



Welcome to

Mathematics for Architects

with Dr Neil Course

Lecture 1

- Information about this course
- 1. Sets
- 2. Symbolic Logic
- 3. Numbers

Information about this course

- \approx 12 classes. Friday evenings 6pm-8pm.

18:00

19:00

20:00

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- Each lecture \approx 60 minutes.

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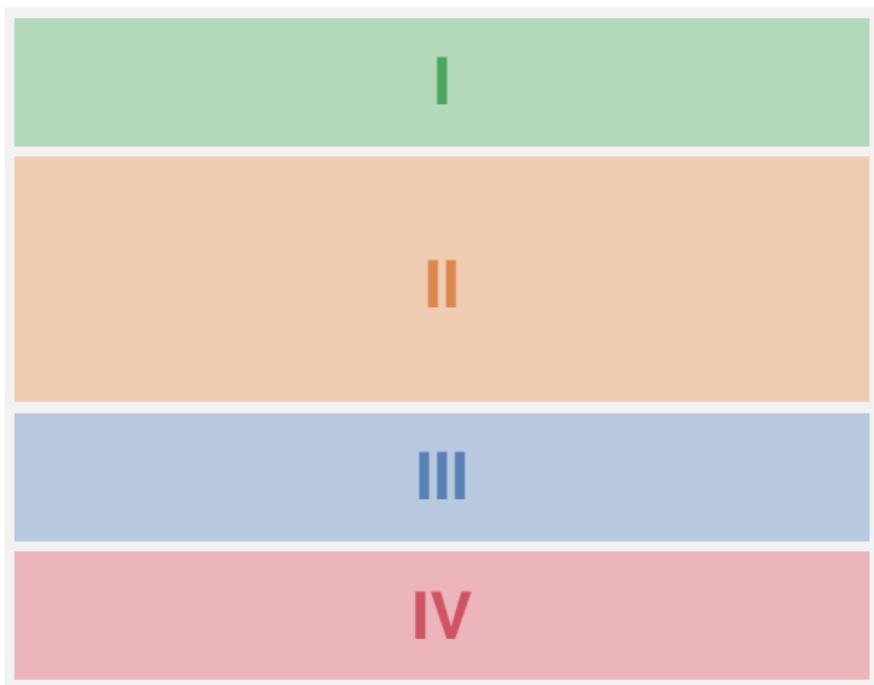
lecture

questions

18:00

19:00

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Introduction

Sets; Symbolic Logic; Numbers; Cartesian Coordinates.

II

III

IV



revision?

Introduction

Sets; Symbolic Logic; Numbers; Cartesian Coordinates.

II

III

IV



revision?

Introduction

Sets; Symbolic Logic; Numbers; Cartesian Coordinates.

The Geometry of Space

Polar Coordinates; Conic Sections;
Three Dimensional Cartesian Coordinates;
Vectors; The Dot Product; The Cross Product;
Lines; Planes; Projections.

III

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revision?

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Sets; Symbolic Logic; Numbers; Cartesian Coordinates.

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Finite Mathematics

Combinatorics; Probability; Graph Theory.

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revision?

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Finite Mathematics

Combinatorics; Probability; Graph Theory.

Calculus

Limits; Continuity; Differentiation; Integration.

revision?

Introduction

Sets; Symbolic Logic; Numbers; Cartesian Coordinates.

2 lectures

The Geometry of Space

Polar Coordinates; Conic Sections;
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Lines; Planes; Projections.

3 lectures

Finite Mathematics

Combinatorics; Probability; Graph Theory.

3 lectures

Calculus

Limits; Continuity; Differentiation; Integration.

4 lectures



Lecture Notes



The image shows the front cover of a lecture notes book. At the top, there is a green horizontal bar with white text: "Logic • Geometry • Combinatorics • Probability • Graph Theory • Calculus". Below this, a yellow diagonal banner contains the text "Autumn 2021-22". The central part of the cover features a close-up photograph of a bee pollinating a yellow flower. At the bottom, there is a white rectangular area with the title "Mathematics for Architects" in large, bold, black font. Below the title, in a smaller black font, are the names "Neil Course • Sezgin Sezer • Asuman Özer".

