

# Week 2

- 4. Intervals
- 5. Cartesian Coordinates
- 6. Functions
- 7. Sigma Notation

# Intervals

### Definition

A subset of  $\mathbb{R}$  is called an *interval* if

- 1 it contains atleast 2 numbers; and
- 2 it doesn't have any holes in it.

## 4. Intervals



### Example

The set  $\{x \mid x \text{ is a real number and } x > 6\}$  is an interval.



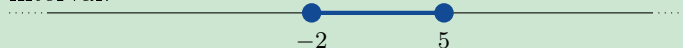
Because 6 is not in this set, we use  $\circ$  at 6.

## 4. Intervals



### Example

The set of all real numbers  $x$  such that  $-2 \leq x \leq 5$  is an interval.



Because  $-2$  and  $5$  are in this set, we use  $\bullet$  at  $-2$  and  $5$ .

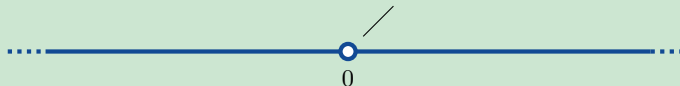
## 4. Intervals



### Example

The set  $\{x \mid x \in \mathbb{R} \text{ and } x \neq 0\}$  is not an interval.

a hole at 0



A finite interval is

- *closed* if it contains both its endpoints;
- *half-open* if it contains one of its endpoints;
- *open* if it does not contain its endpoints;

## 4. Intervals



Notation	Set	Type	Picture
$(a, b)$	$\{x a < x < b\}$	open	A horizontal number line with dots at $a$ and $b$ . Open circles are placed at $a$ and $b$ , and a thick blue line segment connects them. Dotted lines extend from the ends of the number line.
$[a, b]$	$\{x a \leq x \leq b\}$	closed	A horizontal number line with dots at $a$ and $b$ . Closed circles are placed at $a$ and $b$ , and a thick blue line segment connects them. Dotted lines extend from the ends of the number line.
$[a, b)$	$\{x a \leq x < b\}$	half open	A horizontal number line with dots at $a$ and $b$ . A closed circle is placed at $a$ and an open circle is placed at $b$ , with a thick blue line segment connecting them. Dotted lines extend from the ends of the number line.
$(a, b]$	$\{x a < x \leq b\}$	half open	A horizontal number line with dots at $a$ and $b$ . An open circle is placed at $a$ and a closed circle is placed at $b$ , with a thick blue line segment connecting them. Dotted lines extend from the ends of the number line.



An infinite interval is

- *closed* if it contains a finite endpoint;
- *open* if it is not closed.

There is one exception to this rule: The whole real line is called both open and closed.

## 4. Intervals

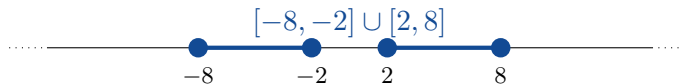


Notation	Set	Type	Picture
$(a, \infty)$	$\{x a < x\}$	open	
$[a, \infty)$	$\{x a \leq x\}$	closed	
$(-\infty, b)$	$\{x x < b\}$	open	
$(-\infty, b]$	$\{x x \leq b\}$	closed	
$(-\infty, \infty)$	$\mathbb{R}$	both open and closed	

## 4. Intervals

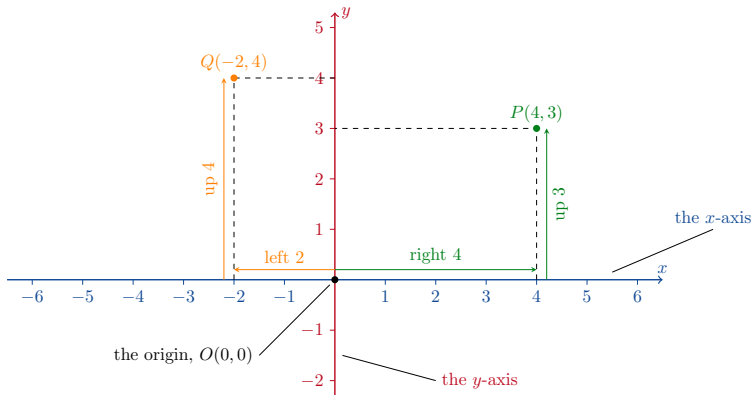


We can combine two (or more) intervals with the notation  $\cup$ . For example,  $[-8, -2] \cup [2, 8]$  is called the *union* of  $[-8, -2]$  and  $[2, 8]$  and is shown below.



# Cartesian Coordinates

## 5. Cartesian Coordinates



## 5. Cartesian Coordinates



### Definition

The set

$$\{(x, y) | x, y \in \mathbb{R}\}$$

is denoted by  $\mathbb{R}^2$ .

### Definition

The point  $O(0, 0)$  is called the *origin*.

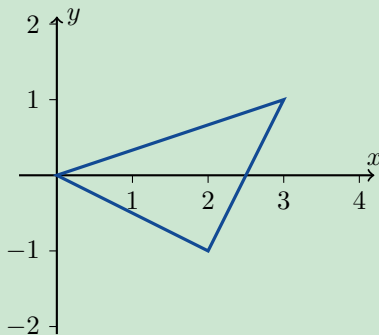
## 5. Cartesian Coordinates



### Example

Let  $A(2, -1)$  and  $B(3, 1)$  be points in  $\mathbb{R}^2$ . Draw the triangle  $OAB$ .

*solution:*



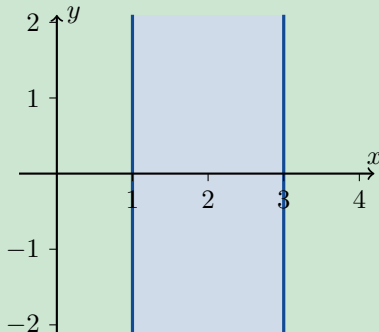
## 5. Cartesian Coordinates



### Example

Draw the region of points which satisfy  $1 \leq x \leq 3$ .

