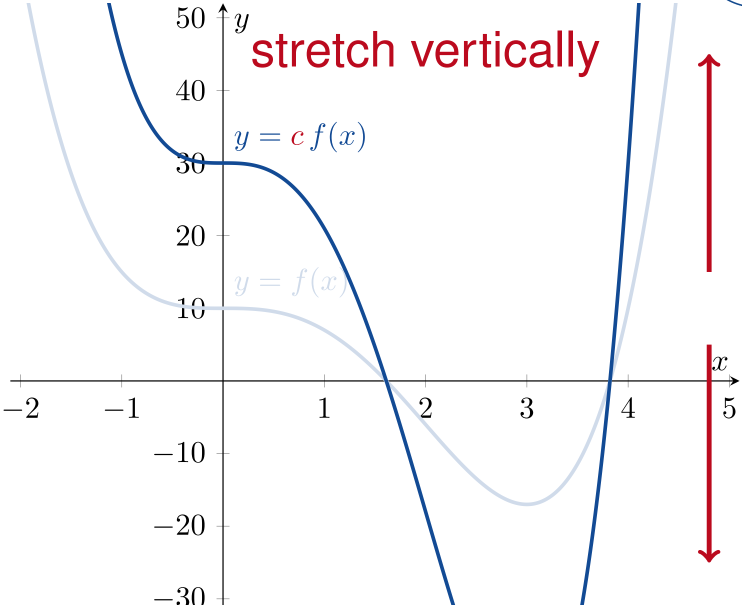
$\updownarrow$	$y = cf(x)$	stretch vertically by a factor of c
	$y = \frac{1}{c}f(x)$	squash vertically by a factor of c
$\rightarrow\leftarrow$	$y = f(cx)$	squash horizontally by a factor of c
$\leftrightarrow$	$y = f\left(\frac{x}{c}\right)$	stretch horizontally by a factor of c

For $c = -1$, the graph is reflected:

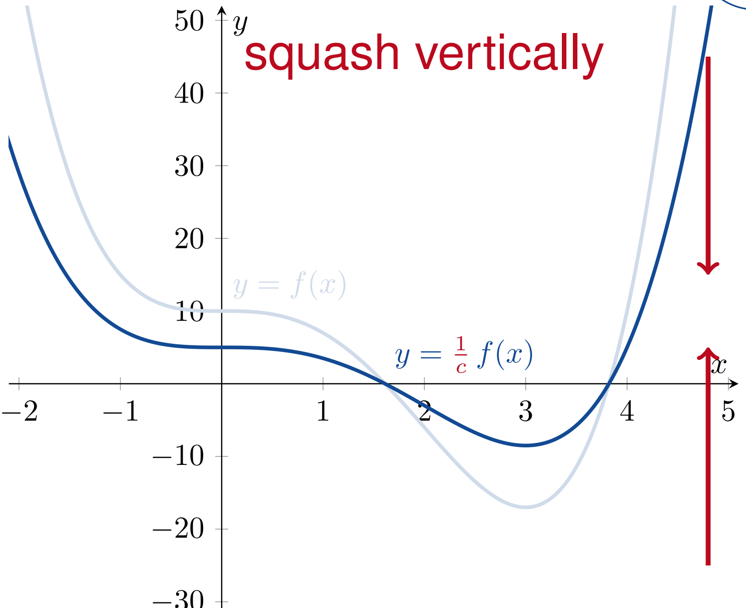
$\xrightarrow{\text{mirror}} x$	$y = -f(x)$	reflect vertically, about the x-axis
mirror $\uparrow y$	$y = f(-x)$	reflect horizontally, about the y-axis