Logic • Geometry • Combinatorics • Probability • Graph Theory • Calculus

Autumn
2021-22

Mathematics
for Architects

Neil Course • Sezgin Sezer • Asuman Özer



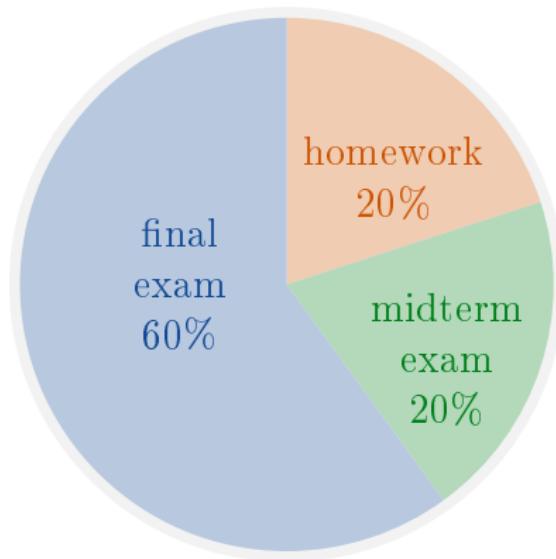
Exams and homework

(This information may change based on the University's decisions)



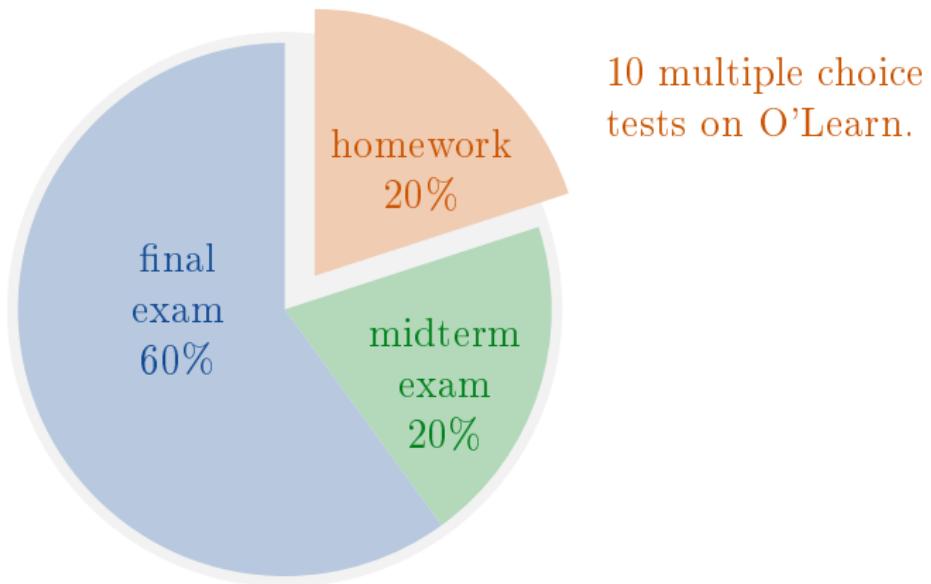
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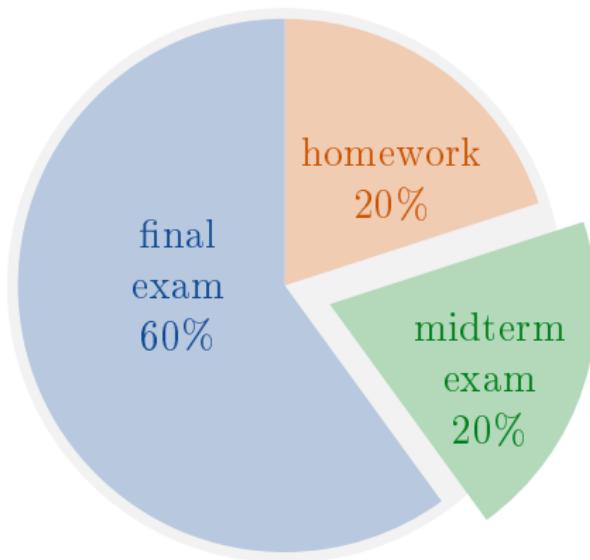
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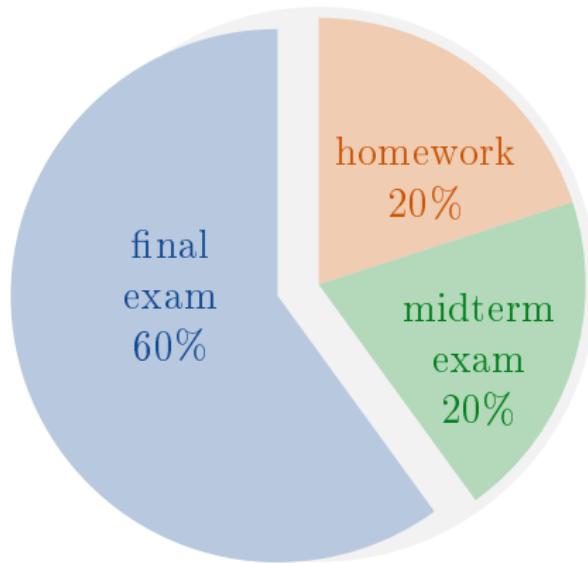
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Expectations