*solution:*



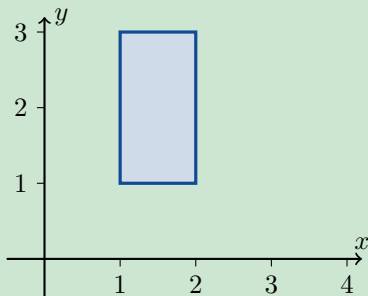


## 5. Cartesian Coordinates

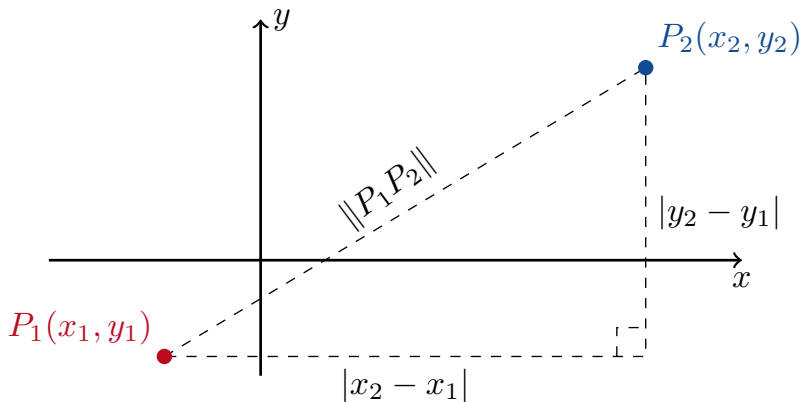


### Example

Draw the region of points which satisfy  $1 \leq x \leq 2$  and  $1 \leq y \leq 3$ .  
*solution:*



## 5. Cartesian Coordinates



## 5. Cartesian Coordinates



### Definition

The *distance* between  $P_1(x_1, y_1)$  and  $P_2(x_2, y_2)$  is

$$\|P_1P_2\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

### Example

The distance between  $A(1, 3)$  and  $B(4, -1)$  is

$$\|AB\| = \sqrt{(4 - 1)^2 + (-1 - 3)^2} = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5.$$

# 6 Functions

$$y = f(x)$$

“ $y$  is equal to  $f$  of  $x$ ”

## 6. Functions



dependent variable

$$y = f(x)$$

function

independent variable

“ $y$  is equal to  $f$  of  $x$ ”

### Definition

A *function* from a set  $D$  to a set  $Y$  is a rule that assigns a unique element of  $Y$  to each element of  $D$ .

### Definition

The set  $D$  of all possible values of  $x$  is called the *domain* of  $f$ .

### Definition

The set  $Y$  is called the *target* of  $f$ .

### Definition

The set of all possible values of  $f(x)$  is called the *range* of  $f$ .

## 6. Functions



If  $f$  is a function with domain  $D$  and target  $Y$ , we can write

$$\begin{array}{ccc} f & : & D \longrightarrow Y \\ & & \swarrow \quad \searrow \\ & & \text{domain} \quad \text{target} \end{array}$$

Example

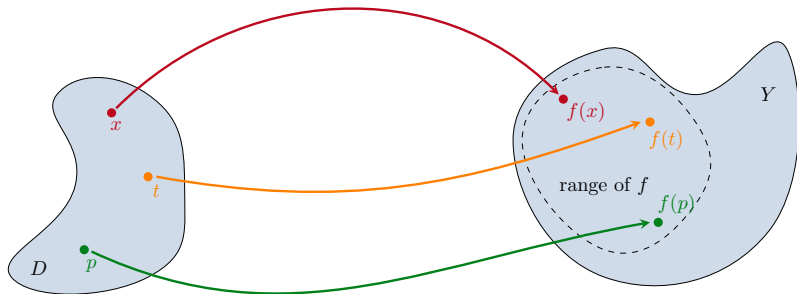
$$f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^2.$$

Example

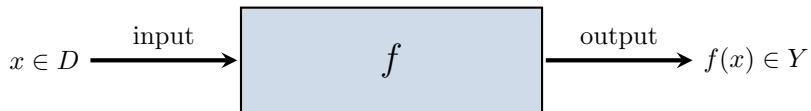
$$f : (-\infty, \infty) \rightarrow [0, \infty), f(x) = x^2.$$



## 6. Functions



## 6. Functions



## 6. Functions



function	domain ( $x$ )	range ( $y$ )
$y = x^2$	$(-\infty, \infty)$	
$y = \frac{1}{x}$	$\{x \mid x \in \mathbb{R}, x \neq 0\}$	
$y = \sqrt{x}$	$[0, \infty)$	
$y = \sqrt{4-x}$		
$y = \sqrt{1-x^2}$		

## 6. Functions



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## 6. Functions



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## 6. Functions



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$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4-x}$	$(-\infty, 4]$	
$y = \sqrt{1-x^2}$		

## 6. Functions



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$y = \sqrt{4-x}$	$(-\infty, 4]$	$[0, \infty)$
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## 6. Functions



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$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4-x}$	$(-\infty, 4]$	$[0, \infty)$
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## 6. Functions



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## 6. Functions



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$y = \sqrt{4-x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1-x^2}$	$[-1, 1]$	$[0, 1]$
$y = x^2$	$[1, 2]$	
$y = x^2$	$[2, \infty)$	
$y = x^2$	$(-\infty, -2]$	

## 6. Functions



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$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4-x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1-x^2}$	$[-1, 1]$	$[0, 1]$
$y = x^2$	$[1, 2]$	$[1, 4]$
$y = x^2$	$[2, \infty)$	
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## 6. Functions



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$y = x^2$	$[1, 2]$	$[1, 4]$
$y = x^2$	$[2, \infty)$	$[4, \infty)$
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## 6. Functions



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$y = x^2$	$[1, 2]$	$[1, 4]$
$y = x^2$	$[2, \infty)$	$[4, \infty)$
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## 6. Functions



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$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4-x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1-x^2}$	$[-1, 1]$	$[0, 1]$
$y = x^2$	$[1, 2]$	$[1, 4]$
$y = x^2$	$[2, \infty)$	$[4, \infty)$
$y = x^2$	$(-\infty, -2]$	$[4, \infty)$
$y = 1 + x^2$	$[1, 3)$	
$y = 1 - \sqrt{x}$	$[0, \infty)$	