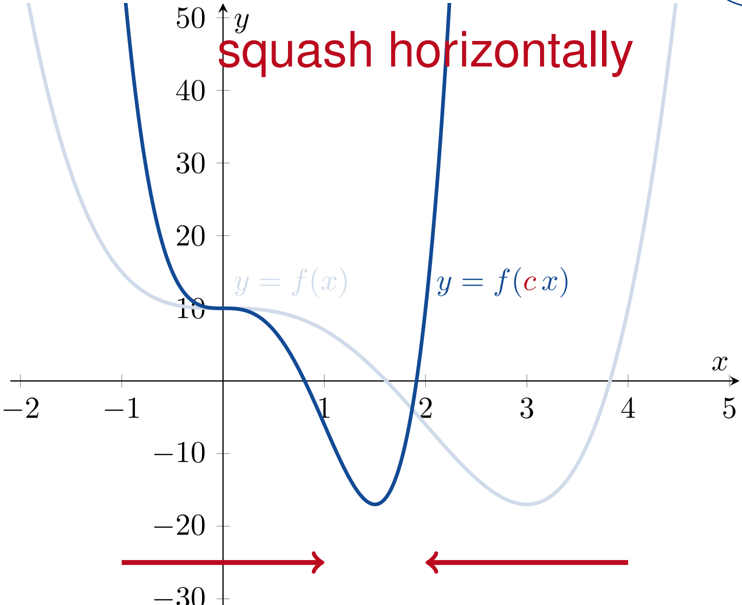


50
40
30
20
10
−10
−20
−30
y
x
−2
−1
1
2
3
4
5
$y = f(x)$

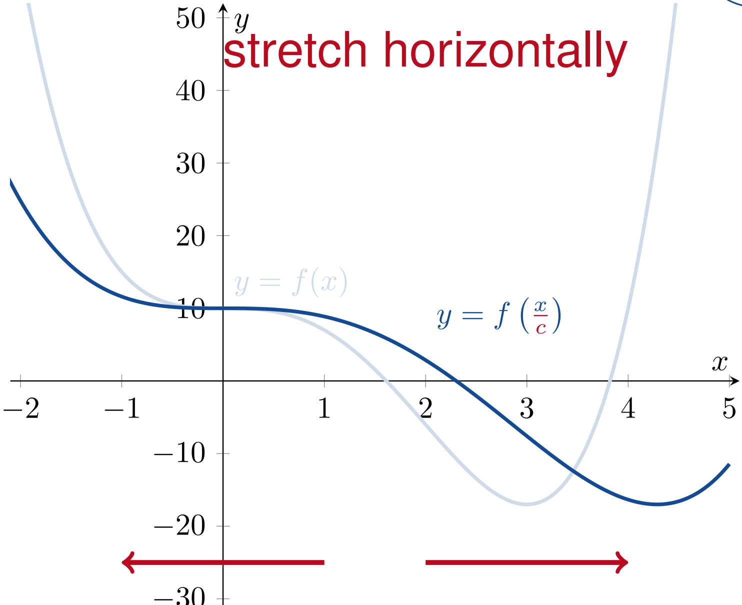


stretch vertically
y
x
50
40
30
20
10
-10
-20
-30
-2
-1
1
2
3
4
5
$y = c\,f(x)$
$y = f(x)$


squash vertically
y
x
50
40
30
20
10
-10
-20
-30
-2
-1
1
2
3
4
5
$y = f(x)$
$y = \frac{1}{c} f(x)$

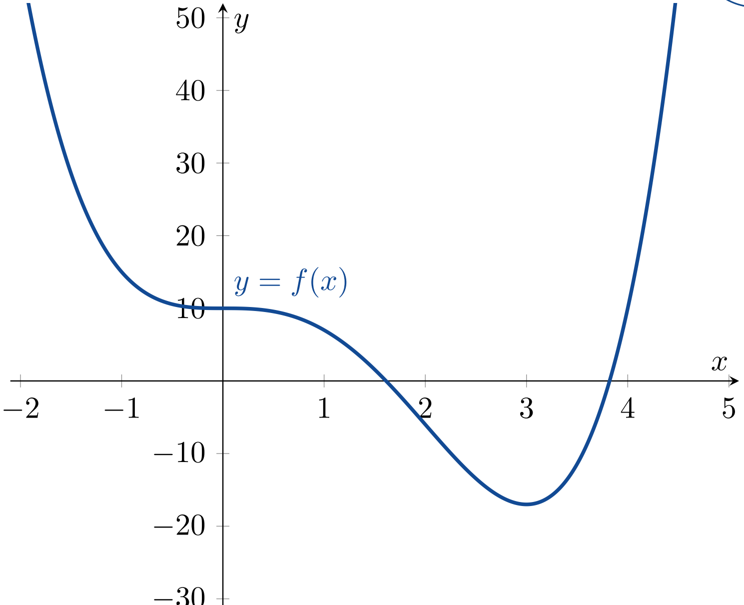


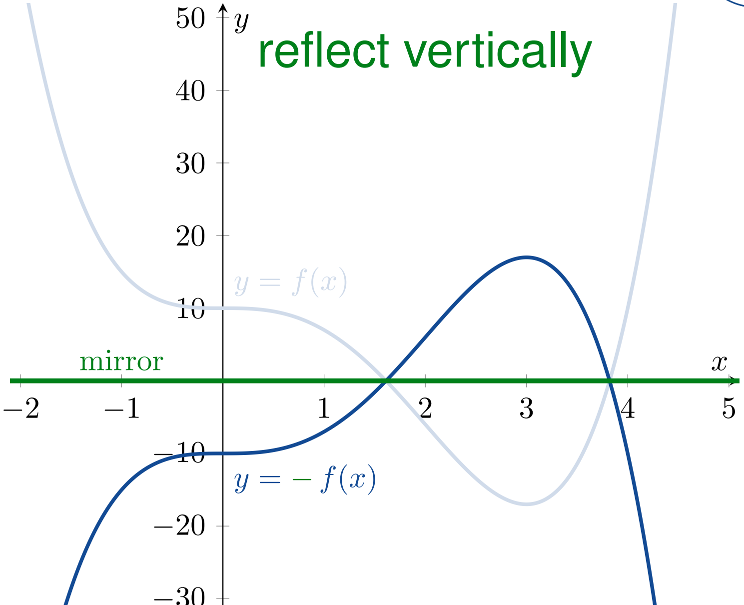
squash horizontally
y
x
50
40
30
20
10
−10
−20
−30
−2
−1
1
2
3
4
5
$y = f(x)$
$y = f(c\,x)$

