



Welcome to **Mathematics I** with Dr Neil Course

Information about this course

- ≈ 12 classes. Friday afternoons 2pm-4:30pm.

14:00

15:00

16:00

16:30

Information about this course

- \approx 12 classes. Friday afternoons 2pm-4:30pm.
- 2 lectures with a break between.

lecture

lecture

14:00

15:00

16:00

16:30

Information about this course

- \approx 12 classes. Friday afternoons 2pm-4:30pm.
- 2 lectures with a break between.
- Then I will answer your questions.

lecture

lecture

questions

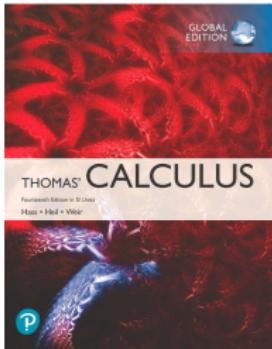
14:00

15:00

16:00

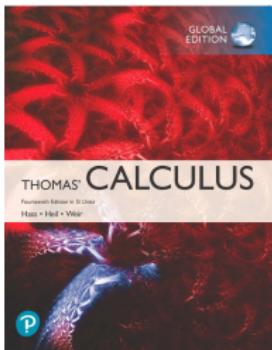
16:30

The Book



Joel R. Hass, Christopher E. Heil and Maurice D. Weir,
Thomas' Calculus,
14th Edition in SI Units, Pearson.

The Book



Joel R. Hass, Christopher E. Heil and Maurice D. Weir,
Thomas' Calculus,
14th Edition in SI Units, Pearson.

This is a required purchase.
You need to have this book to be
able to do the homework.



Your Mathematics courses:

MATH113

MATH114

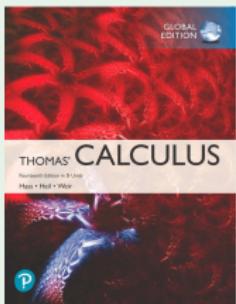
MATH215

MATH216

Your Mathematics courses:

MATH113

MATH114



MATH215

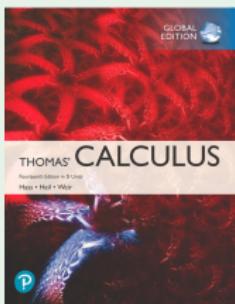
MATH216

Calculus

Your Mathematics courses:

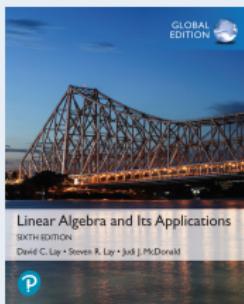
MATH113

MATH114



Calculus

MATH215



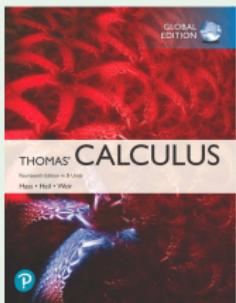
Linear
Algebra

MATH216

Your Mathematics courses:

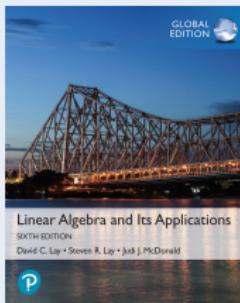
MATH113

MATH114



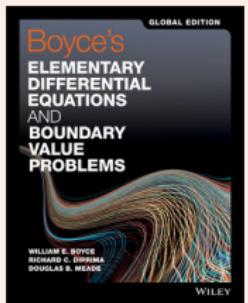
Calculus

MATH215



Linear Algebra

MATH216



Differential Equations

1

2

3

4

5

6

7

1. Functions

2

3

4

5

6

7

1. Functions

2. Limits and Continuity

3

4

5

6

7

1. Functions

2. Limits and Continuity

3. Derivatives

4

5

6

7

1. Functions

2. Limits and Continuity

3. Derivatives

4. Applications of Derivatives

5

6

7

1. Functions

2. Limits and Continuity

3. Derivatives

4. Applications of Derivatives

5. Integrals

6

7

1. Functions

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6. Applications of Integrals

1. Functions

2. Limits and Continuity

3. Derivatives

4. Applications of Derivatives

5. Integrals

6. Applications of Integrals

7. Trancendental Functions

1. Functions

1 week

2. Limits and Continuity

2 weeks

3. Derivatives

4 weeks

4. Applications of Derivatives

5. Integrals

3 weeks

6. Applications of Integrals

7. Trancendental Functions

2 weeks



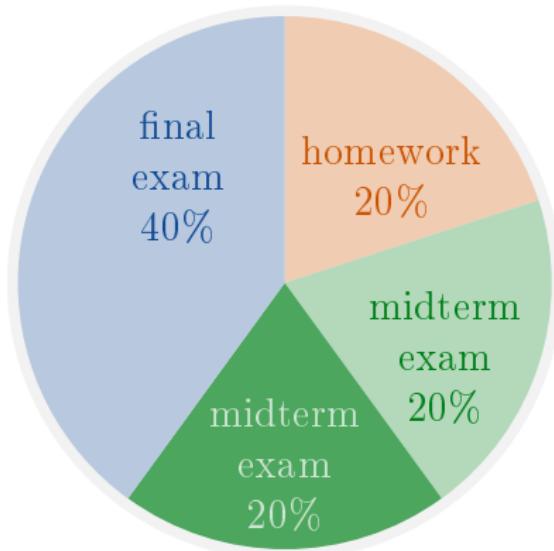
Exams and homework

(This information may change based on the University's decisions)