## 6. Functions



function	domain ( $x$ )	range ( $y$ )
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
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$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4-x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1-x^2}$	$[-1, 1]$	$[0, 1]$
$y = x^2$	$[1, 2]$	$[1, 4]$
$y = x^2$	$[2, \infty)$	$[4, \infty)$
$y = x^2$	$(-\infty, -2]$	$[4, \infty)$
$y = 1 + x^2$	$[1, 3)$	$[2, 10)$
$y = 1 - \sqrt{x}$	$[0, \infty)$	



## 6. Functions



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$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4-x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1-x^2}$	$[-1, 1]$	$[0, 1]$
$y = x^2$	$[1, 2]$	$[1, 4]$
$y = x^2$	$[2, \infty)$	$[4, \infty)$
$y = x^2$	$(-\infty, -2]$	$[4, \infty)$
$y = 1 + x^2$	$[1, 3)$	$[2, 10)$
$y = 1 - \sqrt{x}$	$[0, \infty)$	$(-\infty, 1]$



# Graphs of Functions

### Definition

The *graph* of  $f$  is the set containing all the points  $(x, y)$  which satisfy  $y = f(x)$ .

## 6. Functions



### Example

Graph the function  $y = 1 + x^2$  over the interval  $[-2, 2]$ .

*solution:*

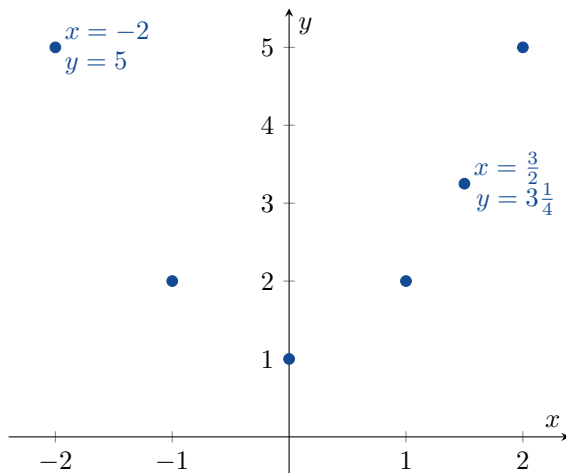
- 1 Make a table of  $(x, y)$  points which satisfy  $y = 1 + x^2$ .

$x$	$y$
-2	5
-1	2
0	1
1	2
$\frac{3}{2}$	$\frac{13}{4} = 3\frac{1}{4}$
2	5

## 6. Functions



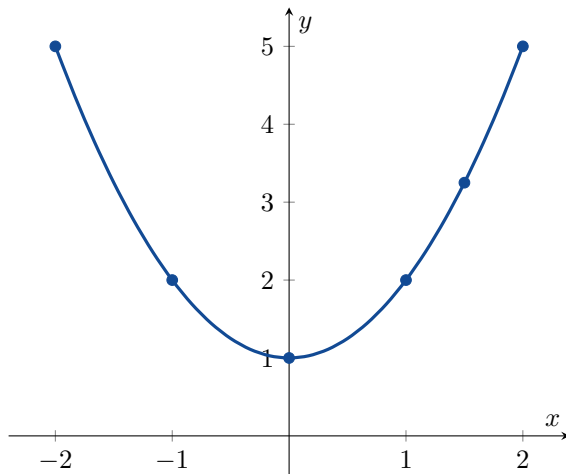
2 Plot these points.



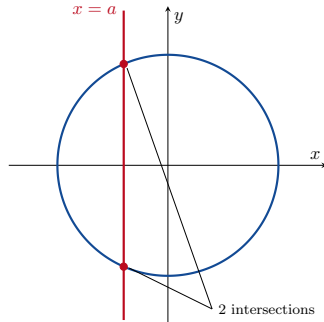
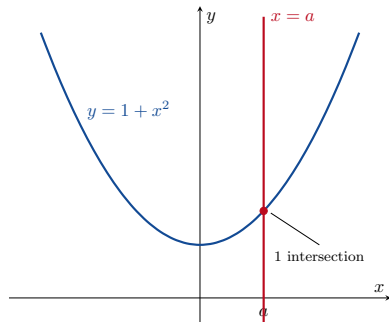
## 6. Functions



- 3 Draw a smooth curve through these points.



### The Vertical Line Test



Not every curve that you draw is a graph of a function.



A function can have only one value  $f(x)$  for each  $x \in D$ . This means that a vertical line can intersect the graph of a function at most once.

A circle can not be the graph of a function because some vertical lines intersect the circle at two points.

If  $a \in D$ , then the vertical line  $x = a$  will intersect the graph of  $f : D \rightarrow Y$  only at the point  $(a, f(a))$ .



# Piecewise-Defined Functions

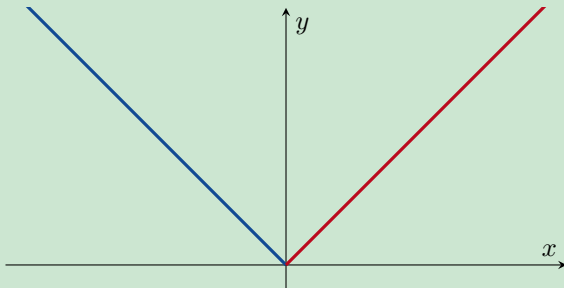


## 6. Functions



### Example

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

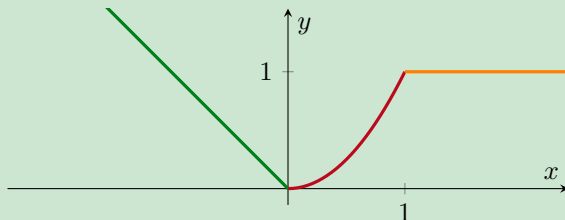


## 6. Functions



### Example

$$f(x) = \begin{cases} -x & x < 0 \\ x^2 & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$



# Increasing and Decreasing Functions

## Definition

Let  $I$  be an interval. Let  $f : I \rightarrow \mathbb{R}$  be a function.

