SON TESLİM TARİHİ: Salı 12 Nisan 2016 saat 16:00'e kadar.

**Egzersiz 12 (Tests for Convergence).** [5 × 20p]
Determine whether each series converges or diverges. Justify (prove) your answer.
[If you are using one of the tests from chapters 10–12, then you must say which one you are using.]

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^3}{(2n)!}$, (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}+1}$, (c) $\displaystyle\sum_{n=1}^{\infty} \log \tfrac{1}{n}$,

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{1+e^{2n}}$, (e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{10}}{\sqrt{(2n)!}}$.

[HINT: $\int \frac{1}{1+u^2}\,du = \tan^{-1} u + c$]

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*Ödev 5'in çözümleri*

10. (a) $a_2 = 17.2$, $a_3 = 21.3691428571$, $a_4 = 23.6662895207$, $a_5 = 25.144189424$ and $a_6 = 26.1747180255$.
(b) Clearly $0 \leq a_1 \leq 30$. Suppose that $0 \leq a_k \leq 30$. Then $a_{k+1} = \frac{1}{70}\left(1200 + a_k^2\right) \leq \frac{1}{70}\left(1200 + 30^2\right) = \frac{2100}{70} = 30$ and $a_{k+1} = \frac{1}{70}\left(1200 + a_n^2\right) \geq \frac{1}{70}\left(1200 + 0\right) = \frac{1200}{70} \geq 0$. By the principle of mathematical induction, $0 \leq a_n \leq 30$ for all $n \in \mathbb{N}$.
(c) $(a_n)$ is an increasing sequence. Proof: For all $n \in \mathbb{N}$, $a_{n+1} - a_n = \frac{1}{70}\left(1200 + a_n^2\right) - a_n = \frac{1}{70}\left(1200 + a_n^2 - 70a_n\right) = \frac{1}{70}(a_n - 30)(a_n - 40) \geq 0$ by (b). Therefore $a_n \leq a_{n+1}$ for all $n \in \mathbb{N}$.
(d) Part (b) tells us that $(a_n)$ is bounded above. Part (c) tells us that $(a_n)$ is an increasing sequence. Therefore, by Theorem 6.1, $(a_n)$ is convergent.
(e) Let $\lim_{n\to\infty} a_n = a$. Then $a \leftarrow a_{n+1} = \frac{1}{70}\left(1200 + a_n^2\right) \to \frac{1}{70}\left(1200 + a^2\right)$. Since the limit is unique, we must have $a = \frac{1}{70}\left(1200 + a^2\right)$. Rearranging gives $0 = (a - 30)(a - 40)$ so $a = 30$ or $a = 40$. Since we know that $a_n \leq 30$ for all $n$, we must have $a = 30$.

11. (a) Let $A > 0$ Since $x_n \to -\infty$ as $n \to \infty$, we know that $\exists N \in \mathbb{N}$ such that $n > N \implies x_n < -A$. But since $n_k \geq k$ for all $k$, it follows that $k > N \implies n_k > N \implies x_{n_k} < -A$. Therefore $x_{n_k} \to -\infty$ as $k \to \infty$.
(b) Let $\varepsilon > 0$. Choose $N \geq \sqrt{\frac{2}{\varepsilon}}$. Then $n > m > N \implies |y_n - y_m| = \left|\frac{2}{m^2} - \frac{2}{n^2}\right| = \frac{2n^2 - 2m^2}{n^2 m^2} \leq \frac{2n^2}{n^2 m^2} = \frac{2}{m^2} < \frac{2}{N^2} \leq \varepsilon$. Therefore $y_n$ is a Cauchy sequence.