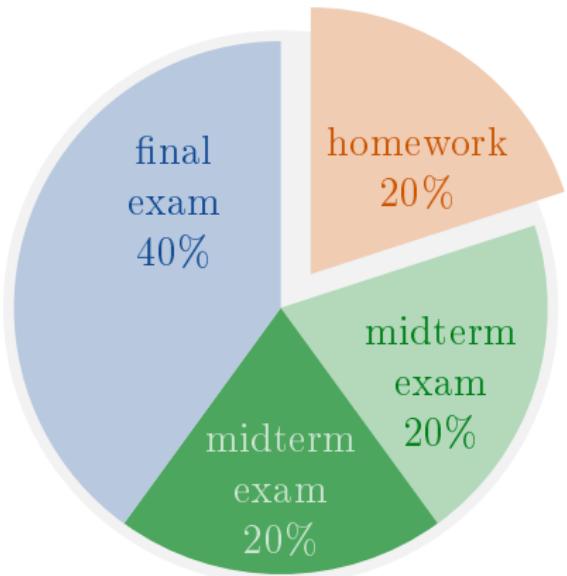
Exams and homework

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Exams and homework

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using Pearson
MyLab Math

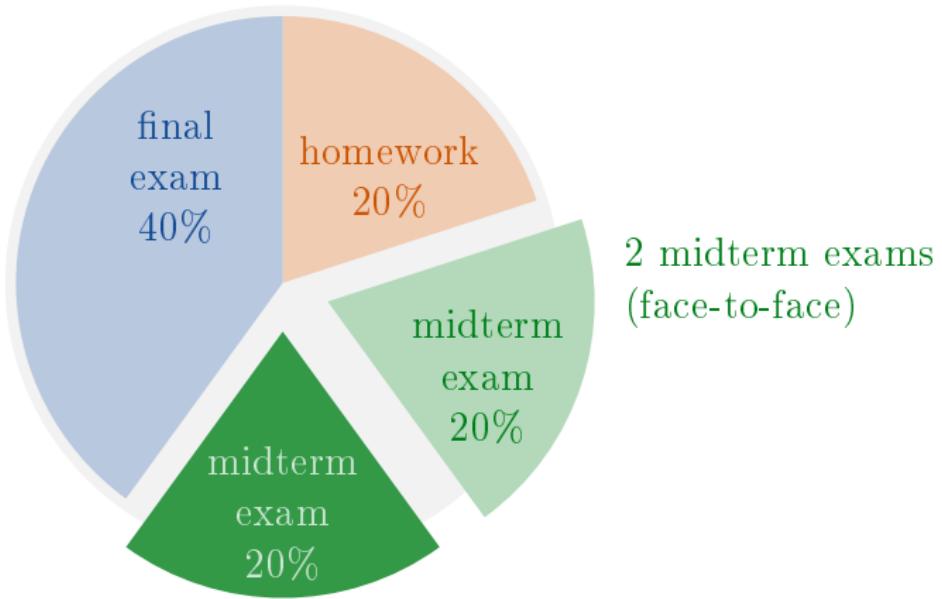
one piece of
homework for
each lesson

deadline = end of
term

more details in
O'Learn

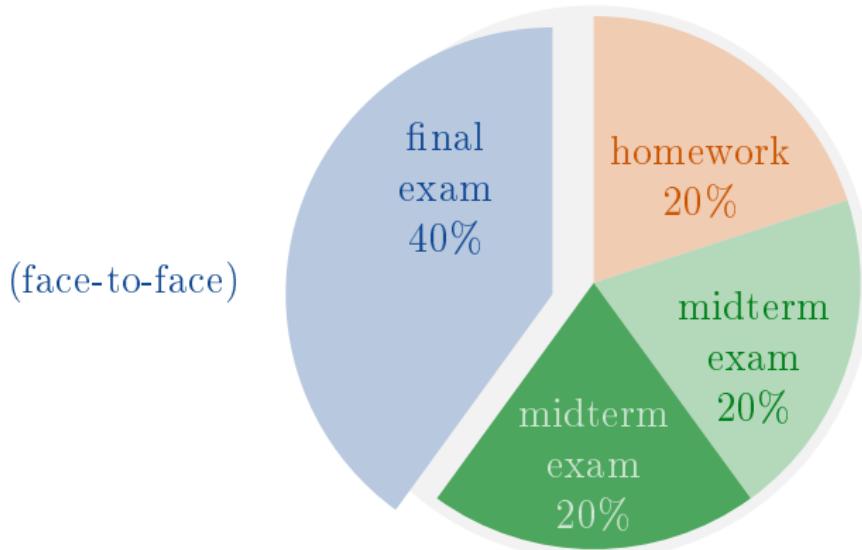
Exams and homework

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Exams and homework

(This information may change based on the University's decisions)



Expectations

If this was a classroom course, the expectation is that for each hour of lectures, you would study an extra 1-2 hours outside of class.

classroom
course

lectures (5 hours)

other study (5-10 hours)

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For an online course, you are still expected to study a total of 10-15 hours each week.

online
course

class
(2.5 hours)

other study (7.5-12.5 hours)

This may include:

- Do the online homework on MyLab;

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- Rewatch the recorded lectures (O'Learn & YouTube);

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- Solve the exercises in the textbook;
- Use the O'Learn Discussion Board;

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- Solve the exercises in the textbook;
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- Read other books?;

This may include:

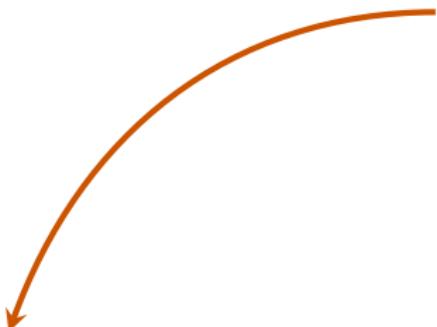
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- Use the O'Learn Discussion Board;
- Read other books?;
- Watch online videos;

⋮

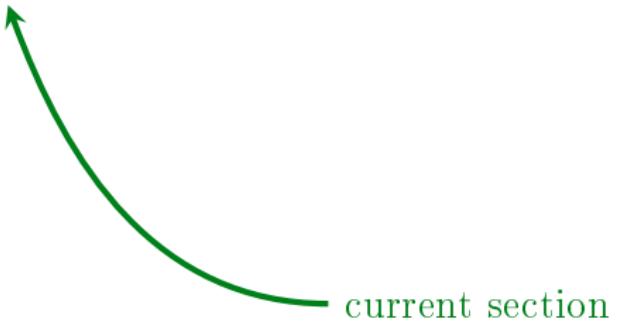
99.9 Section Title



slide number



99.9 Section Title



Lecture 1

- Information about this course
- A.1 Real Numbers and the Real Line
- 1.1 Functions and Their Graphs
- 1.2 Combining Functions; Shifting and Scaling Graphs
- 1.3 Trigonometric Functions

The Natural Numbers

The set

$$\mathbb{N} = \{1, 2, 3, 4, 5, 6, \dots\}$$

is called the set of *natural numbers*. These are the first numbers that children learn. For example

$2 \in \mathbb{N}$ means “2 is a natural number”

$7 \in \mathbb{N}$ means “7 is a natural number”

$\frac{1}{2} \notin \mathbb{N}$ means “ $\frac{1}{2}$ is **not** a natural number”

$0 \notin \mathbb{N}$ means “0 is **not** a natural number”

$-5 \notin \mathbb{N}$ means “−5 is **not** a natural number”

A.1 Real Numbers and the Real Line



In the natural numbers, we can do “+” and “×”

$$2 + 7 = 9 \in \mathbb{N}, \quad 2 \times 7 = 14 \in \mathbb{N}.$$

However we can not do “−” because

$$2 - 7 \notin \mathbb{N}.$$

A.1 Real Numbers and the Real Line



In the natural numbers, we can do “+” and “×”

$$2 + 7 = 9 \in \mathbb{N}, \quad 2 \times 7 = 14 \in \mathbb{N}.$$

However we can not do “−” because

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So we invent new numbers!



The Integers

The set

$$\mathbb{Z} = \{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$$

is called the set of *integers*. We use a \mathbb{Z} for the German word ‘zahlen’ (numbers).

A.1 Real Numbers and the Real Line



The Integers

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$$\mathbb{Z} = \{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$$

is called the set of *integers*. We use a \mathbb{Z} for the German word ‘zahlen’ (numbers). In \mathbb{Z} , we can do “+”, “-” and “ \times ” but we can not do “ \div ”. For example $3 \in \mathbb{Z}$, $4 \in \mathbb{Z}$, $-5 \in \mathbb{Z}$ and

$$3 + 4 \in \mathbb{Z}, \quad 3 - 4 \in \mathbb{Z}, \quad 3 \times 4 \in \mathbb{Z}, \quad 3 \div 4 \notin \mathbb{Z},$$

$$3 + (-5) \in \mathbb{Z}, \quad 3 - (-5) \in \mathbb{Z}, \quad 3 \times (-5) \in \mathbb{Z}, \quad 3 \div (-5) \notin \mathbb{Z}.$$

A.1 Real Numbers and the Real Line



The Integers

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$$3 + (-5) \in \mathbb{Z}, \quad 3 - (-5) \in \mathbb{Z}, \quad 3 \times (-5) \in \mathbb{Z}, \quad 3 \div (-5) \notin \mathbb{Z}.$$

So we invent new numbers!

