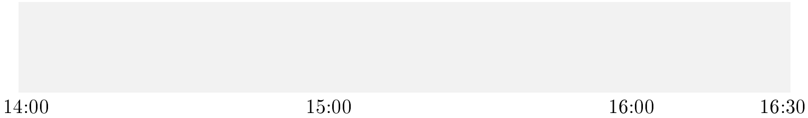
Welcome to

Mathematics I

with Dr Neil Course

Information about this course

- ≈ 12 classes. Friday afternoons 2pm-4:30pm.



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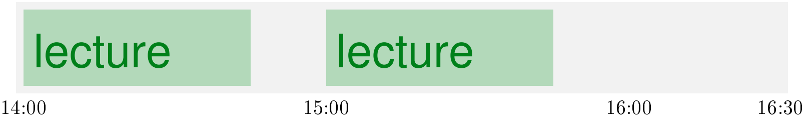
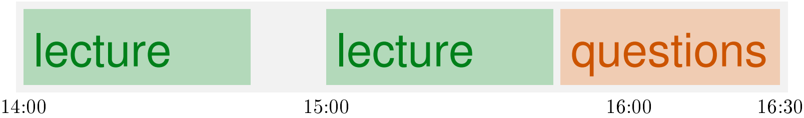
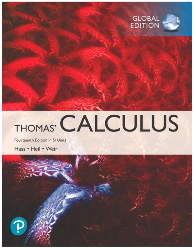


Information about this course

- ≈ 12 classes. Friday afternoons 2pm-4:30pm.
- 2 lectures with a break between.
- Then I will answer your questions.



The Book



Joel R. Hass, Christopher E. Heil and Maurice D. Weir,
Thomas' Calculus,
14th Edition in SI Units, Pearson.

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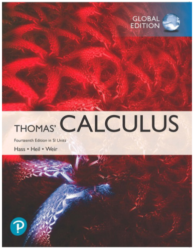

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This is a required purchase.
You need to have this book to be able to do the homework.

Your Mathematics courses:

MATH113 MATH114 MATH215 MATH216

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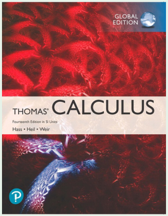
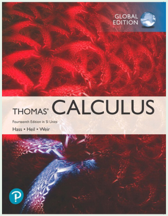
MATH113 MATH114 MATH215 MATH216



Calculus

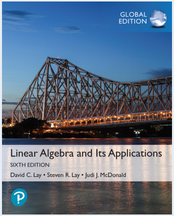
Your Mathematics courses:

MATH113 MATH114



Calculus

MATH215



Linear Algebra

MATH216

Your Mathematics courses:

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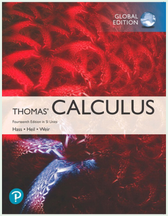

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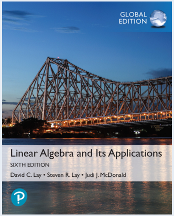
MATH215



Linear Algebra

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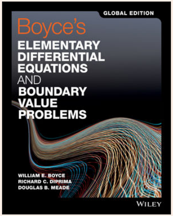

Differential Equations

1
2
3
4
5
6
7

1. Functions

2

3

4

5

6

7

1. Functions

2. Limits and Continuity

3

4

5

6

7

1. Functions

2. Limits and Continuity

3. Derivatives

4

5

6

7

1. Functions

2. Limits and Continuity

3. Derivatives

4. Applications of Derivatives

5

6

7

1. Functions
2. Limits and Continuity
3. Derivatives
4. Applications of Derivatives
5. Integrals
6
7

1. Functions
2. Limits and Continuity
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5. Integrals
6. Applications of Integrals
7

1. Functions
2. Limits and Continuity
3. Derivatives
4. Applications of Derivatives
5. Integrals
6. Applications of Integrals
7. Trancendental Functions

1. Functions	1 week
2. Limits and Continuity	2 weeks
3. Derivatives	4 weeks
4. Applications of Derivatives	
5. Integrals	3 weeks
6. Applications of Integrals	
7. Trancendental Functions	2 weeks

Exams and homework

(This information may change based on the University's decisions)

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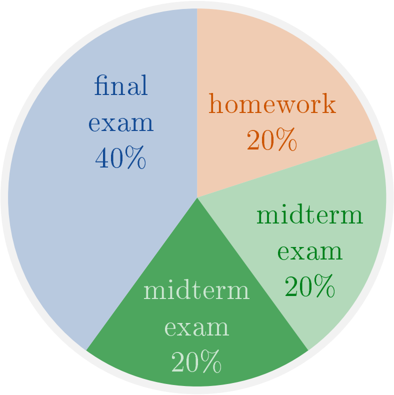

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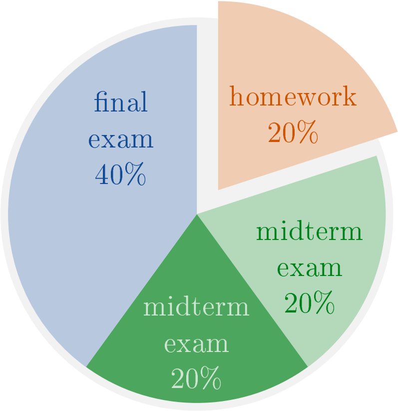

using Pearson MyLab Math

one piece of homework for each lesson

deadline = end of term

more details in O'Learn

Exams and homework

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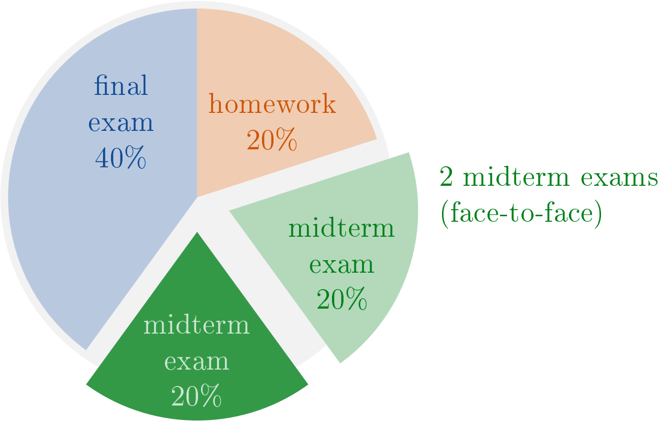

Exams and homework

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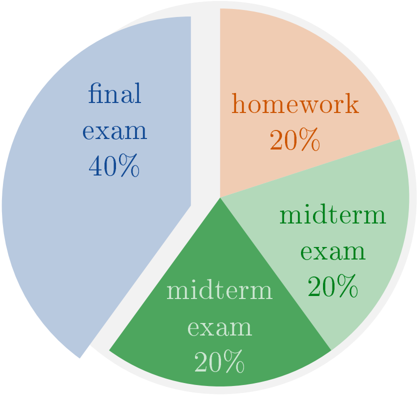
(face-to-face)



Expectations

If this was a classroom course, the expectation is that for each hour of lectures, you would study an extra 1-2 hours outside of class.

classroom course	lectures (5 hours)	other study (5-10 hours)

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For an online course, you are still expected to study a total of 10-15 hours each week.

online course	class (2.5 hours)	other study (7.5-12.5 hours)

This may include:

- Do the online homework on MyLab;

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$\vdots$

slide number

99.9 Section Title

current section

Lecture 1

- Information about this course
- A.1 Real Numbers and the Real Line
- 1.1 Functions and Their Graphs
- 1.2 Combining Functions; Shifting and Scaling Graphs
- 1.3 Trigonometric Functions

The Natural Numbers

The set

$$\mathbb{N} = \{1, 2, 3, 4, 5, 6, \ldots\}$$

is called the set of *natural numbers*. These are the first numbers that children learn. For example

$$\begin{aligned}
2 &\in \mathbb{N} && \text{means "2 is a natural number"} \\
7 &\in \mathbb{N} && \text{means "7 is a natural number"} \\
\tfrac{1}{2} &\notin \mathbb{N} && \text{means "}\tfrac{1}{2}\text{ is \textbf{not} a natural number"} \\
0 &\notin \mathbb{N} && \text{means "0 is \textbf{not} a natural number"} \\
-5 &\notin \mathbb{N} && \text{means "}-5\text{ is \textbf{not} a natural number"}
\end{aligned}$$

In the natural numbers, we can do "+" and "$\times$"

$$2 + 7 = 9 \in \mathbb{N}, \qquad\qquad 2 \times 7 = 14 \in \mathbb{N}.$$

However we can not do "$-$" because

$$2 - 7 \notin \mathbb{N}.$$

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$$2 + 7 = 9 \in \mathbb{N}, \qquad\qquad 2 \times 7 = 14 \in \mathbb{N}.$$

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So we invent new numbers!

The Integers

The set
$$\mathbb{Z} = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$
is called the set of *integers*. We use a $\mathbb{Z}$ for the German word 'zahlen' (numbers).

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$$3+4 \in \mathbb{Z}, \qquad 3-4 \in \mathbb{Z}, \qquad 3 \times 4 \in \mathbb{Z}, \qquad 3 \div 4 \notin \mathbb{Z},$$

$$3+(-5) \in \mathbb{Z}, \quad 3-(-5) \in \mathbb{Z}, \quad 3 \times (-5) \in \mathbb{Z}, \quad 3 \div (-5) \notin \mathbb{Z}.$$

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The Rational Numbers

The set

$$\mathbb{Q} = \{\text{all fractions}\} = \left\{ \frac{a}{b} \;\middle|\; a, b \in \mathbb{Z} \text{ and } b \neq 0 \right\}$$

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$$0 = \frac{0}{1} \in \mathbb{Q} \qquad \frac{100}{13} \in \mathbb{Q}$$

$$1 = \frac{1}{1} \in \mathbb{Q} \qquad \sqrt{2} \notin \mathbb{Q}$$

$$\frac{3}{4} \in \mathbb{Q} \qquad -4 = \frac{8}{-2} \in \mathbb{Q}$$

$$\pi \notin \mathbb{Q} \qquad 0.12345 = \frac{12345}{100000} \in \mathbb{Q}.$$

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$$\pi \notin \mathbb{Q} \qquad 0.12345 = \frac{12345}{100000} \in \mathbb{Q}.$$

In $\mathbb{Q}$ we can do "$+$", "$-$", "$\times$" and "$\div$(by a number $\neq 0$)".

Are we happy now?

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No!

Are we happy now?

No!

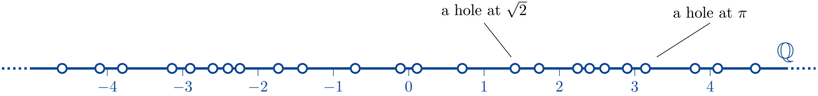
Why?

Are we happy now?

No!

Why?

Because if we draw all the rational numbers in a line, then the line has lots of holes in it. In fact, $\mathbb{Q}$ has ∞ many holes in it.



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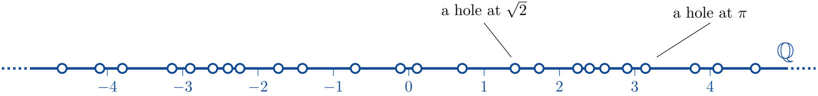

So we invent new numbers!

The Real Numbers

The set

$$\mathbb{R} = \{\text{all numbers which can be written as a decimal}\}$$

is called the set of *real numbers*. For example

$$\begin{aligned} 0 &= 0.0 \in \mathbb{R} & \frac{100}{13} &= 7.692307\ldots \in \mathbb{R} \\ \frac{23}{99} &= 0.232323\ldots \in \mathbb{R} & \sqrt{2} &= 1.414213\ldots \in \mathbb{R} \\ \frac{3}{4} &= 0.75 \in \mathbb{R} & \frac{123}{999} &= 0.123123\ldots \in \mathbb{R} \\ \pi &= 3.141592\ldots \in \mathbb{R} & \frac{12345}{100000} &= 0.12345 \in \mathbb{R}. \end{aligned}$$

A.1 Real Numbers and the Real Line

The real numbers are complete – this means that if we draw all the real numbers in a line, then there are no holes in the line.

Are we happy now?

Are we happy now?

Yes! Now we have enough numbers to do Calculus.

Intervals

Definition

A subset of $\mathbb{R}$ is called an *interval* if

1. it contains atleast 2 numbers; and
2. it doesn't have any holes in it.

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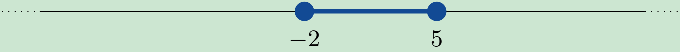
Example

The set $\{x \mid x \text{ is a real number and } x > 6\}$ is an interval.

Because 6 is not in this set, we use $\circ$ at 6.

Example

The set of all real numbers x such that $-2 \leq x \leq 5$ is an interval.



Because -2 and 5 are in this set, we use ● at -2 and 5.

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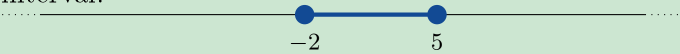

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Example

The set $\{x \mid x \in \mathbb{R} \text{ and } x \neq 0\}$ is not an interval.

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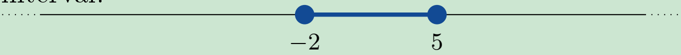

Because -2 and 5 are in this set, we use ● at -2 and 5.

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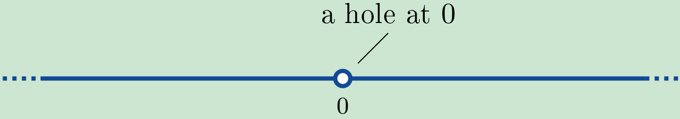
The set $\{x \mid x \in \mathbb{R} \text{ and } x \neq 0\}$ is not an interval.



A finite interval is

- *closed* if it contains both its endpoints;
- *half-open* if it contains one of its endpoints;
- *open* if it does not contain its endpoints;

Notation	Set	Type	Picture
(a, b)	$\{x \vert a < x < b\}$	open	
$[a, b]$	$\{x \vert a \leq x \leq b\}$	closed	
$[a, b)$	$\{x \vert a \leq x < b\}$	half open	
$(a, b]$	$\{x \vert a < x \leq b\}$	half open	

An infinite interval is

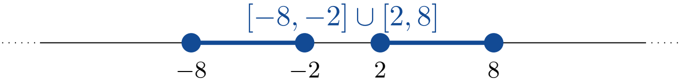
- *closed* if it contains a finite endpoint;
- *open* if it is not closed.

There is one exception to this rule: The whole real line is called both open and closed.

A.1 Real Numbers and the Real Line

Notation	Set	Type	Picture
(a, ∞)	$\{x \mid a < x\}$	open	
$[a, \infty)$	$\{x \mid a \leq x\}$	closed	
$(-\infty, b)$	$\{x \mid x < b\}$	open	
$(-\infty, b]$	$\{x \mid x \leq b\}$	closed	
$(-\infty, \infty)$	$\mathbb{R}$	both open and closed	

We can combine two (or more) intervals with the notation $\cup$.
For example, $[-8, -2] \cup [2, 8]$ is called the *union* of $[-8, -2]$ and $[2, 8]$ and is shown below.



Functions and Their Graphs

$$y = f(x)$$

“y eşittir f x”

“y is equal to f of x”

dependent variable

$$y = f(x)$$

function

independent variable

"y eşittir f x"

"y is equal to f of x"

Definition

A *function* from a set D to a set Y is a rule that assigns a unique element of Y to each element of D.

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Definition

The set D of all possible values of x is called the *domain* of f.

Definition

The set Y is called the *target* of f.

Definition

The set of all possible values of $f(x)$ is called the *range* of f.