- 1  $f$  is called *increasing on  $I$*  if

$$f(x_1) < f(x_2)$$

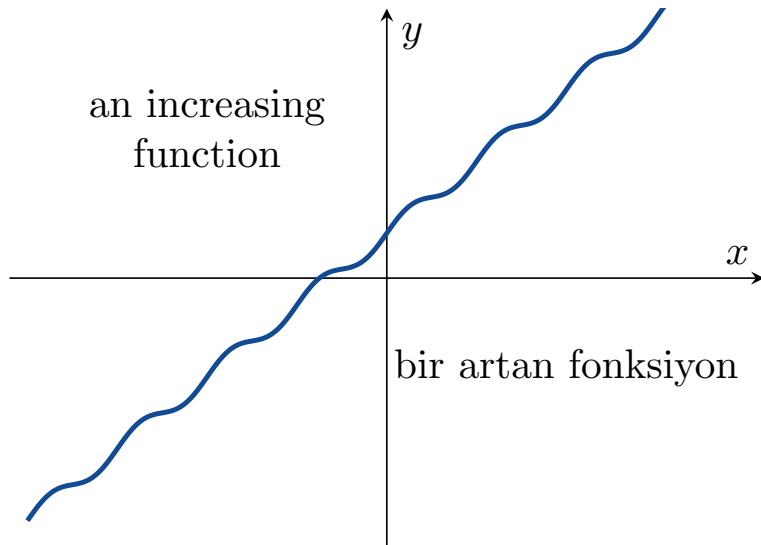
for all  $x_1, x_2 \in I$  which satisfy  $x_1 < x_2$ ;

- 2  $f$  is called *decreasing on  $I$*  if

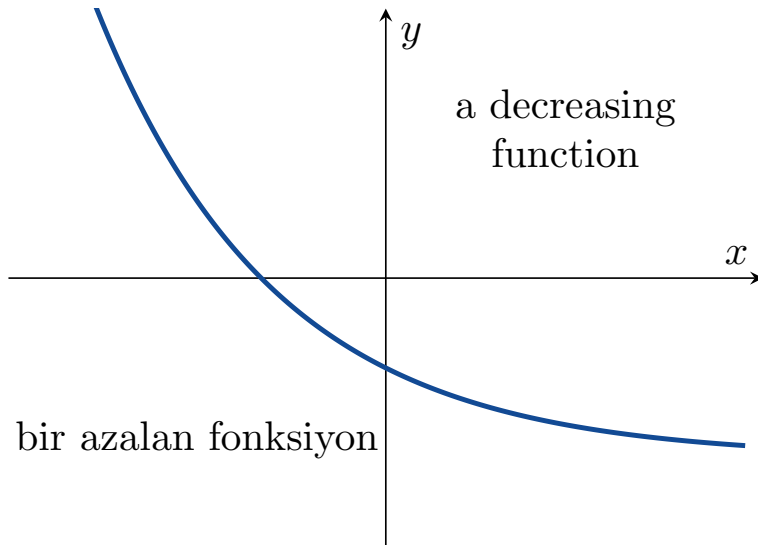
$$f(x_1) > f(x_2)$$

for all  $x_1, x_2 \in I$  which satisfy  $x_1 < x_2$ .

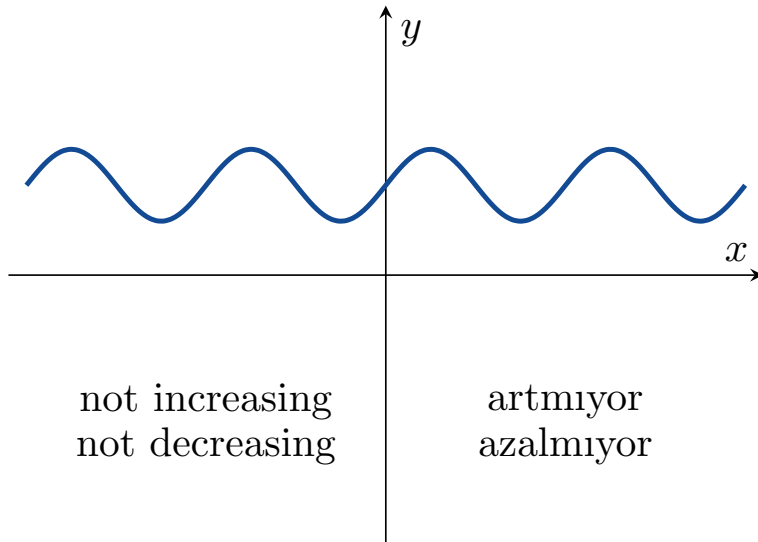
## 6. Functions



## 6. Functions



## 6. Functions



not increasing  
not decreasing

artmıyor  
azalmıyor

# Even Functions and Odd Functions

Recall that

- 2, 4, 6, 8, 10, ... are even numbers; and
- 1, 3, 5, 7, 9, ... are odd numbers.

### Definition

- 1  $f : D \rightarrow \mathbb{R}$  is an *even function* if  $f(-x) = f(x)$  for all  $x \in D$ ;
- 2  $f : D \rightarrow \mathbb{R}$  is an *odd function* if  $f(-x) = -f(x)$  for all  $x \in D$ .

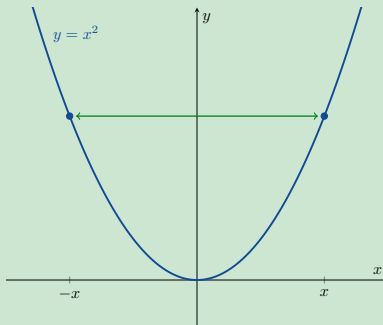
## 6. Functions



### Example

$f(x) = x^2$  is an even function because

$$f(-x) = (-x)^2 = x^2 = f(x).$$



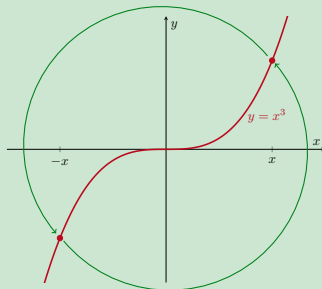


## 6. Functions

### Example

$f(x) = x^3$  is an odd function because

$$f(-x) = (-x)^3 = -x^3 = -f(x).$$



### Example

Is  $f(x) = x^2 + 1$  even, odd or neither?

*solution:* Since

$$f(-x) = (-x)^2 + 1 = x^2 + 1 = f(x),$$

$f$  is an even function.