The Rational Numbers

The set

$$\mathbb{Q} = \{\text{all fractions}\} = \left\{ \frac{a}{b} \mid a, b \in \mathbb{Z} \text{ and } b \neq 0 \right\}$$

is called the set of *rational numbers*. We use a \mathbb{Q} for the word ‘quotient’.

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$$0 = \frac{0}{1} \in \mathbb{Q}$$

$$\frac{100}{13} \in \mathbb{Q}$$

$$1 = \frac{1}{1} \in \mathbb{Q}$$

$$\sqrt{2} \notin \mathbb{Q}$$

$$\frac{3}{4} \in \mathbb{Q}$$

$$-4 = \frac{8}{-2} \in \mathbb{Q}$$

$$\pi \notin \mathbb{Q}$$

$$0.12345 = \frac{12345}{100000} \in \mathbb{Q}.$$

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$$\pi \notin \mathbb{Q}$$

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In \mathbb{Q} we can do “+”, “−”, “ \times ” and “ \div (by a number $\neq 0$)”.

A.1 Real Numbers and the Real Line



Are we happy now?

A.1 Real Numbers and the Real Line



Are we happy now?

No!

A.1 Real Numbers and the Real Line



Are we happy now?

No!

Why?

A.1 Real Numbers and the Real Line

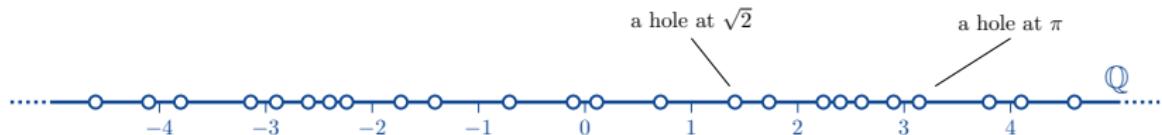


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Why?

Because if we draw all the rational numbers in a line, then the line has lots of holes in it. In fact, \mathbb{Q} has ∞ many holes in it.



A.1 Real Numbers and the Real Line

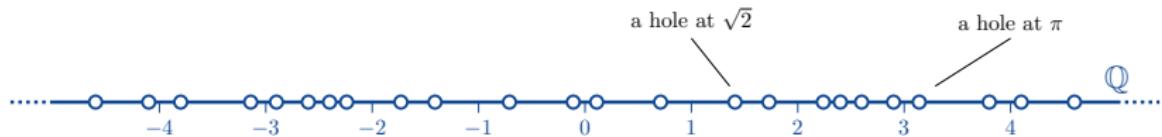


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So we invent new numbers!

The Real Numbers

The set

$$\mathbb{R} = \{\text{all numbers which can be written as a decimal}\}$$

is called the set of *real numbers*. For example

$$\begin{array}{ll} 0 = 0.0 \in \mathbb{R} & \frac{100}{13} = 7.692307\ldots \in \mathbb{R} \\ \frac{23}{99} = 0.232323\ldots \in \mathbb{R} & \sqrt{2} = 1.414213\ldots \in \mathbb{R} \\ \frac{3}{4} = 0.75 \in \mathbb{R} & \frac{123}{999} = 0.123123\ldots \in \mathbb{R} \\ \pi = 3.141592\ldots \in \mathbb{R} & \frac{12345}{100000} = 0.12345 \in \mathbb{R}. \end{array}$$

A.1 Real Numbers and the Real Line



The real numbers are complete – this means that if we draw all the real numbers in a line, then there are no holes in the line.



A.1 Real Numbers and the Real Line



Are we happy now?

A.1 Real Numbers and the Real Line



Are we happy now?

Yes! Now we have enough numbers to do Calculus.



Intervals

Definition

A subset of \mathbb{R} is called an *interval* if

- 1 it contains atleast 2 numbers; and
- 2 it doesn't have any holes in it.

A.1 Real Numbers and the Real Line



Intervals

Definition

A subset of \mathbb{R} is called an *interval* if

- 1 it contains atleast 2 numbers; and
- 2 it doesn't have any holes in it.

Example

The set $\{x \mid x \text{ is a real number and } x > 6\}$ is an interval.



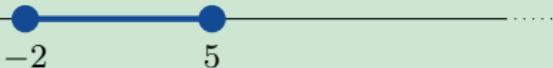
Because 6 is not in this set, we use **○** at 6.

A.1 Real Numbers and the Real Line



Example

The set of all real numbers x such that $-2 \leq x \leq 5$ is an interval.



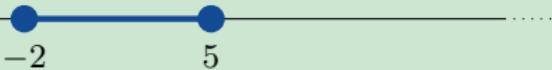
Because -2 and 5 are in this set, we use \bullet at -2 and 5 .

A.1 Real Numbers and the Real Line



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A.1 Real Numbers and the Real Line



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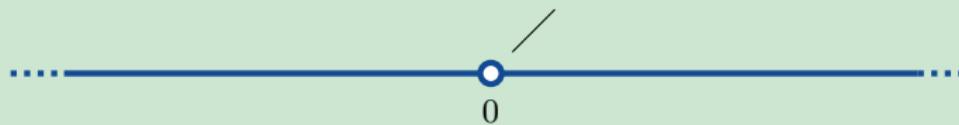


Because -2 and 5 are in this set, we use \bullet at -2 and 5 .

Example

The set $\{x \mid x \in \mathbb{R} \text{ and } x \neq 0\}$ is not an interval.

a hole at 0



A.1 Real Numbers and the Real Line



A finite interval is

- *closed* if it contains both its endpoints;
- *half-open* if it contains one of its endpoints;
- *open* if it does not contain its endpoints;

A.1 Real Numbers and the Real Line

Notation	Set	Type	Picture
(a, b)	$\{x a < x < b\}$	open	 A horizontal number line with two open circles at points a and b . The line segment between them is shaded blue, representing the open interval (a, b) .
$[a, b]$	$\{x a \leq x \leq b\}$	closed	 A horizontal number line with two solid black dots at points a and b . The line segment between them is shaded blue, representing the closed interval $[a, b]$.
$[a, b)$	$\{x a \leq x < b\}$	half open	 A horizontal number line with a solid black dot at point a and an open circle at point b . The line segment between them is shaded blue, representing the half-open interval $[a, b)$.
$(a, b]$	$\{x a < x \leq b\}$	half open	 A horizontal number line with an open circle at point a and a solid black dot at point b . The line segment between them is shaded blue, representing the half-open interval $(a, b]$.

A.1 Real Numbers and the Real Line



An infinite interval is

- *closed* if it contains a finite endpoint;
- *open* if it is not closed.

There is one exception to this rule: The whole real line is called both open and closed.