If f is a function with domain D and target Y, we can write

$$\underbrace{f : D}_{\text{domain}} \to \underbrace{Y}_{\text{target}}$$

1.1 Functions and Their Graphs

If f is a function with domain D and target Y, we can write

$$f : \underset{\text{domain}}{D} \to \underset{\text{target}}{Y}$$

Example

$f : \mathbb{R} \to \mathbb{R},\ f(x) = x^2.$

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$$\underbrace{f : D}_{\text{domain}} \to \underbrace{Y}_{\text{target}}$$

Example

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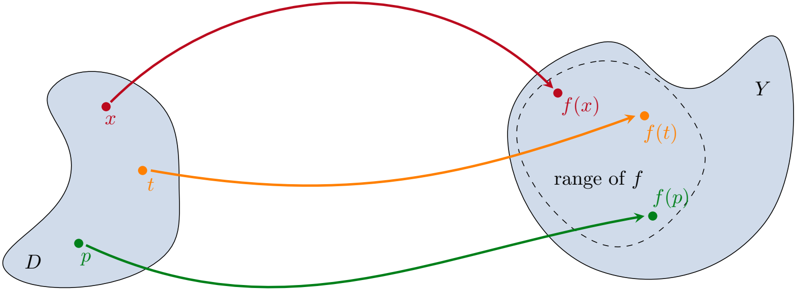
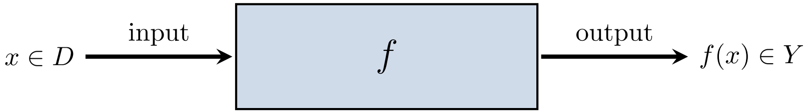
Example

$f : (-\infty, \infty) \to [0, \infty),\ f(x) = x^2.$



x
t
p
D
$f(x)$
$f(t)$
$f(p)$
range of f
Y


$x \in D$
input
f
output
$f(x) \in Y$

Sometimes we want to use the largest possible domain for a function. This is called the *natural domain* of the function.

Sometimes we will want to use a smaller domain.

function	domain (x)	range (y)
$y = x^2$	$(-\infty, \infty)$	
$y = \frac{1}{x}$	$\{x \mid x \in \mathbb{R}, x \neq 0\}$	
$y = \sqrt{x}$	$[0, \infty)$	
$y = \sqrt{4 - x}$		
$y = \sqrt{1 - x^2}$		

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$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
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$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	
$y = \sqrt{1 - x^2}$		

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function	domain (x)	range (y)
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = \frac{1}{x}$	$\{x \mid x \in \mathbb{R}, x \neq 0\}$	$\{y \mid y \in \mathbb{R}, y \neq 0\}$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	

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$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
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$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$
$y = x^2$	$[1, 2]$	
$y = x^2$	$[2, \infty)$	
$y = x^2$	$(-\infty, -2]$	

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$y = \frac{1}{x}$	$\{x \mid x \in \mathbb{R}, x \neq 0\}$	$\{y \mid y \in \mathbb{R}, y \neq 0\}$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$
$y = x^2$	$[1, 2]$	$[1, 4]$
$y = x^2$	$[2, \infty)$	
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$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$
$y = x^2$	$[1, 2]$	$[1, 4]$
$y = x^2$	$[2, \infty)$	$[4, \infty)$
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$y = x^2$	$[1, 2]$	$[1, 4]$
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$y = x^2$	$[1, 2]$	$[1, 4]$
$y = x^2$	$[2, \infty)$	$[4, \infty)$
$y = x^2$	$(-\infty, -2]$	$[4, \infty)$
$y = 1 + x^2$	$[1, 3)$	
$y = 1 - \sqrt{x}$	$[0, \infty)$	

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$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$
$y = x^2$	$[1, 2]$	$[1, 4]$
$y = x^2$	$[2, \infty)$	$[4, \infty)$
$y = x^2$	$(-\infty, -2]$	$[4, \infty)$
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$y = x^2$	$[1, 2]$	$[1, 4]$
$y = x^2$	$[2, \infty)$	$[4, \infty)$
$y = x^2$	$(-\infty, -2]$	$[4, \infty)$
$y = 1 + x^2$	$[1, 3)$	$[2, 10)$
$y = 1 - \sqrt{x}$	$[0, \infty)$	$(-\infty, 1]$

Graphs of Functions

Definition

The *graph* of f is the set containing all the points (x, y) which satisty $y = f(x)$.

Example

Graph the function $y = 1 + x^2$ over the interval $[-2, 2]$.

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STEP 1: Make a table of (x, y) points which satisfy $y = 1 + x^2$.

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STEP 1: Make a table of (x, y) points which satisfy $y = 1 + x^2$.

x	y
-2	5
-1	2
0	1
1	2
$\frac{3}{2}$	$\frac{13}{4} = 3\frac{1}{4}$
2	5

STEP 2: Plot these points.

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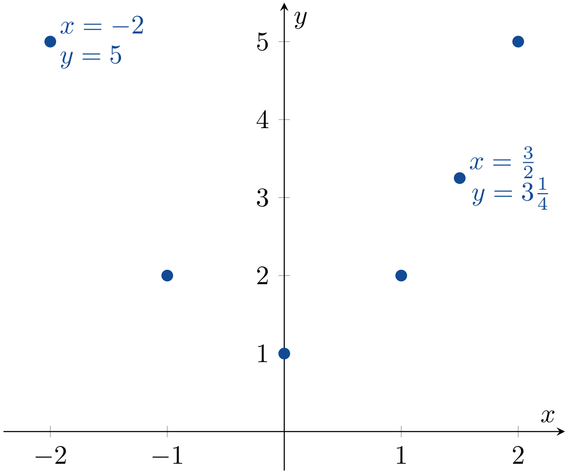
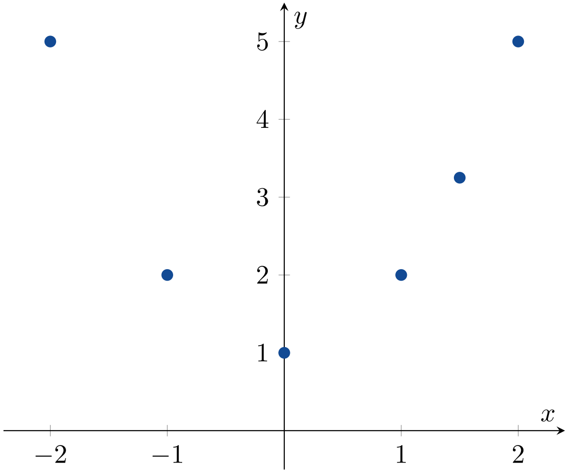


STEP 3: Draw a smooth curve through these points.



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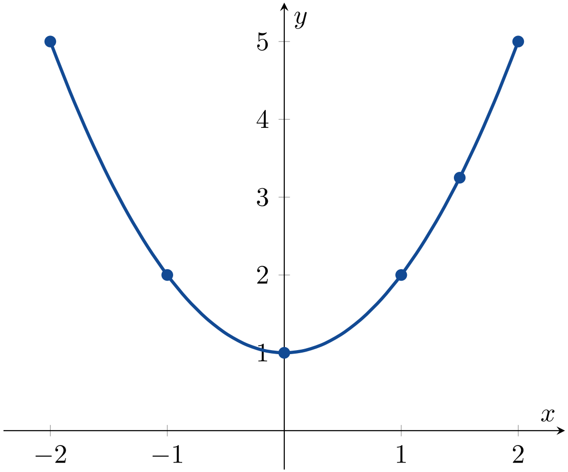
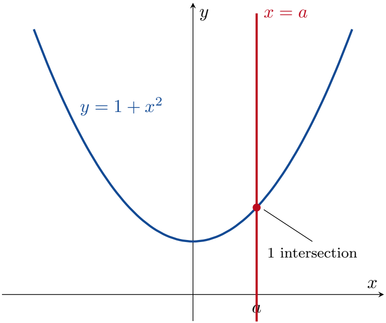
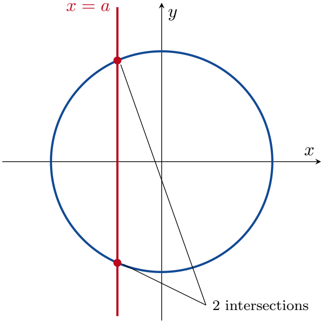


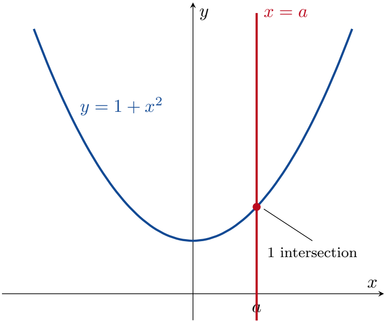
The Vertical Line Test

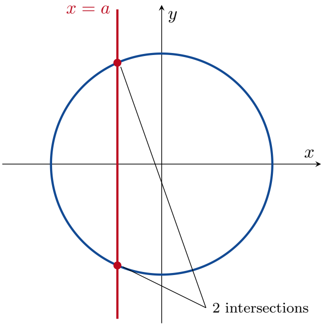
Not every curve that you draw is a graph of a function.



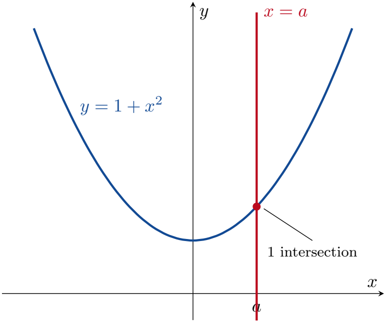


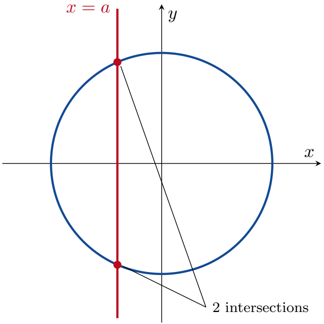
A function can have only one value $f(x)$ for each $x \in D$. This means that a vertical line can intersect the graph of a function at most once.





A circle can not be the graph of a function because some vertical lines intersect the circle at two points.



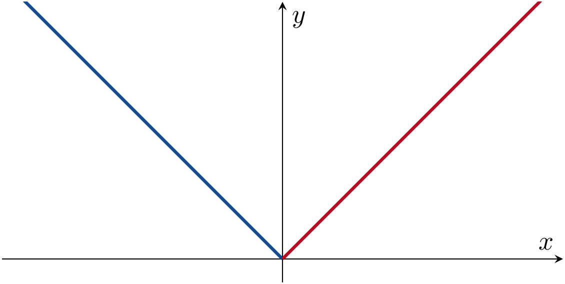


If $a \in D$, then the vertical line $x = a$ will intersect the graph of $f : D \to Y$ only at the point $(a, f(a))$.

Piecewise-Defined Functions

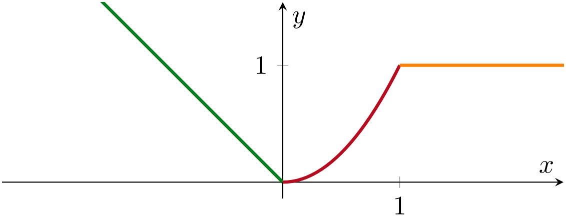
Example

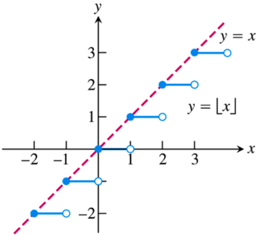
$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$



Example

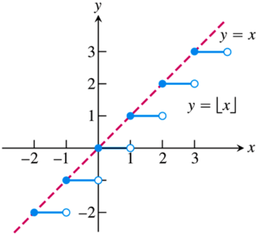
$$f(x) = \begin{cases} -x & x < 0 \\ x^2 & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

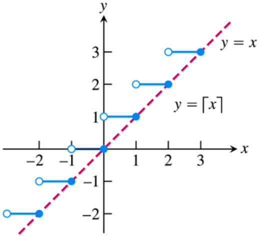




EXAMPLE 5 The function whose value at any number x is the *greatest integer less than or equal to x* is called the **greatest integer function** or the **integer floor function**. It is denoted $\lfloor x \rfloor$. Figure 1.10 shows the graph. Observe that

$$\lfloor 2.4 \rfloor = 2, \quad \lfloor 1.9 \rfloor = 1, \quad \lfloor 0 \rfloor = 0, \quad \lfloor -1.2 \rfloor = -2,$$
$$\lfloor 2 \rfloor = 2, \quad \lfloor 0.2 \rfloor = 0, \quad \lfloor -0.3 \rfloor = -1, \quad \lfloor -2 \rfloor = -2.$$





EXAMPLE 5 The function whose value at any number x is the *greatest integer less than or equal to* x is called the **greatest integer function** or the **integer floor function**. It is denoted $\lfloor x \rfloor$. Figure 1.10 shows the graph. Observe that

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$$\lfloor 2 \rfloor = 2, \quad \lfloor 0.2 \rfloor = 0, \quad \lfloor -0.3 \rfloor = -1, \quad \lfloor -2 \rfloor = -2.$$

EXAMPLE 6 The function whose value at any number x is the *smallest integer greater than or equal to* x is called the **least integer function** or the **integer ceiling function**. It is denoted $\lceil x \rceil$. Figure 1.11 shows the graph. For positive values of x, this function might represent, for example, the cost of parking x hours in a parking lot that charges \$1 for each hour or part of an hour.

Increasing and Decreasing Functions

Definition

Let I be an interval. Let $f : I \to \mathbb{R}$ be a function.

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for all $x_1, x_2 \in I$ which satisfy $x_1 < x_2$;

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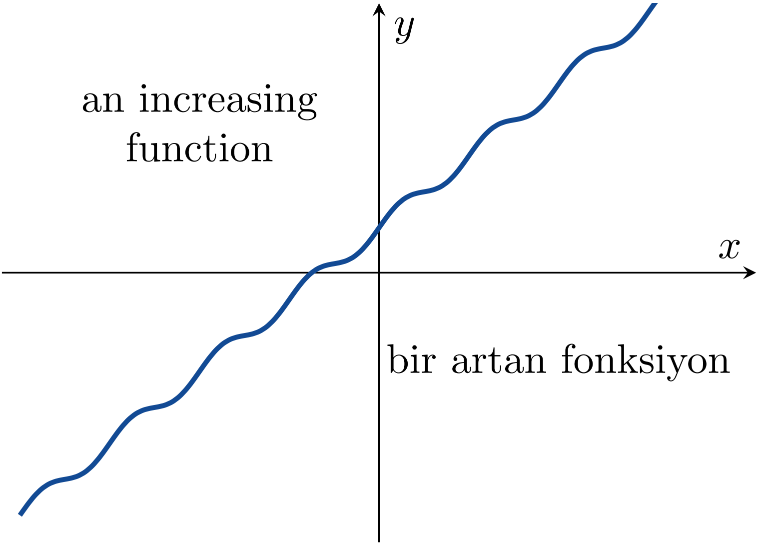
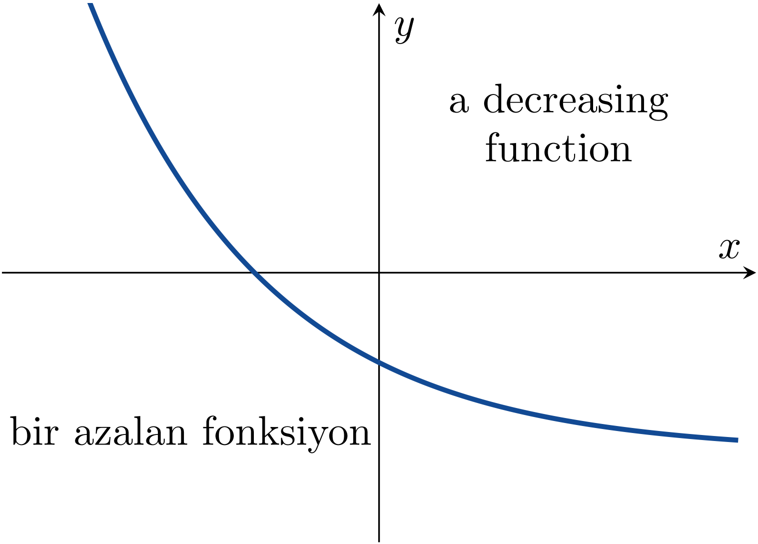
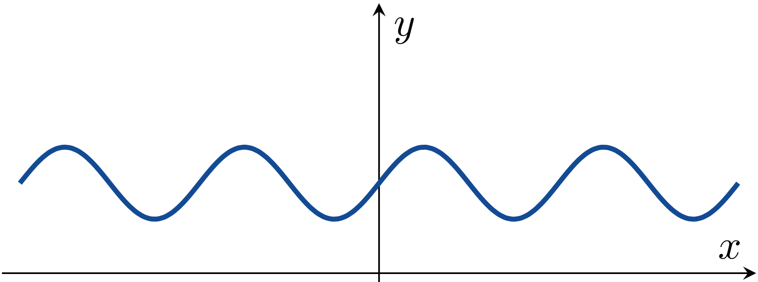
for all $x_1, x_2 \in I$ which satisfy $x_1 < x_2$;
2. f is called *decreasing on I* if
$$f(x_1) > f(x_2)$$
for all $x_1, x_2 \in I$ which satisfy $x_1 < x_2$.


y
an increasing
function
x
bir artan fonksiyon


y
a decreasing function
x
bir azalan fonksiyon



not increasing
not decreasing

artmıyor
azalmıyor

Even Functions and Odd Functions

Recall that

- $2, 4, 6, 8, 10, \dots$ are even numbers; and
- $1, 3, 5, 7, 9, \dots$ are odd numbers.

