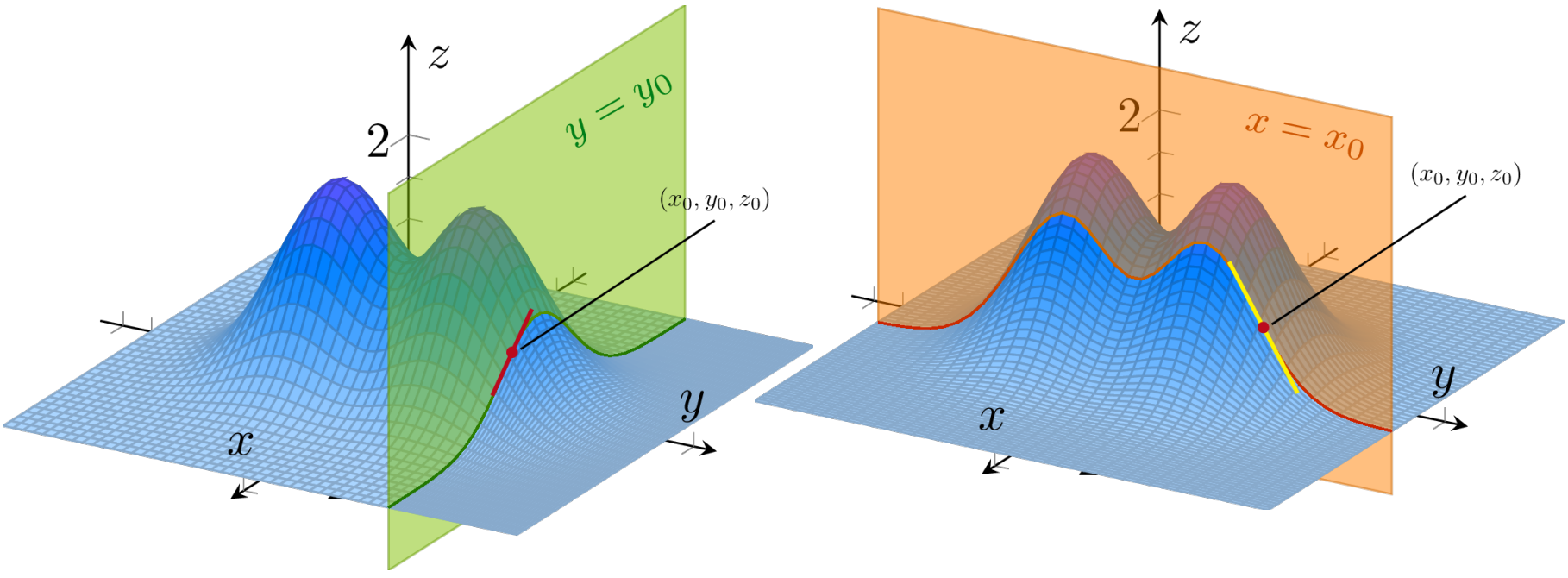
Lecture 6

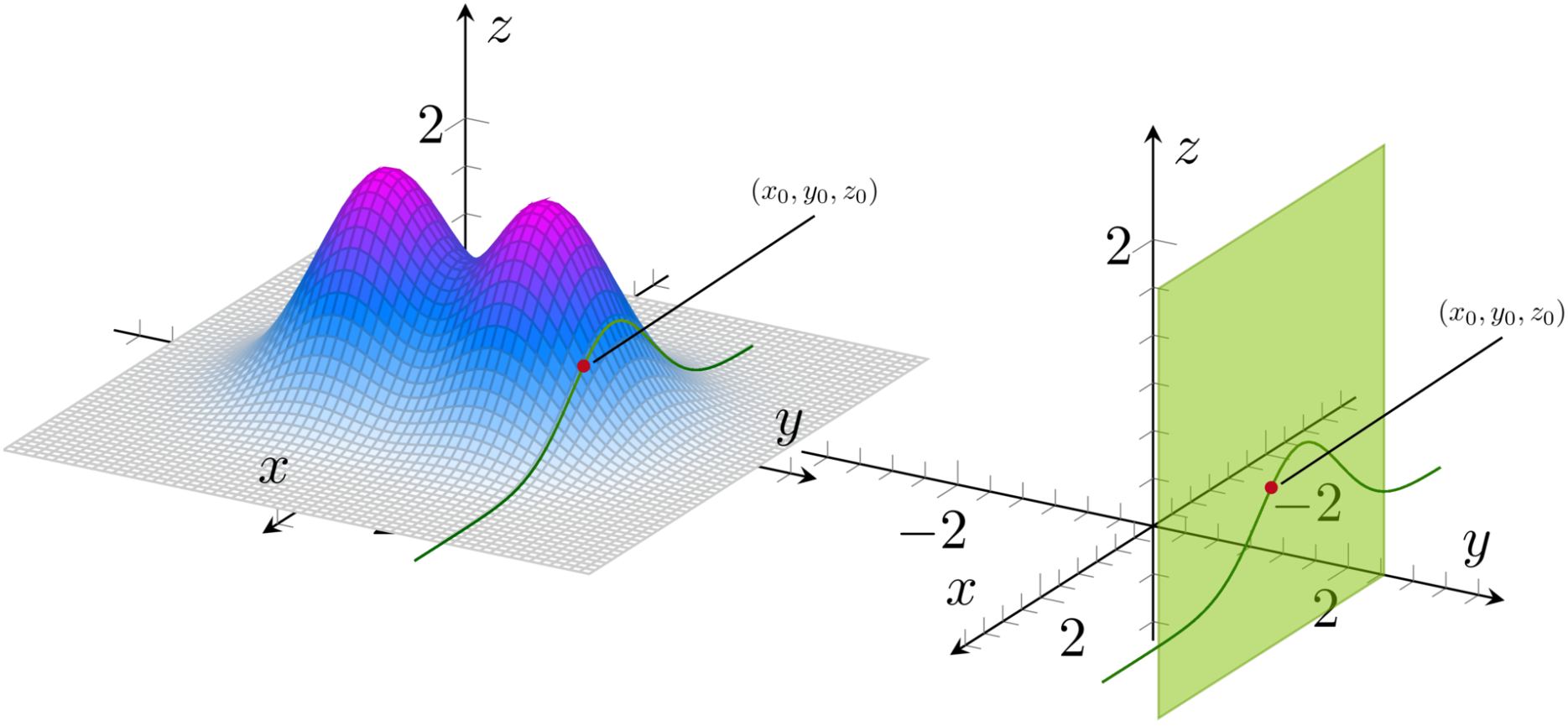
- 13.5 Directional Derivatives and Gradient Vectors
- 13.6 Tangent Planes and Differentials
- 13.7 Extreme Values and Saddle Points
- 13.8 Lagrange Multipliers

Directional Derivatives and Gradient Vectors

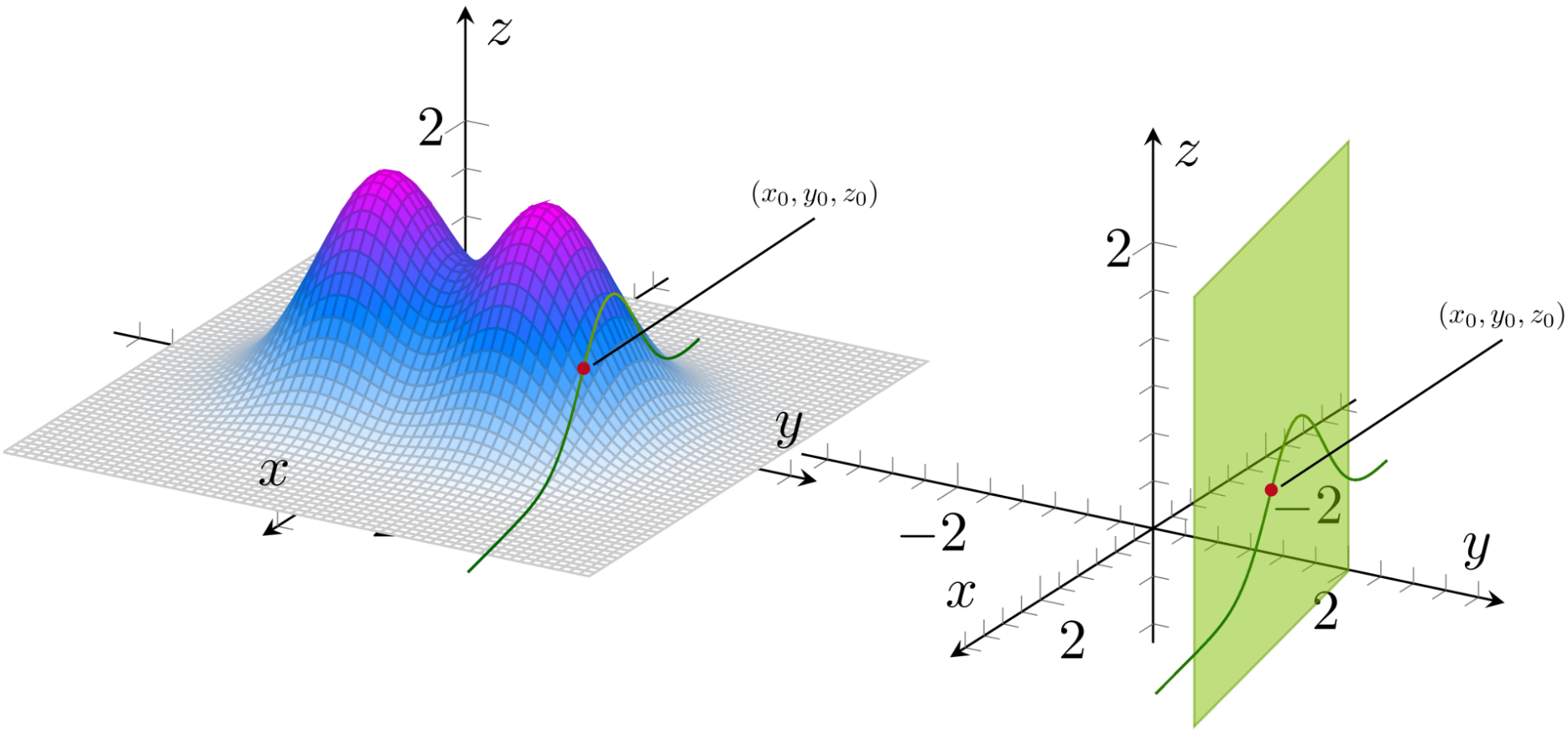
Partial Derivatives (revision)