A.1 Real Numbers and the Real Line

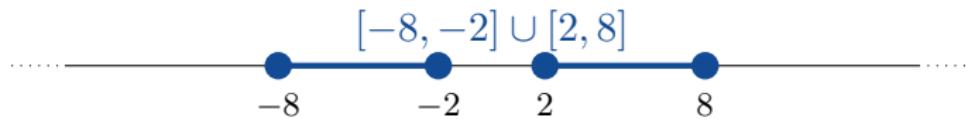


Notation	Set	Type	Picture
(a, ∞)	$\{x a < x\}$	open	A horizontal blue line with a gap at point a. The gap is indicated by two small dots above and below the line, and the letter 'a' is labeled below the gap.
$[a, \infty)$	$\{x a \leq x\}$	closed	A horizontal blue line with a solid dot at point a, indicating it is included in the set. Two small dots are at the ends of the line, and the letter 'a' is labeled below the dot.
$(-\infty, b)$	$\{x x < b\}$	open	A horizontal blue line with a gap at point b. Two small dots are at the ends of the line, and the letter 'b' is labeled below the gap.
$(-\infty, b]$	$\{x x \leq b\}$	closed	A horizontal blue line with a solid dot at point b, indicating it is included in the set. Two small dots are at the ends of the line, and the letter 'b' is labeled below the dot.
$(-\infty, \infty)$	\mathbb{R}	both open and closed	A continuous horizontal blue line extending infinitely in both directions, representing the set of all real numbers.

A.1 Real Numbers and the Real Line



We can combine two (or more) intervals with the notation \cup .
For example, $[-8, -2] \cup [2, 8]$ is called the *union* of $[-8, -2]$ and $[2, 8]$ and is shown below.





Functions and Their Graphs

1.1 Functions and Their Graphs



$$y = f(x)$$

“ y eşittir f x ”

“ y is equal to f of x ”

1.1 Functions and Their Graphs



dependent variable

$$y = f(x)$$

function

independent variable

“ y eşittir f x ”

“ y is equal to f of x ”

1.1 Functions and Their Graphs



Definition

A *function* from a set D to a set Y is a rule that assigns a unique element of Y to each element of D .

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A *function* from a set D to a set Y is a rule that assigns a unique element of Y to each element of D .

Definition

The set D of all possible values of x is called the *domain* of f .

Definition

The set Y is called the *target* of f .

Definition

The set of all possible values of $f(x)$ is called the *range* of f .

1.1 Functions and Their Graphs



If f is a function with domain D and target Y , we can write

$$f : D \rightarrow Y$$

/ \
domain target

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Example

$$f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^2.$$

1.1 Functions and Their Graphs



If f is a function with domain D and target Y , we can write

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/ \
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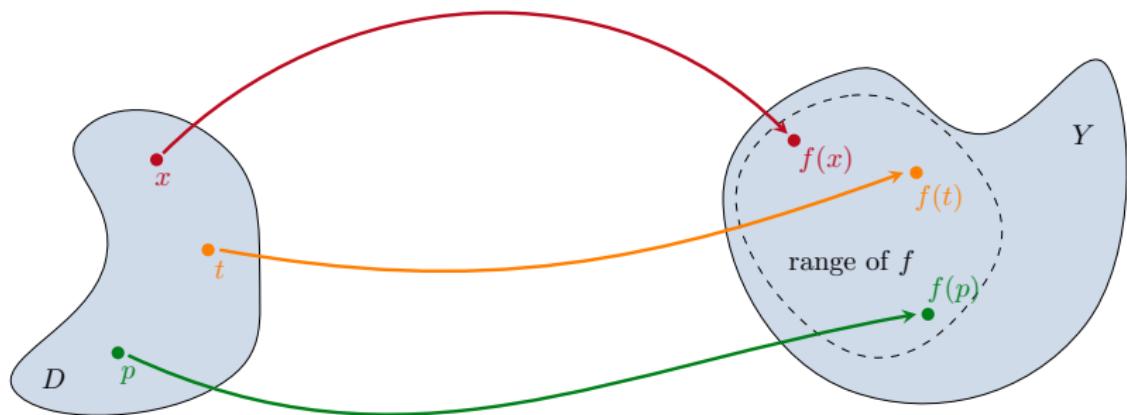
Example

$$f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^2.$$

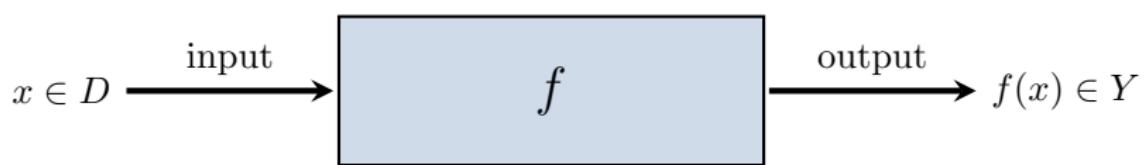
Example

$$f : (-\infty, \infty) \rightarrow [0, \infty), f(x) = x^2.$$

1.1 Functions and Their Graphs



1.1 Functions and Their Graphs



1.1 Functions and Their Graphs



Sometimes we want to use the largest possible domain for a function. This is called the *natural domain* of the function.

Sometimes we will want to use a smaller domain.

1.1 Functions and Their Graphs



function	domain (x)	range (y)
$y = x^2$	$(-\infty, \infty)$	
$y = \frac{1}{x}$	$\{x \mid x \in \mathbb{R}, x \neq 0\}$	
$y = \sqrt{x}$	$[0, \infty)$	
$y = \sqrt{4 - x}$		
$y = \sqrt{1 - x^2}$		

1.1 Functions and Their Graphs



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$y = \sqrt{x}$		$[0, \infty)$
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1.1 Functions and Their Graphs



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$y = \sqrt{x}$	$[0, \infty)$	
$y = \sqrt{4 - x}$		
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1.1 Functions and Their Graphs



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$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	
$y = \sqrt{1 - x^2}$		

1.1 Functions and Their Graphs



function	domain (x)	range (y)
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$y = \frac{1}{x}$	$\{x \mid x \in \mathbb{R}, x \neq 0\}$	$\{y \mid y \in \mathbb{R}, y \neq 0\}$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$		

1.1 Functions and Their Graphs



function	domain (x)	range (y)
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = \frac{1}{x}$	$\{x \mid x \in \mathbb{R}, x \neq 0\}$	$\{y \mid y \in \mathbb{R}, y \neq 0\}$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	

1.1 Functions and Their Graphs



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$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$
$y = x^2$	$[1, 2]$	
$y = x^2$	$[2, \infty)$	
$y = x^2$	$(-\infty, -2]$	

1.1 Functions and Their Graphs



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1.1 Functions and Their Graphs



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$y = x^2$	$[2, \infty)$	$[4, \infty)$
$y = x^2$	$(-\infty, -2]$	$[4, \infty)$
$y = 1 + x^2$	$[1, 3)$	
$y = 1 - \sqrt{x}$	$[0, \infty)$	

1.1 Functions and Their Graphs



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$y = x^2$	$[2, \infty)$	$[4, \infty)$
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1.1 Functions and Their Graphs



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$y = 1 - \sqrt{x}$	$[0, \infty)$	$(-\infty, 1]$

1.1 Functions and Their Graphs



Graphs of Functions

Definition

The *graph* of f is the set containing all the points (x, y) which satisfy $y = f(x)$.