Even Functions and Odd Functions

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- $2, 4, 6, 8, 10, \ldots$ are even numbers; and
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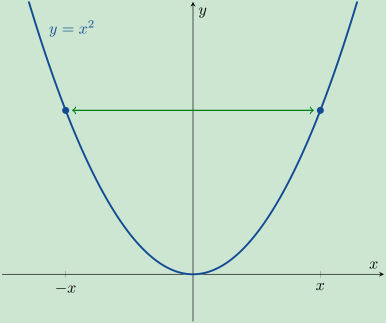
Definition

1. $f : D \to \mathbb{R}$ is an *even function* if $f(-x) = f(x)$ for all $x \in D$;
2. $f : D \to \mathbb{R}$ is an *odd function* if $f(-x) = -f(x)$ for all $x \in D$.

Example

$f(x) = x^2$ is an even function because

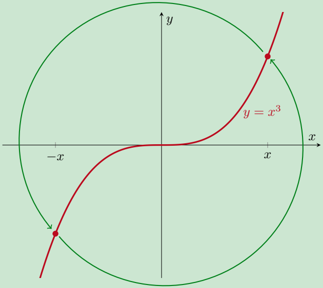
$$f(-x) = (-x)^2 = x^2 = f(x).$$



Example

$f(x) = x^3$ is an odd function because

$$f(-x) = (-x)^3 = -x^3 = -f(x).$$



Example

Is $f(x) = x^2 + 1$ even, odd or neither?

Example

Is $f(x) = x^2 + 1$ even, odd or neither?

Since
$$f(-x) = (-x)^2 + 1 = x^2 + 1 = f(x),$$
f is an even function.

Example

Is $g(x) = x + 1$ even, odd or neither?

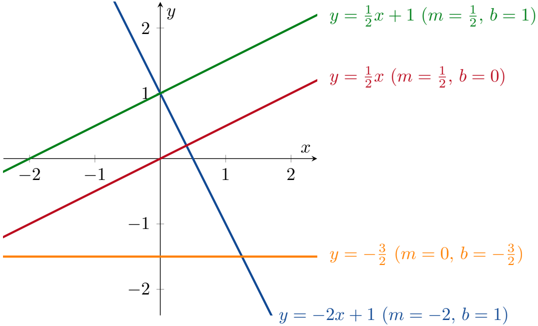
Example

Is $g(x) = x + 1$ even, odd or neither?

Since $g(-2) = -2 + 1 = -1$ and $g(2) = 3$, we have $g(-2) \neq g(2)$ and $g(-2) \neq -g(2)$. Hence g is neither even nor odd.

Linear Functions

$$f(x) = mx + b$$
$$(m, b \in \mathbb{R})$$

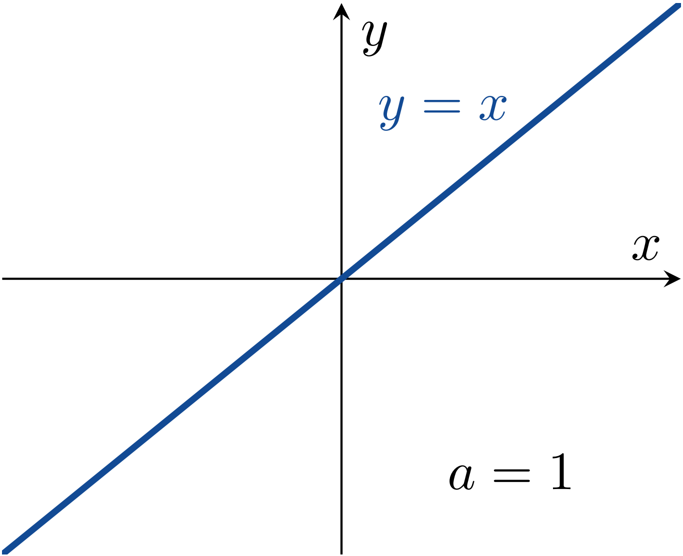


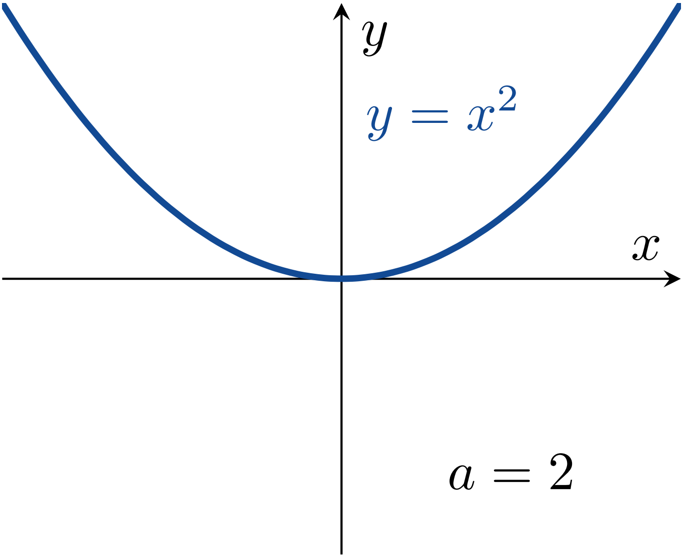
Power Functions

$$f(x) = x^a$$

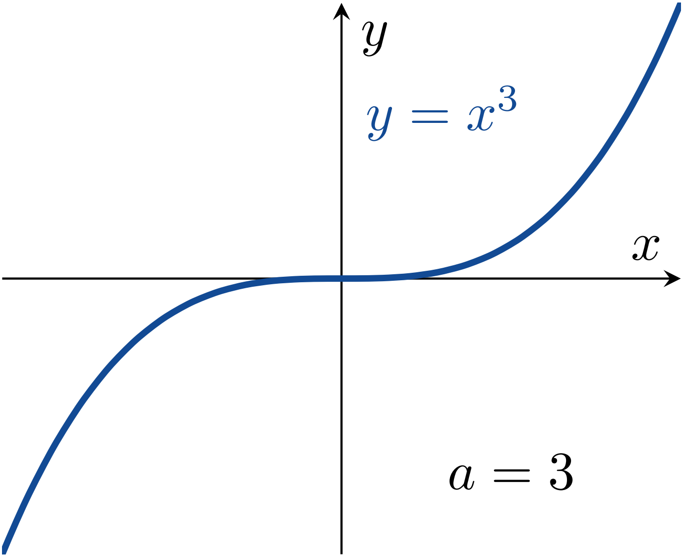
$(a \in \mathbb{R})$

"x to the power of a"

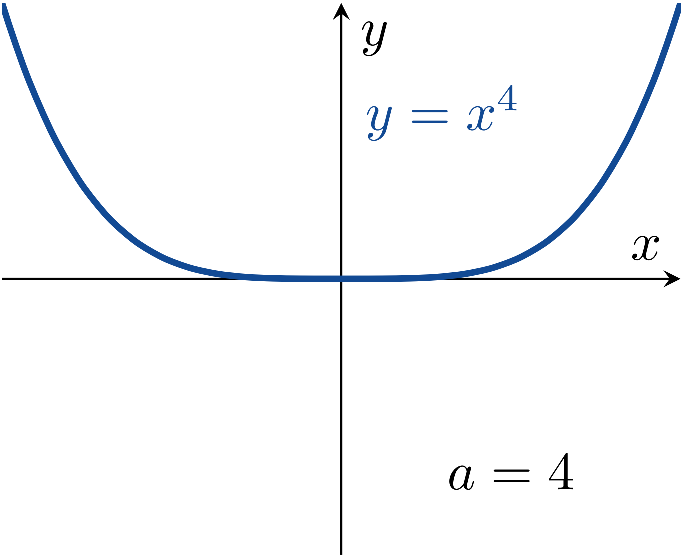

y
$y = x$
x
$a = 1$



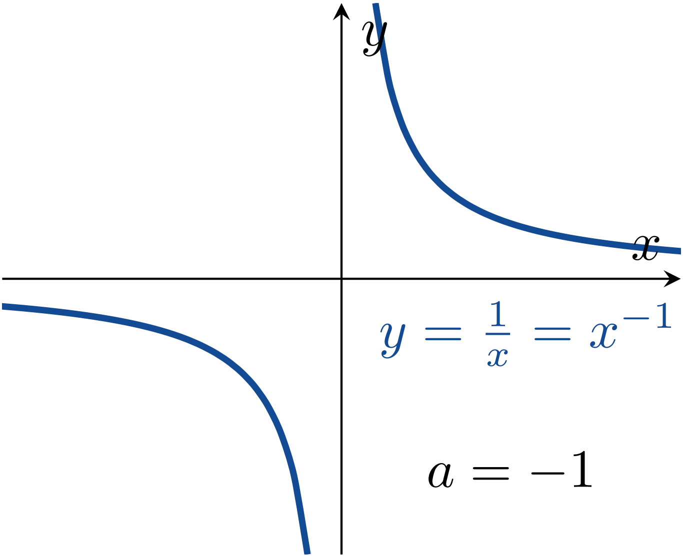
$a = 2$



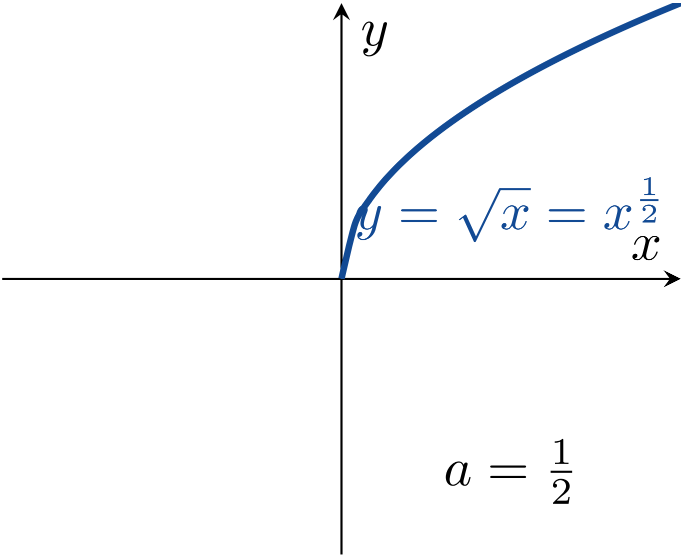
y
$y = x^3$
x
$a = 3$



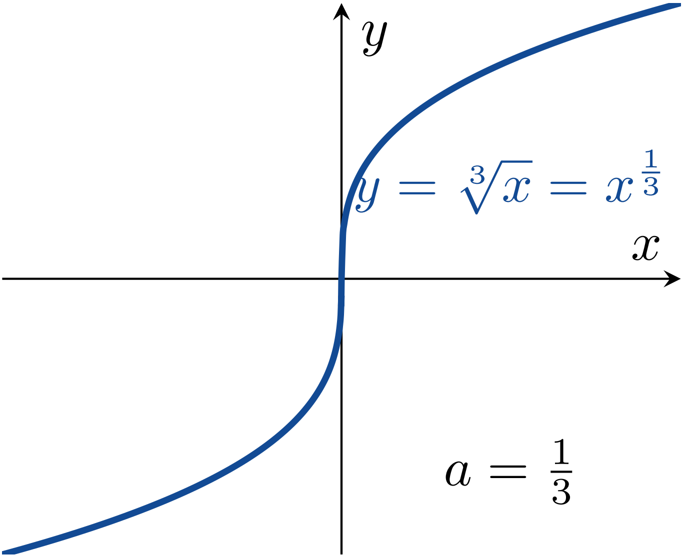
$a = 4$

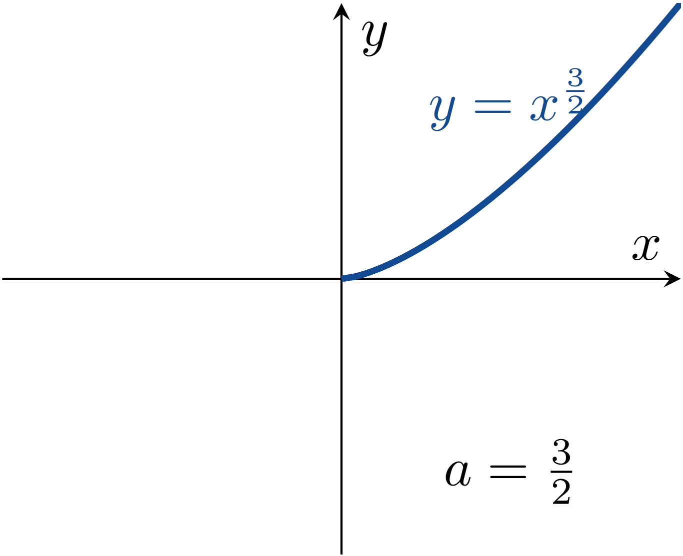


$a = -1$

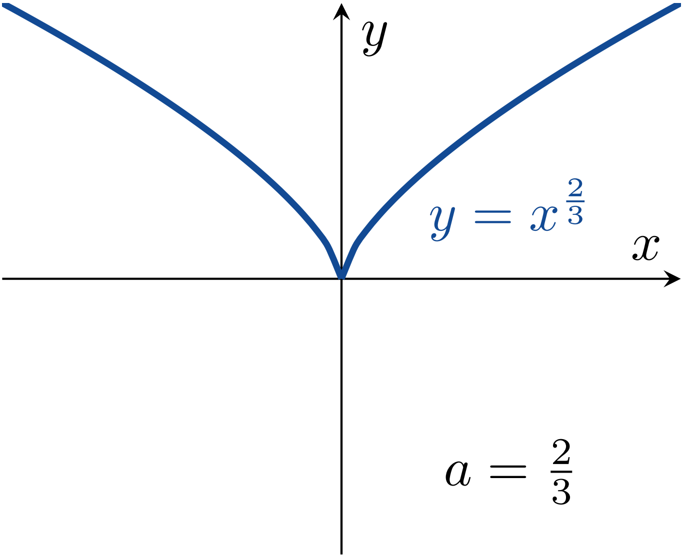


$$a = \frac{1}{2}$$


y
$y = \sqrt[3]{x} = x^{\frac{1}{3}}$
x
$a = \frac{1}{3}$



$$a = \frac{3}{2}$$



$$a = \frac{2}{3}$$

Polynomials

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

$$(n \in \mathbb{N} \cup \{0\},\ a_j \in \mathbb{R}).$$

Polynomials

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The (natural) domain of a polynomial is always $(-\infty, \infty)$. If $n > 0$ and $a_n \neq 0$, then n is called the *degree* of $p(x)$.

