2019–20 **MATH216 Mathematics IV – Exercise Sheet 10** N. Course

Exercise 34 (Non-Homogeneous Systems of Equations).
Use the Method of Undetermined Coefficients to solve the following systems of ODEs:

(a) $\mathbf{x}' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} e^t \\ t \end{bmatrix}$

(b) $\mathbf{x}' = \begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} e^t \\ \sqrt{3}e^{-t} \end{bmatrix}$

Use the Method of Diagonalisation (use the substitution $\mathbf{x} = T\mathbf{y}$) to solve the following systems of ODEs:

(c) $\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} e^{-2t} \\ -2e^t \end{bmatrix}$

(d) $\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} -\cos t \\ \sin t \end{bmatrix}$

Use the Method of Variation of Parameters ($\mathbf{x}(t) = \Psi(t) \int \Psi^{-1}(s)\mathbf{g}(s)\, ds$) to solve the following systems of ODEs:

(e) $\mathbf{x}' = \begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} t^{-1} \\ 2t^{-1} + 4 \end{bmatrix}, \quad t > 0$

(f) $\mathbf{x}' = \begin{bmatrix} 4 & -2 \\ 8 & -4 \end{bmatrix} \mathbf{x} + \begin{bmatrix} t^{-3} \\ -t^{-2} \end{bmatrix}, \quad t > 0$

Exercise 35 (The Laplace Transform). Use the Laplace Transform to solve the following IVPs:

(a) $\begin{cases} x' = x - 2y \\ y' = 5x - y \\ x(0) = -1 \\ y(0) = 2 \end{cases}$

(b) $\begin{cases} x' = -x + y \\ y' = 2x \\ x(0) = 0 \\ y(0) = 1 \end{cases}$

(c) $\begin{cases} 2x' + y' - 2x = 1 \\ x' + y' - 3x - 3y = 2 \\ x(0) = 0 \\ y(0) = 0 \end{cases}$

(d) $\begin{cases} 2x' + y' - y - t = 0 \\ x' + y' - t^2 = 0 \\ x(0) = 1 \\ y(0) = 0 \end{cases}$

[Hint: For (c) and (d), you must first rearrange the ODEs to the form $\begin{cases} x' = f_1(x, y) \\ y' = f_2(x, y) \end{cases}$.]