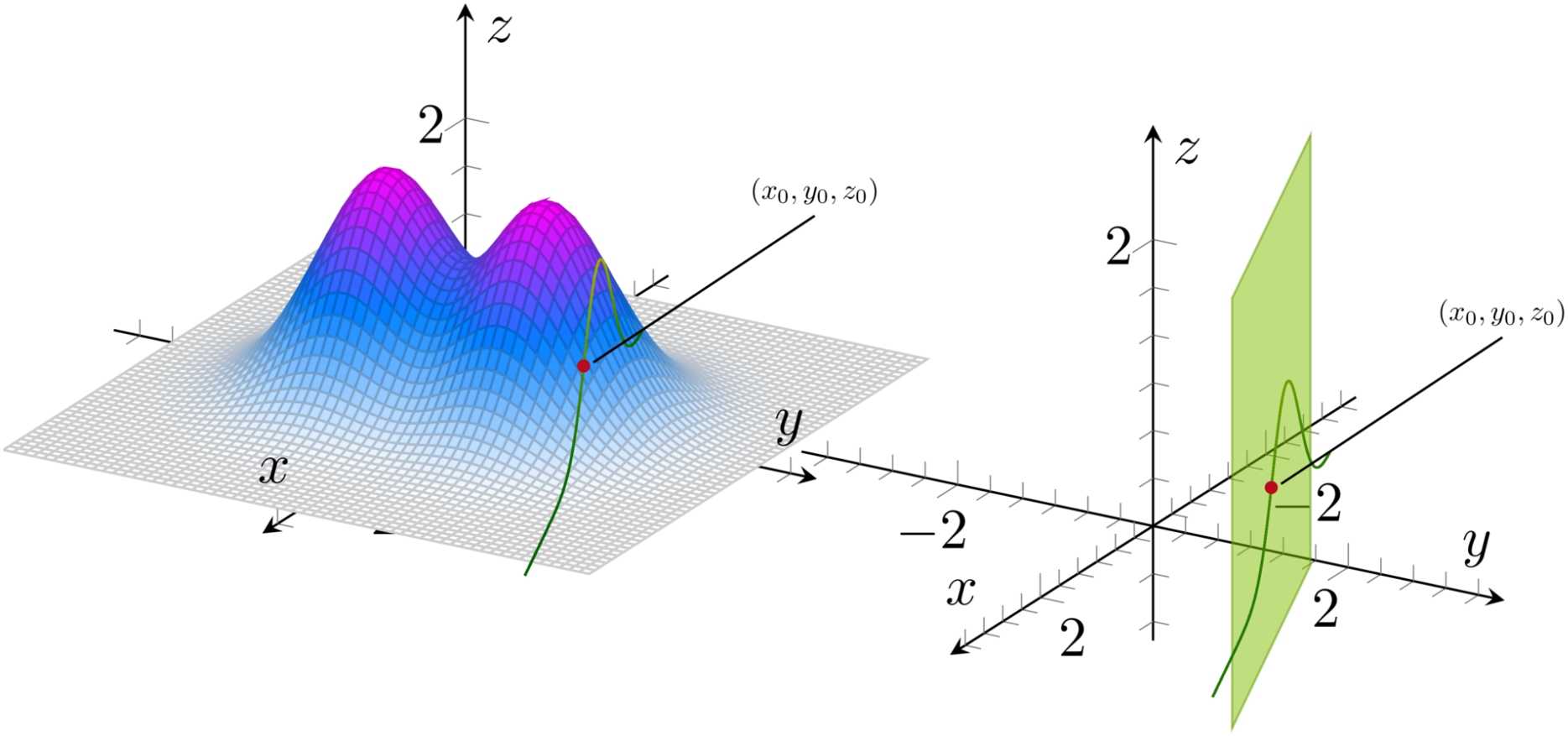
Directional Derivatives



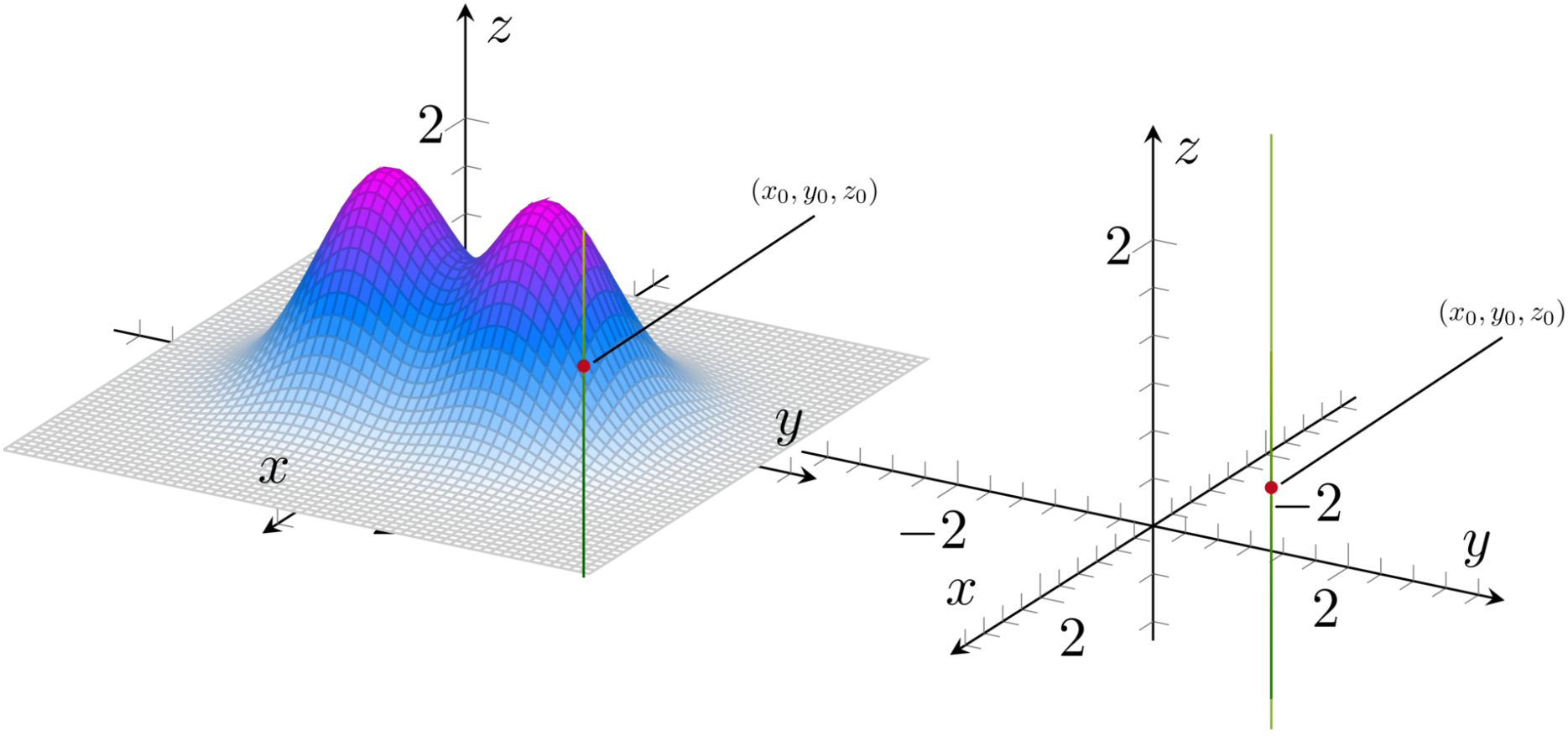
Directional Derivatives



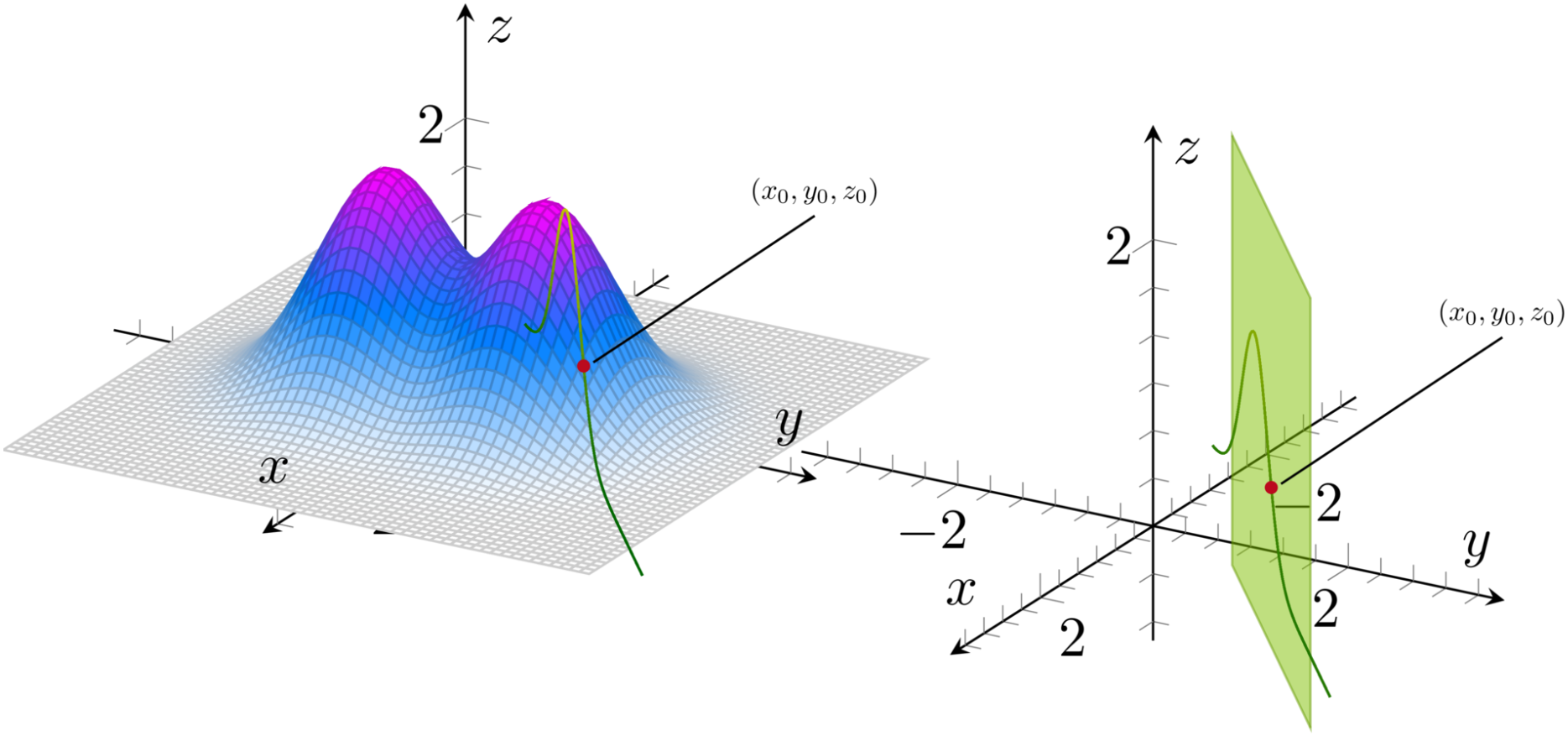
Directional Derivatives



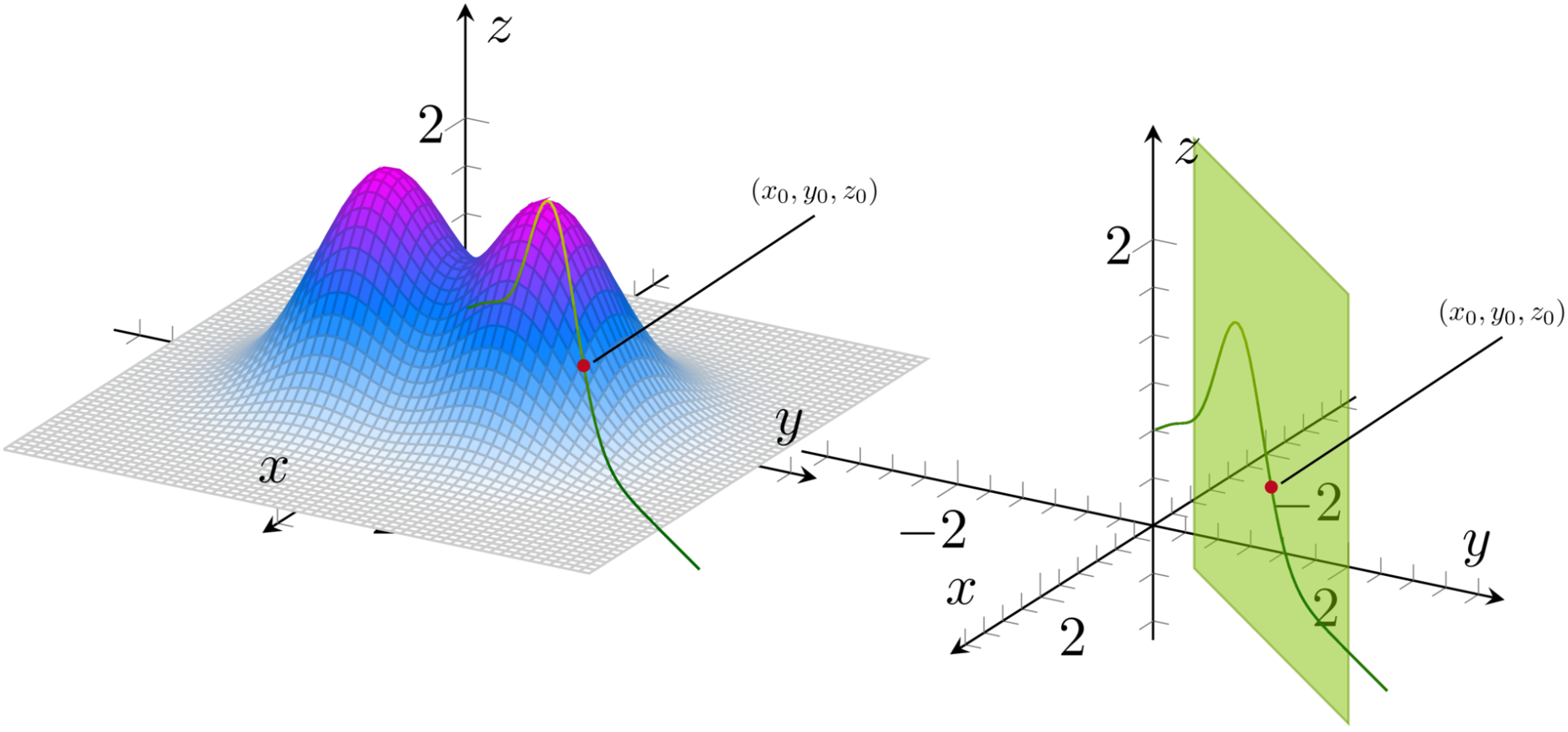
Directional Derivatives



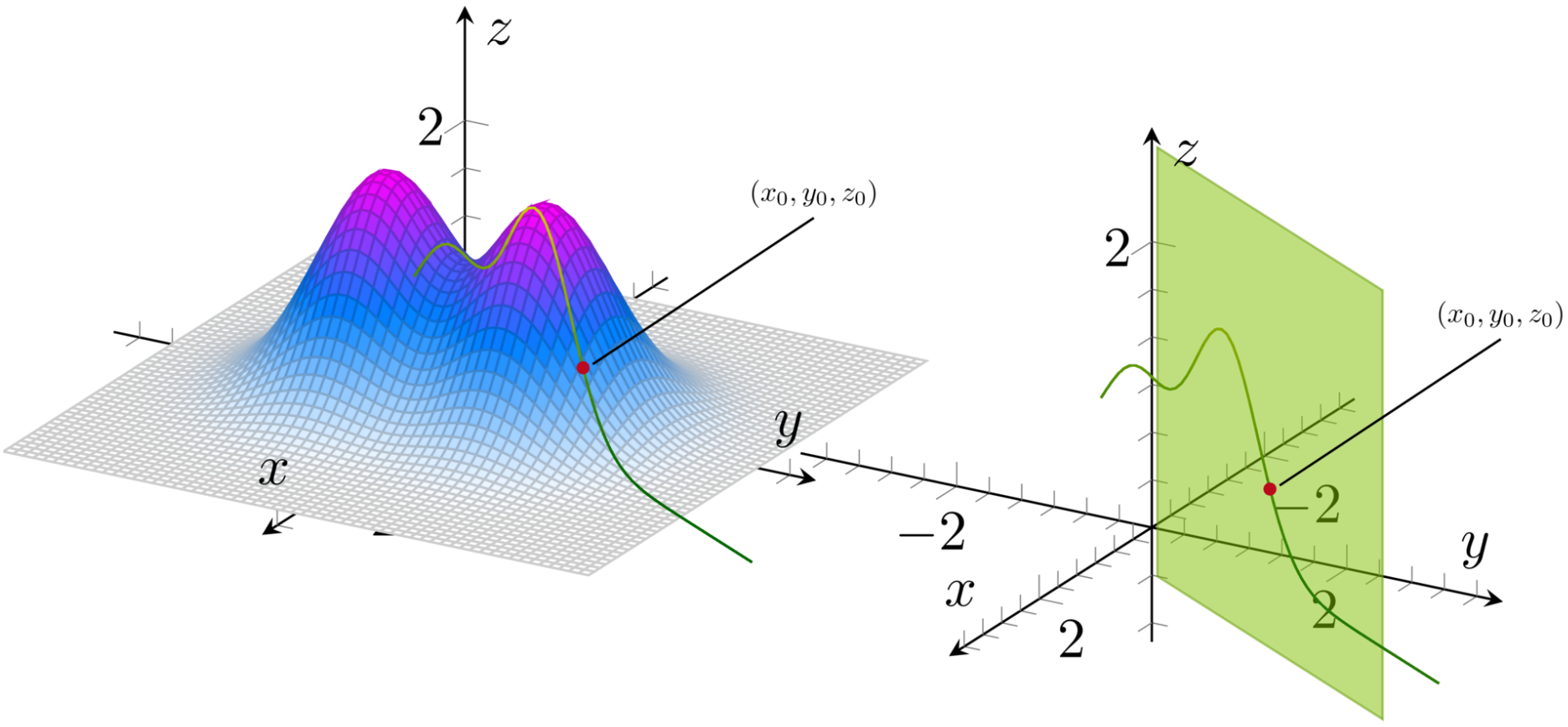
Directional Derivatives



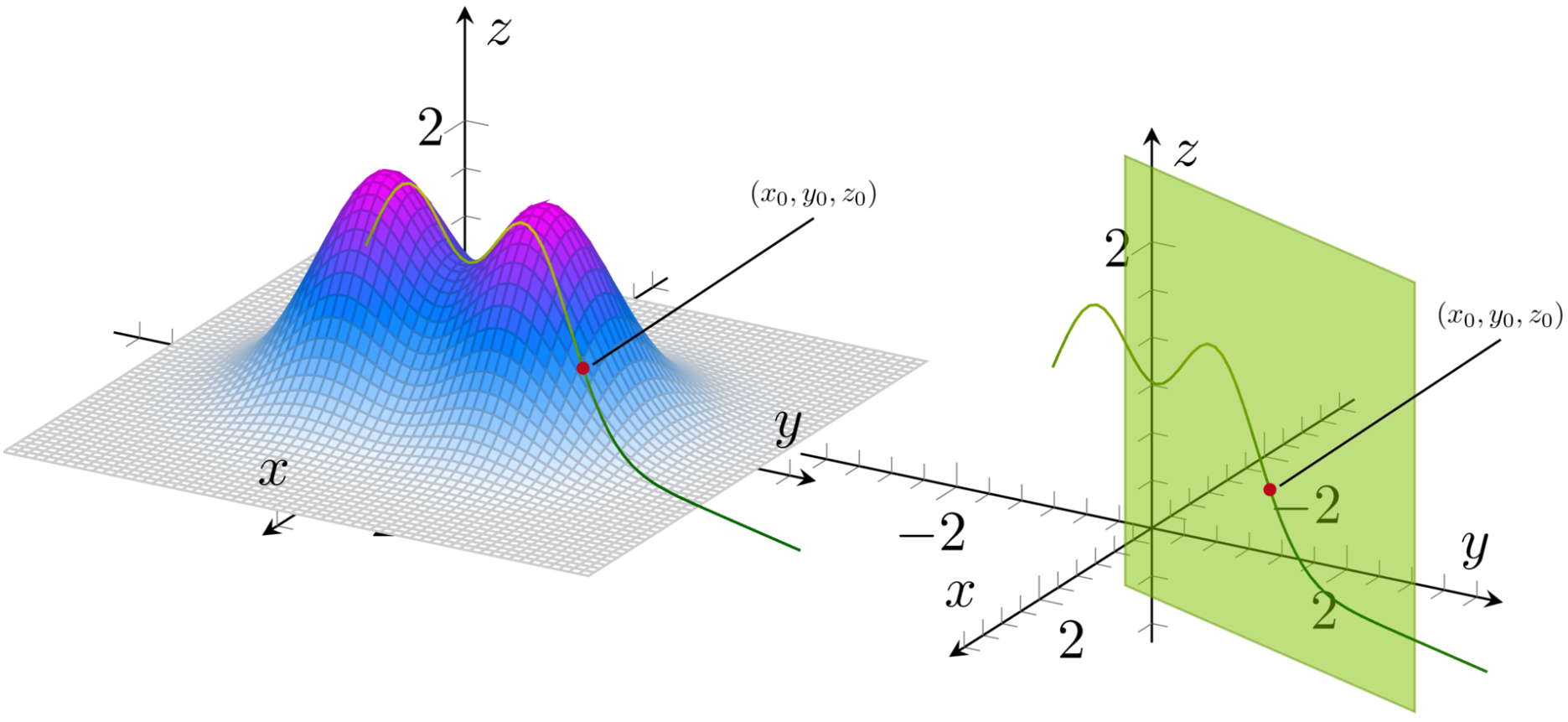
Directional Derivatives



