2019–20 **MATH216 Mathematics IV – Second Makeup Midterm Exam** N. Course

Question 1. [15 pts] If $y(t)$ solves

$$\begin{cases} y'' - 9y = e^{3t} \\ y(0) = 1 \\ y'(0) = 0, \end{cases}$$

find $Y(s) = \mathcal{L}\left[y\right](s)$.

(A). $\dfrac{s^2+3s+1}{(s-3)(s+3)}$

(B). $\dfrac{s^2-3s+1}{(s-3)^2(s+3)}$

(C). $\dfrac{1}{(s-3)(s+3)^2}$

(D). $\dfrac{1}{(s-5)(s+5)}$

(E). $\dfrac{1}{19}e^{3s}$

Question 2. [10 pts] Write $\frac{s+1}{(s+2)(s+3)}$ in partial fractions.

(A). $\dfrac{3}{s+3} + \dfrac{2}{s+2}$

(B). $\dfrac{2}{s+3} - \dfrac{1}{s+2}$

(C). $\dfrac{1}{s+1} - \dfrac{3}{s+2}$

(D). $\dfrac{2s+2}{s+3} + \dfrac{s+1}{s+2}$

(E). $47e^{99t}$

Question 3. [15 pts] Find the inverse Laplace Transform of $F(s) = \frac{2e^{-7s}}{s^3}$.

(A). $u_3(t)\cos(t-3)$

(B). $u_7(t)(t-7)^2$

(C). $u_7(t)e^{t-7}$

(D). $u_3(t)e^t$

(E). $u_7(t)t^2$

Question 4. [10 pts] Write the function

$$f(t) = \begin{cases} 0 & t < 2 \\ t & 2 \le t < 3 \\ t^2 & t \ge 3. \end{cases}$$

in terms of the unit step function $u_c(t)$.

(A). $f(t) = tu_2(t) + (t^2 - t)u_3(t)$

(B). $f(t) = tu_2(t) + t^2u_3(t)$

(C). $f(t) = 2u_2(t) + 6u_3(t)$

(D). $f(t) = 2u_2(t) - 6u_1(t)$

(E). $f(t) = u_1(t) + u_2(t) + u_3(t) + \ldots$

Question 5. [10 pts] The matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ has eigenvalues $r_1 = -1$ and $r_2 = 4$. Solve $\mathbf{x}' = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \mathbf{x}$.

(A). $\mathbf{x}(t) = c_1 e^{-t} + c_2 e^{4t}$

(B). $\mathbf{x}(t) = c_1 \begin{bmatrix} 2 \\ 3 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{4t}$

(C). $\mathbf{x}(t) = c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 2 \\ 2 \end{bmatrix} e^{4t}$

(D). $\mathbf{x}(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^{4t}$

(E). $\mathbf{x}(t) = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 2 \\ 3 \end{bmatrix} e^{4t}$

Question 6. [10 pts] Which of the following matrices is a *fundamental matrix* for $\mathbf{x}' = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \mathbf{x}$?

(A). $\Psi(t) = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

(B). $\Psi(t) = \begin{bmatrix} e^{-t} & 2e^{4t} \\ -e^{-t} & 3e^{4t} \end{bmatrix}$

(C). $\Psi(t) = \begin{bmatrix} 2e^{-t} & 2e^{4t} \\ 3e^{-t} & e^{4t} \end{bmatrix}$

(D). $\Psi(t) = \begin{bmatrix} e^{-t} & 2e^{4t} \\ 3e^{-t} & 2e^{4t} \end{bmatrix}$

(E). $\Psi(t) = \begin{bmatrix} e^{-t} & 3e^{4t} \\ e^{-t} & 2e^{4t} \end{bmatrix}$

Question 7. [15 pts] The matrix $A = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$ has eigenvalues $r_1 = 1 + 2i$ and $r_2 = 1 - 2i$; and corresponding eigenvectors $\boldsymbol{\xi}^{(1)} = \begin{bmatrix} 1+i \\ 2 \end{bmatrix}$ and $\boldsymbol{\xi}^{(2)} = \begin{bmatrix} 1-i \\ 2 \end{bmatrix}$. Solve $\mathbf{x}' = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix} \mathbf{x}$.

(A). $\mathbf{x}(t) = c_1 e^{t} \begin{bmatrix} \cos 2t - \sin 2t \\ 2\cos 2t \end{bmatrix} + c_2 e^{t} \begin{bmatrix} \cos 2t + \sin 2t \\ 2\sin 2t \end{bmatrix}$

(B). $\mathbf{x}(t) = c_1 e^{2t} \begin{bmatrix} \cos 2t \\ \cos 2t \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} \cos 2t \\ \sin 2t \end{bmatrix}$

(C). $\mathbf{x}(t) = c_1 e^{t} \cos 2t + c_2 e^{t} \sin 2t$

(D). $\mathbf{x}(t) = c_1 e^{t} \begin{bmatrix} \cos 2t + 2\sin 2t \\ 2\cos 2t \end{bmatrix} + c_2 e^{t} \begin{bmatrix} \cos 2t + 3\sin 2t \\ 2\sin 2t \end{bmatrix}$

(E). $\mathbf{x}(t) = c_1 e^{t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^{t} \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

Question 8. [15 pts] The matrix $A = \begin{bmatrix} 7 & 1 \\ -4 & 3 \end{bmatrix}$ has repeated eigenvalue $r_1 = r_2 = 5$ and and only one linearly independent eigenvectors $\boldsymbol{\xi} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$. Solve $\begin{cases} \mathbf{x}' = \begin{bmatrix} 7 & 1 \\ -4 & 3 \end{bmatrix} \mathbf{x} \\ \mathbf{x}(0) = \begin{bmatrix} 2 \\ -5 \end{bmatrix} \end{cases}$.

(A). $\mathbf{x}(t) = e^{-25t} \begin{bmatrix} 2+3 \\ 1+2t \end{bmatrix}$

(B). $\mathbf{x}(t) = \begin{bmatrix} 2+3 \\ 1+2t \end{bmatrix} e^{-25t}$

(C). $\mathbf{x}(t) = e^{5t} \begin{bmatrix} 2-3t \\ -5+t \end{bmatrix}$

(D). $\mathbf{x}(t) = e^{5t} \begin{bmatrix} 2-5t \\ -5-8t \end{bmatrix}$

(E). $\mathbf{x}(t) = e^{5t} \begin{bmatrix} 2-t \\ -5+2t \end{bmatrix}$