If this was a classroom course, the expectation is that for each hour of lectures, you would study an extra 1-2 hours outside of class.

classroom
course

lectures (4 hours)

other study (4-8 hours)

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classroom
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other study (4-8 hours)

For an online course, you are still expected to study a total of 8-12 hours each week.

online
course

class
(2 hours)

other study (6-10 hours)

Your other study may include:

- Do the online homework tests each week;

⋮

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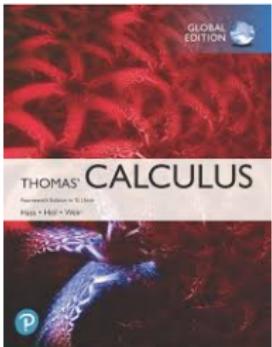
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- Solve the problems in the lecture notes;
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- Read books;
- Watch online videos (e.g. blackpenredpen on YouTube has good Calculus videos);

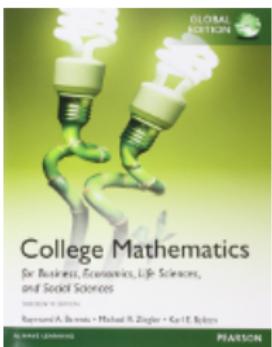
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Two good books



George B. Thomas Jr., Maurice D. Weir and Joel Hass,
Thomas' Calculus,
Pearson.

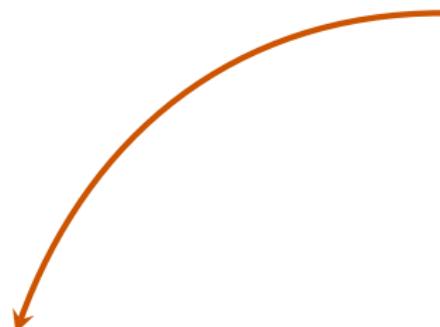


Raymond A. Barnett, Michael R. Ziegler and Karl E. Byleen,
College Mathematics for Business, Economics, Life Sciences, and Social Sciences,
Pearson.

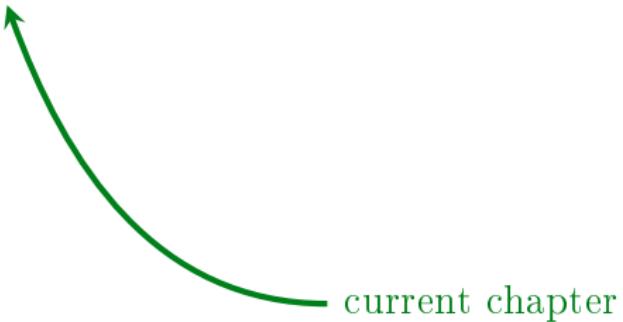
99. Chapter Title



slide number



99. Chapter Title



1 Sets

1. Sets



Definition

A set is a collection of objects, specified in such a way that we can tell whether any given object is or is not in the collection.

1. Sets

Example

For example

$$A = \{1, 2, 3, 4, 5\},$$

$$B = \{\text{apple, banana, cherry}\}$$

and

$$C = \{n, e, i, l\}$$

are sets.

1. Sets



Definition

The symbol \in means “is in the set”.

Example

If

$$B = \{\text{apple, banana, cherry}\}$$

then

$$\text{banana} \in B$$

and

$$\text{date} \notin B$$

Definition

Each object in a set is called an *element* of the set.

Definition

A set without any elements is called the *empty set* and is denoted by \emptyset .

1. Sets

Definition

The symbol | means “such that”.

Example

$$\{x \mid x \text{ is a weekend day}\} = \{\text{Saturday, Sunday}\}$$

$$\{x \mid x^2 = 4\} = \{-2, 2\}$$

$$\{\text{all the people who are } > 5\text{m tall}\} = \emptyset.$$

1. Sets

Definition

If every element of a set A is also in a set B , then we say that A is a *subset* of B , and we write $A \subseteq B$.