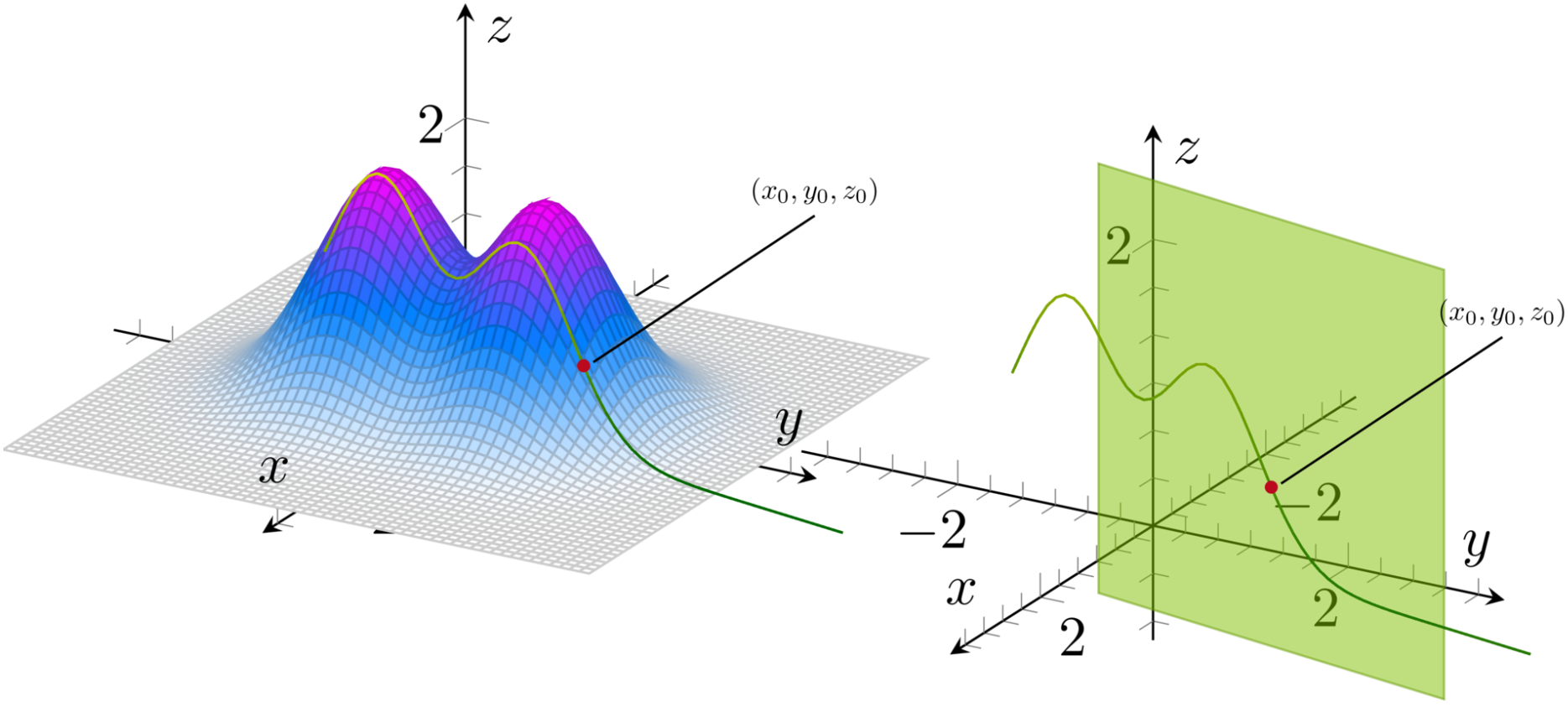
Directional Derivatives



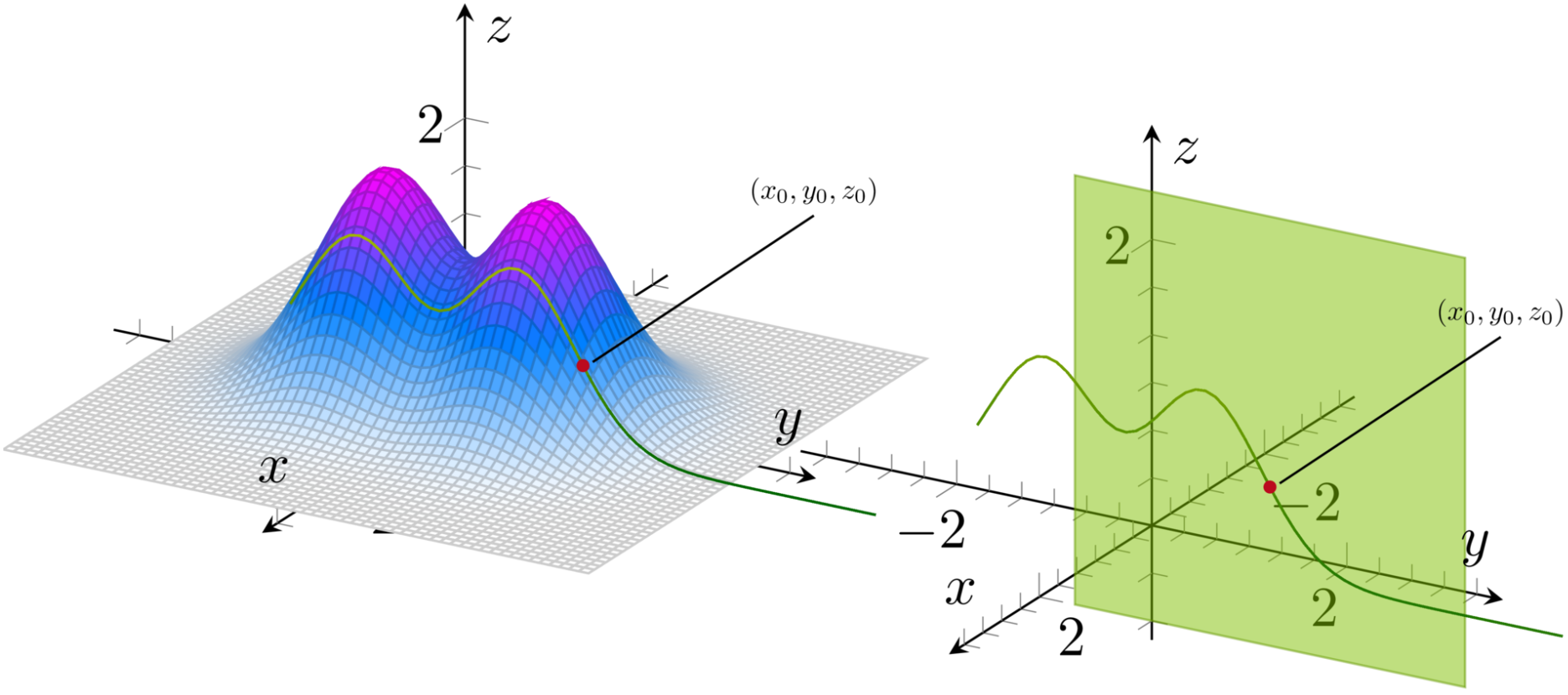
Directional Derivatives



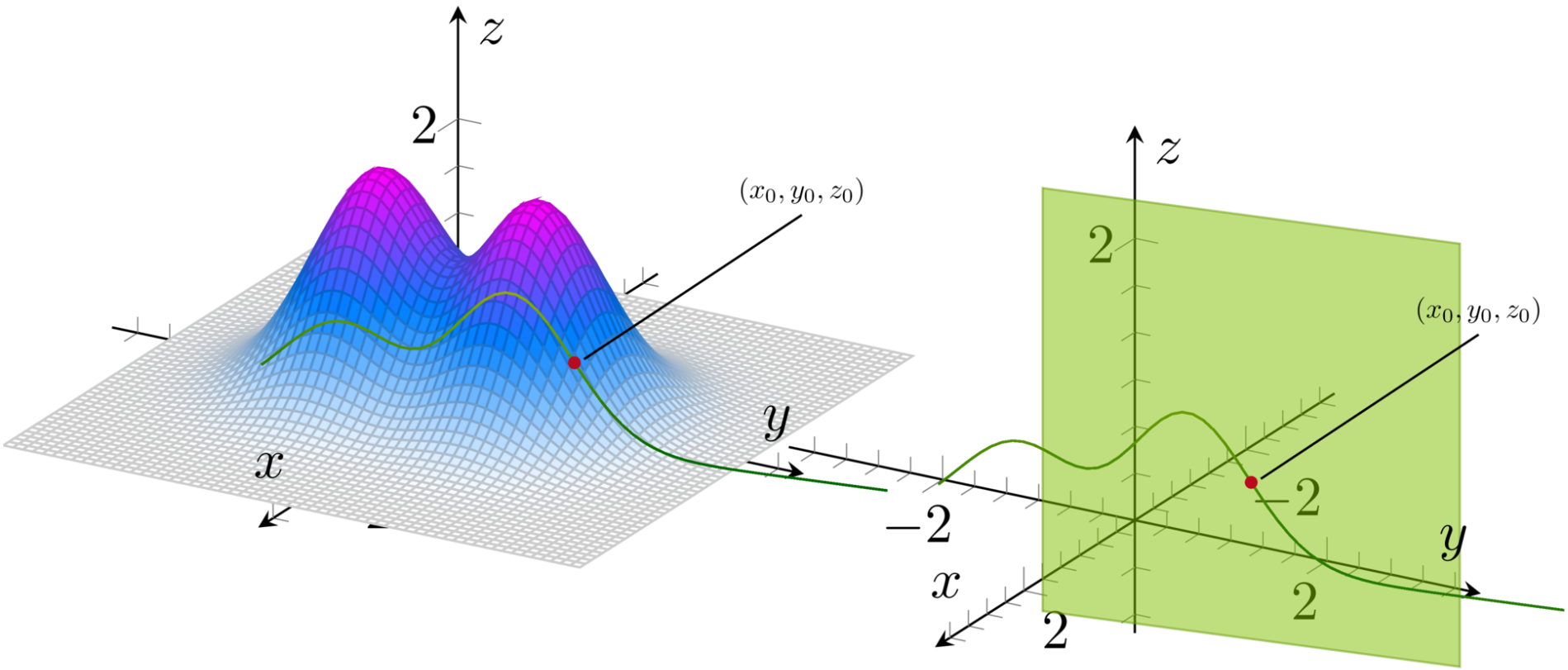
Directional Derivatives



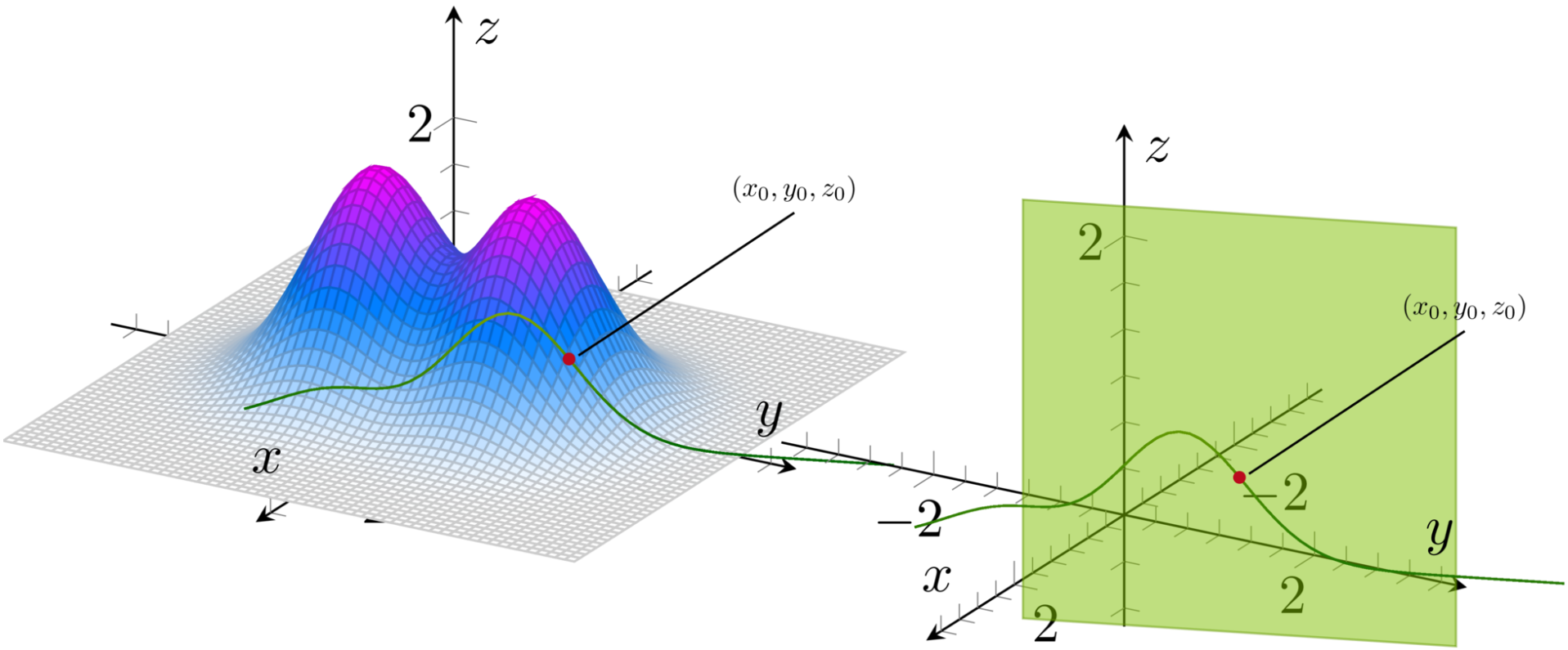
Directional Derivatives



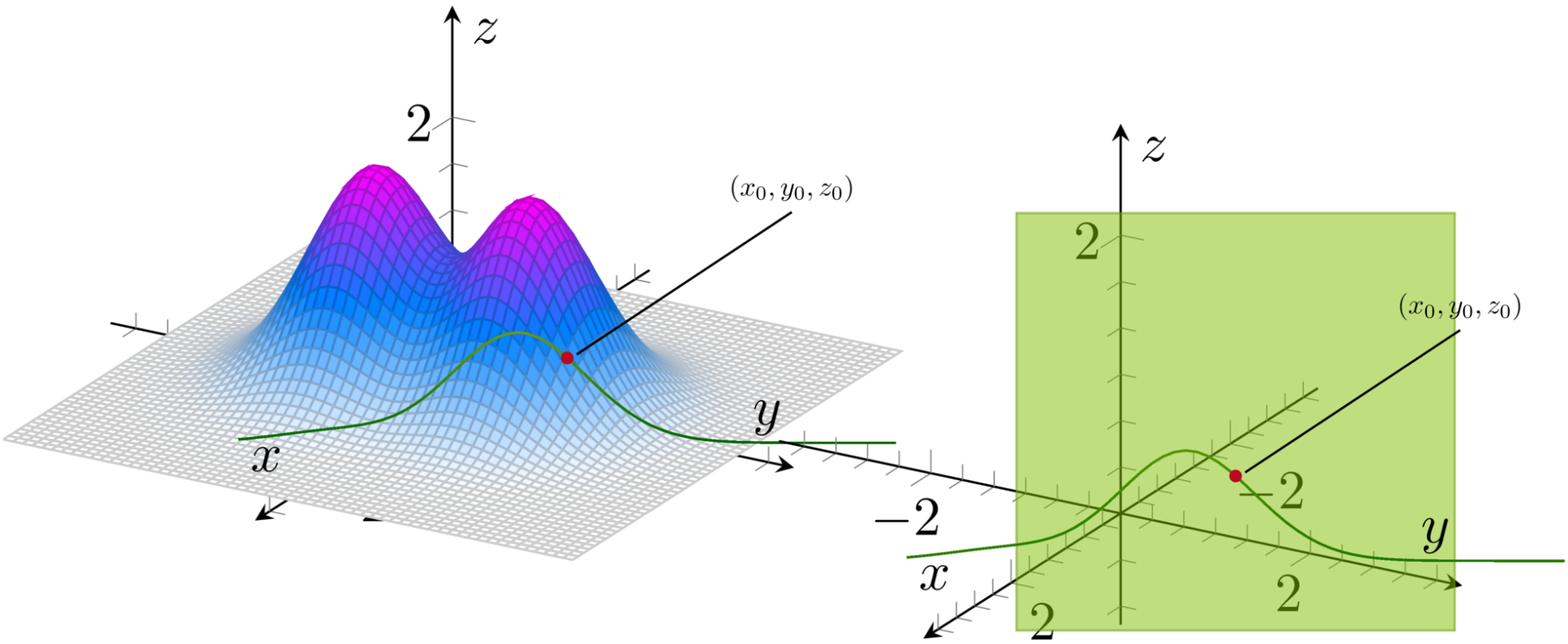
Directional Derivatives



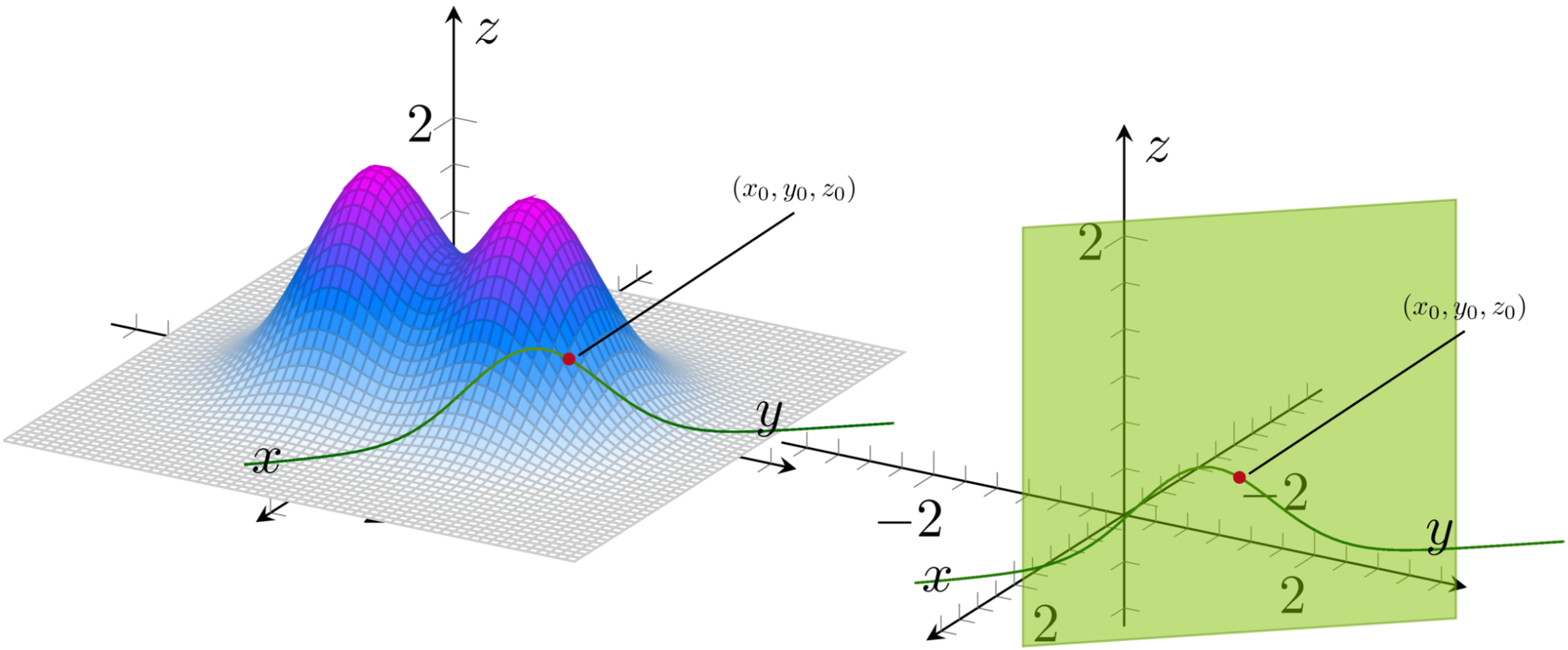
Directional Derivatives



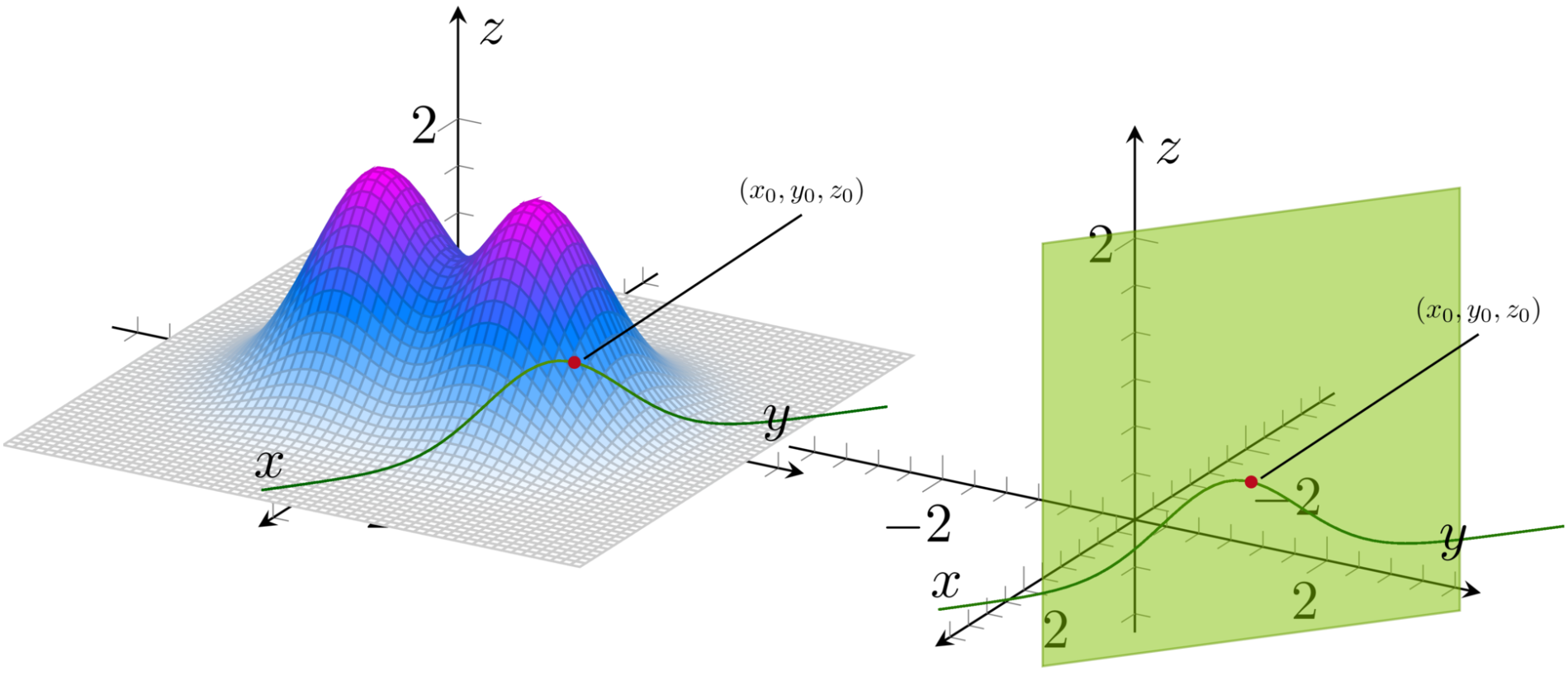