Rational Functions

$$\underbrace{f(x)}_{\text{rational function}} = \frac{p(x)}{q(x)}$$

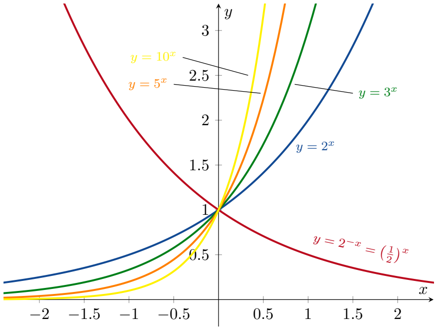
polynomial

Example

$$f(x) = \frac{2x^3 - 3}{7x + 4}$$

Exponential Functions

$$f(x) = a^x$$
$$(a \in \mathbb{R},\ a > 0,\ a \neq 1)$$



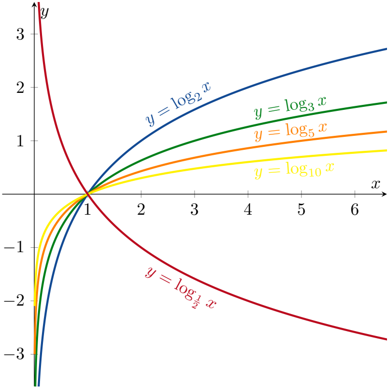
The (natural) domain of an exponential function is $(-\infty, \infty)$.

Logarithmic Functions

$$y = \log_a x \iff x = a^y$$

$$(a \in \mathbb{R},\ a > 0,\ a \neq 1)$$

"log base a of x"



Break

We will continue at 3pm

Combining Functions; Shifting and Scaling Graphs

Sums, Differences, Products and Quotients

Consider $f : D(f) \to \mathbb{R}$ and $g : D(g) \to \mathbb{R}$.

Sums, Differences, Products and Quotients

Consider $f : D(f) \to \mathbb{R}$ and $g : D(g) \to \mathbb{R}$. We can define 4 new functions:

$$
\begin{aligned}
(f+g)(x) &= f(x) + g(x) \\
(f-g)(x) &= f(x) - g(x) \\
(fg)(x) &= f(x)g(x) \\
\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}
\end{aligned}
$$

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\end{aligned}
$$

The domain of $(f+g)$, $(f-g)$ and (fg) is $D(f) \cap D(g)$.

Sums, Differences, Products and Quotients

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$$(f+g)(x) = f(x) + g(x)$$
$$(f-g)(x) = f(x) - g(x)$$
$$(fg)(x) = f(x)g(x)$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

The domain of $(f+g)$, $(f-g)$ and (fg) is $D(f) \cap D(g)$.
The domain of $\left(\frac{f}{g}\right)$ is $D(f) \cap \{x \in D(g) \mid g(x) \neq 0\}$.

Example

Consider $f : [0, \infty) \to \mathbb{R}$, $f(x) = \sqrt{x}$ and $g : (-\infty, 1] \to \mathbb{R}$, $g(x) = \sqrt{1 - x}$.

Example

Consider $f : [0, \infty) \to \mathbb{R}$, $f(x) = \sqrt{x}$ and $g : (-\infty, 1] \to \mathbb{R}$, $g(x) = \sqrt{1-x}$. Find $(f+g)$, $(f-g)$, (fg), $\left(\frac{f}{g}\right)$ and $\left(\frac{g}{f}\right)$ and find the domains of these 5 functions.

$$D(f) = [0, \infty),\ f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1],\ g(x) = \sqrt{1-x}$$

Function	Formula	Domain
$f+g$		
$f-g$		
fg		
$\frac{f}{g}$		
$\frac{g}{f}$		

$$D(f) = [0, \infty),\ f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1],\ g(x) = \sqrt{1-x}$$

Function	**Formula**	**Domain**
$f+g$	$\sqrt{x} + \sqrt{1-x}$	
$f-g$		
fg		
$\frac{f}{g}$		
$\frac{g}{f}$		

$$D(f) = [0, \infty),\ f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1],\ g(x) = \sqrt{1-x}$$

Function	**Formula**	**Domain**
$f+g$	$\sqrt{x} + \sqrt{1-x}$	$[0, 1]$
$f-g$		
fg		
$\frac{f}{g}$		
$\frac{g}{f}$		

1.2 Combining Functions; Shifting and Scaling Graphs

$$D(f) = [0, \infty),\ f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1],\ g(x) = \sqrt{1-x}$$

Function	**Formula**	**Domain**
$f+g$	$\sqrt{x} + \sqrt{1-x}$	$[0, 1]$
$f-g$	$\sqrt{x} - \sqrt{1-x}$	$[0, 1]$
fg	$\sqrt{x}\sqrt{1-x} = \sqrt{x - x^2}$	$[0, 1]$
$\frac{f}{g}$		
$\frac{g}{f}$		

$$D(f) = [0, \infty),\ f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1],\ g(x) = \sqrt{1-x}$$

Function	**Formula**	**Domain**
$f+g$	$\sqrt{x} + \sqrt{1-x}$	$[0, 1]$
$f-g$	$\sqrt{x} - \sqrt{1-x}$	$[0, 1]$
fg	$\sqrt{x}\sqrt{1-x} = \sqrt{x-x^2}$	$[0, 1]$
$\frac{f}{g}$	$\sqrt{\frac{x}{1-x}}$	
$\frac{g}{f}$	$\sqrt{\frac{1-x}{x}}$	

$D(f) = [0, \infty),\ f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1],\ g(x) = \sqrt{1-x}$

Function	**Formula**	**Domain**
$f+g$	$\sqrt{x} + \sqrt{1-x}$	$[0, 1]$
$f-g$	$\sqrt{x} - \sqrt{1-x}$	$[0, 1]$
fg	$\sqrt{x}\sqrt{1-x} = \sqrt{x-x^2}$	$[0, 1]$
$\frac{f}{g}$	$\sqrt{\frac{x}{1-x}}$	$[0, 1)$
$\frac{g}{f}$	$\sqrt{\frac{1-x}{x}}$	

$D(f) = [0, \infty),\ f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1],\ g(x) = \sqrt{1-x}$

Function	**Formula**	**Domain**
$f+g$	$\sqrt{x} + \sqrt{1-x}$	$[0,1]$
$f-g$	$\sqrt{x} - \sqrt{1-x}$	$[0,1]$
fg	$\sqrt{x}\sqrt{1-x} = \sqrt{x - x^2}$	$[0,1]$
$\frac{f}{g}$	$\sqrt{\frac{x}{1-x}}$	$[0,1)$
$\frac{g}{f}$	$\sqrt{\frac{1-x}{x}}$	$(0,1]$

Composite Functions

$$f \circ g(x) = f(g(x))$$

is the *composite* of f and g.

We read $f \circ g$ as "f composed with g".

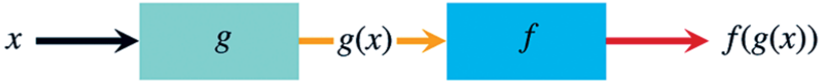
Composite Functions

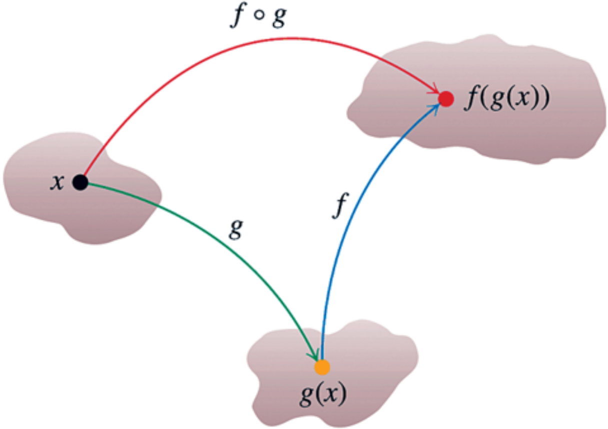
$$f \circ g(x) = f(g(x))$$

is the *composite* of f and g.

We read $f \circ g$ as "f composed with g".

The domain of $f \circ g$ consists of all numbers x in the domain of g for which the number $g(x)$ lies in the domain of f.


x
g
$g(x)$
f
$f(g(x))$


$f \circ g$
$f(g(x))$
x
f
g
$g(x)$

Example

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$, and find the domains of these 4 functions.

Note that the domain of f is $[0, \infty)$ and the domain of g is $(-\infty, \infty)$.

Function	**Formula**	**Domain**
$f \circ g$		
$g \circ f$		
$f \circ f$		
$g \circ g$		

Example

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$, and find the domains of these 4 functions.

Note that the domain of f is $[0, \infty)$ and the domain of g is $(-\infty, \infty)$.

Function	Formula	Domain
$f \circ g$	$f(g(x)) = \sqrt{x+1}$	
$g \circ f$		
$f \circ f$		
$g \circ g$		

Example

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$, and find the domains of these 4 functions.

Note that the domain of f is $[0, \infty)$ and the domain of g is $(-\infty, \infty)$.

Function	**Formula**	**Domain**
$f \circ g$	$f(g(x)) = \sqrt{x+1}$	$[-1, \infty)$
$g \circ f$		
$f \circ f$		
$g \circ g$		

Example

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$, and find the domains of these 4 functions.

Note that the domain of f is $[0, \infty)$ and the domain of g is $(-\infty, \infty)$.

Function	Formula	Domain
$f \circ g$	$f(g(x)) = \sqrt{x+1}$	$[-1, \infty)$
$g \circ f$	$g(f(x)) = \sqrt{x} + 1$	$[0, \infty)$
$f \circ f$		
$g \circ g$		

Example

If $f(x)=\sqrt{x}$ and $g(x)=x+1$, find $f\circ g$, $g\circ f$, $f\circ f$ and $g\circ g$, and find the domains of these 4 functions.

Note that the domain of f is $[0,\infty)$ and the domain of g is $(-\infty,\infty)$.

Function	Formula	Domain
$f\circ g$	$f(g(x))=\sqrt{x+1}$	$[-1,\infty)$
$g\circ f$	$g(f(x))=\sqrt{x}+1$	$[0,\infty)$
$f\circ f$	$f(f(x))=\sqrt{\sqrt{x}}=x^{\frac{1}{4}}$	$[0,\infty)$
$g\circ g$		

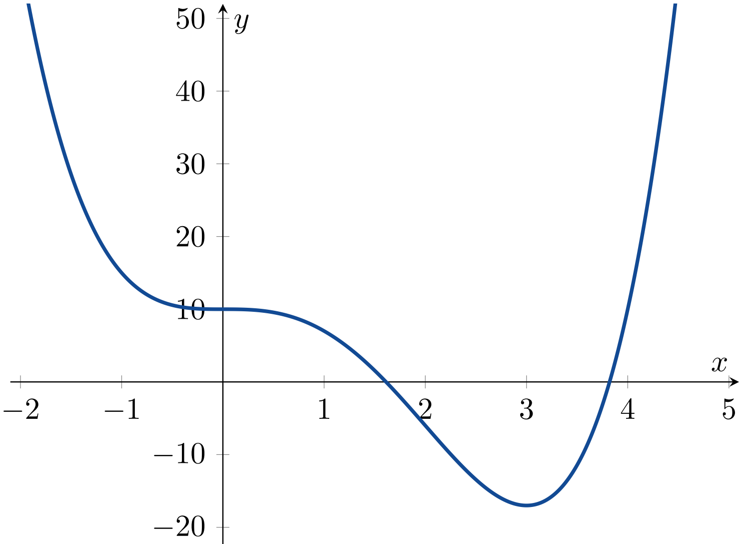
Example

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$, and find the domains of these 4 functions.

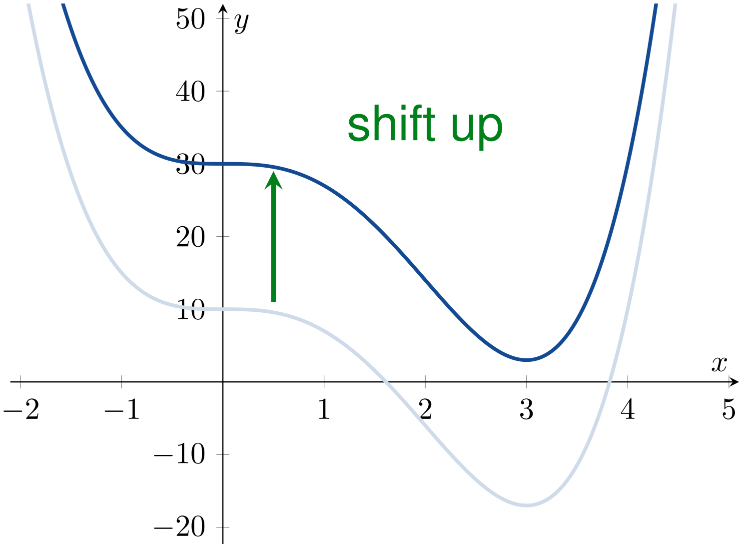
Note that the domain of f is $[0, \infty)$ and the domain of g is $(-\infty, \infty)$.

