2015–16 **MAT234 Matematik IV – Ödev 3** N. Course

SON TESLİM TARİHİ: Salı 8 Mart 2016 saat 16:00'e kadar.

False Lemma. $n+1 < n$ *for all* $n \in \mathbb{N}$.

Proof. Let $P_n = (n+1 < n)$. Suppose that P_k is true. Then we know that $k+1 < k$. It follows that

$$(k+1)+1 < (k)+1 = k+1$$

and hence P_{k+1} is true. So $P_k \implies P_{k+1}$.
By the principle of mathematical induction, it follows that $n+1 < n$ for all $n \in \mathbb{N}$. □

Egzersiz 6 (Proof by Induction). [20p] The false lemma above is clearly not true (e.g. we know that $7 < 6$ is not true), so the proof must be wrong. Find all the mistakes in the above proof.

Definition. A sequence (a_n) of real numbers *tends to* l ($a_n \to l$ as $n \to \infty$) iff, given any $\varepsilon > 0$, there exists $N = N(\varepsilon) \in \mathbb{N}$ such that

$$n > N \implies |a_n - l| < \varepsilon.$$

Example. Define $g_n = 3 + 3^{-n}$. Use the definition to show that $g_n \to 3$ as $n \to \infty$.

solution: Let $\varepsilon > 0$. Choose $N \geq -\frac{\log \varepsilon}{\log 3}$. Then

$$n > N \implies |g_n - 3| = \left|3^{-n}\right| = \frac{1}{3^n} < \frac{1}{3^N} = 3^{-N} = e^{-N\log 3} \leq e^{\log \varepsilon} = \varepsilon.$$

Therefore $g_n \to 3$ as $n \to \infty$.

Egzersiz 7 (Sequences tending to a finite limit). Let

$$y_n = \begin{cases} \frac{1}{n^2} & n = 1, 4, 9, 16, 25, 36, \dots \\ 0 & \text{otherwise} \end{cases} \qquad \text{and} \qquad z_n = \frac{3n+1}{n+2}.$$

(a) [10p] Find the first 10 terms of (y_n).
[HINT: The first 10 terms of $a_n = \frac{1}{n}$ are $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}$ and $\frac{1}{10}$.]

(b) [10p] Plot the first 10 terms of (y_n) on a graph.

(c) [30p] *Use the definition* to prove that $y_n \to 0$ as $n \to \infty$.

(d) [30p] *Use the definition* to prove that $z_n \to 3$ as $n \to \infty$.

Ödev 2'nin çözümleri

4. (a) Let $A > 0$. Choose $N > \max\{3, A\}$. Then $n > N \implies |u_n| \geq n! - n^2 = n((n-1)! - n) \geq n(1) > N > A$. Therefore $u_n \to \infty$ as $n \to \infty$.

(b) Let $A > 0$. Choose $N > 3A$. Then $n > N \implies |v_n| = \left|\frac{n+7}{2+\sin n}\right| \geq \frac{n+7}{3} \geq \frac{n}{3} > \frac{N}{3} > A$. Therefore $v_n \to \infty$ as $n \to \infty$.

(c)First note that since $1 < (1+\frac{1}{n}) \leq 2$ for all $n \in \mathbb{N}$, we know that $0 < \log(1+\frac{1}{n}) \leq \log 2 < 1$ for all n.
Let $A > 0$. Choose $N \geq A+2$. Then for all $n > N$, $w_n = n - \log(1+\frac{1}{n}) > n-2 > N-2 \geq A$. Therefore $w_n \to \infty$ as $n \to \infty$.

5. Let $A > 0$. Since $a_n \to \infty$ as $n \to \infty$, there exists $N \in \mathbb{N}$ such that $n > N \implies a_n > A + c$. But then $n > N \implies a_n - c > A$. Therefore $a_n - c \to \infty$ as $n \to \infty$.