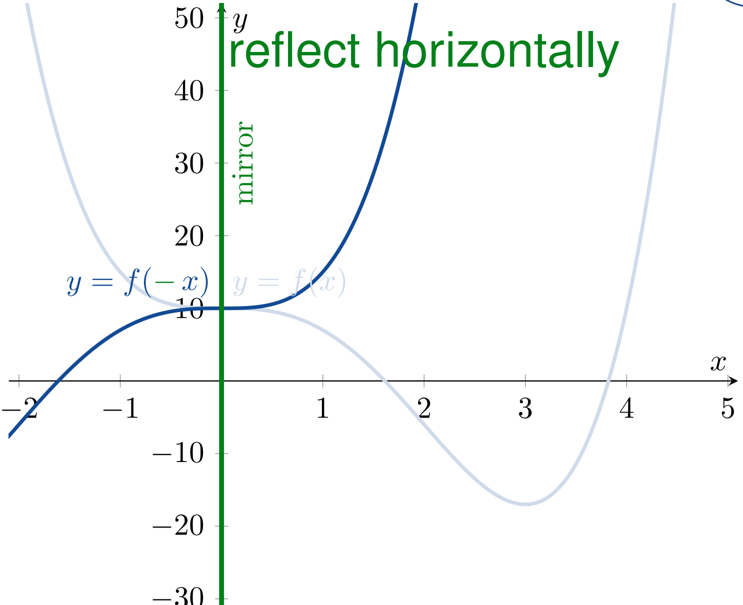


stretch horizontally
y
x
50
40
30
20
10
−10
−20
−30
−2
−1
1
2
3
4
5
$y = f(x)$
$y = f\left(\frac{x}{c}\right)$

1.2 Combining Functions; Shifting and Scaling Graphs




reflect vertically
y
x
50
40
30
20
10
−10
−20
−30
−2
−1
1
2
3
4
5
$y = f(x)$
$y = -f(x)$
mirror


reflect horizontally
mirror
$y = f(-x)$
$y = f(x)$
x
y
-2
-1
1
2
3
4
5
50
40
30
20
10
-10
-20
-30

Please read Example 5 in your textbook.

1.3 Trigonometric Functions

Angles

There are two ways to measure angles. Using degrees or using radians.

Angles

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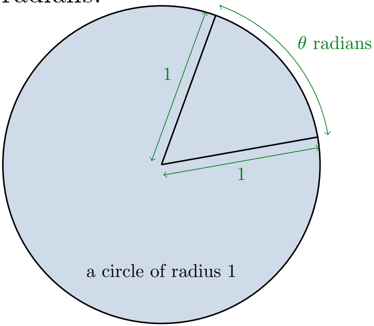
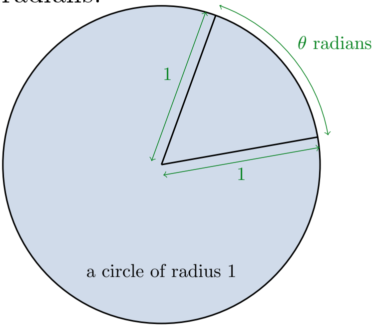
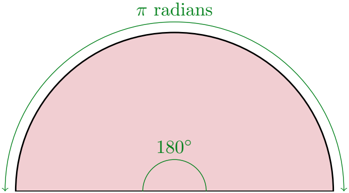


Angles

There are two ways to measure angles. Using degrees or using radians.





We have that

$$\pi \text{ radians} = 180 \text{ degrees}$$

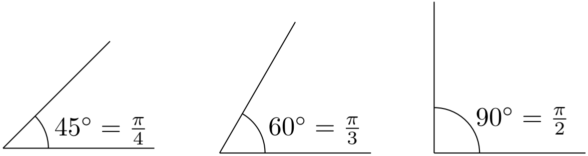
We have that

$$\begin{aligned} \pi \text{ radians} &= 180 \text{ degrees} \\ 1 \text{ radian} &= \frac{180}{\pi} \text{ degrees} \\ 1 \text{ degree} &= \frac{\pi}{180} \text{ radians.} \end{aligned}$$