Directional Derivatives



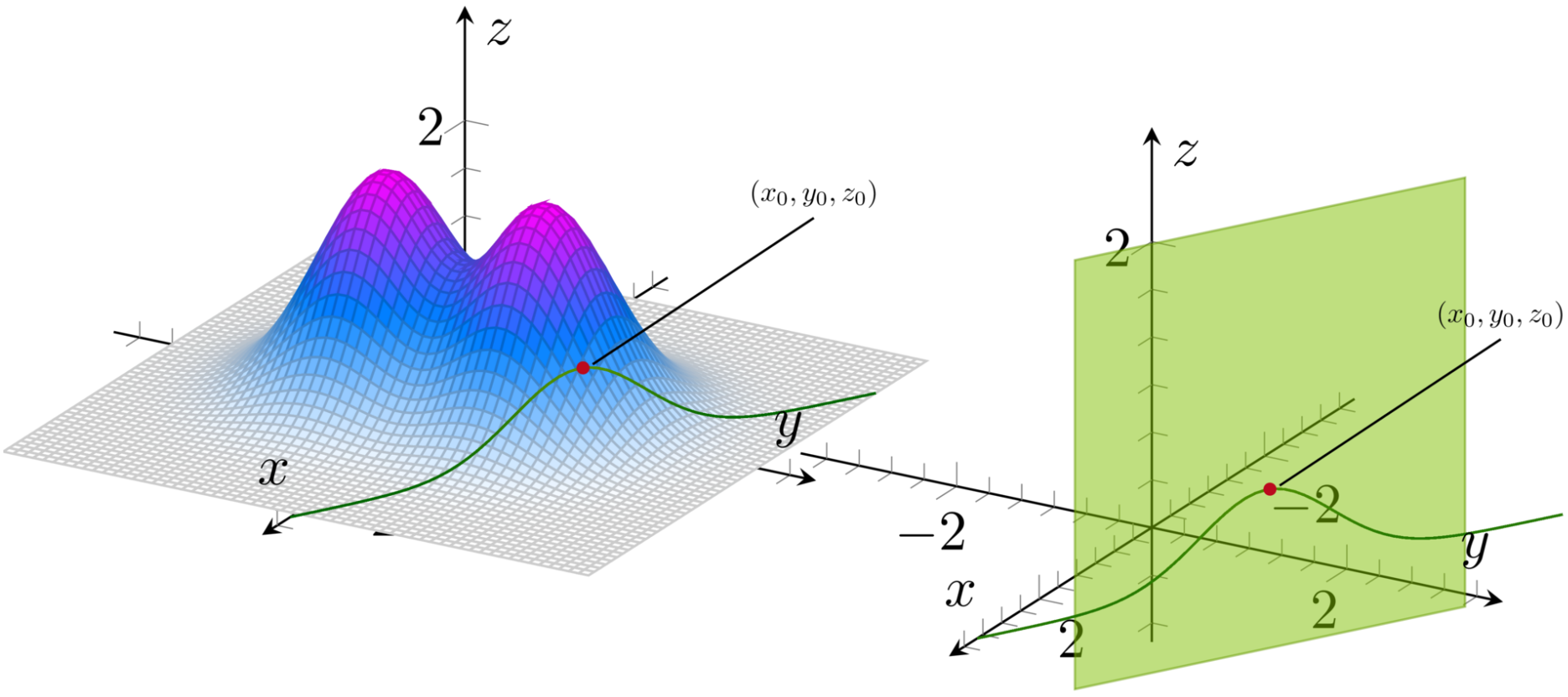
Directional Derivatives



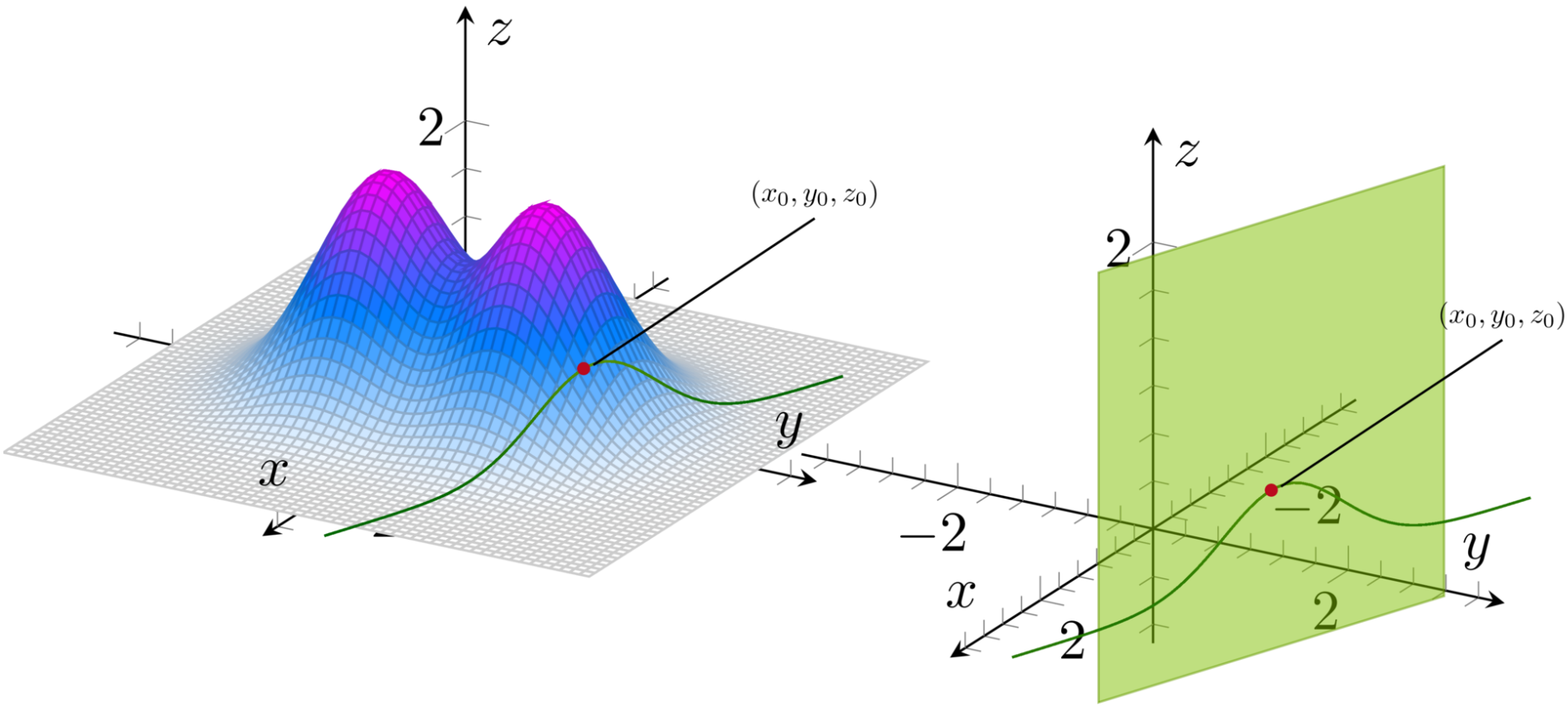
Directional Derivatives

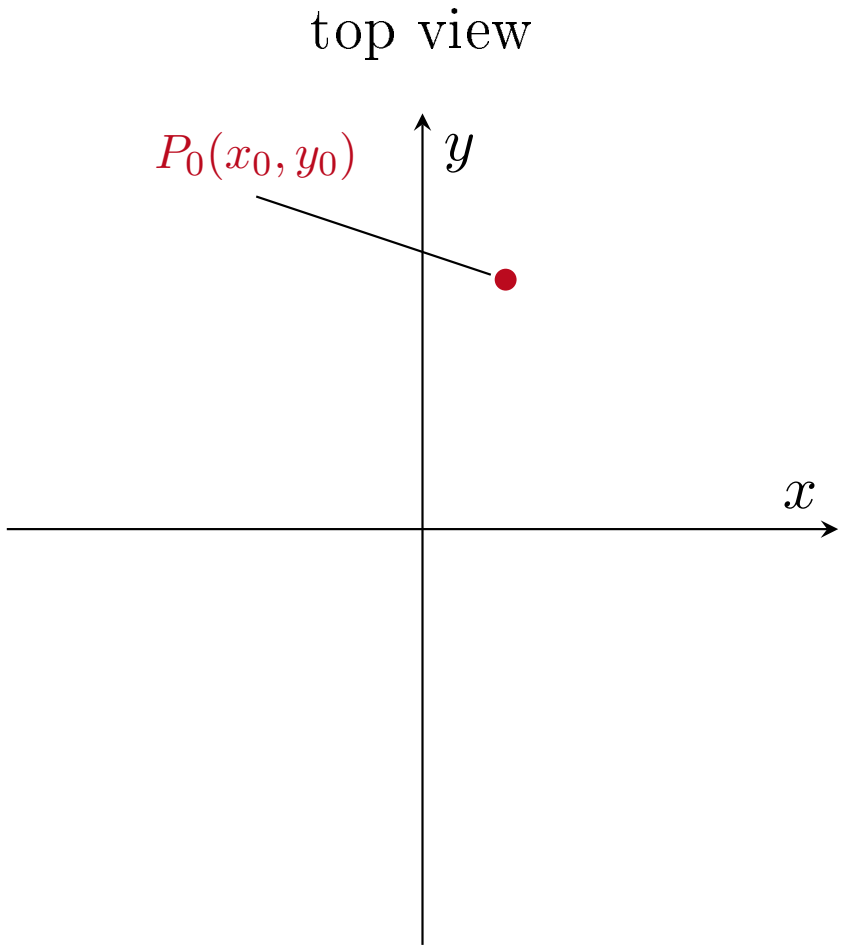


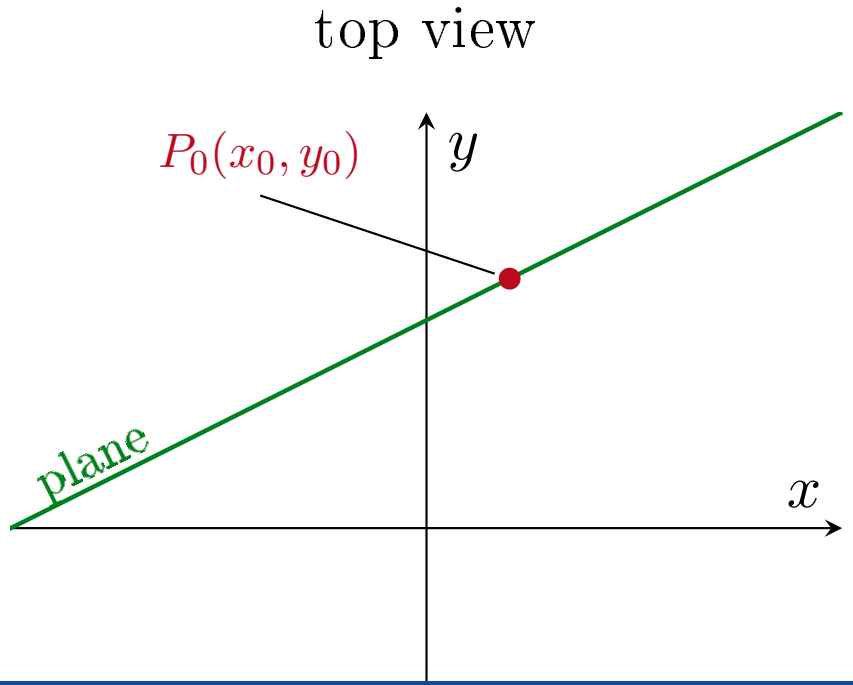
Directional Derivatives



Directional Derivatives

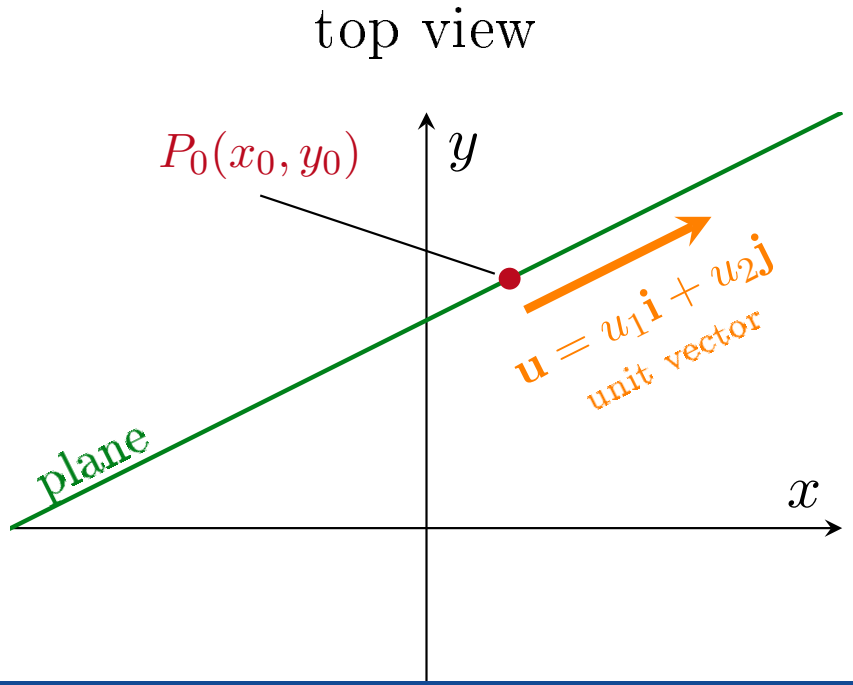



top view
$P_0(x_0, y_0)$
y
x



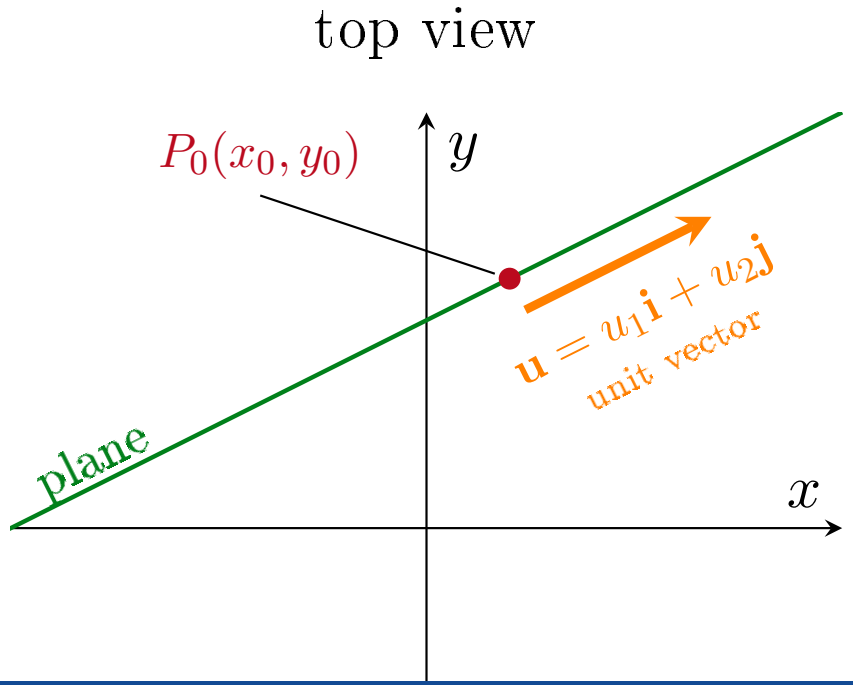
Definition