Function	**Formula**	**Domain**
$f \circ g$	$f(g(x)) = \sqrt{x+1}$	$[-1, \infty)$
$g \circ f$	$g(f(x)) = \sqrt{x} + 1$	$[0, \infty)$
$f \circ f$	$f(f(x)) = \sqrt{\sqrt{x}} = x^{\frac{1}{4}}$	$[0, \infty)$
$g \circ g$	$g(g(x)) = (x+1) + 1 = x + 2$	$(-\infty, \infty)$

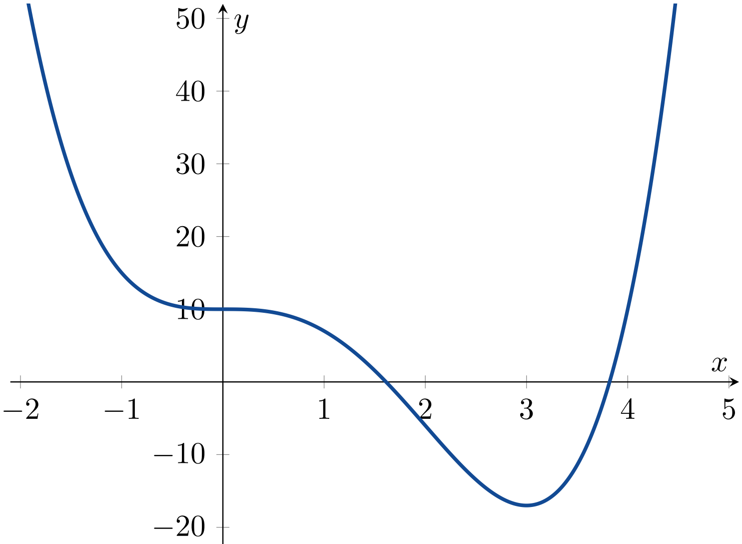
Shifting



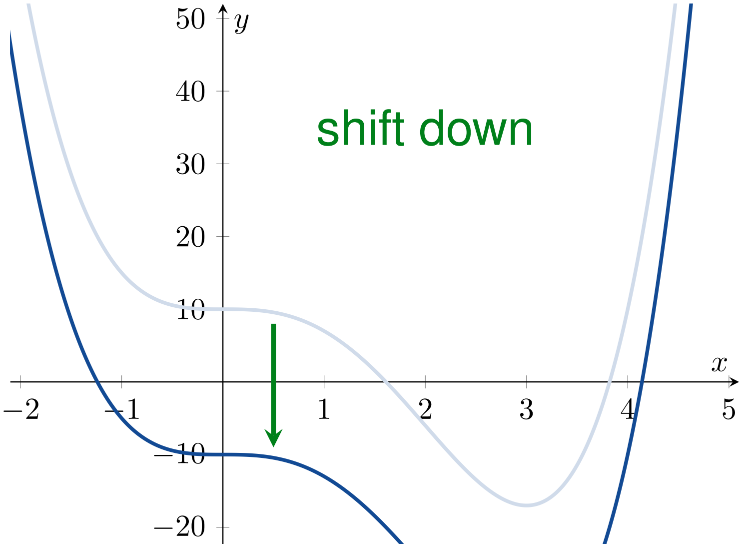
Shifting



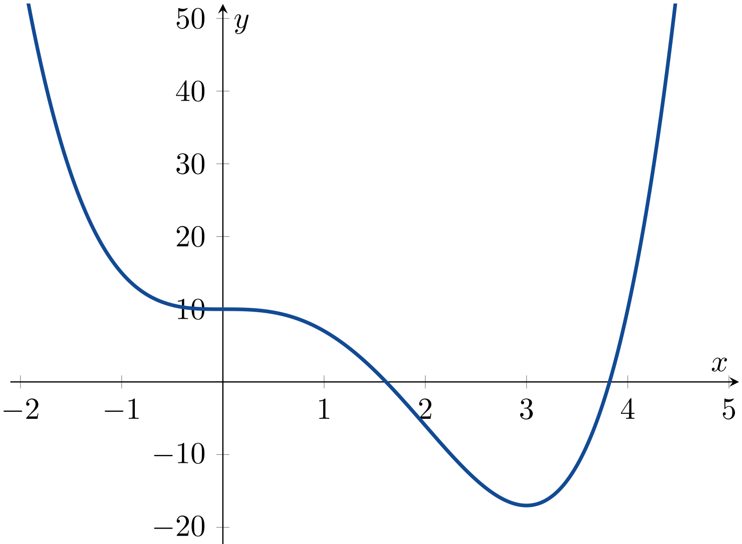
Shifting



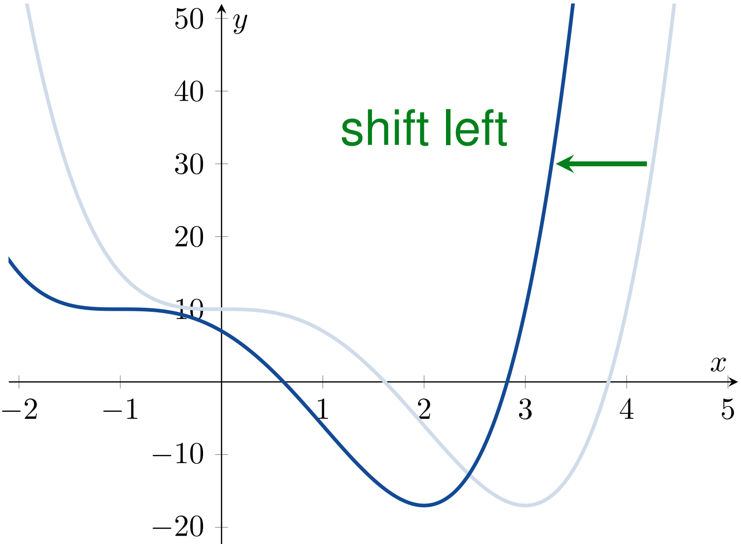
Shifting



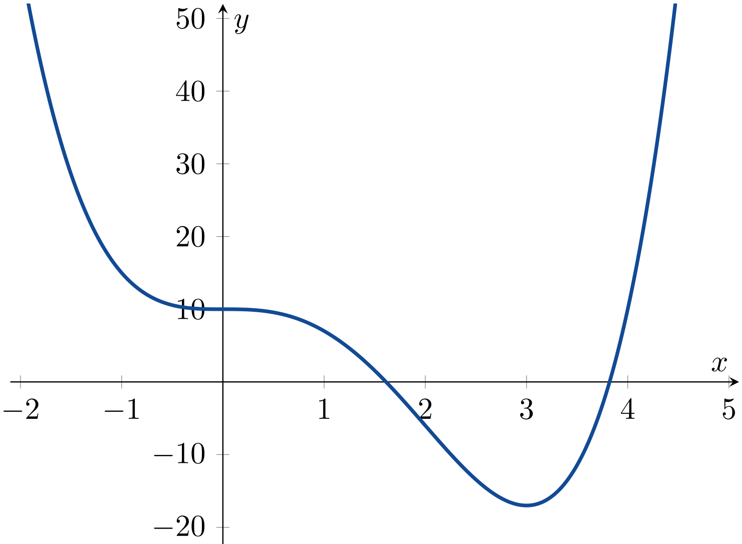
Shifting



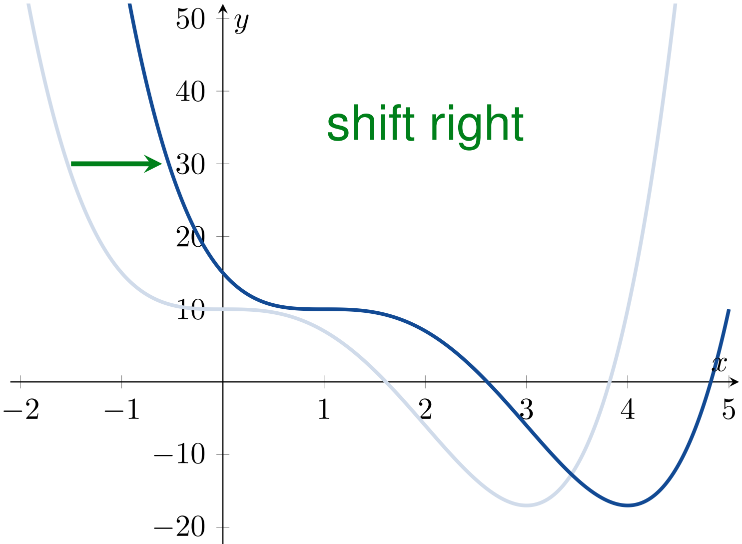
Shifting



Shifting



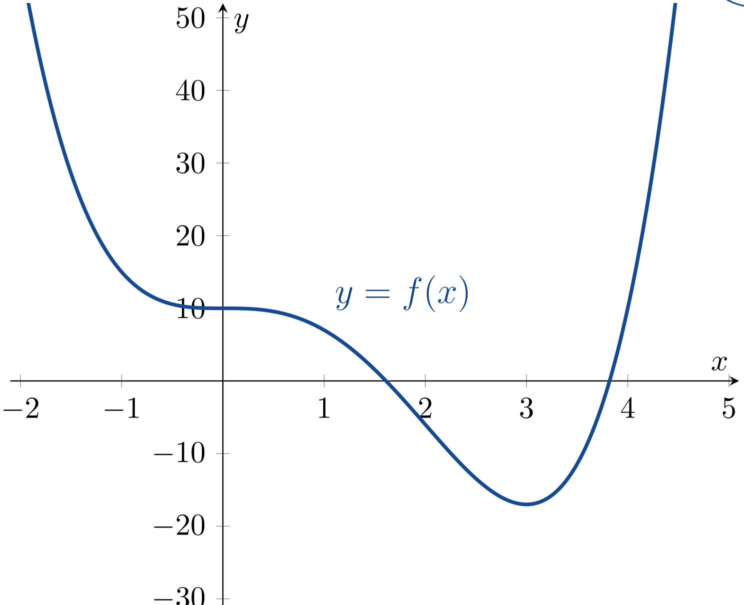
Shifting



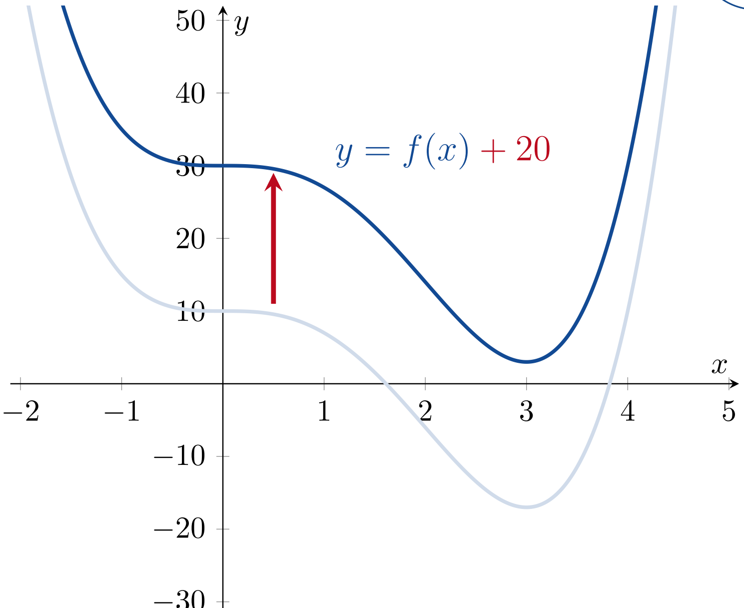
$$y = f(x) + k \quad \text{shifts the graph up } k \text{ units.}$$
$$\text{(or down } |k| \text{ units if } k < 0)$$

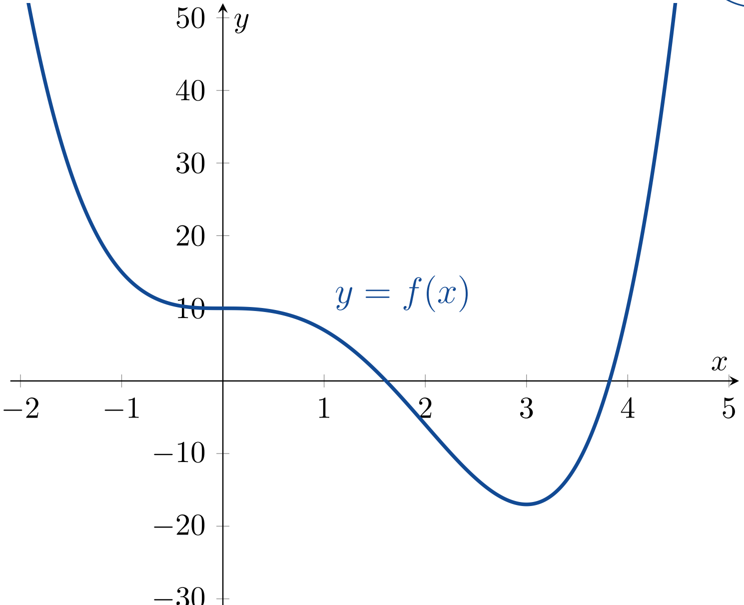
$y = f(x) + k$ shifts the graph up k units.
(or down $|k|$ units if $k < 0$)

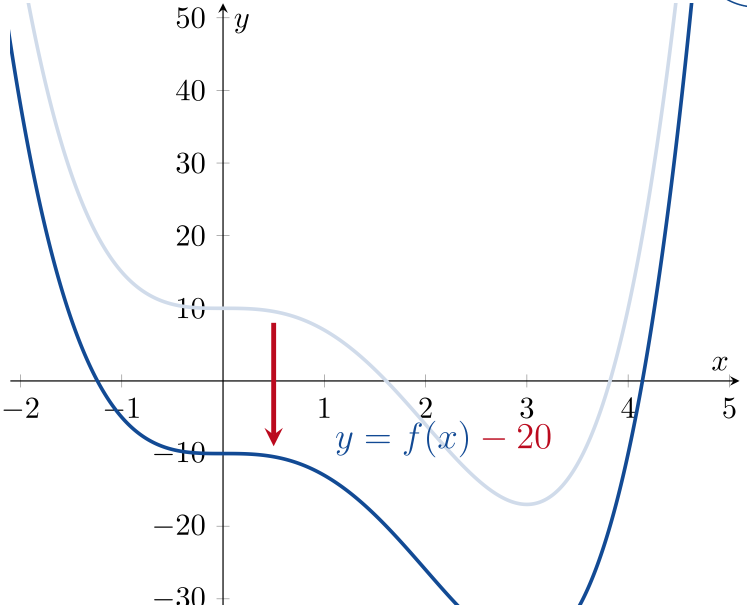
$y = f(x + k)$ shifts the graph left k units.
(or right $|k|$ units if $k < 0$)



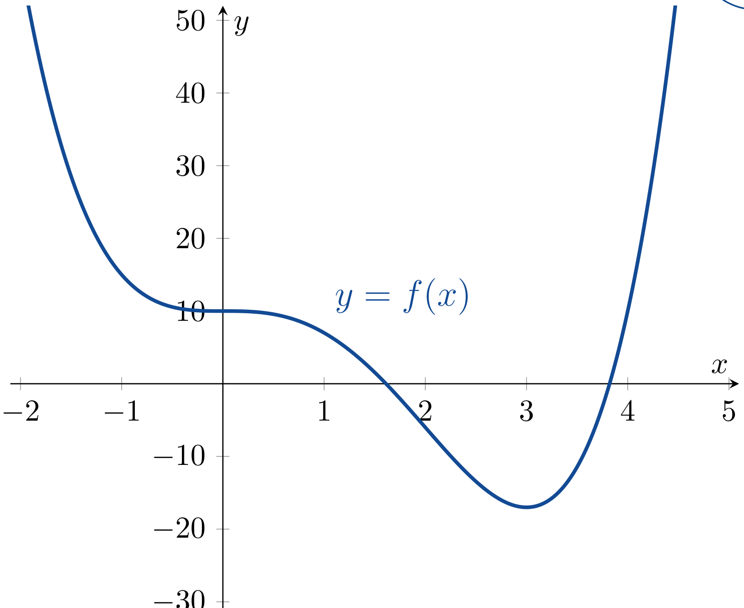
y
50
40
30
20
10
$y = f(x)$
x
−2
−1
1
2
3
4
5
−10
−20
−30


y
50
40
30
20
10
−10
−20
−30
x
−2
−1
1
2
3
4
5
$y = f(x) + 20$

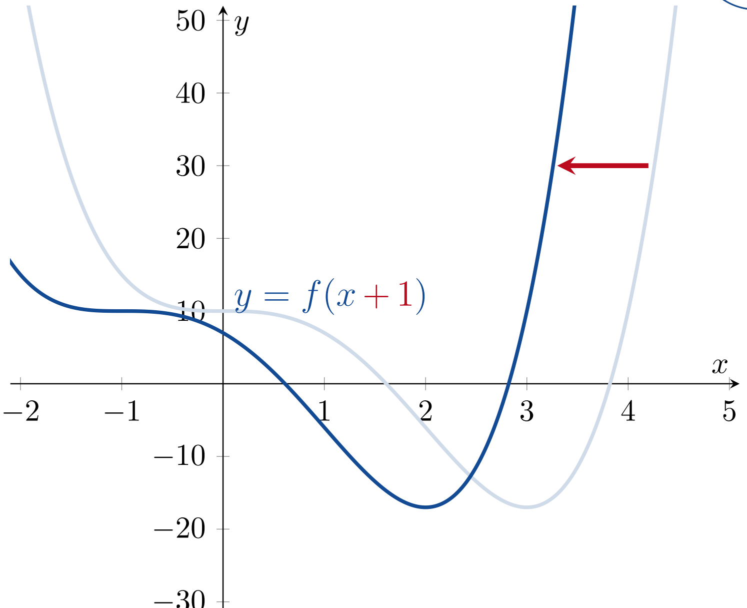

50
40
30
20
10
−10
−20
−30
y
x
−2
−1
1
2
3
4
5
$y = f(x)$



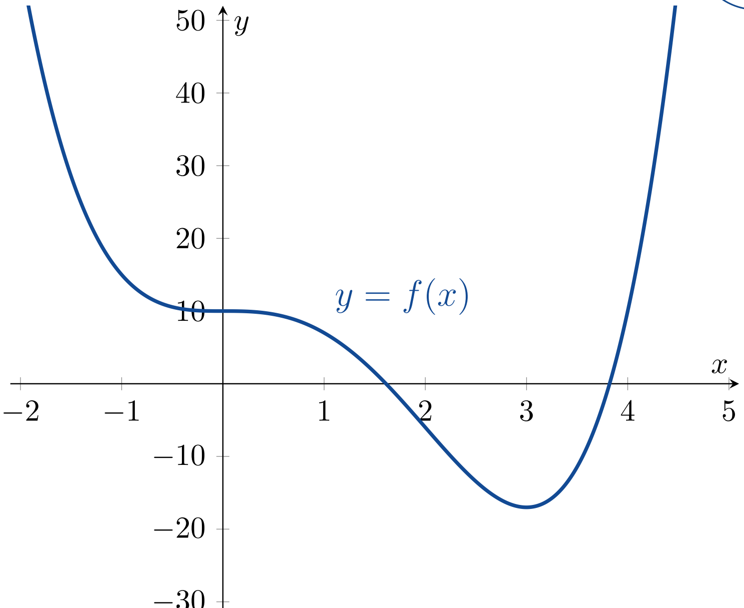
50
40
30
20
10
−10
−20
−30
y
x
−2
−1
1
2
3
4
5
$y = f(x) - 20$



