16. The perimeter of the window is

$$2 = 2h + 2r + \pi r.$$

Rearranging this gives $h = 1 - r - \frac{1}{2}\pi r$.

The area of the rectangle is $2rh$. It we say that 1 unit area of glass lets through 1 unit of light, then the rectangle admits $2rh$ units of light. The area of the semicircle is $\frac{1}{2}\pi r^2$. Because tinted glass admits only half as much light as clear glass, the semicircle lets $\frac{1}{2}\left(\frac{1}{2}\pi r^2\right) = \frac{1}{4}\pi r^2$ units of light through. Hence, the window admits

$$L(r) = 2rh + \frac{1}{4}\pi r^2 = 2r\left(1 - r - \frac{1}{2}\pi r\right) + \frac{1}{4}\pi r^2 = 2r - 2r^2 - \pi r^2 + \frac{1}{4}\pi r^2 = 2r - 2r^2 - \frac{3}{4}\pi r^2$$

units of light. We wish to maximise $L(r)$.

Since $0 = L'(r) = 2 - 4r - \frac{3}{2}\pi r$, we must have $2 = (4 + \frac{3}{2}\pi)r$ and $r = \frac{2}{4+\frac{3}{2}\pi} = \frac{4}{8+3\pi} \approx 0.230$ metres. Thus

$$h = 1 - r - \frac{1}{2}\pi r = 1 - \left(1 + \frac{1}{2}\pi\right)r = 1 - \left(\frac{2+\pi}{2}\right)\left(\frac{4}{8+3\pi}\right) = 1 - \frac{4+2\pi}{8+3\pi} = \frac{4+\pi}{8+3\pi} \approx 0.410$$

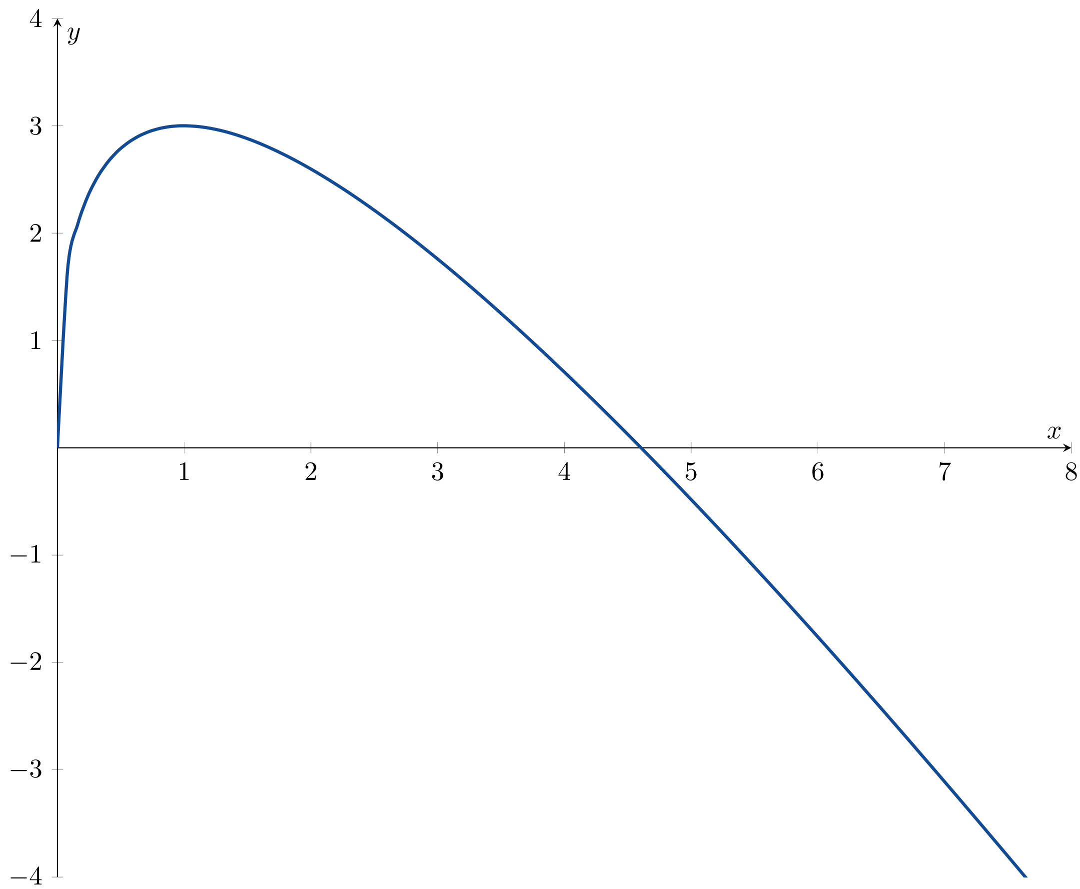
metres.

17. (a) f has a critical point at $x = 1$.

(b) f is increasing on $(0, 1)$. f is decreasing on $(1, \infty)$.

(c) Since $f''(x)$ exists and is non-zero for all $x \in (0, \infty)$, f is concave down everywhere.

(d)



18. (a) $G(x) = \frac{x^4}{4} + \frac{1}{2x^2}$ is an antiderivative of $g(x) = x^3 - \frac{1}{x^3}$, because

$$G'(x) = \frac{d}{dx}\left(\frac{x^4}{4} + \frac{1}{2}x^{-2}\right) = \frac{4x^3}{4} + \frac{1}{2}(-2x^{-3}) = x^3 - \frac{1}{x^3} = g(x).$$

(b) $H(x) = \frac{4}{3}\sec 3x$ is an antiderivative of $h(x) = 4\sec 3x \tan 3x$, because

$$H'(x) = \frac{d}{dx}\left(\frac{4}{3}\sec 3x\right) = \frac{4}{3}\left(\frac{d}{du}\sec u\right)\left(\frac{d}{dx}3x\right) = \frac{4}{3}(\sec u \tan u)(3) = 4\sec 3x \tan 3x = h(x)$$

by the chain rule.

(c) $L(x) = \frac{1}{2}(e^x - e^{-x})$ is an antiderivative of $l(x) = \frac{1}{2}(e^x + e^{-x})$, because

$$L'(x) = \frac{d}{dx}\left(\frac{1}{2}(e^x - e^{-x})\right) = \frac{1}{2}(e^x - (-1)e^{-x}) = l(x)$$

by the chain rule.

19. This is wrong because

$$\begin{aligned}\frac{d}{dx}(xe^x + 3\cot 3x) &= (x)'e^x + x(e^x)' + 3\left(\frac{d}{du}\cot u\right)\left(\frac{d}{dx}3x\right)\\ &= e^x + xe^x + 3(-\operatorname{cosec}^2 u)(3)\\ &= e^x + xe^x - 9\operatorname{cosec}^2 3x\end{aligned}$$

by the product rule and the chain rule.

20. (a) $\displaystyle\int\left(8t^3 - \frac{t^2}{2} + t\right)\,dt = 2t^4 - \frac{t^3}{6} + \frac{t^2}{2} + C.$

(b) $\displaystyle\int\left(\sec^2 \pi\theta\right)\,d\theta = \frac{1}{\pi}\tan \pi\theta + C.$

(c) $\displaystyle\int\left(x + \frac{1}{x}\right)^2\,dx = \int\left(x^2 + 2 + \frac{1}{x^2}\right)\,dx = \frac{1}{3}x^3 + 2x - \frac{1}{x} + C.$