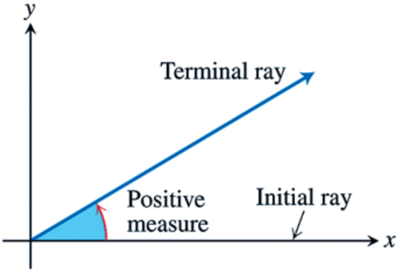
1.3 Trigonometric Functions

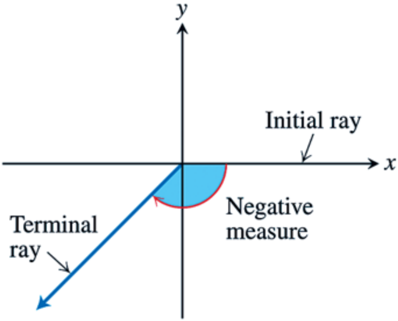
We have that

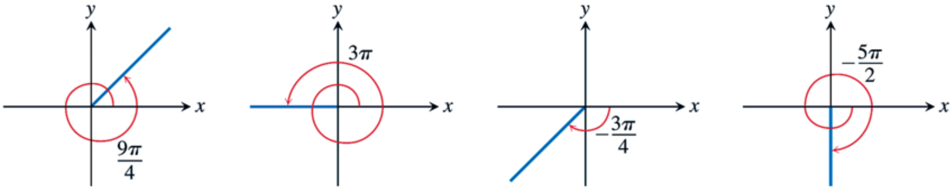
$$\pi \text{ radians} = 180 \text{ degrees}$$
$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$
$$1 \text{ degree} = \frac{\pi}{180} \text{ radians.}$$



Degrees	**−180**	**−135**	**−90**	**−45**	**0**	**30**	**45**	**60**	**90**	**120**	**135**	**150**	**180**	**270**	**360**
θ (radians)	$-\pi$	$\frac{-3\pi}{4}$	$\frac{-\pi}{2}$	$\frac{-\pi}{4}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π


y
Terminal ray
Positive
measure
Initial ray
x

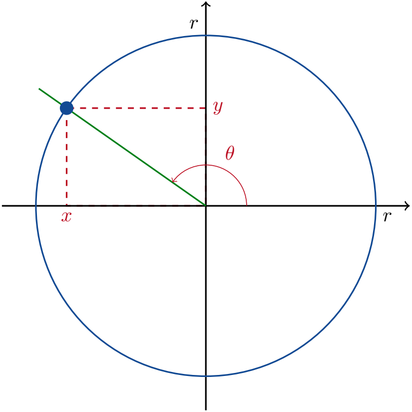

y
Initial ray
x
Negative
measure
Terminal
ray


y
x
9π/4
y
x
3π
y
x
−3π/4
y
x
−5π/2

Remark

In Calculus, we use radians!!!! If you see an angle in this course, it will be in radians. Calculus doesn't work with degrees!!

6 Trigonometric Functions



sine $\qquad \sin\theta = \dfrac{y}{r}$

cosine $\qquad \cos\theta = \dfrac{x}{r}$

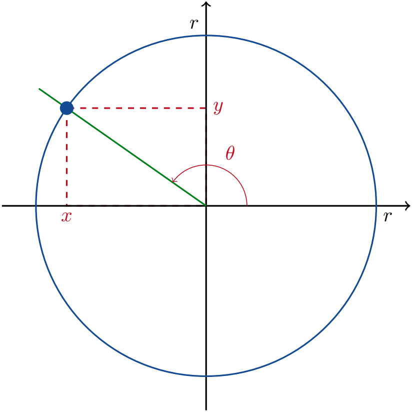
tangent

secant

cosecant

cotangent

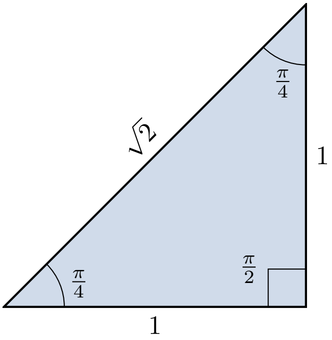
6 Trigonometric Functions



sine	$\sin\theta = \dfrac{y}{r}$
cosine	$\cos\theta = \dfrac{x}{r}$
tangent	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
secant	$\sec\theta = \dfrac{1}{\cos\theta}$
cosecant	$\operatorname{cosec}\theta = \csc\theta = \dfrac{1}{\sin\theta}$
cotangent	$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Remark

Note that $\tan\theta$ and $\sec\theta$ are only defined if $\cos\theta \neq 0$; and $\operatorname{cosec}\theta$ and $\cot\theta$ are only defined if $\sin\theta \neq 0$.



$$\sin 45^\circ = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

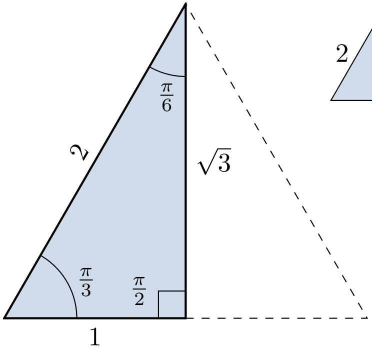
$$\cos 45^\circ = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