Example

- $$\{1, 2, 3\} \subseteq \{1, 2, 3, 4\},$$
- $$\{\text{banana}\} \subseteq \{\text{apple, banana, cherry}\},$$
- $$\{\text{Neil, Sezgin}\} \subseteq \{\text{Neil, Sezgin}\}.$$

1. Sets

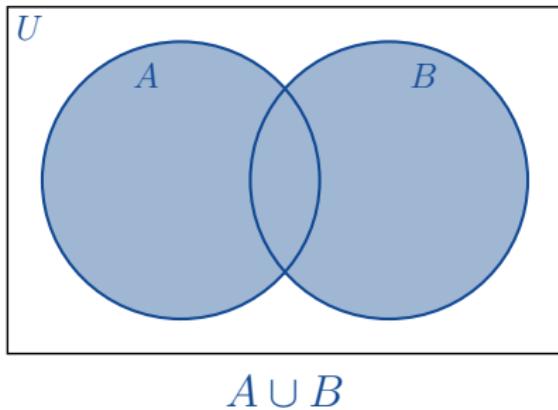


Definition

The *universal set* is the set of all elements under consideration.
We call this set U .

1. Sets

Suppose that A and B are subsets of U .

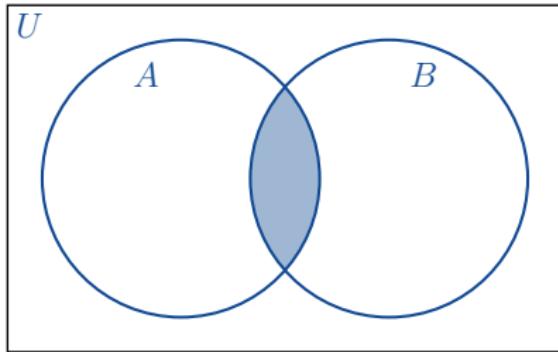


Definition

The *union* of A and B is

$$A \cup B = \{e \in U \mid e \in A \text{ or } e \in B\}.$$

1. Sets



$$A \cap B$$

Definition

The *intersection* of A and B is

$$A \cap B = \{e \in U \mid e \in A \text{ and } e \in B\}.$$

1. Sets

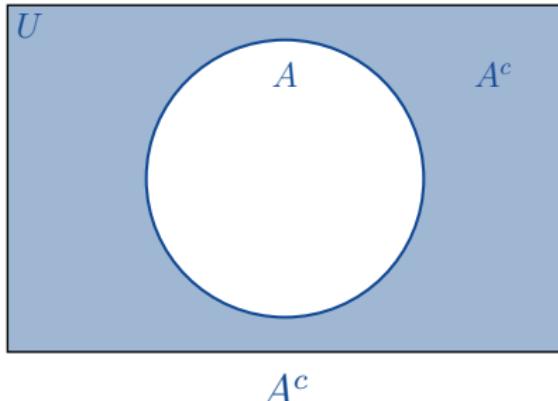


Example

$$\{a, b, c\} \cup \{b, c, d\} = \{a, b, c, d\}$$

$$\{a, b, c\} \cap \{b, c, d\} = \{b, c\}$$

1. Sets



Definition

The *complement* of a subset A of U is

$$A^c = \{e \in U \mid e \notin A\}.$$

Example

If U is the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = \{1, 3, 5, 7, 9\}$, then

$$A^c = \{2, 4, 6, 8, 10\}.$$



Symbolic Logic

2. Symbolic Logic



Definition

A *proposition* is a statement which is either *true* or *false* (but not both).

2. Symbolic Logic

Example

- “Grass is green” (true)
- “ $2+5=5$ ” (false)
- “My name is Neil” (true)

are propositions, but

- “Close the door”
- “Is it cold today?”
- “1”

are not propositions.

2. Symbolic Logic

Notation

The symbol for *or* (veya) is \vee .

Example

If P is the proposition “It is snowing today” and Q is the proposition “It is raining today”, then $P \vee Q$ is the proposition “It is snowing or raining today”.

Example

If $M = (x \in A)$ and $N = (x \in B)$, then $M \vee N = (x \in A \cup B)$

2. Symbolic Logic



Truth Table

(T = true, F = false)

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

2. Symbolic Logic

Notation

The symbol for *and* (ve) is \wedge .

Example

If P = “I am hungry” and Q = “I am sleepy”, then $P \wedge Q$ = “I am hungry and sleepy”.