### Example

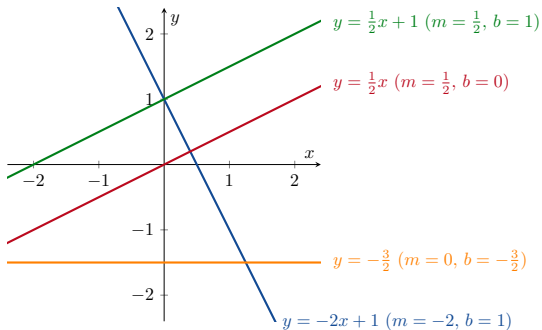
Is  $g(x) = x + 1$  even, odd or neither?

*solution:* Since  $g(-2) = -2 + 1 = -1$  and  $g(2) = 3$ , we have  $g(-2) \neq g(2)$  and  $g(-2) \neq -g(2)$ . Hence  $g$  is neither even nor odd.

### Linear Functions

$$f(x) = mx + b$$

$(m, b \in \mathbb{R})$



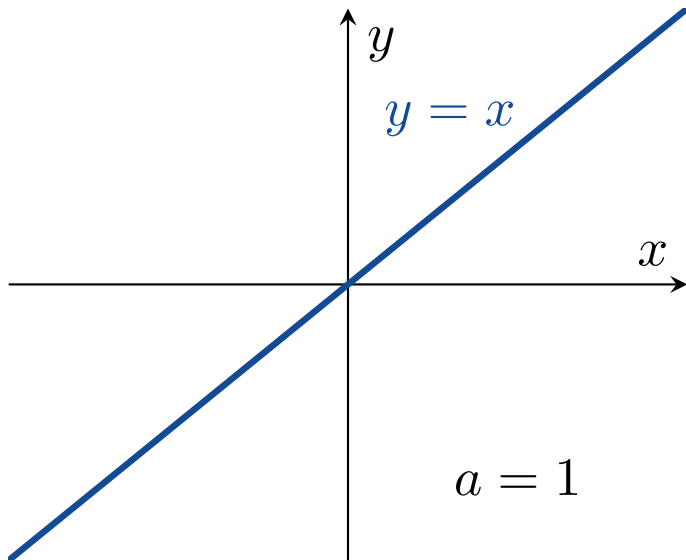
### Power Functions

$$f(x) = x^a$$

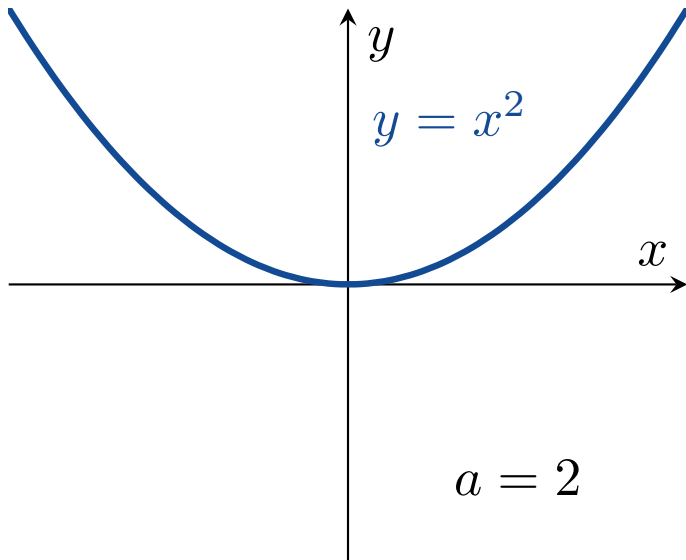
$(a \in \mathbb{R})$

“ $x$  to the power of  $a$ ”