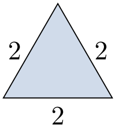
$$\tan 45^\circ = \tan\frac{\pi}{4} = 1$$

$$\sec 45^\circ = \sec\frac{\pi}{4} = \sqrt{2}$$

$$\operatorname{cosec} 45^\circ = \operatorname{cosec}\frac{\pi}{4} = \sqrt{2}$$

$$\cot 45^\circ = \cot\frac{\pi}{4} = 1$$





$$\sin 60^\circ = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\tan 60^\circ = \tan\frac{\pi}{3} = \sqrt{3}$$

$$\sec 60^\circ = \sec\frac{\pi}{3} = 2$$

$$\operatorname{cosec} 60^\circ = \operatorname{cosec}\frac{\pi}{3} = \frac{2}{\sqrt{3}}$$

$$\cot 60^\circ = \cot\frac{\pi}{3} = \frac{1}{\sqrt{3}}$$

Degrees	**−180**	**−135**	**−90**	**−45**	**0**	**30**	**45**	**60**	**90**	**120**	**135**	**150**	**180**	**270**	**360**
θ (radians)	$-\pi$	$\frac{-3\pi}{4}$	$\frac{-\pi}{2}$	$\frac{-\pi}{4}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin θ	0	$\frac{-\sqrt{2}}{2}$	-1	$\frac{-\sqrt{2}}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
cos θ	-1	$\frac{-\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{3}}{2}$	-1	0	1
tan θ	0	1		-1	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$		$-\sqrt{3}$	-1	$\frac{-\sqrt{3}}{3}$	0		0

Periodicy

Definition

A function $f(x)$ is called *periodic* if there exists a positive number p such that

$$f(x+p) = f(x)$$

for all x.

Periodicy

Definition

A function $f(x)$ is called *periodic* if there exists a positive number p such that

$$f(x+p) = f(x)$$

for all x.

The smallest such $p > 0$ is called the *period* of f.

Example

The period of $\tan x$ is π since

$$\tan(x+\pi) = \tan x$$

for all x, and this is the smallest $p > 0$ which works.

Example

The period of $\tan x$ is π since

$$\tan(x + \pi) = \tan x$$

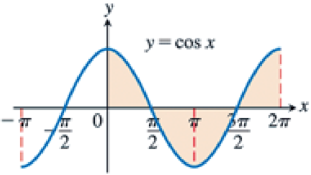
for all x, and this is the smallest $p > 0$ which works.

Example

The period of $\sin x$ is 2π since

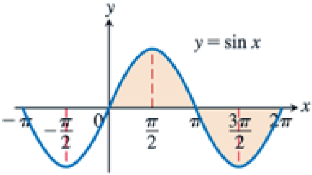
$$\sin(x + 2\pi) = \sin x$$

for all x, and this is the smallest $p > 0$ which works.



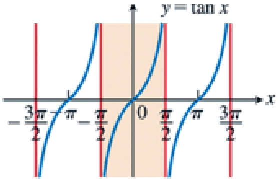
Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Period: 2π

(a)



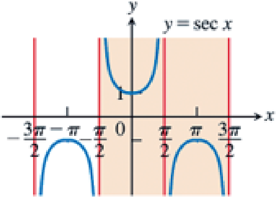
Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Period: 2π

(b)



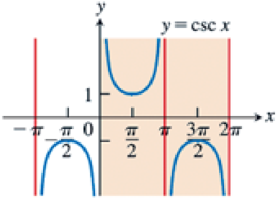
Domain: $x \ne \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
Range: $-\infty < y < \infty$
Period: π

(c)



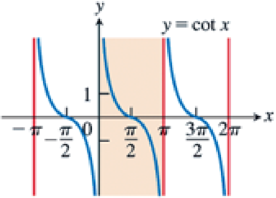
Domain: $x \ne \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
Range: $y \le -1$ or $y \ge 1$
Period: 2π

(d)



Domain: $x \ne 0, \pm\pi, \pm 2\pi, \ldots$
Range: $y \le -1$ or $y \ge 1$
Period: 2π

(e)



Domain: $x \ne 0, \pm\pi, \pm 2\pi, \ldots$
Range: $-\infty < y < \infty$
Period: π

(f)

Trigonometric Identities

$$
\begin{aligned}
\cos\theta &= \sin\left(\frac{\pi}{2}-\theta\right) \\
\cos^2\theta+\sin^2\theta &= 1 \\
1+\tan^2\theta &= \sec^2\theta \\
1+\cot^2\theta &= \operatorname{cosec}^2\theta \\
\cos(A+B) &= \cos A\cos B-\sin A\sin B \\
\sin(A+B) &= \sin A\cos B+\cos A\sin B \\
\cos 2\theta &= \cos^2\theta-\sin^2\theta \\
\sin 2\theta &= 2\sin\theta\cos\theta \\
\cos^2\theta &= \frac{1}{2}(1+\cos 2\theta) \\
\sin^2\theta &= \frac{1}{2}(1-\cos 2\theta)
\end{aligned}
$$