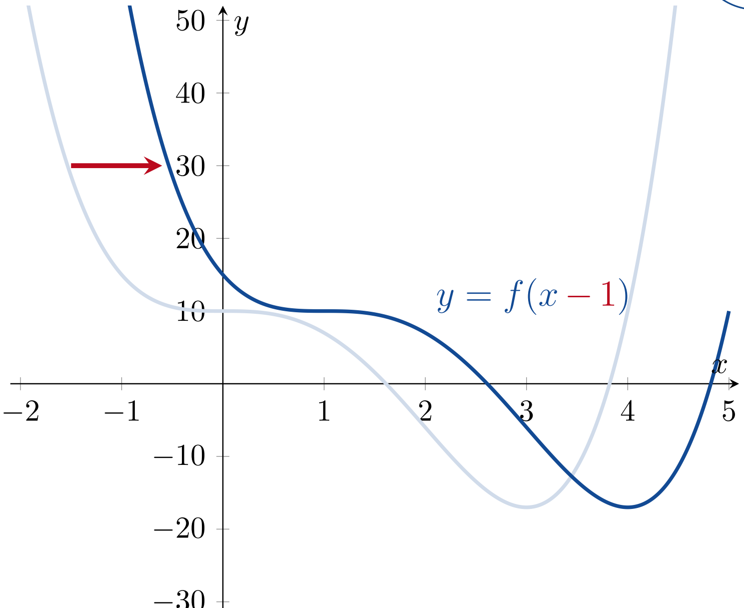
y
x
50
40
30
20
10
−10
−20
−30
−2
−1
1
2
3
4
5
$y = f(x)$



y
50
40
30
20
10
$y = f(x+1)$
x
-2
-1
1
2
3
4
5
-10
-20
-30



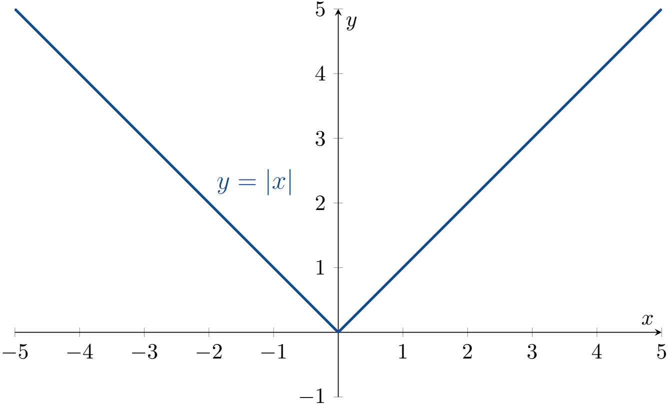
y
x
50
40
30
20
10
−10
−20
−30
−2
−1
1
2
3
4
5
$y = f(x)$



50
40
30
20
10
−10
−20
−30
y
x
−2
−1
1
2
3
4
5
y = f(x − 1)

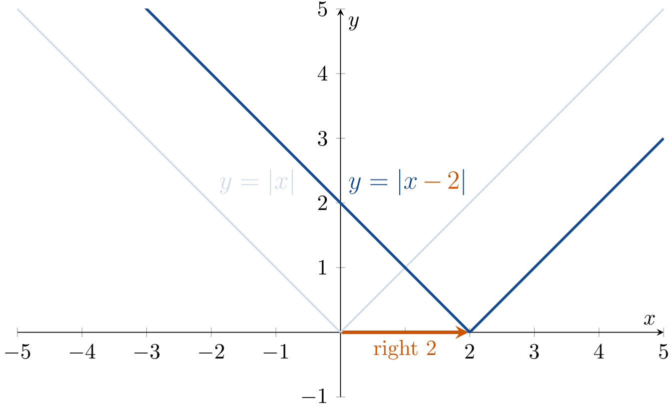
Example

Shift the function $y = |x|$ by 2 units to the right and 1 unit up.



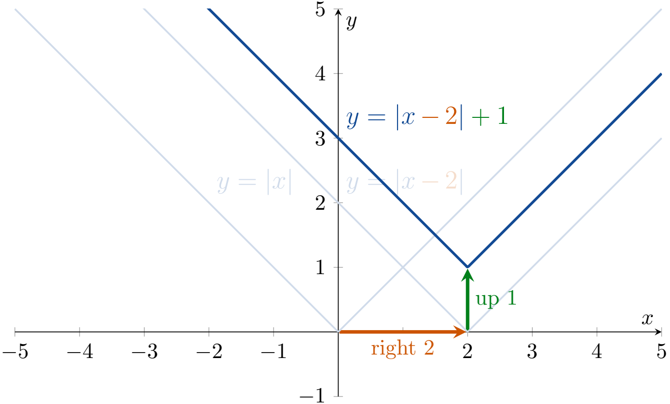
Example

Shift the function $y = |x|$ by 2 units to the right and 1 unit up.



Example

Shift the function $y = |x|$ by 2 units to the right and 1 unit up.



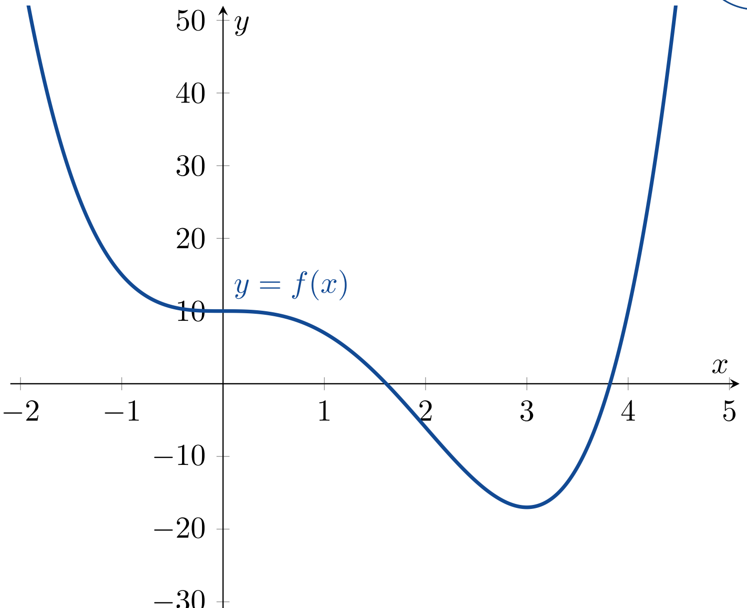
Scaling and Reflecting

For $c > 1$, the graph is scaled:

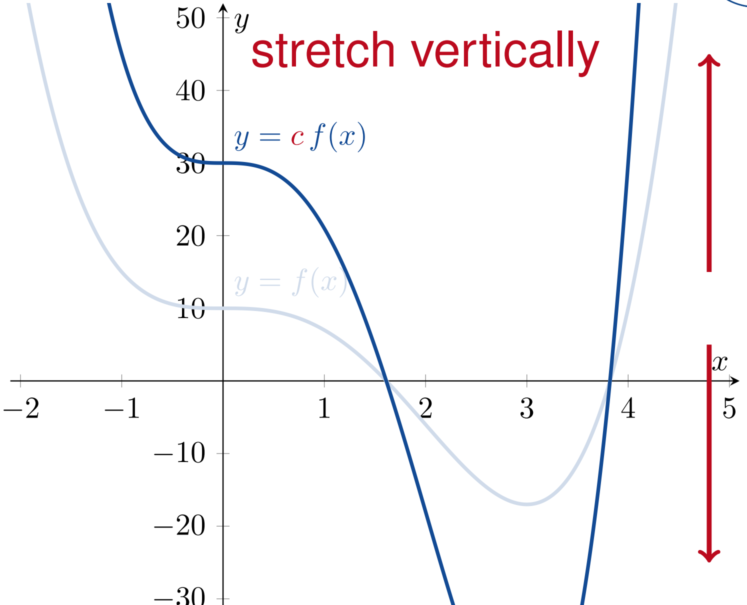
	$y = cf(x)$	stretch vertically by a factor of c
	$y = \frac{1}{c}f(x)$	squash vertically by a factor of c
	$y = f(cx)$	squash horizontally by a factor of c
	$y = f\left(\frac{x}{c}\right)$	stretch horizontally by a factor of c

For $c = -1$, the graph is reflected:

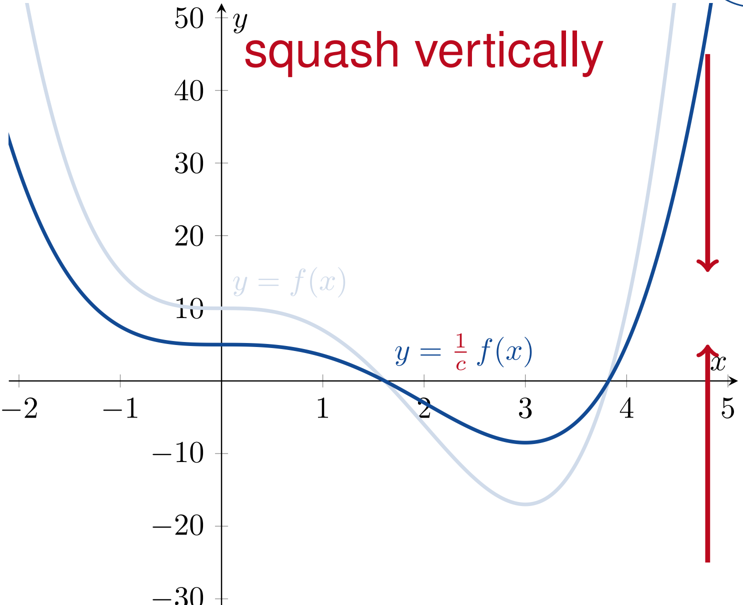
mirror x	$y = -f(x)$	reflect vertically, about the x-axis
mirror y	$y = f(-x)$	reflect horizontally, about the y-axis

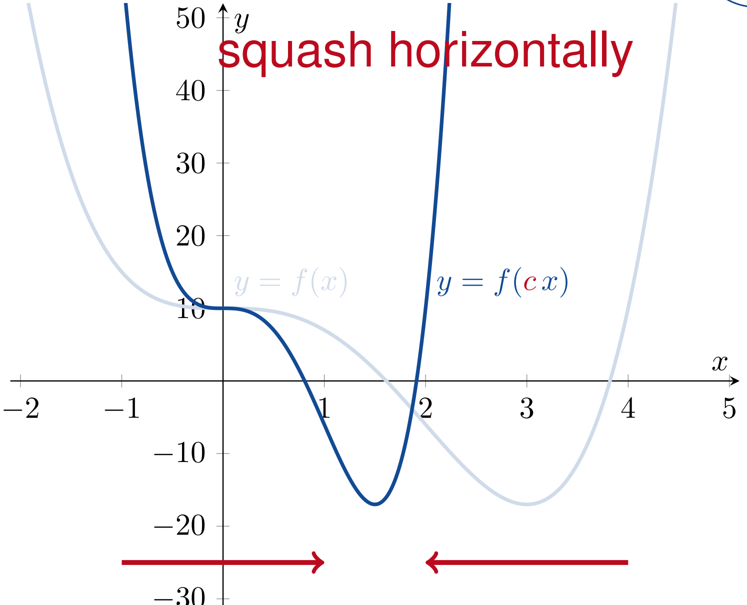


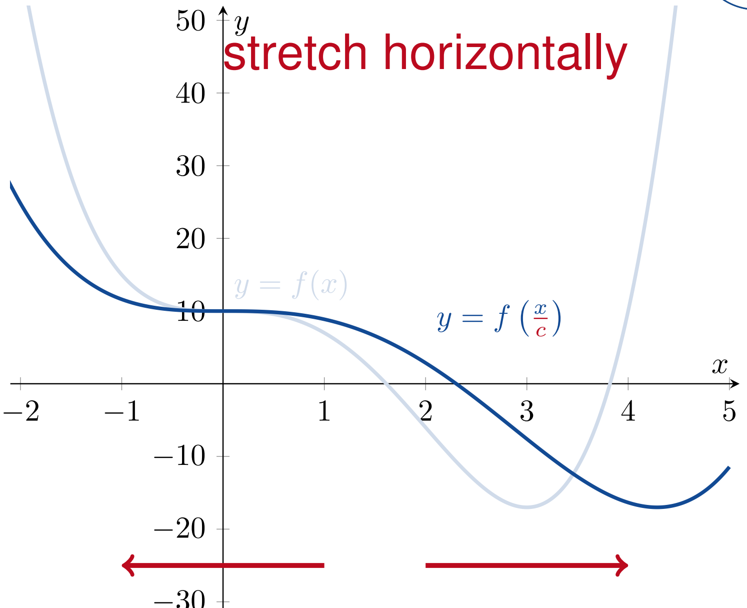
y
50
40
30
20
10
y = f(x)
x
−2
−1
1
2
3
4
5
−10
−20
−30



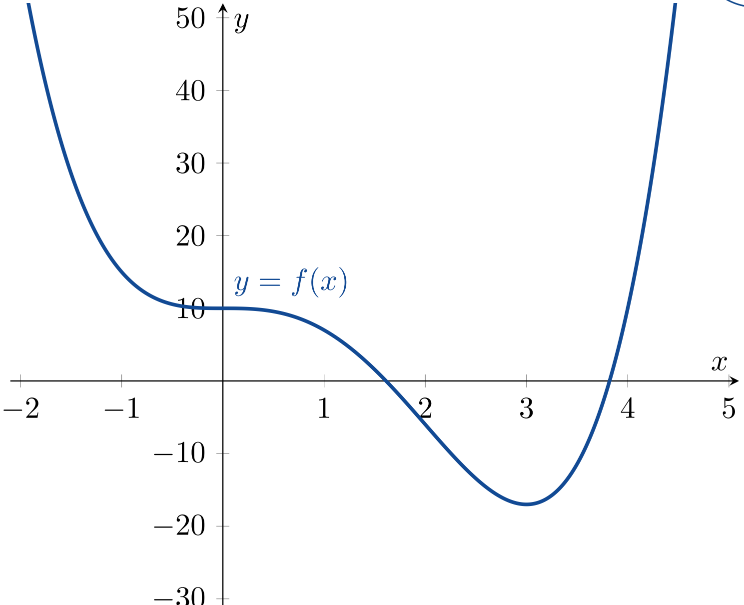
stretch vertically
$y = c\,f(x)$
$y = f(x)$
y
x
50
40
30
20
10
−10
−20
−30
−2
−1
1
2
3
4
5

