İSTANBUL OKAN ÜNİVERSİTESİ
MÜHENDİSLİK FAKÜLTESİ
MÜHENDİSLİK TEMEL BİLİMLERİ BÖLÜMÜ

2019–20 **MATH216 Mathematics IV – Exercise Sheet 6** N. Course

Exercise 26 (The Laplace Transform). Find the Laplace Transform of the following functions:

(a) $f(t) = e^{-2t}$

(b) $f(t) = 3t^2$

(c) $f(t) = \cos^2 2t$

(d) $f(t) = t\cos t + te^t$

(e) $f(t) = \frac{\sinh t}{t}$

(f) $f(t) = t^2 \cos 2t$

(g) $f(t) = \frac{e^{3t}-1}{t}$

(h) $f(t) = te^{-t}\sin^2 t$

(i) $f(t) = \begin{cases} 2 & 0 < t \leq 3 \\ 0 & t > 3 \end{cases}$

(j) $f(t) = \begin{cases} \sin 2t & \pi \leq t \leq 2\pi \\ 0 & t < \pi \text{ or } t > 2\pi \end{cases}$

Exercise 27 (The Inverse Laplace Transform). Find the inverse Laplace Transform of the following functions:

(a) $F(s) = \frac{1}{s-2}$

(b) $F(s) = \frac{1}{s} - \frac{2}{s^{5/2}}$

(c) $F(s) = \frac{3s+1}{s^2+4}$

(d) $F(s) = \frac{2e^{-3s}}{s}$

(e) $F(s) = \frac{1}{s(s-3)}$

(f) $F(s) = \frac{2s+1}{s(s^2+9)}$

(g) $F(s) = \frac{s^3}{(s-4)^4}$

(h) $F(s) = \frac{s^2-2s}{s^4+5s^2+4}$

(i) $F(s) = \frac{2s^3-s^2}{(4s^2-4s+5)^2}$

(j) $F(s) = \ln\left(1+\frac{1}{s^2}\right)$

(k) $F(s) = \arctan\left(\frac{3}{s+2}\right)$

(l) $F(s) = \frac{s}{(s^2+1)^3}$

(m) $F(s) = \frac{e^{-s}}{s+2}$

Hint for 27(j)-(k): Note that since $\mathcal{L}[tf(t)] = (-1)\frac{dF}{ds}$, we have that $-\mathcal{L}^{-1}\left[\frac{dF}{ds}\right] = tf(t)$ and thus $f(t) = -\frac{1}{t}\mathcal{L}^{-1}\left[\frac{dF}{ds}\right]$.

$f(t)$	$F(s) = \mathcal{L}[f](s)$	
1	$\frac{1}{s}$	$s > 0$
e^{at}	$\frac{1}{s-a}$	$s > a$
$t^n \quad (n \in \mathbb{N})$	$\frac{n!}{s^{n+1}}$	$s > 0$
$\sin at$	$\frac{a}{s^2+a^2}$	$s > 0$
$\cos at$	$\frac{s}{s^2+a^2}$	$s > 0$
$\sinh at$	$\frac{a}{s^2-a^2}$	$s > \lvert a \rvert$
$\cosh at$	$\frac{s}{s^2-a^2}$	$s > \lvert a \rvert$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$	$s > a$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$	$s > a$
$t^n e^{at} \quad (n \in \mathbb{N})$	$\frac{n!}{(s-a)^{n+1}}$	$s > a$
$u_c(t)$	$\frac{e^{-cs}}{s}$	$s > 0$
$u_c(t)f(t-c)$	$e^{-cs}F(s)$	
$e^{ct}f(t)$	$F(s-c)$	
$f(ct) \quad (c > 0)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$	
$\int_0^t f(t-\tau)g(\tau)d\tau$	$F(s)G(s)$	
$t^n f(t)$	$(-1)^n F^{(n)}(s)$	

$\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$

$\mathcal{L}[f''](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$

$\mathcal{L}[f^{(n)}](s) = s^n\mathcal{L}[f](s) - s^{n-1}f(0) - \ldots - sf^{(n-2)}(0) - f^{(n-1)}(0)$