2019–20 **MATH216 Mathematics IV – Exercise Sheet 5** N. Course

Exercise 21 (The Method of Undetermined Coefficients). Find the general solutions of the following ODEs:

(a) $y'' - 2y' - 3y = 3e^{2t}$
(b) $y'' + 2y' + 5y = 3\cos 2t$
(c) $y'' - 2y' - 3y = 2 - 3te^{-t}$
(d) $y'' + 2y' = 3 + 4\sin 2t$
(e) $y'' + 9y = t^2e^{3t} + 6$
(f) $y'' + 2y' + y = 2e^{-t}$
(g) $2y'' + 3y' + y = t^3 + 3\sin t$
(h) $y'' + y = 3\sin 2t + t\cos 2t$
(i) $y'' + y' + 4y = 2\sinh t$

Exercise 22 (The Method of Undetermined Coefficients). Solve the following IVPs:

(a) $\begin{cases} y'' + y' - 2y = 2t \\ y(0) = 0 \\ y'(0) = 1 \end{cases}$

(b) $\begin{cases} y'' - 2y' + y = te^t + 4 \\ y(0) = 1 \\ y'(0) = 1 \end{cases}$

(c) $\begin{cases} y'' + 4y = t^2 + 3e^t \\ y(0) = 0 \\ y'(0) = 2 \end{cases}$

(d) $\begin{cases} -y'' + 6y' - 16y = 1 + 6e^{3t}\sin(2t) \\ y(0) = \frac{15}{16} \\ y'(0) = -1 \end{cases}$

Exercise 23 (The Method of Variation of Parameters). Find the general solutions of the following ODEs:

(a) $y'' + y = \tan t, \quad 0 < t < \frac{\pi}{2}$
(b) $y'' + 4y = 3\operatorname{cosec} 2t, \quad 0 < t < \frac{\pi}{2}$
(c) $y'' + 4y' + 4y = t^{-2}e^{-2t}, \quad t > 0$
(d) $y'' - 2y' + y = \dfrac{e^t}{1+t^2}$

Exercise 24 (Going Backwards). Find linear, homogeneous ODEs with constant coefficients, which have general solutions equal to the functions given below. The first one is done for you.

(ω) $y(t) = c_1e^t + c_2e^{2t} + c_3e^{3t}$.

Clearly $r_1 = 1$, $r_2 = 2$ and $r_3 = 3$. We need to give an ODE which has characteristic equation $0 = (r - r_1)(r - r_2)(r - r_3) = (r-1)(r-2)(r-3) = r^3 - 6r^2 + 11r - 6$. One possible answer is $y''' - 6y'' + 11y' - 6y = 0$.

(a) $y(t) = c_1 + c_2t + c_3e^{3t}\sin t + c_4e^{3t}\cos t + c_5e^{3t}\sin 2t + c_6e^{3t}\cos 2t$
(b) $y(t) = c_1e^t + c_2te^t + c_3e^{2t}\sin t + c_4e^{2t}\cos t + c_5e^{2t}t\sin t + c_6e^{2t}t\cos t$
(c) $y(t) = c_1e^{2t} + c_2te^{2t} + c_3t^2e^{2t} + c_4e^{-t}\sin 3t + c_5e^{-t}\cos 3t$

Exercise 25 (Higher Order Linear ODEs).

(a) Given that $\sin t$ is a solution of $y^{(4)} + 2y''' + 6y'' + 2y' + 5y = 0$, find the general solution of this ODE.
(b) Find the general solution of $y^{(4)} + y'' = 3x^2 + 4\sin x - 2\cos x$.
(c) Solve $\begin{cases} \frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 0 \\ y(0) = 2 \\ y'(0) = 0 \\ y''(0) = 0. \end{cases}$