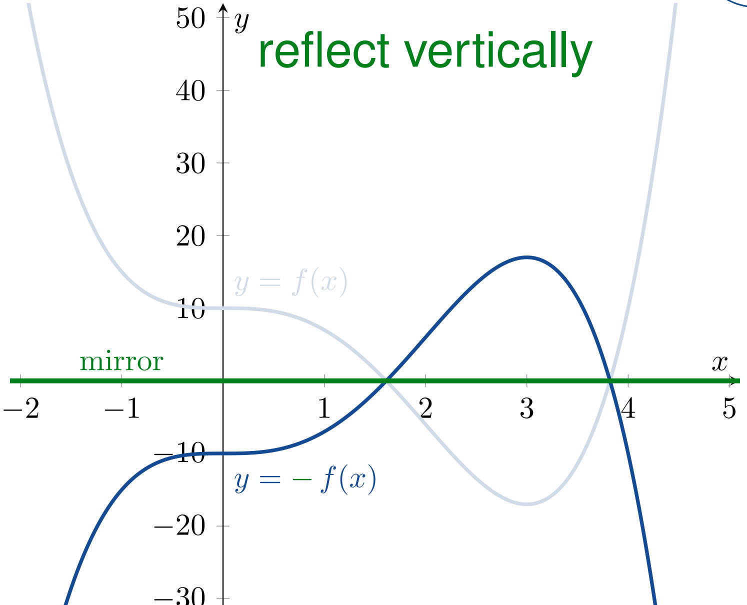

squash vertically
y
x
50
40
30
20
10
-10
-20
-30
-2
-1
1
2
3
4
5
$y = f(x)$
$y = \frac{1}{c}\,f(x)$

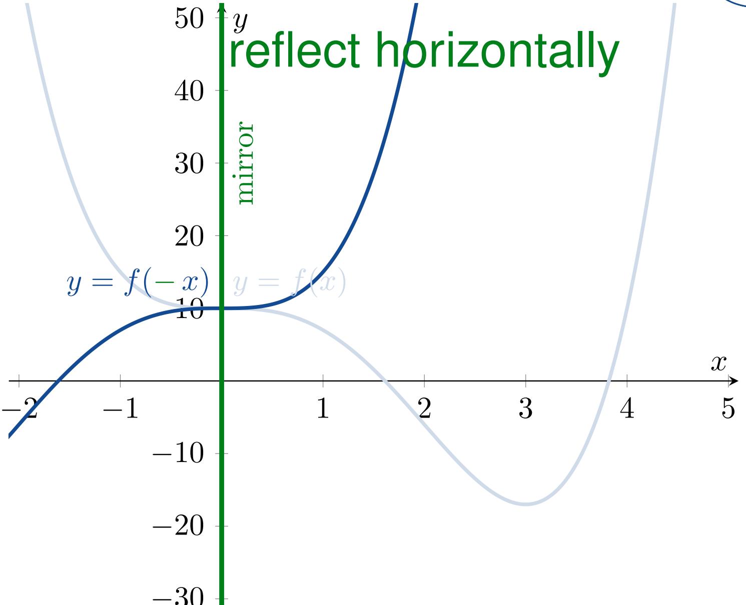

squash horizontally
y
x
50
40
30
20
10
-10
-20
-30
-2
-1
1
2
3
4
5
$y = f(x)$
$y = f(c\,x)$



stretch horizontally
y
x
50
40
30
20
10
-10
-20
-30
-2
-1
1
2
3
4
5
$y = f(x)$
$y = f\left(\frac{x}{c}\right)$


y
x
50
40
30
20
10
−10
−20
−30
−2
−1
1
2
3
4
5
y = f(x)


reflect vertically
y
50
40
30
20
10
−10
−20
−30
x
−2
−1
1
2
3
4
5
mirror
$y = f(x)$
$y = -f(x)$


reflect horizontally
mirror
$y = f(-x)$
$y = f(x)$
x
y
50
40
30
20
10
−10
−20
−30
−2
−1
1
2
3
4
5

Please read Example 5 in your textbook.

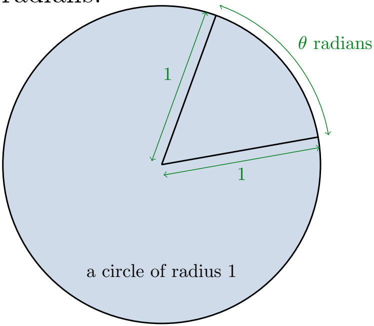
Trigonometric Functions

Angles

There are two ways to measure angles. Using degrees or using radians.

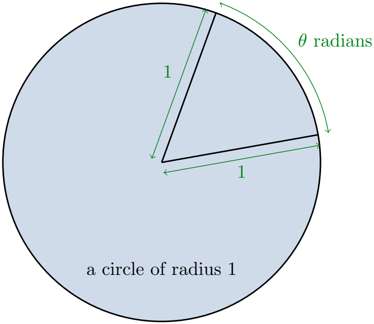
Angles

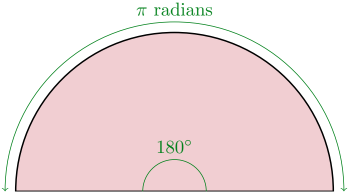
There are two ways to measure angles. Using degrees or using radians.



Angles

There are two ways to measure angles. Using degrees or using radians.





We have that

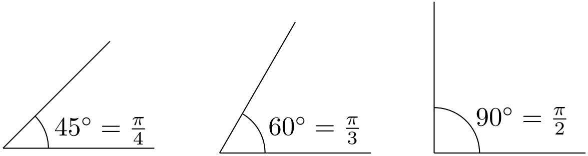
$$\pi \text{ radians} = 180 \text{ degrees}$$

We have that

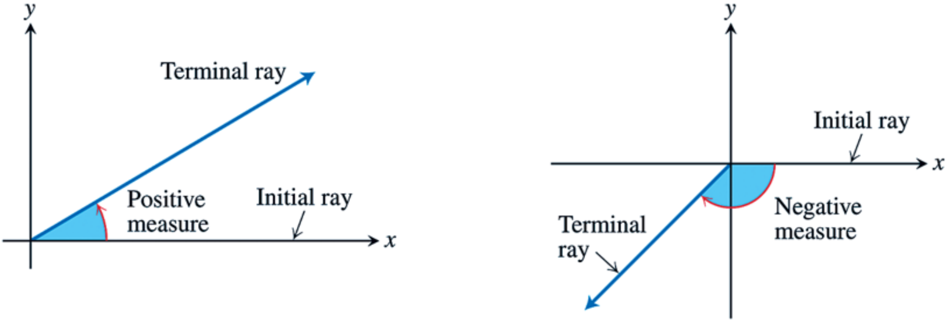
$$\begin{aligned} \pi \text{ radians} &= 180 \text{ degrees} \\ 1 \text{ radian} &= \frac{180}{\pi} \text{ degrees} \\ 1 \text{ degree} &= \frac{\pi}{180} \text{ radians.} \end{aligned}$$

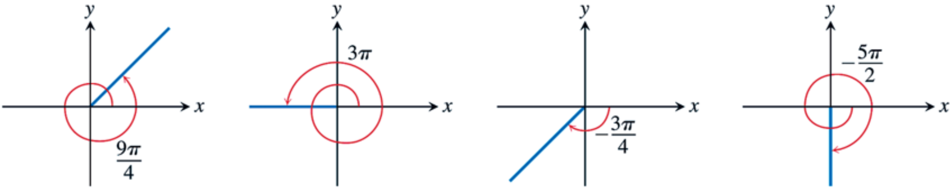
We have that

$$\pi \text{ radians} = 180 \text{ degrees}$$
$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$
$$1 \text{ degree} = \frac{\pi}{180} \text{ radians.}$$



Degrees	−180	−135	−90	−45	0	30	45	60	90	120	135	150	180	270	360
θ (radians)	$-\pi$	$\frac{-3\pi}{4}$	$\frac{-\pi}{2}$	$\frac{-\pi}{4}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π

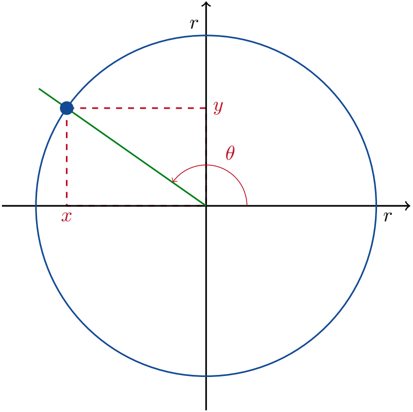

y
Terminal ray
Positive
measure
Initial ray
x
y
Initial ray
x
Negative
measure
Terminal
ray


y
x
9π/4
y
x
3π
y
x
−3π/4
y
x
−5π/2

Remark

In Calculus, we use radians!!!! If you see an angle in this course, it will be in radians. Calculus doesn't work with degrees!!

6 Trigonometric Functions



sine $\sin\theta = \dfrac{y}{r}$

cosine $\cos\theta = \dfrac{x}{r}$

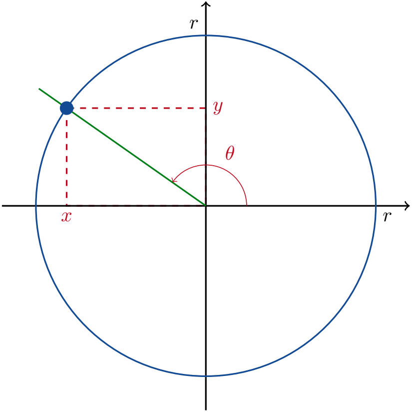
tangent

secant

cosecant

cotangent

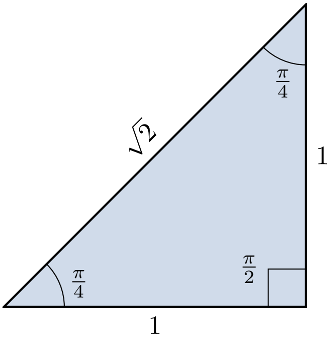
6 Trigonometric Functions



sine	$\sin\theta = \dfrac{y}{r}$
cosine	$\cos\theta = \dfrac{x}{r}$
tangent	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
secant	$\sec\theta = \dfrac{1}{\cos\theta}$
cosecant	$\operatorname{cosec}\theta = \csc\theta = \dfrac{1}{\sin\theta}$
cotangent	$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Remark

Note that $\tan\theta$ and $\sec\theta$ are only defined if $\cos\theta \neq 0$; and $\operatorname{cosec}\theta$ and $\cot\theta$ are only defined if $\sin\theta \neq 0$.



$$\sin 45^\circ = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

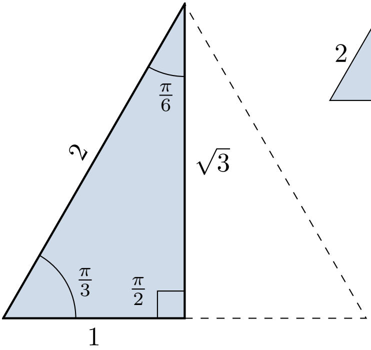
$$\cos 45^\circ = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

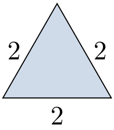
$$\tan 45^\circ = \tan\frac{\pi}{4} = 1$$

$$\sec 45^\circ = \sec\frac{\pi}{4} = \sqrt{2}$$

$$\operatorname{cosec} 45^\circ = \operatorname{cosec}\frac{\pi}{4} = \sqrt{2}$$

$$\cot 45^\circ = \cot\frac{\pi}{4} = 1$$





$$\sin 60^\circ = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\tan 60^\circ = \tan\frac{\pi}{3} = \sqrt{3}$$

$$\sec 60^\circ = \sec\frac{\pi}{3} = 2$$

$$\operatorname{cosec} 60^\circ = \operatorname{cosec}\frac{\pi}{3} = \frac{2}{\sqrt{3}}$$

$$\cot 60^\circ = \cot\frac{\pi}{3} = \frac{1}{\sqrt{3}}$$

Degrees	**−180**	**−135**	**−90**	**−45**	**0**	**30**	**45**	**60**	**90**	**120**	**135**	**150**	**180**	**270**	**360**
θ (radians)	$-\pi$	$\frac{-3\pi}{4}$	$\frac{-\pi}{2}$	$\frac{-\pi}{4}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
$\sin\theta$	0	$\frac{-\sqrt{2}}{2}$	-1	$\frac{-\sqrt{2}}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\cos\theta$	-1	$\frac{-\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{3}}{2}$	-1	0	1
$\tan\theta$	0	1		-1	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$		$-\sqrt{3}$	-1	$\frac{-\sqrt{3}}{3}$	0		0

Periodicy

Definition

A function $f(x)$ is called *periodic* if there exists a positive number p such that

$$f(x+p) = f(x)$$

for all x.

Periodicy

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A function $f(x)$ is called *periodic* if there exists a positive number p such that

$$f(x+p) = f(x)$$

for all x.

The smallest such $p > 0$ is called the *period* of f.

Example

The period of $\tan x$ is π since

$$\tan(x + \pi) = \tan x$$

for all x, and this is the smallest $p > 0$ which works.

Example

The period of $\tan x$ is π since

$$\tan(x + \pi) = \tan x$$

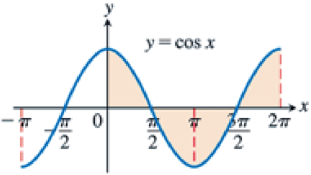
for all x, and this is the smallest $p > 0$ which works.

Example

The period of $\sin x$ is 2π since

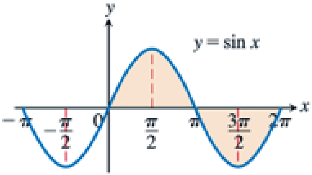
$$\sin(x + 2\pi) = \sin x$$

for all x, and this is the smallest $p > 0$ which works.