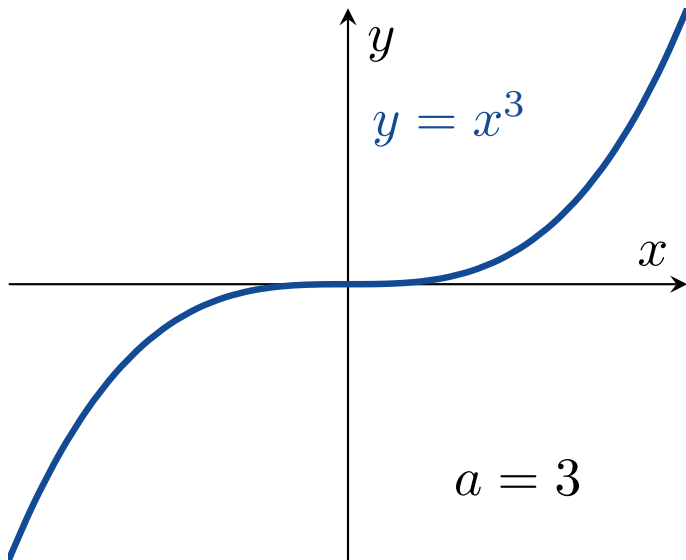
## 6. Functions



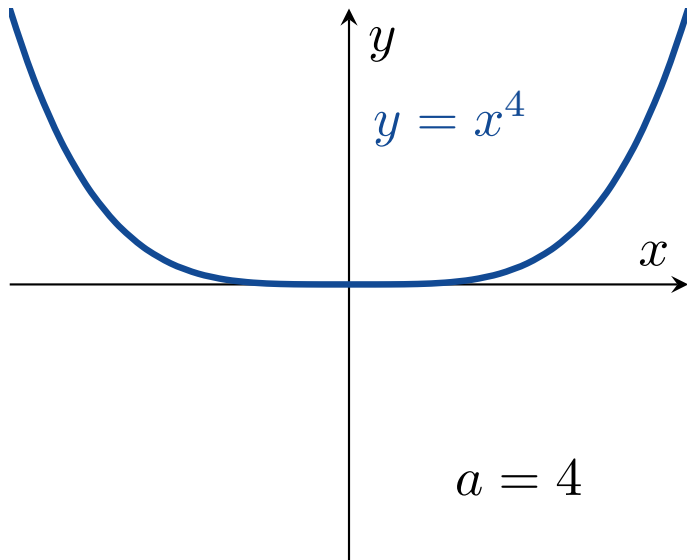
## 6. Functions



## 6. Functions

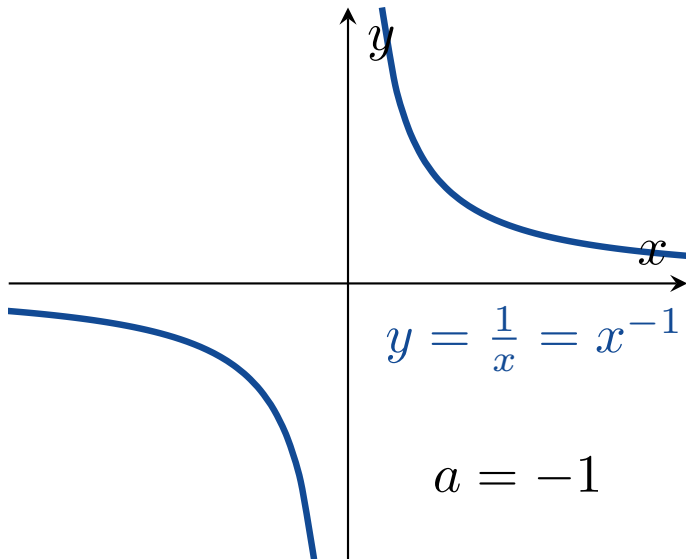


## 6. Functions

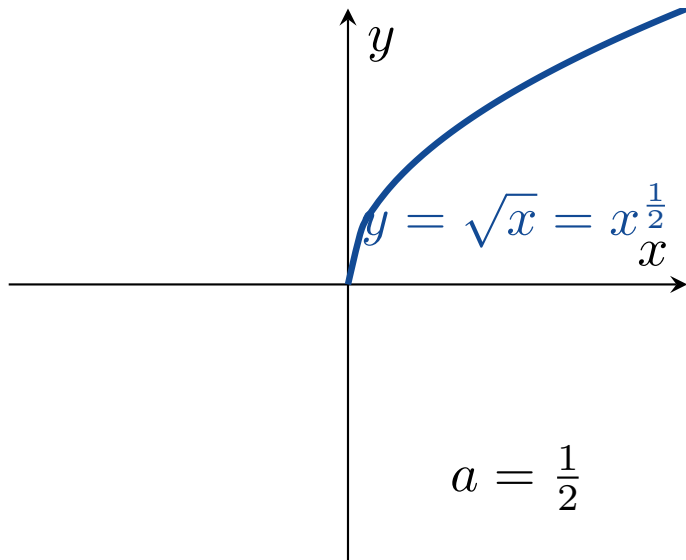




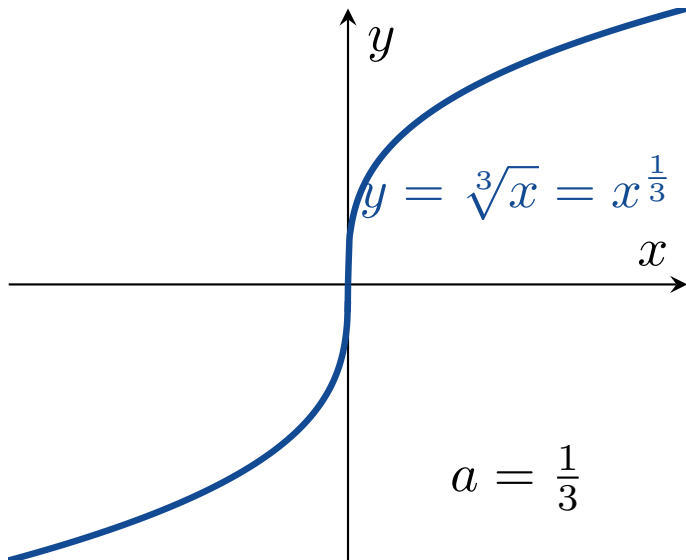
## 6. Functions



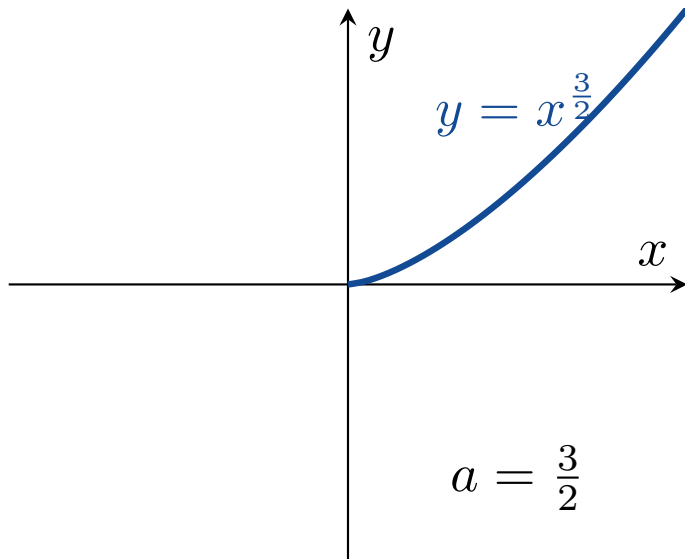
## 6. Functions



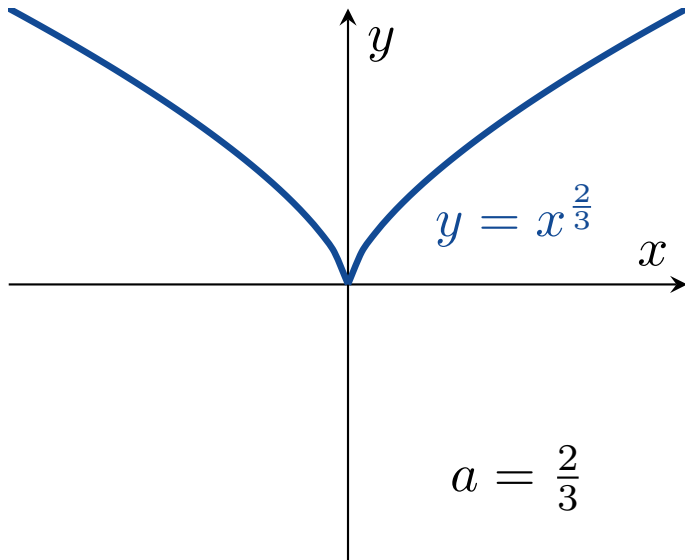
## 6. Functions



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## 6. Functions



# Polynomials

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

$$(n \in \mathbb{N} \cup \{0\}, a_j \in \mathbb{R}).$$

The domain of a polynomial is always  $(-\infty, \infty)$ . If  $n > 0$  and  $a_n \neq 0$ , then  $n$  is called the *degree* of  $p(x)$ .

