2019–20 **MATH216 Mathematics IV – First Makeup Midterm Exam** N. Course

Question 1. [10 pts] Solve $y' + 3y = t^2e^{-3t}$.

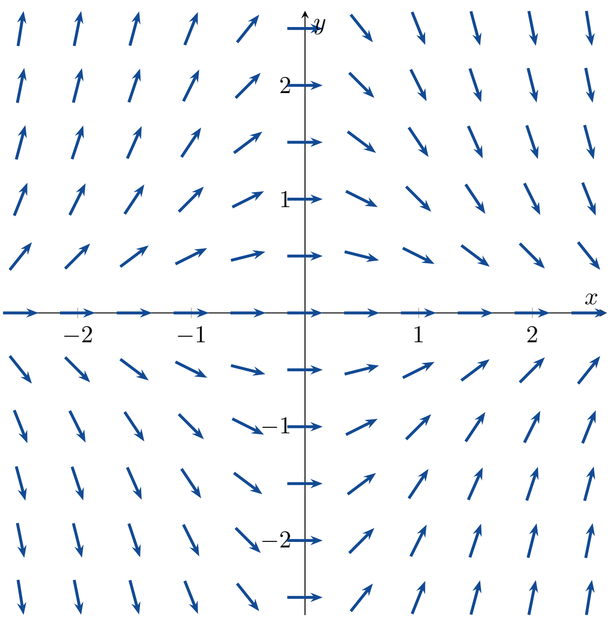
(A). $y(t) = ce^{3t} + t^3$

(B). $y(t) = ce^{-3t} + \frac{1}{3}e^{-3t}t^3$

(C). $y(t) = ce^{-3t} + 3e^{3t}t$

(D). $y(t) = ce^{3t} - 3e^{3t}t$

(E). $y(t) = c_1 \sin 3t + c_2 \cos 3t$



Question 2. [10 pts] Match the direction field shown above with one of the following five functions.

(A). $\frac{dy}{dx} = y - 2x$

(B). $\frac{dy}{dx} = -xy$

(C). $\frac{dy}{dx} = y + 2x$

(D). $\frac{dy}{dx} = xy$

(E). $\frac{dy}{dx} = e^x + \sin(\tan x)$

Question 3. [10 pts] Consider

$$x^6 y^{(5)} + y''' = 6y'' - x^6 y + \cos x$$

where $y^{(n)} = \frac{d^n y}{dx^n}$. This differential equation is

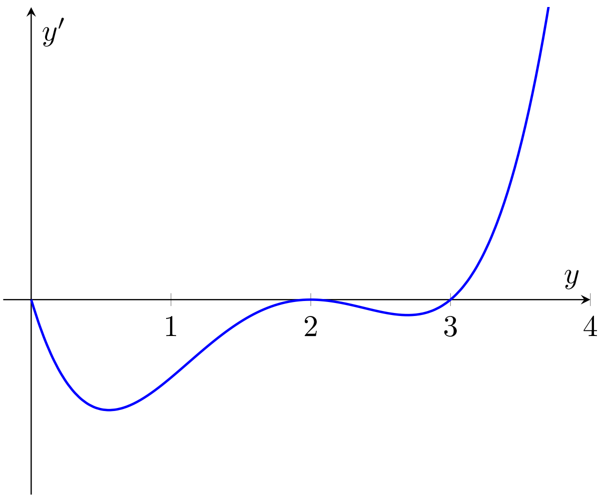
(A). 5th order and non-linear

(B). 5th order and linear

(C). 6th order and linear

(D). 6th order and non-linear

(E). 99th order and non-linear



Question 4. [15 pts] The critical points/equalibrium solutions of

$$\frac{dy}{dt} = x(2x-4)(2-x)(3-x)$$

(graph shown above) are $y = 0$, $y = 2$ and $y = 3$. Which of the following are true (choose up to 3).

(A). $y = 0$ is asymptotically stable

(B). $y = 0$ is unstable

(C). $y = 0$ is semistable

(D). $y = 2$ is asymptotically stable

(E). $y = 2$ is unstable

(F). $y = 2$ is semistable

(G). $y = 3$ is asymptotically stable

(H). $y = 3$ is unstable

(I). $y = 3$ is semistable

Question 5. [10 pts] True or False? The following equation is an exact equation:

$$(ye^{xy}\cos x) + (xe^{xy}\cos x)\, y' = 0.$$

(T). True

(F). False

Question 6. [15 pts] Solve $y'' + 2y' - 3y = 0$.

(A). $y(t) = c_1 e^{3t} + c_2 e^{-t}$

(B). $y(t) = c_1 e^{-3t} + c_2 e^{t}$

(C). $y(t) = c_1 e^{2t} + c_2 e^{3t}$

(D). $y(t) = c_1 e^{2t} + c_2 e^{-3t}$

(E). $y(t) = c_1 e^{2t}\cos 3t + c_2 e^{2t}\sin 3t$

Question 7. [15 pts] If I know that $y_1(t)$ solves $y'' + p(t)y' + q(t)y = 0$ and I want to find a second linearly independent solution, what do I do?

(A). Try $y_2(t) = v(t)y_1(t)$

(B). Find the eigenvalues

(C). Use the substitution $v(x) = \frac{y}{x}$

(D). Try $y_2(t) = Ap(t) + Bq(t)$

(E). Give up.

Question 8. [15 pts] Given that $(r-1)^2 = r^2 - 2r + 1$, solve

$$y'' - 2y' + y = e^t.$$

(A). $y(t) = c_1 e^{t} + c_2 e^{t} + 3e^{t}$

(B). $y(t) = c_1 e^{t} + c_2 te^{t} - 3e^{t}$

(C). $y(t) = c_1 e^{t} + c_2 e^{2t} + \dfrac{3}{2}te^{t}$

(D). $y(t) = c_1 e^{t} + c_2 te^{2} - t^2 e^{t}$

(E). $y(t) = c_1 e^{t} + c_2 te^{t} + \dfrac{1}{2}t^2 e^{t}$