1.1 Functions and Their Graphs



Example

Graph the function $y = 1 + x^2$ over the interval $[-2, 2]$.

1.1 Functions and Their Graphs

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STEP 1: Make a table of (x, y) points which satisfy $y = 1 + x^2$.

1.1 Functions and Their Graphs



Example

Graph the function $y = 1 + x^2$ over the interval $[-2, 2]$.

STEP 1: Make a table of (x, y) points which satisfy $y = 1 + x^2$.

x	y
-2	5
-1	2
0	1
1	2
$\frac{3}{2}$	$\frac{13}{4} = 3\frac{1}{4}$
2	5

1.1 Functions and Their Graphs

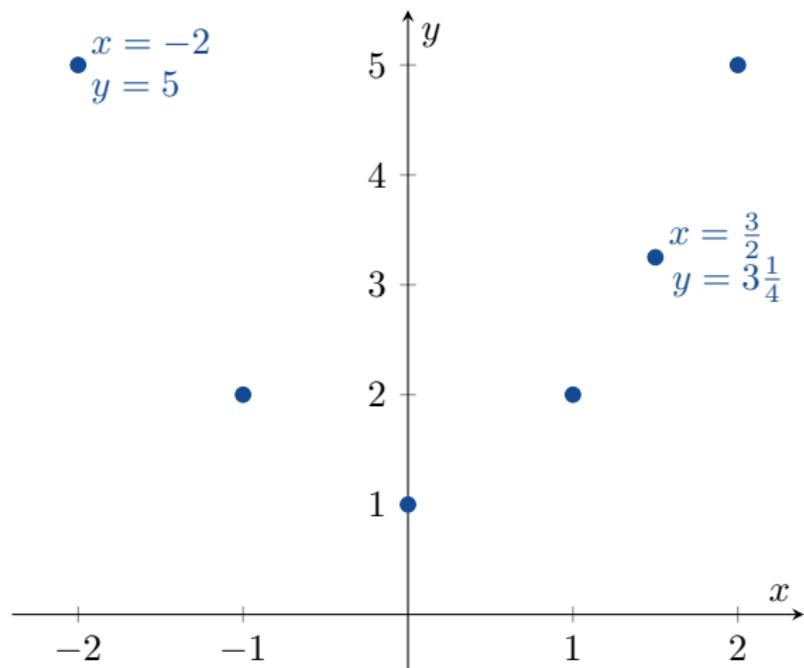


STEP 2: Plot these points.

1.1 Functions and Their Graphs



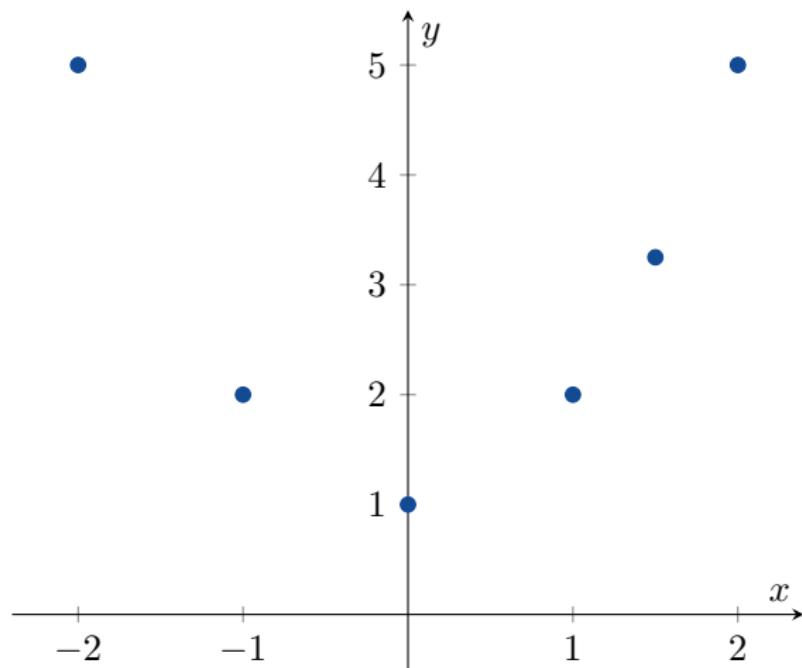
STEP 2: Plot these points.



1.1 Functions and Their Graphs



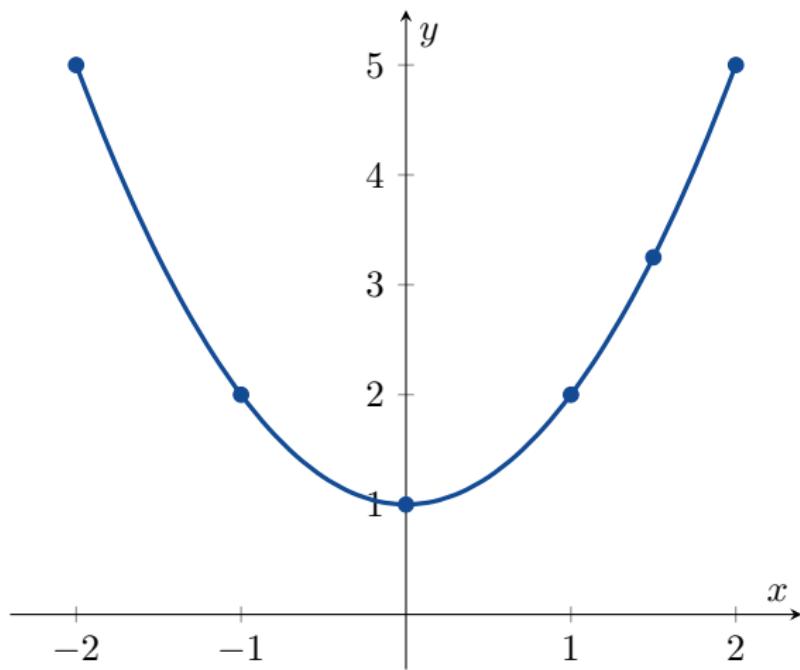
STEP 3: Draw a smooth curve through these points.



1.1 Functions and Their Graphs



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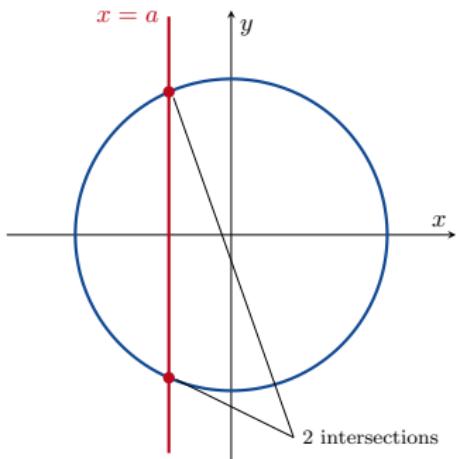
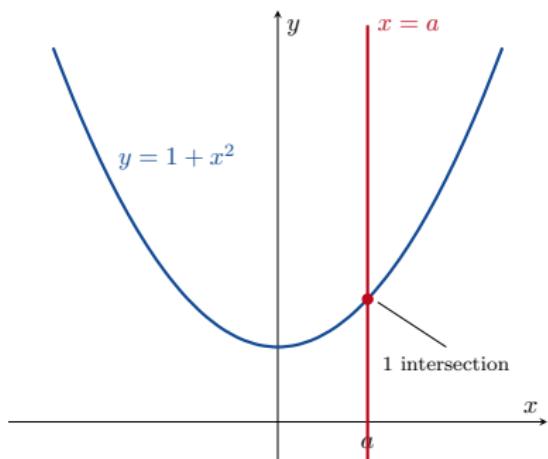
1.1 Functions and Their Graphs



