2015–16 **MAT234 Matematik IV – Ödev 8** N. Course

SON TESLİM TARİHİ: Salı 3 Mayıs 2016 saat 16:00'e kadar.

Egzersiz 15 (Taylor Series). Let $f(x) = \cos x$ and let $a = 2\pi$.

(a) [35p] Show that the "remainder term" tends to zero. In other words; show that

$$\frac{f^{(n)}(c)\ (x-a)^n}{n!} \to 0$$

as $n \to \infty$, for all $x \in \mathbb{R}$ and for all c between a and x ($a < c < x$ or $x < c < a$).

(b) [65p] Calculate the Taylor Series for $f(x) = \cos x$, centred at $a = 2\pi$.

Ödev 7'nin çözümleri

13. (a) Since $\frac{1}{\cosh n} = \frac{2}{e^n + e^{-n}} < \frac{2}{e^n} = 2e^{-n}$ and since $\sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^n$ converges, it follows by the Comparison Test that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\cosh n}$ converges absolutely.
(b) Since $\frac{\log n}{n - \log n} > \frac{\log n}{n} > \frac{1}{n}$ (for $n \geq 3$), it follows by the Comparison Test that $\sum_{n=1}^{\infty} \left| \frac{(-1)^n \log n}{n - \log n} \right| = \sum_{n=1}^{\infty} \frac{\log n}{n - \log n}$ diverges.

If $f(x) = \frac{\log x}{x - \log x}$, then $f'(x) = \frac{x^{-1}(x - \log x) - \log x (1 - x^{-1})}{(x - \log x)^2} = \frac{1 - \log x}{(x - \log x)^2} < 0$ if $x > e$. Therefore $a_n = \frac{\log n}{n - \log n}$ is a decreasing sequence (for $n \geq 3$). Clearly $a_n > 0$ for all $n \geq 3$ and $a_n = \frac{\log n}{n - \log n} \to 0$ as $n \to \infty$. It follows by the Alternating Series Test that $\sum_{n=1}^{\infty} \frac{(-1)^n \log n}{n - \log n}$ converges.

Therefore $\sum_{n=1}^{\infty} \frac{(-1)^n \log n}{n - \log n}$ converges conditionally.

14. (a) Here $a_n = \frac{2^n}{\sqrt{n^2+3}}$. Since $\left| \frac{a_n}{a_{n+1}} \right| = \frac{2^n}{\sqrt{n^2+3}} \frac{\sqrt{(n+1)^2+3}}{2^{n+1}} = \frac{1}{2} \sqrt{\frac{(n+1)^2+3}{n^2+3}} \to \frac{1}{2}$, it follows by a theorem from the course that $R = \frac{1}{2}$. The open interval of convergence is $(-\frac{1}{2}, \frac{1}{2})$.
(b) $R = \infty$. The interval is $(-\infty, \infty)$.
(c) $R = e^2$. The interval is $(-e^2, e^2)$.