2015–16 | **MAT234 Matematik IV – Ödev 4** | N. Course

SON TESLİM TARİHİ: Salı 15 Mart 2016 saat 16:00'e kadar.

NEW RULE: Poor quality photos of answers sent by email will no longer be accepted.

I prefer to receive your answers in hard copy. If you must email your answers, then you must either (1) prepare them with LaTeX; (2) use a word processor; (3) write them on paper, then use a proper flatbed scanner to scan them; or (4) write them on paper, then use a "scanner" app on your mobile phone to scan them. Make sure your name and student number are clearly visible on every page that you email.

Egzersiz 8 (Bounded Sequences).

(a) [20p] Let $z_n = \left(\frac{2n}{1+n}\right)\cos(n\pi)$ for all $n \in \mathbb{N}$. Show that (z_n) is a *bounded* sequence.

(b) [20p] The sequence (g_n) is defined by $g_1 = \cos 1$ and $g_{n+1} := \max\{g_n, \cos(n+1)\}$. For example, since $\cos 2 \approx -0.41614 < 0.54030 \approx \cos 1 = g_1$, we have $g_2 = \max\{g_1, \cos 2\} = g_1$.

Is (g_n) a convergent sequence, or a divergent sequence? Prove your answer.
[You do not need to calculate $\lim_{n\to\infty} g_n$ if you think that it exists.]

Egzersiz 9 (Limits of Sequences). [6×10p] Determine whether each of the following sequences has a limit or does not have a limit. If the limit exists, then find it. If the limit does not exist, then prove that it does not exist. The first one is done for you.

(ω) $\omega_n = \frac{n!+8^n}{7^n+n!}$

Solution: Since $\frac{a^n}{n!} \to 0$ as $n \to \infty$ for any $a \in \mathbb{R}$, it follows that $\omega_n = \frac{n!+8^n}{7^n+n!} = \frac{1+\frac{8^n}{n!}}{\frac{7^n}{n!}+1} \to \frac{1+0}{0+1} = 1$ as $n \to \infty$ by a theorem from the course.

(a) $a_n = \cos\left(\frac{n}{2^n}\right)$

(b) $b_n = \frac{n+(-1)^n\sqrt{n}}{(n^2+1)^{1/2}}$

(c) $c_n = \sqrt[n]{n^2}$

(d) $d_n = \frac{1}{n}\int_1^n \frac{1}{x}dx$

(e) $e_n = \frac{n^2-n^3\cos n+2}{4n^3+n^2-4\sin n}$

(f) $f_n = ((-1)^n+1)\left(\frac{n+1}{n}\right)$

You must prove your answers. If you use a result from the course, then you must write which result you are using; e.g if you use the Sandwich Rule then write "...by the Sandwich Rule". Moreover, make sure that you use "$\Longrightarrow$", "=" and "$\to$" correctly.

Ödev 3'ün çözümleri

6. I "forgot" to prove that P_1 is true. In fact, P_1 is false.

7. (a) $1, 0, 0, \frac{1}{16}, 0, 0, 0, 0, \frac{1}{81}, 0$. (b) Omitted.
(c) Let $\varepsilon > 0$ (this should always be the first sentence for questions like this!!! -5 points if you forget to write this.). Choose $N \geq \frac{1}{\sqrt{\varepsilon}}$. Then for all $n > N$, $|y_n| \leq \frac{1}{n^2} < \frac{1}{N^2} \leq \varepsilon$. Therefore $y_n \to 0$ as $n \to \infty$.
(d) Let $\varepsilon > 0$. Note first that $z_n - 3 = \frac{3n+1}{n+2} - \frac{3n+6}{n+2} = -\frac{5}{n+2}$, so $|z_n - 3| = \frac{5}{n+2} < \frac{5}{n}$. Choose $N \geq \frac{5}{\varepsilon}$. Then for all $n > N$, $|z_n - 3| < \frac{5}{n} < \frac{5}{N} \leq \varepsilon$. Therefore $z_n \to 3$ as $n \to \infty$.