Example

If $M = (x \in A)$ and $N = (x \in B)$, then $M \wedge N = (x \in A \cap B)$

2. Symbolic Logic



Truth Table

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

2. Symbolic Logic

Notation

The symbol for *not* (değil) is \neg .

Example

If P = “Sizin hocanız kahve seviyor”, then $\neg P$ = “Sizin hocanız kahve sevmiyor”.

Example

If M = $(x \geq 7)$, then $\neg M$ = $(x < 7)$

2. Symbolic Logic

Truth Table

P	$\neg P$
T	F
F	T

2. Symbolic Logic

Notation

The symbol for *if and only if* (iff/ancak ve ancak) is \iff .

Truth Table

P	Q	$P \iff Q$
T	T	T
T	F	F
F	T	F
F	F	T

2. Symbolic Logic

Notation

The symbol for *implies* (ise) is \Rightarrow .

Example

Let P = “I am in London” and Q = “I am in the UK.” Then
 $P \Rightarrow Q$.

Truth Table

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

2. Symbolic Logic

Remark

We must only write “ $P \Rightarrow Q$ ” if both P and Q are propositions. I don’t want to see nonsense like

$$\int_0^1 3x^2 \, dx = [x^3]_0^1 \Rightarrow 1$$

in your work. Yes, “ $\int_0^1 3x^2 \, dx = [x^3]_0^1$ ” is a proposition. In fact, it is a *true* proposition. But “1” is not a proposition.

If you mean “=”, then write “=”.

Remark

If P and Q are propositions, then $(P \vee Q)$, $(P \wedge Q)$, $(\neg P)$, $(P \Rightarrow Q)$ and $(P \Leftrightarrow Q)$ are also propositions.

2. Symbolic Logic

Definition

The *converse* (zıt) of $(P \Rightarrow Q)$ is $(Q \Rightarrow P)$.

Definition

The *contrapositive* (devrik) of $(P \Rightarrow Q)$ is $(\neg Q \Rightarrow \neg P)$.

Example

P = “It is raining”

Q = “I get wet”

$(P \Rightarrow Q)$ = “If it is raining, then I get wet”

converse: $(Q \Rightarrow P)$ = “If I get wet, then it is raining”

contrapositive: $(\neg Q \Rightarrow \neg P)$ = “If I do not get wet, then it is not raining”

The 22 Identities

1. $(P \vee P) = P$
2. $(P \wedge P) = P$
3. $(P \vee Q) = (Q \vee P)$
4. $(P \wedge Q) = (Q \wedge P)$
5. $((P \vee Q) \vee R) = (P \vee (Q \vee R))$
6. $((P \wedge Q) \wedge R) = (P \wedge (Q \wedge R))$
7. $\neg(P \vee Q) = (\neg P \wedge \neg Q)$
8. $\neg(P \wedge Q) = (\neg P \vee \neg Q)$
9. $(P \wedge (Q \vee R)) = ((P \wedge Q) \vee (P \wedge R))$
10. $(P \vee (Q \wedge R)) = ((P \vee Q) \wedge (P \vee R))$
11. $(P \vee \text{true}) = \text{true}$

2. Symbolic Logic

$$12 \quad (P \wedge \text{false}) = \text{false}$$

$$13 \quad (P \vee \text{false}) = P$$

$$14 \quad (P \wedge \text{true}) = P$$

$$15 \quad (P \vee \neg P) = \text{true}$$

$$16 \quad (P \wedge \neg P) = \text{false}$$

$$17 \quad \neg(\neg P) = P$$

$$18 \quad (P \implies Q) = (\neg P \vee Q)$$

$$19 \quad (P \iff Q) = ((P \implies Q) \wedge (Q \implies P))$$

$$20 \quad ((P \wedge Q) \implies R) = (P \implies (Q \implies R))$$

$$21 \quad ((P \implies Q) \wedge (P \implies \neg Q)) = \neg P$$

$$22 \quad (P \implies Q) = (\neg Q \implies \neg P)$$

2. Symbolic Logic

Proof of Identity 18.

P	Q	$P \Rightarrow Q$	$\neg P$	Q	$\neg P \vee Q$
T	T	T	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	F	T

Note that the 3rd and 6th columns are the same:

$$T, F, T, T.$$

Therefore $(P \Rightarrow Q) = (\neg P \vee Q)$. □

2. Symbolic Logic

Proof of Identity 22.