The *derivative of* f *at* $P_0(x_0, y_0)$



Definition

The *derivative of* f *at* $P_0(x_0, y_0)$ *in the direction of the unit vector* $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j}$



Definition

The *derivative of* f *at* $P_0(x_0, y_0)$ *in the direction of the unit vector* $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j}$ is

$$D_{\mathbf{u}} f(P_0) = \lim_{s \to 0} \frac{f(x_0 + su_1, y_0 + su_2) - f(x_0, y_0)}{s}$$

provided the limit exists.

$$D_{\mathbf{u}}f(P_0) = \left(\frac{df}{ds}\right)_{\mathbf{u},\ P_0}$$

EXAMPLE 1 Using the definition, find the derivative of

$$f(x, y) = x^2 + xy$$

at $P_0(1, 2)$ in the direction of the unit vector $\mathbf{u} = \left(1/\sqrt{2}\right)\mathbf{i} + \left(1/\sqrt{2}\right)\mathbf{j}$.

Solution Applying the definition in Equation (1), we obtain

$$\left(\frac{df}{ds}\right)_{\mathbf{u},P_0} = \lim_{s\to 0} \frac{f(x_0 + su_1, y_0 + su_2) - f(x_0, y_0)}{s} \qquad \text{Eq. (1)}$$

$$= \lim_{s\to 0} \frac{f\left(1 + s\cdot\frac{1}{\sqrt{2}}, 2 + s\cdot\frac{1}{\sqrt{2}}\right) - f(1, 2)}{s}$$

$$= \lim_{s\to 0} \frac{\left(1 + \frac{s}{\sqrt{2}}\right)^2 + \left(1 + \frac{s}{\sqrt{2}}\right)\left(2 + \frac{s}{\sqrt{2}}\right) - (1^2 + 1\cdot 2)}{s}$$

$$= \lim_{s\to 0} \frac{\left(1 + \frac{2s}{\sqrt{2}} + \frac{s^2}{2}\right) + \left(2 + \frac{3s}{\sqrt{2}} + \frac{s^2}{2}\right) - 3}{s}$$

$$= \lim_{s\to 0} \frac{\frac{5s}{\sqrt{2}} + s^2}{s} = \lim_{s\to 0}\left(\frac{5}{\sqrt{2}} + s\right) = \frac{5}{\sqrt{2}}.$$

The rate of change of $f(x, y) = x^2 + xy$ at $P_0(1, 2)$ in the direction $\mathbf{u}$ is $5/\sqrt{2}$.

Remark

But it is easier to calculate directional derivatives if we use gradient vectors.