Trigonometric Identities

$$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

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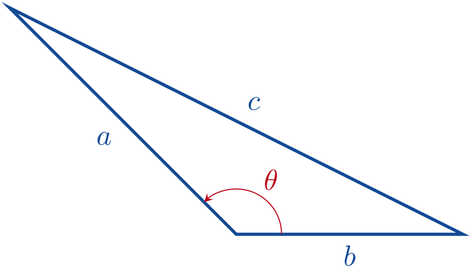
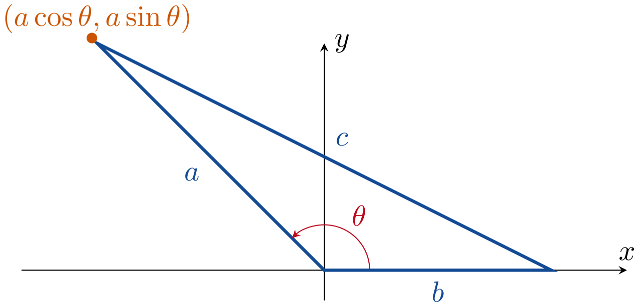
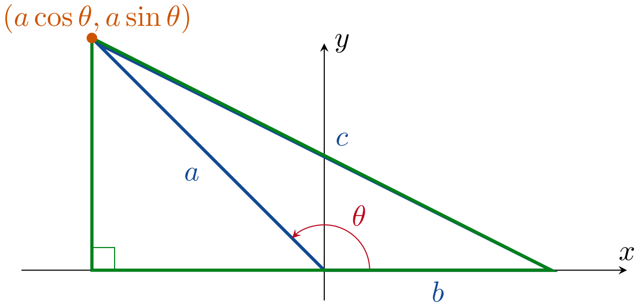
The Law of Cosines



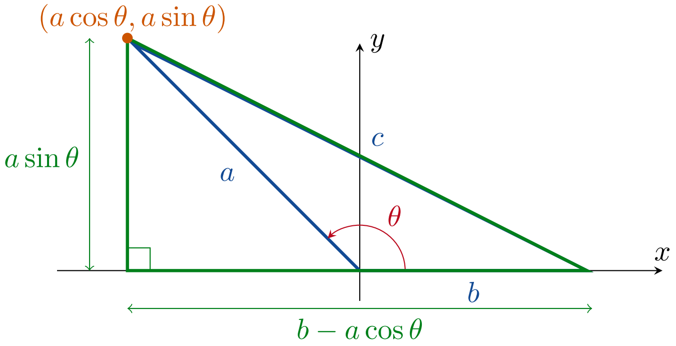
The Law of Cosines



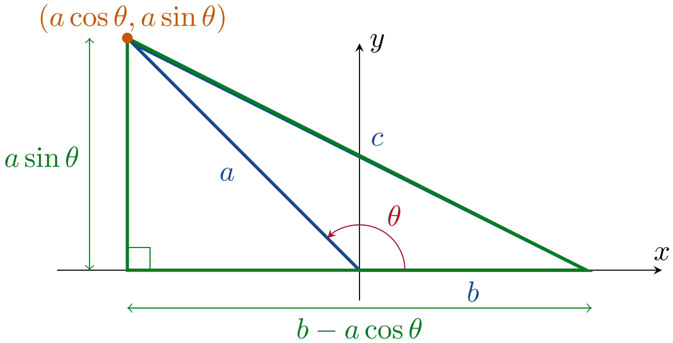
The Law of Cosines



The Law of Cosines

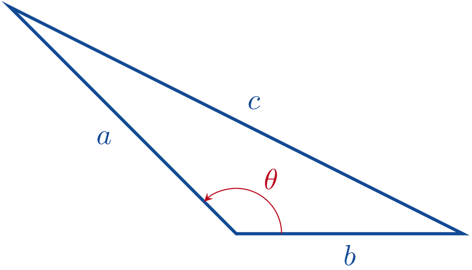


The Law of Cosines

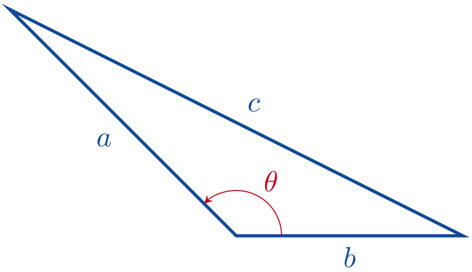


By Pythagoras we have that

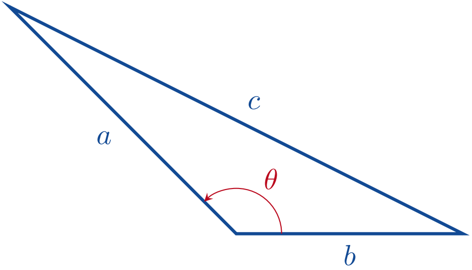
$$c^2 = (b - a\cos\theta)^2 + (a\sin\theta)^2$$



$$\begin{aligned} c^2 &= (b - a\cos\theta)^2 + (a\sin\theta)^2 \\ &= \\ &= \end{aligned}$$



$$
\begin{aligned}
c^2 &= (b - a\cos\theta)^2 + (a\sin\theta)^2 \\
&= b^2 - 2a\cos\theta + a^2\cos^2\theta + a^2\sin^2\theta \\
&=
\end{aligned}
$$