### Rational Functions

$$f(x) = \frac{p(x)}{q(x)}$$

Diagram illustrating the components of a rational function:

- The term  $f(x)$  is labeled as the **rational function**.
- The term  $p(x)$  is labeled as **polynomial**.
- The term  $q(x)$  is labeled as **polynomial**.

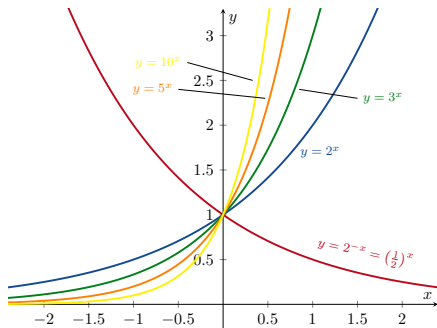
#### Example

$$f(x) = \frac{2x^3 - 3}{7x + 4}$$

## Exponential Functions

$$f(x) = a^x$$

$(a \in \mathbb{R}, a > 0, a \neq 1)$



The domain of an exponential function is  $(-\infty, \infty)$ .

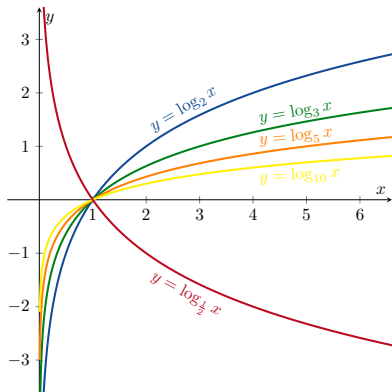


## Logarithmic Functions

$$y = \log_a x \iff x = a^y$$

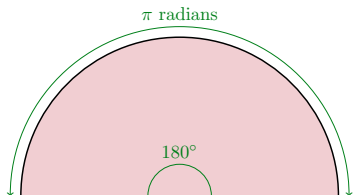
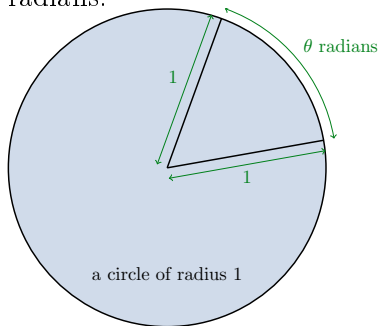
$$(a \in \mathbb{R}, a > 0, a \neq 1)$$

“log base  $a$  of  $x$ ”



### Angles

There are two ways to measure angles. Using degrees or using radians.



## 6. Functions

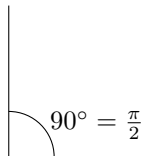
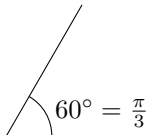
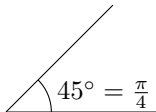


We have that

$$\pi \text{ radians} = 180 \text{ degrees}$$

$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$

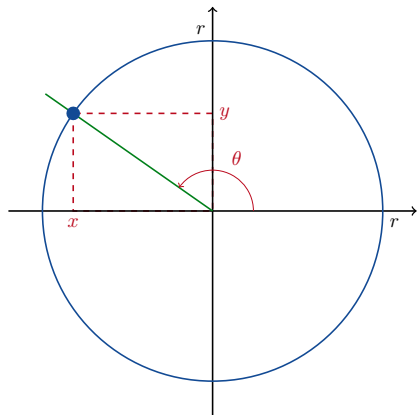
$$1 \text{ degree} = \frac{\pi}{180} \text{ radians.}$$



### Remark

In Calculus, we use radians!!!! If you see an angle in Part IV of this course, it will be in radians. Calculus doesn't work with degrees!!

# Trigonometric Functions



sine

$$\sin \theta = \frac{y}{r}$$

cosine

$$\cos \theta = \frac{x}{r}$$

tangent

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

secant

$$\sec \theta = \frac{1}{\cos \theta}$$

cosecant

$$\operatorname{cosec} \theta = \csc \theta = \frac{1}{\sin \theta}$$

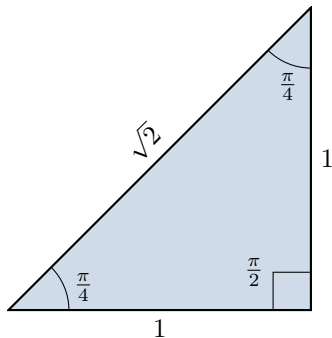
cotangent

$$\cot \theta = \frac{1}{\tan \theta}$$

### Remark

Note that  $\tan \theta$  and  $\sec \theta$  are only defined if  $\cos \theta \neq 0$ ; and  $\operatorname{cosec} \theta$  and  $\cot \theta$  are only defined if  $\sin \theta \neq 0$ .

## 6. Functions



$$\sin 45^\circ = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

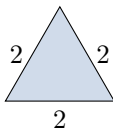
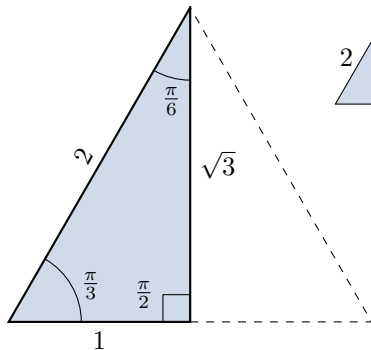
$$\tan 45^\circ = \tan \frac{\pi}{4} = 1$$

$$\sec 45^\circ = \sec \frac{\pi}{4} = \sqrt{2}$$

$$\operatorname{cosec} 45^\circ = \operatorname{cosec} \frac{\pi}{4} = \sqrt{2}$$