The Vertical Line Test

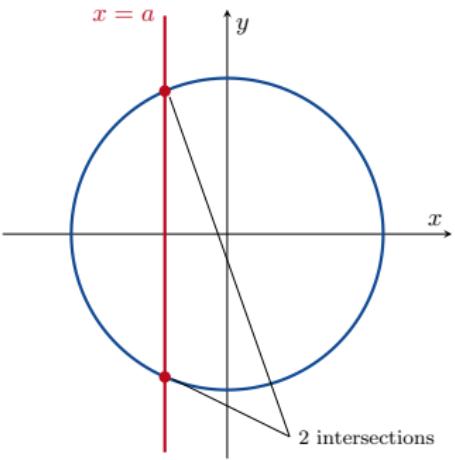
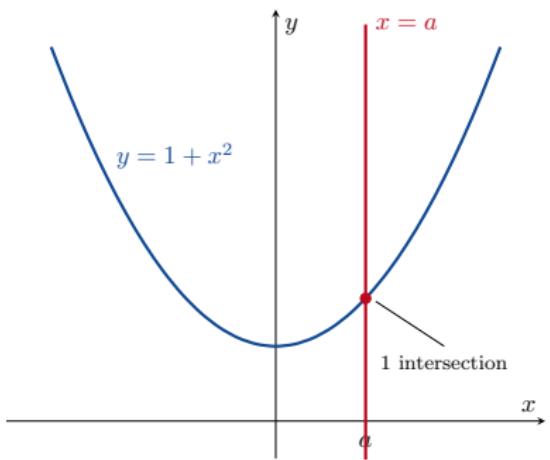
Not every curve that you draw is a graph of a function.

1.1 Functions and Their Graphs



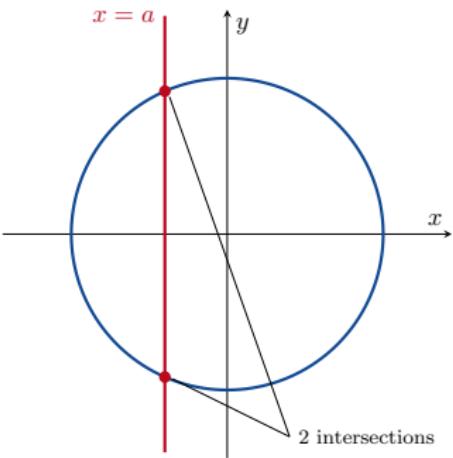
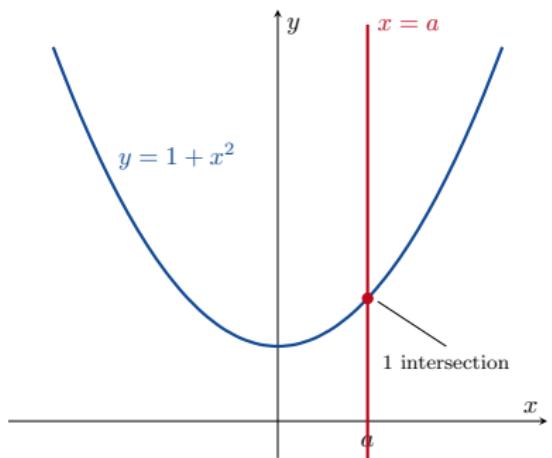
A function can have only one value $f(x)$ for each $x \in D$. This means that a vertical line can intersect the graph of a function at most once.

1.1 Functions and Their Graphs



A circle can not be the graph of a function because some vertical lines intersect the circle at two points.

1.1 Functions and Their Graphs

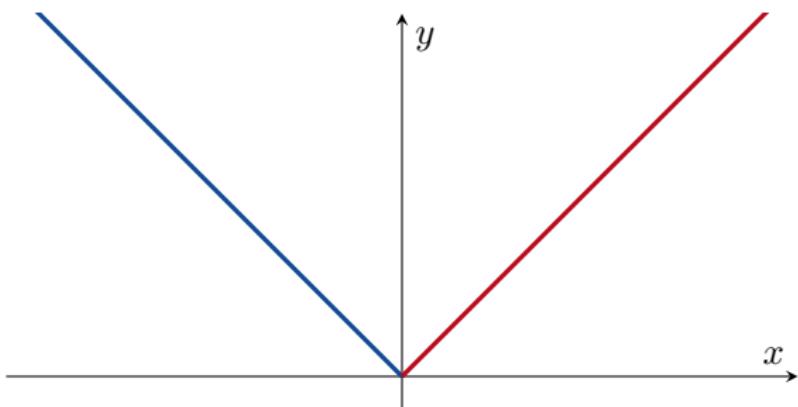


If $a \in D$, then the vertical line $x = a$ will intersect the graph of $f : D \rightarrow Y$ only at the point $(a, f(a))$.

Piecewise-Defined Functions

Example

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

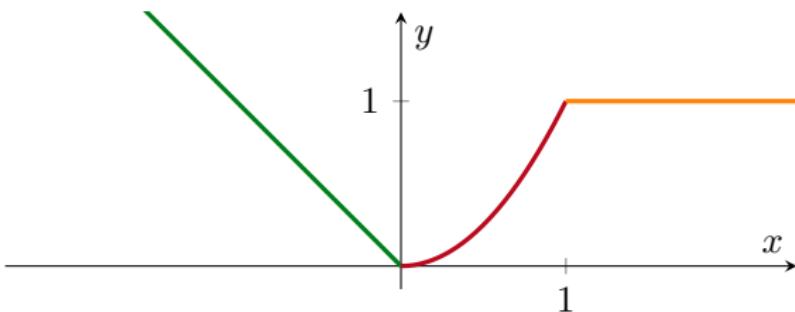


1.1 Functions and Their Graphs

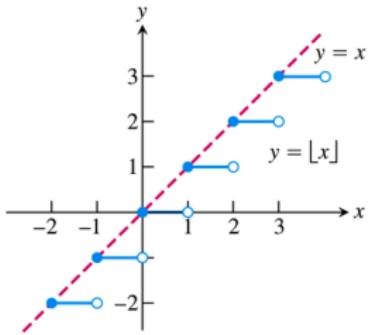


Example

$$f(x) = \begin{cases} -x & x < 0 \\ x^2 & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$



1.1 Functions and Their Graphs

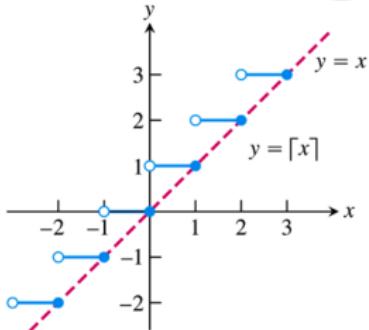
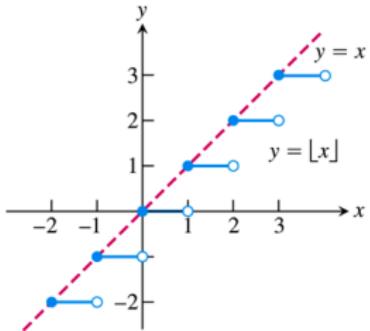