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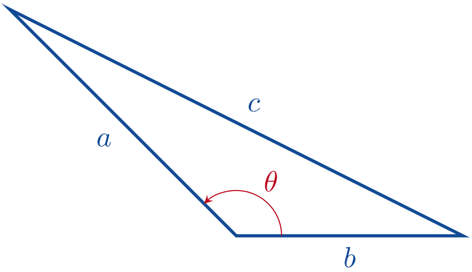
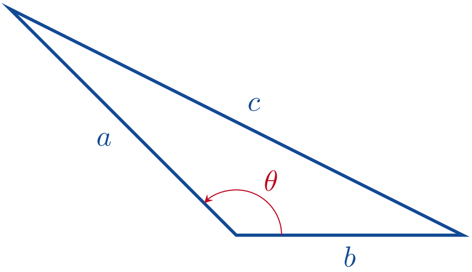


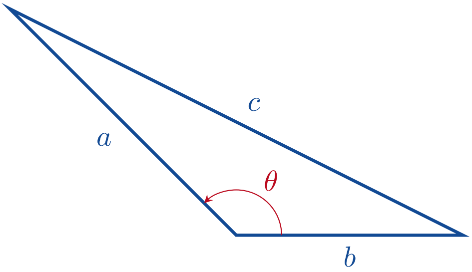
$$
\begin{aligned}
c^2 &= (b - a\cos\theta)^2 + (a\sin\theta)^2 \\
&= b^2 - 2a\cos\theta + a^2\cos^2\theta + a^2\sin^2\theta \\
&= a^2 + b^2 - 2a\cos\theta
\end{aligned}
$$

1.3 Trigonometric Functions



Theorem (The Law of Cosines)

$$c^2 = a^2 + b^2 - 2a\cos\theta$$



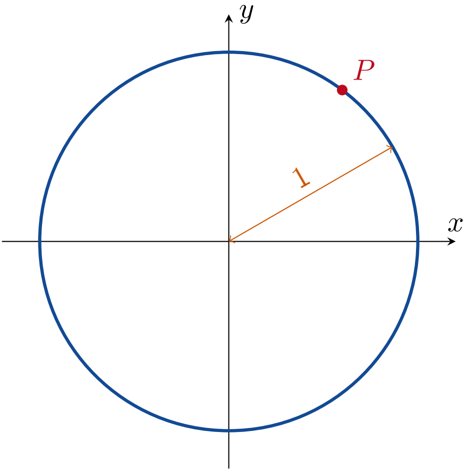
Theorem (The Law of Cosines)

$$c^2 = a^2 + b^2 - 2a\cos\theta$$

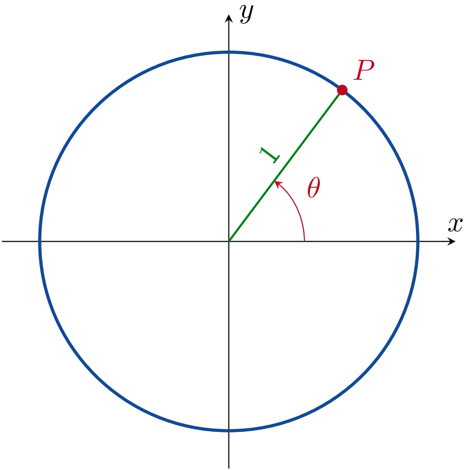
Remark

If $\theta = \frac{\pi}{2}$, then we just get $c^2 = a^2 + b^2$.

2 Special Inequalities



2 Special Inequalities



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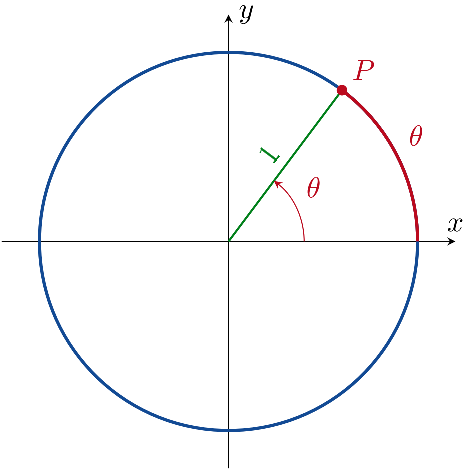

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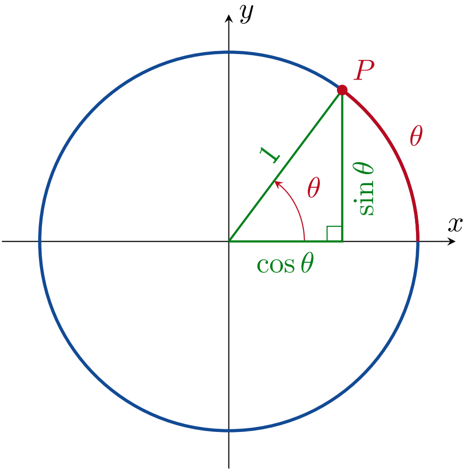