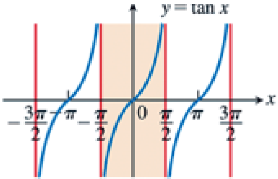
Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Period: 2π

(a)



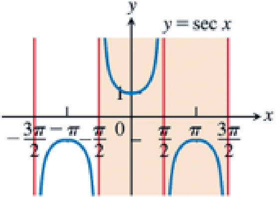
Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Period: 2π

(b)



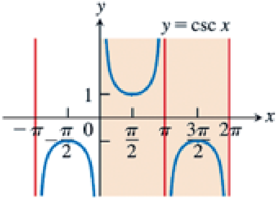
Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
Range: $-\infty < y < \infty$
Period: π

(c)



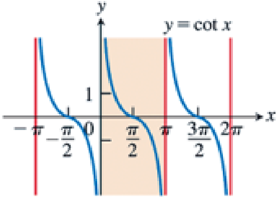
Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
Range: $y \le -1$ or $y \ge 1$
Period: 2π

(d)



Domain: $x \neq 0, \pm\pi, \pm 2\pi, \ldots$
Range: $y \le -1$ or $y \ge 1$
Period: 2π

(e)



Domain: $x \neq 0, \pm\pi, \pm 2\pi, \ldots$
Range: $-\infty < y < \infty$
Period: π

(f)

Trigonometric Identities

$$
\begin{aligned}
\cos\theta &= \sin\left(\frac{\pi}{2}-\theta\right) \\
\cos^2\theta+\sin^2\theta &= 1 \\
1+\tan^2\theta &= \sec^2\theta \\
1+\cot^2\theta &= \operatorname{cosec}^2\theta \\
\cos(A+B) &= \cos A\cos B-\sin A\sin B \\
\sin(A+B) &= \sin A\cos B+\cos A\sin B \\
\cos 2\theta &= \cos^2\theta-\sin^2\theta \\
\sin 2\theta &= 2\sin\theta\cos\theta \\
\cos^2\theta &= \frac{1}{2}(1+\cos 2\theta) \\
\sin^2\theta &= \frac{1}{2}(1-\cos 2\theta)
\end{aligned}
$$

Trigonometric Identities

$$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\cos^2\theta + \sin^2\theta = 1$$

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\end{aligned}$$

Trigonometric Identities

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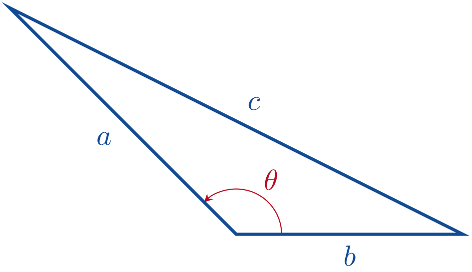
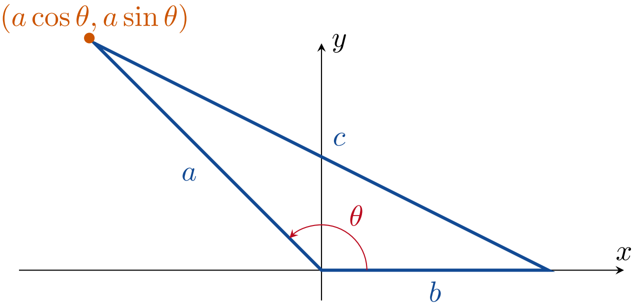
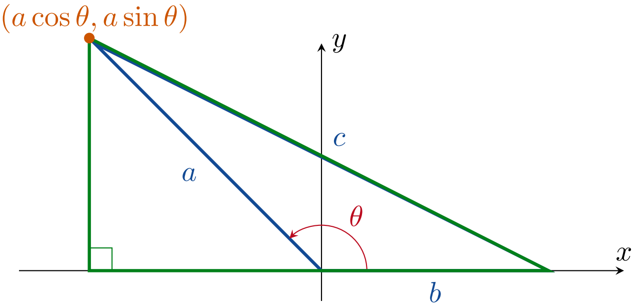
The Law of Cosines



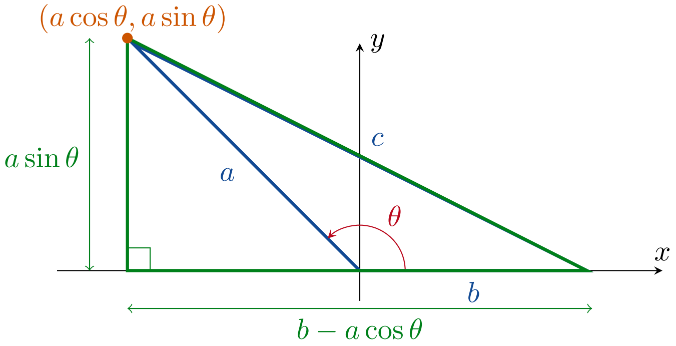
The Law of Cosines



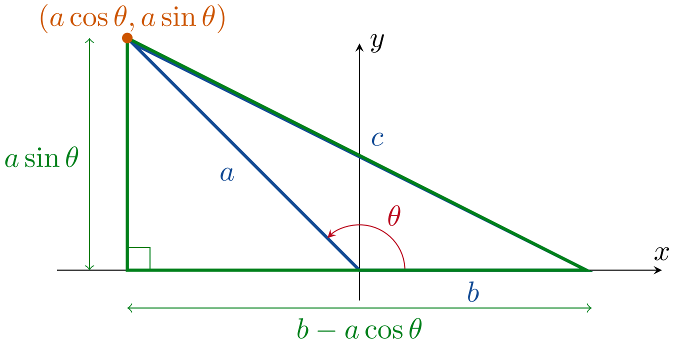
The Law of Cosines



The Law of Cosines

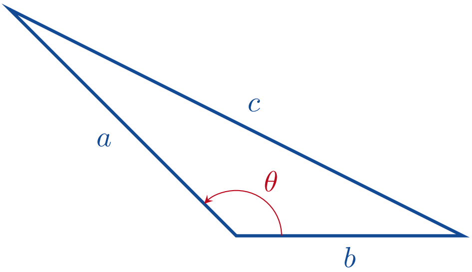


The Law of Cosines

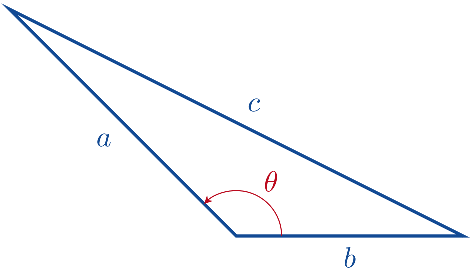


By Pythagoras we have that

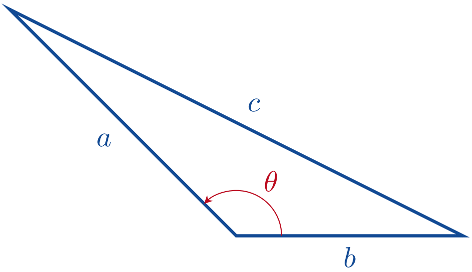
$$c^2 = (b - a\cos\theta)^2 + (a\sin\theta)^2$$



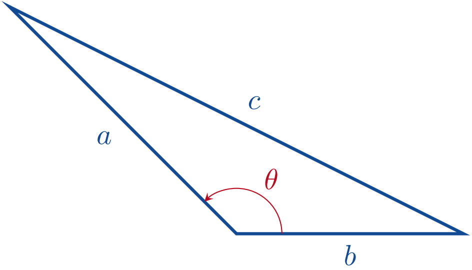
$$
\begin{aligned}
c^2 &= (b - a\cos\theta)^2 + (a\sin\theta)^2 \\
&= \\
&=
\end{aligned}
$$



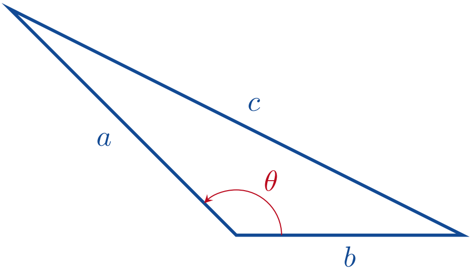
$$
\begin{aligned}
c^2 &= (b - a\cos\theta)^2 + (a\sin\theta)^2 \\
&= b^2 - 2a\cos\theta + a^2\cos^2\theta + a^2\sin^2\theta \\
&=
\end{aligned}
$$



$$\begin{aligned} c^2 &= (b - a\cos\theta)^2 + (a\sin\theta)^2 \\ &= b^2 - 2a\cos\theta + a^2\cos^2\theta + a^2\sin^2\theta \\ &= \end{aligned}$$

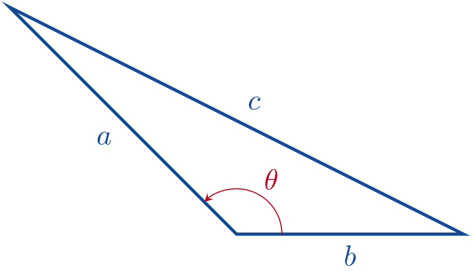


$$
\begin{aligned}
c^2 &= (b - a\cos\theta)^2 + (a\sin\theta)^2 \\
&= b^2 - 2a\cos\theta + a^2\cos^2\theta + a^2\sin^2\theta \\
&= a^2 + b^2 - 2a\cos\theta
\end{aligned}
$$



Theorem (The Law of Cosines)

$$c^2 = a^2 + b^2 - 2a\cos\theta$$



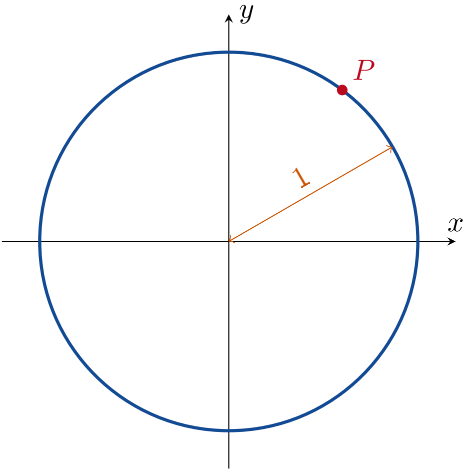
Theorem (The Law of Cosines)

$$c^2 = a^2 + b^2 - 2a\cos\theta$$

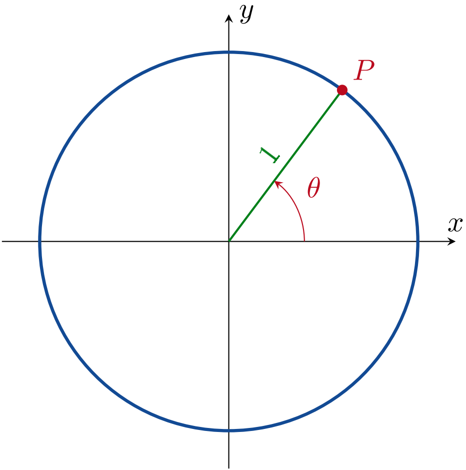
Remark

If $\theta = \frac{\pi}{2}$, then we just get $c^2 = a^2 + b^2$.

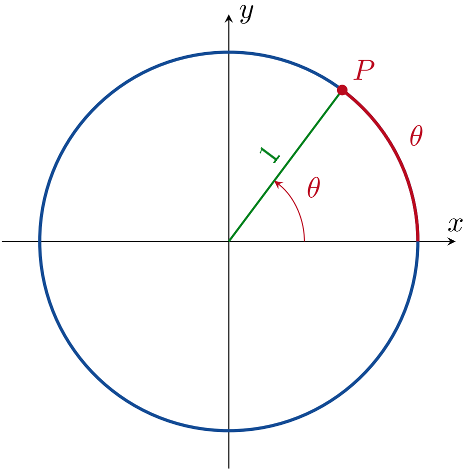
2 Special Inequalities



2 Special Inequalities



2 Special Inequalities



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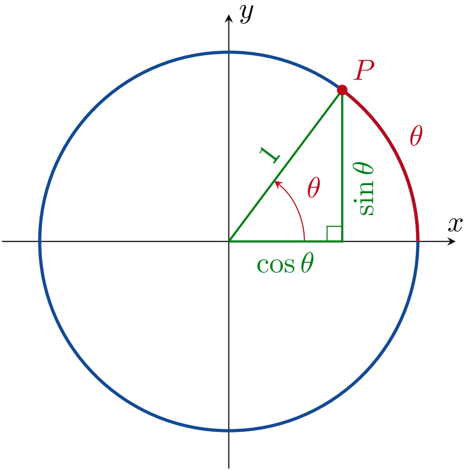

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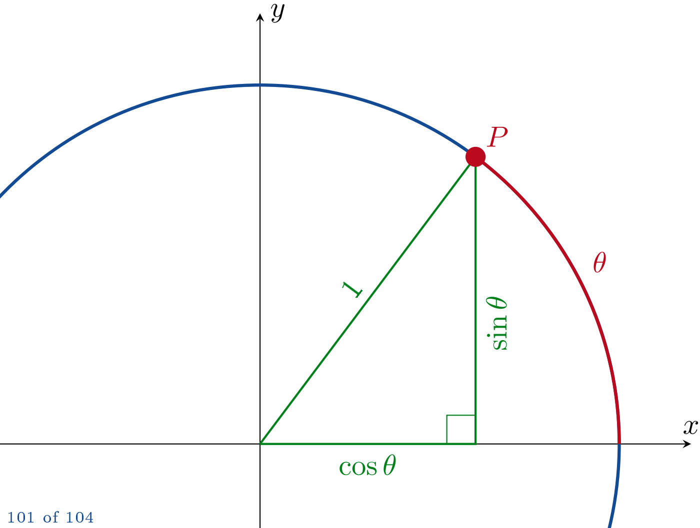

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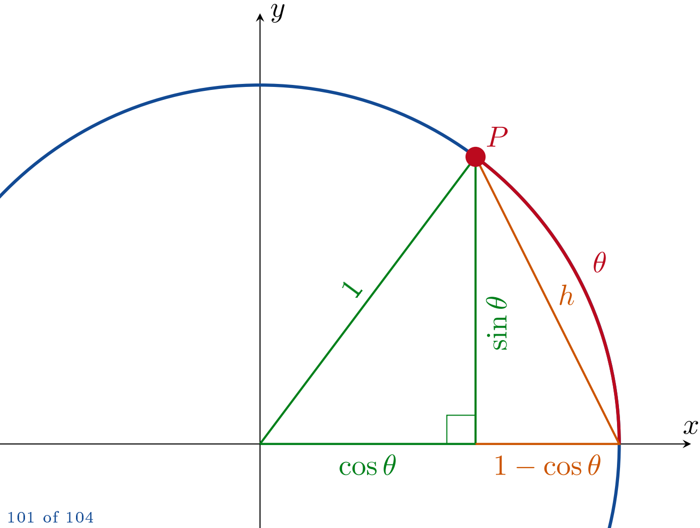

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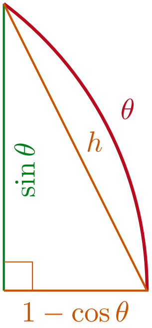
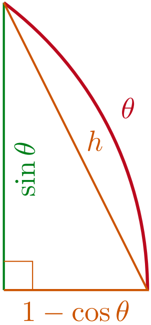


Since

$$h \le \theta$$

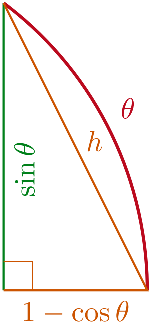
2 Special Inequalities



Since

$$h \le \theta$$
$$h^2 \le \theta^2$$

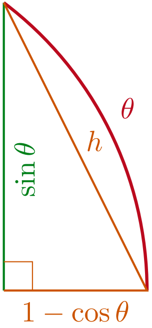
2 Special Inequalities



Since

$$h \leq \theta$$
$$h^2 \leq \theta^2$$
$$(1-\cos\theta)^2 + \sin^2\theta \leq \theta^2$$

2 Special Inequalities



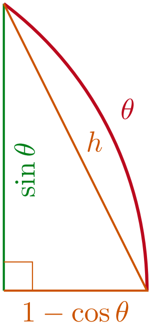
Since

$$h \leq \theta$$
$$h^2 \leq \theta^2$$
$$(1-\cos\theta)^2 + \sin^2\theta \leq \theta^2$$

we have that

$$(1-\cos\theta)^2 \leq \theta^2 \quad \text{and} \quad \sin^2\theta \leq \theta^2.$$

2 Special Inequalities



Since

$$h \leq \theta$$
$$h^2 \leq \theta^2$$
$$(1 - \cos\theta)^2 + \sin^2\theta \leq \theta^2$$

we have that

$$(1-\cos\theta)^2 \leq \theta^2 \quad \text{and} \quad \sin^2\theta \leq \theta^2.$$

It follows that

$$\boxed{-|\theta| \leq \sin\theta \leq |\theta| \qquad \text{and} \qquad -|\theta| \leq 1 - \cos\theta \leq |\theta|.}$$

Remark

We will need

$$-|\theta| \leq \sin\theta \leq |\theta| \qquad \text{and} \qquad -|\theta| \leq 1-\cos\theta \leq |\theta|$$

next week.

Next Time

- 2.1 Rates of Change and Tangents to Curves
- 2.2 Limit of a Function and Limit Laws
- 2.3 The Precise Definition of a Limit