EXAMPLE 5 The function whose value at any number x is the *greatest integer less than or equal to x* is called the **greatest integer function** or the **integer floor function**. It is denoted $\lfloor x \rfloor$. Figure 1.10 shows the graph. Observe that

$$\begin{aligned}\lfloor 2.4 \rfloor &= 2, & \lfloor 1.9 \rfloor &= 1, & \lfloor 0 \rfloor &= 0, & \lfloor -1.2 \rfloor &= -2, \\ \lfloor 2 \rfloor &= 2, & \lfloor 0.2 \rfloor &= 0, & \lfloor -0.3 \rfloor &= -1, & \lfloor -2 \rfloor &= -2.\end{aligned}$$

■

1.1 Functions and Their Graphs



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EXAMPLE 6 The function whose value at any number x is the *smallest integer greater than or equal to x* is called the **least integer function** or the **integer ceiling function**. It is denoted $\lceil x \rceil$. Figure 1.11 shows the graph. For positive values of x , this function might represent, for example, the cost of parking x hours in a parking lot that charges \$1 for each hour or part of an hour.

Increasing and Decreasing Functions

Definition

Let I be an interval. Let $f : I \rightarrow \mathbb{R}$ be a function.

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$$f(x_1) < f(x_2)$$

for all $x_1, x_2 \in I$ which satisfy $x_1 < x_2$;

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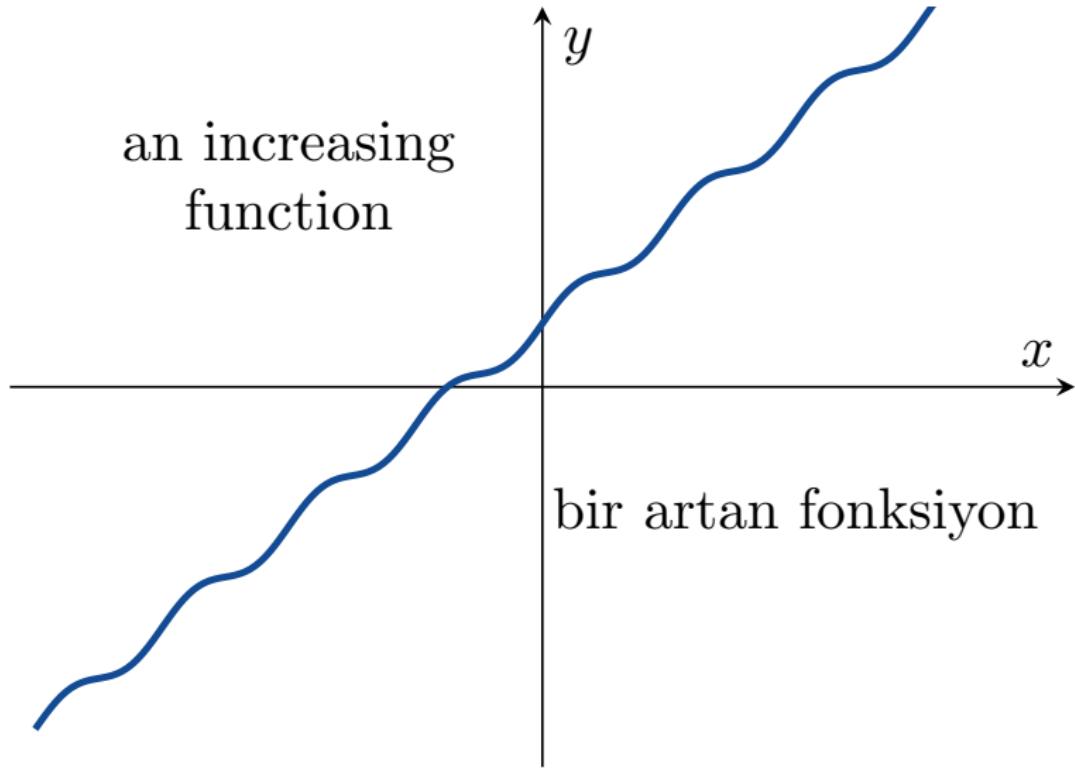
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for all $x_1, x_2 \in I$ which satisfy $x_1 < x_2$.