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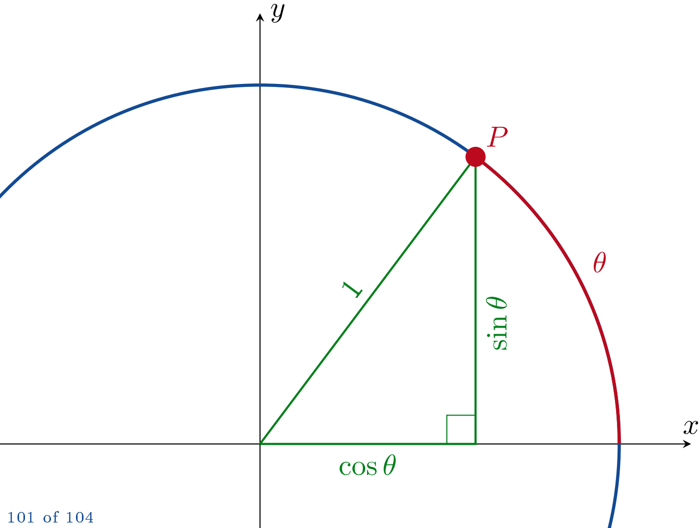

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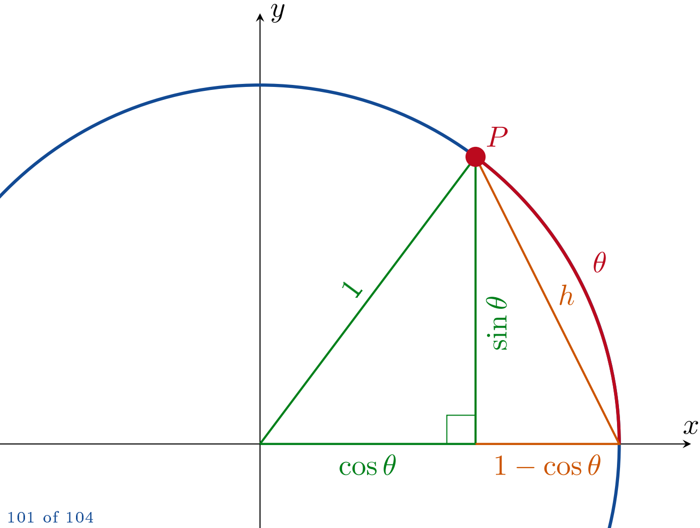

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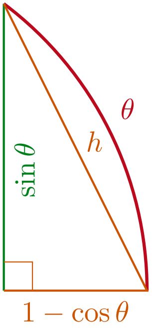
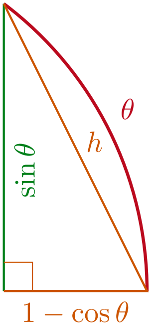


Since

$$h \leq \theta$$

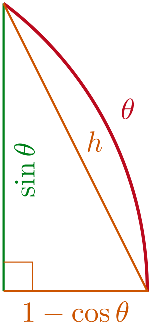
2 Special Inequalities



Since

$$h \leq \theta$$
$$h^2 \leq \theta^2$$

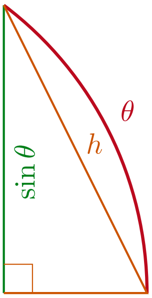
2 Special Inequalities



Since

$$
\begin{aligned}
h &\le \theta \\
h^2 &\le \theta^2 \\
(1-\cos\theta)^2 + \sin^2\theta &\le \theta^2
\end{aligned}
$$

2 Special Inequalities



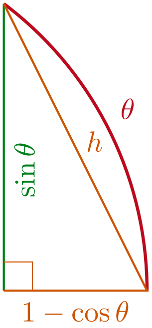
Since

$$\begin{aligned} h &\leq \theta \\ h^2 &\leq \theta^2 \\ (1-\cos\theta)^2 + \sin^2\theta &\leq \theta^2 \end{aligned}$$

we have that

$$(1-\cos\theta)^2 \leq \theta^2 \quad \text{and} \quad \sin^2\theta \leq \theta^2.$$

2 Special Inequalities



Since

$$h \leq \theta$$
$$h^2 \leq \theta^2$$
$$(1-\cos\theta)^2 + \sin^2\theta \leq \theta^2$$

we have that

$$(1-\cos\theta)^2 \leq \theta^2 \quad \text{and} \quad \sin^2\theta \leq \theta^2.$$

It follows that

$$\boxed{-|\theta| \leq \sin\theta \leq |\theta| \qquad \text{and} \qquad -|\theta| \leq 1-\cos\theta \leq |\theta|\,.}$$

Remark

We will need

$$-|\theta| \leq \sin\theta \leq |\theta| \qquad \text{and} \qquad -|\theta| \leq 1 - \cos\theta \leq |\theta|$$

next week.

Next Time

- 2.1 Rates of Change and Tangents to Curves
- 2.2 Limit of a Function and Limit Laws
- 2.3 The Precise Definition of a Limit