15. (a) Since $\frac{d^n}{dx^n}\cos x$ is either $\cos x$, $\sin x$, $-\cos x$ or $-\sin x$, we can say that $\left|\frac{d^n}{dx^n}\cos x\big|_{x=c}\right| \leq 1$ for all n. It follows that $\left|\frac{f^{(n)}(c)\ (x-a)^n}{n!}\right| \leq \frac{|x-2\pi|^n}{n!} \to 0$ as $n \to \infty$ for each fixed $x \in \mathbb{R}$ and for all $c \in \mathbb{R}$.

(b) Since $f^{(n)}(2\pi) = \begin{cases} 1 & n = 0, 4, 8, \ldots \\ -1 & n = 2, 6, 10, \ldots \\ 0 & \text{otherwise} \end{cases}$ we have that

$$\cos x = 1 - \frac{(x-2\pi)^2}{2} + \frac{(x-2\pi)^4}{24} - \frac{(x-2\pi)^6}{720} + \frac{(x-2\pi)^8}{40320} - \frac{(x-2\pi)^{10}}{3628800} + \frac{(x-2\pi)^{12}}{479001600}$$
$$- \frac{(x-2\pi)^{14}}{87178291200} + \frac{(x-2\pi)^{16}}{20922789888000} - \frac{(x-2\pi)^{18}}{6402373705728000} + \ldots.$$