P	Q	$P \Rightarrow Q$	$\neg Q$	$\neg P$	$\neg Q \Rightarrow \neg P$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Therefore $(P \Rightarrow Q) = (\neg Q \Rightarrow \neg P)$.



2. Symbolic Logic



Notation

The symbol for *for all* (her) is \forall .

Notation

The symbol for *there exists* (vardır) is \exists .



Numbers

3. Numbers



The set

$$\mathbb{N} = \{1, 2, 3, 4, 5, 6, \dots\}$$

is called the set of *natural numbers*. These are the first numbers that children learn. For example

$2 \in \mathbb{N}$ means “2 is a natural number”

$7 \in \mathbb{N}$ means “7 is a natural number”

$\frac{1}{2} \notin \mathbb{N}$ means “ $\frac{1}{2}$ is **not** a natural number”

$0 \notin \mathbb{N}$ means “0 is **not** a natural number”

$-5 \notin \mathbb{N}$ means “-5 is **not** a natural number”

3. Numbers



In the natural numbers, we can do “+” and “×”

$$2 + 7 = 9 \in \mathbb{N}, \quad 2 \times 7 = 14 \in \mathbb{N}.$$

However we can not do “−” because

$$2 - 7 \notin \mathbb{N}.$$

So we invent new numbers!

3. Numbers



The set

$$\mathbb{Z} = \{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$$

is called the set of *integers*. We use a \mathbb{Z} for the German word ‘zahlen’ (numbers). In \mathbb{Z} , we can do “+”, “−” and “ \times ” but we can not do “ \div ”. For example $3 \in \mathbb{Z}$, $4 \in \mathbb{Z}$, $-5 \in \mathbb{Z}$ and

$$3 + 4 \in \mathbb{Z}, \quad 3 - 4 \in \mathbb{Z}, \quad 3 \times 4 \in \mathbb{Z}, \quad 3 \div 4 \notin \mathbb{Z},$$

$$3 + (-5) \in \mathbb{Z}, \quad 3 - (-5) \in \mathbb{Z}, \quad 3 \times (-5) \in \mathbb{Z}, \quad 3 \div (-5) \notin \mathbb{Z}.$$

So we invent new numbers!

3. Numbers



The set

$$\mathbb{Q} = \{\text{all fractions}\} = \left\{ \frac{a}{b} \mid a, b \in \mathbb{Z} \text{ and } b \neq 0 \right\}$$

is called the set of *rational numbers*. We use a \mathbb{Q} for the word ‘quotient’. For example

$$0 = \frac{0}{1} \in \mathbb{Q}$$

$$\frac{100}{13} \in \mathbb{Q}$$

$$1 = \frac{1}{1} \in \mathbb{Q}$$

$$\sqrt{2} \notin \mathbb{Q}$$

$$\frac{3}{4} \in \mathbb{Q}$$

$$-4 = \frac{8}{-2} \in \mathbb{Q}$$

$$\pi \notin \mathbb{Q}$$

$$0.12345 = \frac{12345}{100000} \in \mathbb{Q}.$$

In \mathbb{Q} we can do “+”, “-”, “ \times ” and “ \div (by a number $\neq 0$)”.

3. Numbers



Are we happy now? No!

Why? Because if we draw all the rational numbers in a line, then the line has lots of holes in it. In fact, \mathbb{Q} has ∞ many holes in it.



So we invent new numbers!

3. Numbers



The set

$$\mathbb{R} = \{\text{all numbers which can be written as a decimal}\}$$

is called the set of *real numbers*. For example

$$0 = 0.0 \in \mathbb{R}$$

$$\frac{23}{99} = 0.232323\dots \in \mathbb{R}$$

$$\frac{3}{4} = 0.75 \in \mathbb{R}$$

$$\pi = 3.141592\dots \in \mathbb{R}$$

$$\frac{100}{13} = 7.692307\dots \in \mathbb{R}$$

$$\sqrt{2} = 1.414213\dots \in \mathbb{R}$$

$$\frac{123}{999} = 0.123123\dots \in \mathbb{R}$$

$$\frac{12345}{100000} = 0.12345 \in \mathbb{R}.$$

3. Numbers



The real numbers are complete – this means that if we draw all the real numbers in a line, then there are no holes in the line.



3. Numbers



Are we happy now?

3. Numbers



Are we happy now?

Yes!



Next Time

- 4. Intervals
- 5. Cartesian Coordinates
- 6. Functions
- 7. Sigma Notation