$$\cot 45^\circ = \cot \frac{\pi}{4} = 1$$

## 6. Functions



$$\sin 60^\circ = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \cos \frac{\pi}{3} = \frac{1}{2}$$

$$\tan 60^\circ = \tan \frac{\pi}{3} = \sqrt{3}$$

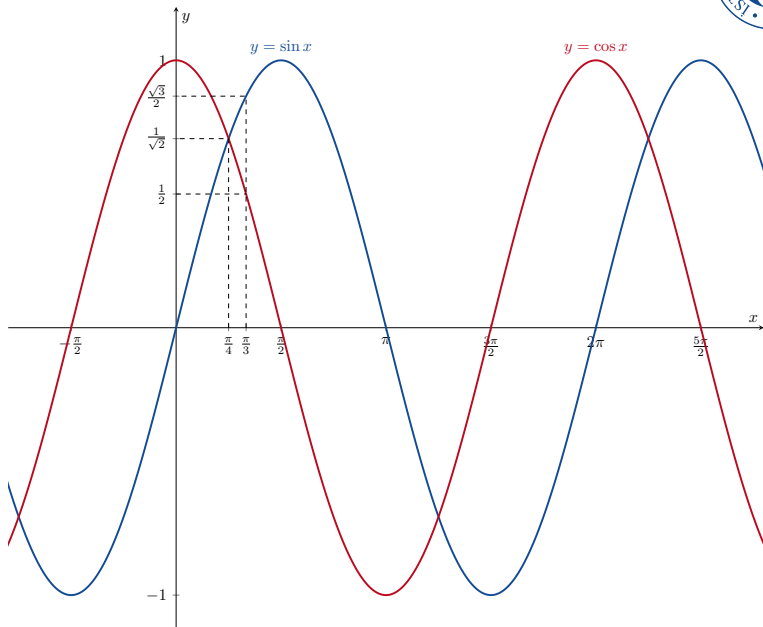
$$\sec 60^\circ = \sec \frac{\pi}{3} = 2$$

$$\operatorname{cosec} 60^\circ = \operatorname{cosec} \frac{\pi}{3} = \frac{2}{\sqrt{3}}$$

$$\cot 60^\circ = \cot \frac{\pi}{3} = \frac{1}{\sqrt{3}}$$



## 6. Functions



# Sigma Notation

## 7. Sigma Notation



$$\sum_{k=1}^n a_k = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + \dots + a_{n-1} + a_n$$

## 7. Sigma Notation



$$\sum_{k=1}^n a_k = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + \dots + a_{n-1} + a_n$$


$$\sum_{k=1}^n a_k$$

## 7. Sigma Notation



$$\sum_{k=1}^n a_k = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + \dots + a_{n-1} + a_n$$

the Greek  
letter Sigma


$$\sum_{k=1}^n a_k$$

## 7. Sigma Notation



$$\sum_{k=1}^n a_k = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + \dots + a_{n-1} + a_n$$

the Greek  
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$$\sum_{k=1}^n a_k$$

the sum starts  
at  $k = 1$

## 7. Sigma Notation



$$\sum_{k=1}^n a_k = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + \dots + a_{n-1} + a_n$$

the Greek  
letter Sigma

$$\sum_{k=1}^n a_k$$

the sum finishes  
at  $k = n$

the sum starts  
at  $k = 1$

## 7. Sigma Notation



### Example

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 + 11^2 = \sum_{k=1}^{11} k^2$$

$$f(1) + f(2) + f(3) + \dots + f(99) + f(100) = \sum_{k=1}^{100} f(k)$$

$$\sum_{k=1}^5 k = 1 + 2 + 3 + 4 + 5 = 15$$



## 7. Sigma Notation



### Example

$$\sum_{k=1}^3 (-1)^k k = (-1)(1) + (-1)^2(2) + (-1)^3(3) = -1 + 2 - 3 = -2$$

$$\sum_{k=1}^2 \frac{k}{k+1} = \frac{1}{1+1} + \frac{2}{2+1} = \frac{1}{2} + \frac{2}{3} = \frac{7}{6}$$

$$\sum_{k=4}^5 \frac{k^2}{k-1} = \frac{4^2}{4-1} + \frac{5^2}{5-1} = \frac{16}{3} + \frac{25}{4} = \frac{139}{12}$$

## 7. Sigma Notation



### Example

I want to find a formula for  $1 + 2 + 3 + \dots + n$ .

## 7. Sigma Notation



### Example

I want to find a formula for  $1 + 2 + 3 + \dots + n$ .

Note that

$$\begin{aligned} & 2(1+2+3+4+5+\dots+(n-1)+n) \\ &= \begin{array}{ccccccccccc} 1 & + & 2 & + & 3 & + & 4 & + & 5 & + & \dots & + & (n-1) & + & n \\ + & n & + & (n-1) & + & (n-2) & + & (n-3) & + & (n-4) & + & \dots & + & 2 & + & 1 \end{array} \\ &= (n+1) + (n+1) + (n+1) + (n+1) + (n+1) + \dots + (n+1) + (n+1) \\ &= n(n+1). \end{aligned}$$

## 7. Sigma Notation



### Example

I want to find a formula for  $1 + 2 + 3 + \dots + n$ .

Note that

$$\begin{aligned} & 2(1+2+3+4+5+\dots+(n-1)+n) \\ &= \begin{array}{ccccccccccc} 1 & + & 2 & + & 3 & + & 4 & + & 5 & + & \dots & + & (n-1) & + & n \\ + & n & + & (n-1) & + & (n-2) & + & (n-3) & + & (n-4) & + & \dots & + & 2 & + & 1 \end{array} \\ &= (n+1) + (n+1) + (n+1) + (n+1) + (n+1) + \dots + (n+1) + (n+1) \\ &= n(n+1). \end{aligned}$$

Therefore

$$\boxed{\sum_{k=1}^n k = \frac{n(n+1)}{2}.}$$

## 7. Sigma Notation



Similarly (but more difficult) we can find that

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

and

$$\sum_{k=1}^n k^3 = \left( \frac{n(n+1)}{2} \right)^2.$$

# Next Week

- 8. Polar Coordinates
- 9. Conic Sections
- 10. Three Dimensional Cartesian Coordinates