SON TESLİM TARİHİ: Salı 23 Şubat 2016 saat 16:00'e kadar.

Definition. A sequence (a_n) of real numbers *tends to infinity* ($a_n \to \infty$ as $n \to \infty$) iff $\forall\ A > 0$, $\exists\ N = N(A) \in \mathbb{N}$ such that

$$n > N \qquad \Longrightarrow \qquad a_n > A.$$

Egzersiz 4 (Examples of sequences which tend to infinity).

(a) [20p] Let $u_n = n! - n^2 \sin n$ for all $n \in \mathbb{N}$. *Use the definition* to show that $u_n \to \infty$ as $n \to \infty$.

(b) [20p] Let $v_n = \frac{n+7}{2+\sin n}$ for all $n \in \mathbb{N}$. *Use the definition* to show that $v_n \to \infty$ as $n \to \infty$.

(c) [20p] Let $w_n = n - 2\log\left(1 + \frac{1}{n}\right)$ for all $n \in \mathbb{N}$. *Use the definition* to show that $w_n \to \infty$ as $n \to \infty$.

Egzersiz 5 (Sequences tending to infinity). [40p] Suppose that

- $(a_n)_{n=1}^{\infty}$ is a sequence of real numbers;
- $a_n \to \infty$ as $n \to \infty$;
- $c > 0$ is a real number; and
- $b_n := a_n - c$ for all $n \in \mathbb{N}$.

Show that $b_n \to \infty$ as $n \to \infty$.

Ödev 1'in çözümleri

1. (a)

P	Q	R	$Q \vee R$	$(P \wedge (Q \vee R))$	$P \wedge Q$	$P \wedge R$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

(b) $(\exists \varepsilon > 0)(\forall N \in \mathbb{N})(\exists n \in \mathbb{N})((n > N) \wedge (|a_n| \geq \varepsilon))$.

2. Let P_n denote the proposition $1 + 2 + 3 + \ldots + n = \frac{1}{2}n(n+1)$.
First $1 = \frac{1(1+1)}{2}$ so P_1 is true. Next assume that P_k is true. Then $1 + 2 + 3 + 4 + 5 + \ldots + k = \frac{k(k+1)}{2}$. It follows that $1+2+3+4+5+\ldots+k+(k+1) = (1+2+3+4+5+\ldots+k)+(k+1) = \frac{k(k+1)}{2} + (k+1) = \frac{k(k+1)+2k+2}{2} = \frac{k^2+3k+2}{2} = \frac{(k+1)(k+2)}{2}$ and hence P_{k+1} is also true.
By the principle of mathematical induction, the proposition is true for all $n \in \mathbb{N}$.

3. Suppose that $x \in \mathbb{Q}$ and $y \in \mathbb{Q}$. Then we can write $x = \frac{a}{b}$ and $y = \frac{c}{d}$ for $a, b, c, d \in \mathbb{Z}$, $b \neq 0 \neq d$. But then $xy = \frac{ac}{bd}$ so $xy \in \mathbb{Q}$.