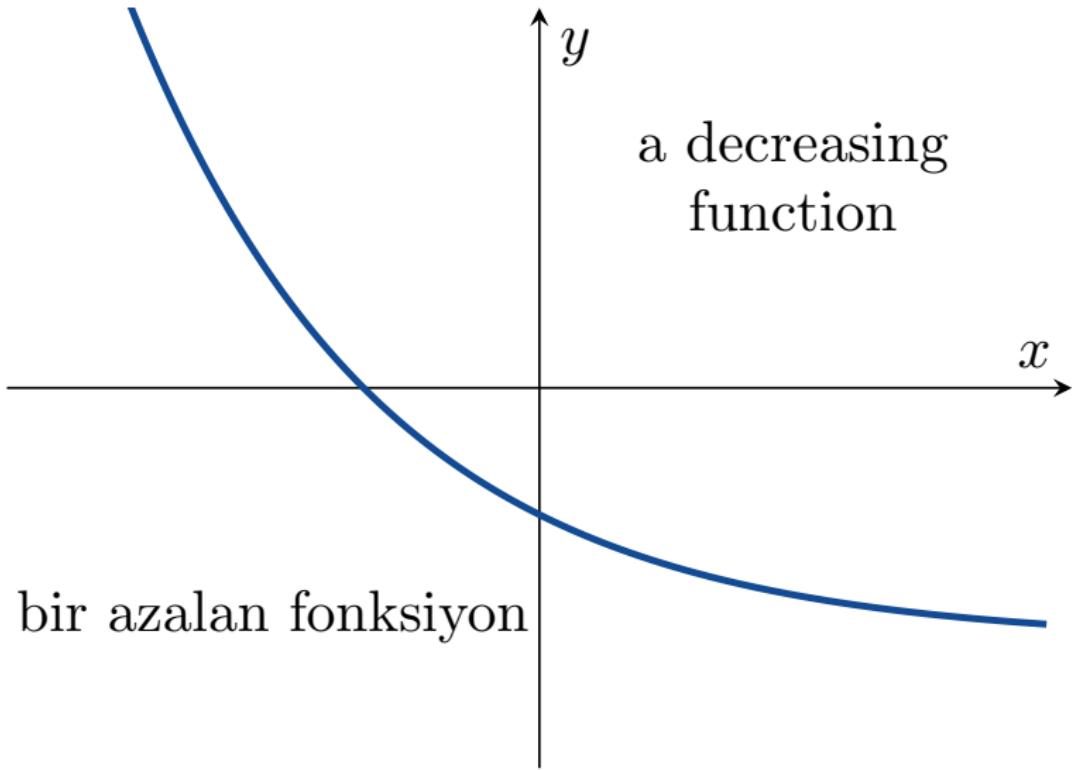
1.1 Functions and Their Graphs



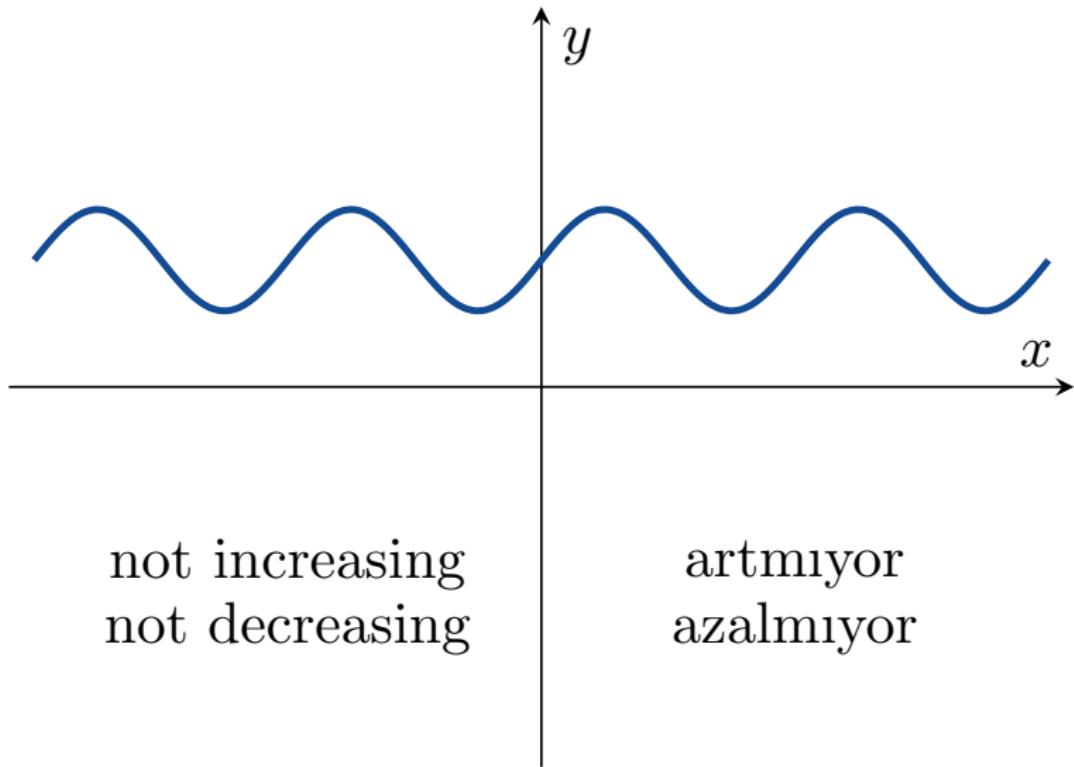
an increasing
function



1.1 Functions and Their Graphs



1.1 Functions and Their Graphs



Even Functions and Odd Functions

Recall that

- 2, 4, 6, 8, 10, ... are even numbers; and
- 1, 3, 5, 7, 9, ... are odd numbers.

Even Functions and Odd Functions

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- 2, 4, 6, 8, 10, ... are even numbers; and
- 1, 3, 5, 7, 9, ... are odd numbers.

Definition

- 1 $f : D \rightarrow \mathbb{R}$ is an *even function* if $f(-x) = f(x)$ for all $x \in D$;
- 2 $f : D \rightarrow \mathbb{R}$ is an *odd function* if $f(-x) = -f(x)$ for all $x \in D$.

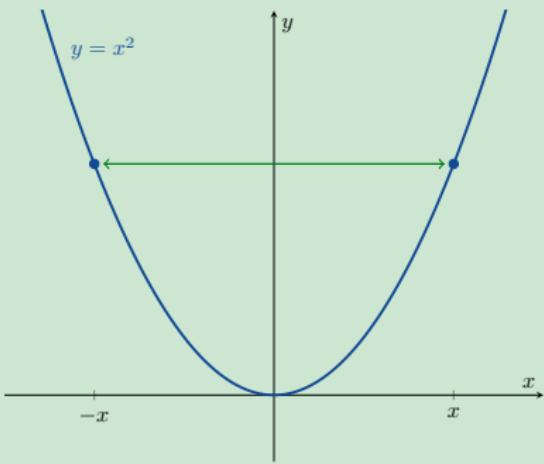
1.1 Functions and Their Graphs



Example

$f(x) = x^2$ is an even function because

$$f(-x) = (-x)^2 = x^2 = f(x).$$



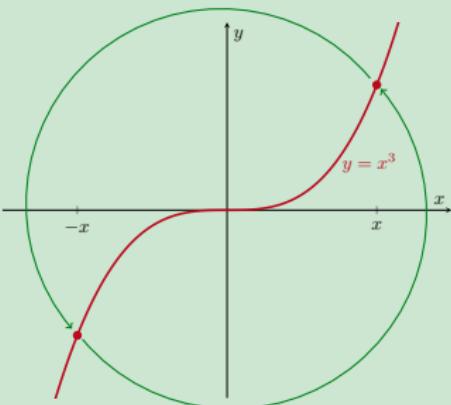
1.1 Functions and Their Graphs



Example

$f(x) = x^3$ is an odd function because

$$f(-x) = (-x)^3 = -x^3 = -f(x).$$



1.1 Functions and Their Graphs



Example

Is $f(x) = x^2 + 1$ even, odd or neither?

1.1 Functions and Their Graphs



Example

Is $f(x) = x^2 + 1$ even, odd or neither?

Since

$$f(-x) = (-x)^2 + 1 = x^2 + 1 = f(x),$$

f is an even function.

1.1 Functions and Their Graphs



Example

Is $g(x) = x + 1$ even, odd or neither?

1.1 Functions and Their Graphs



Example

Is $g(x) = x + 1$ even, odd or neither?

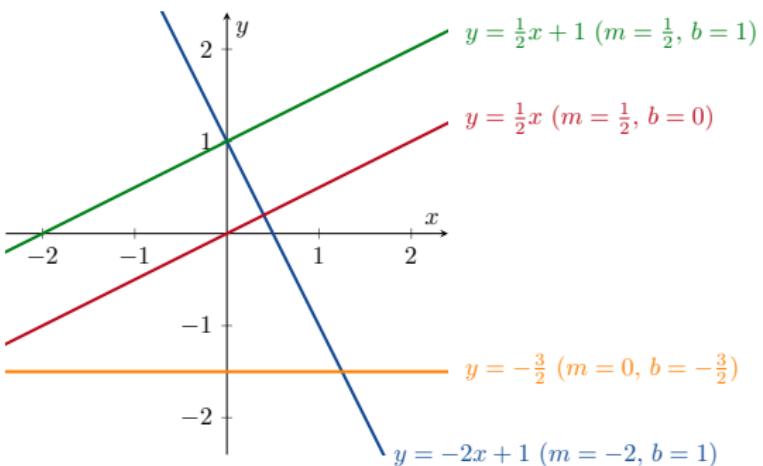
Since $g(-2) = -2 + 1 = -1$ and $g(2) = 3$, we have $g(-2) \neq g(2)$ and $g(-2) \neq -g(2)$. Hence g is neither even nor odd.

1.1 Functions and Their Graphs



Linear Functions

$$f(x) = mx + b \quad (m, b \in \mathbb{R})$$



1.1 Functions and Their Graphs



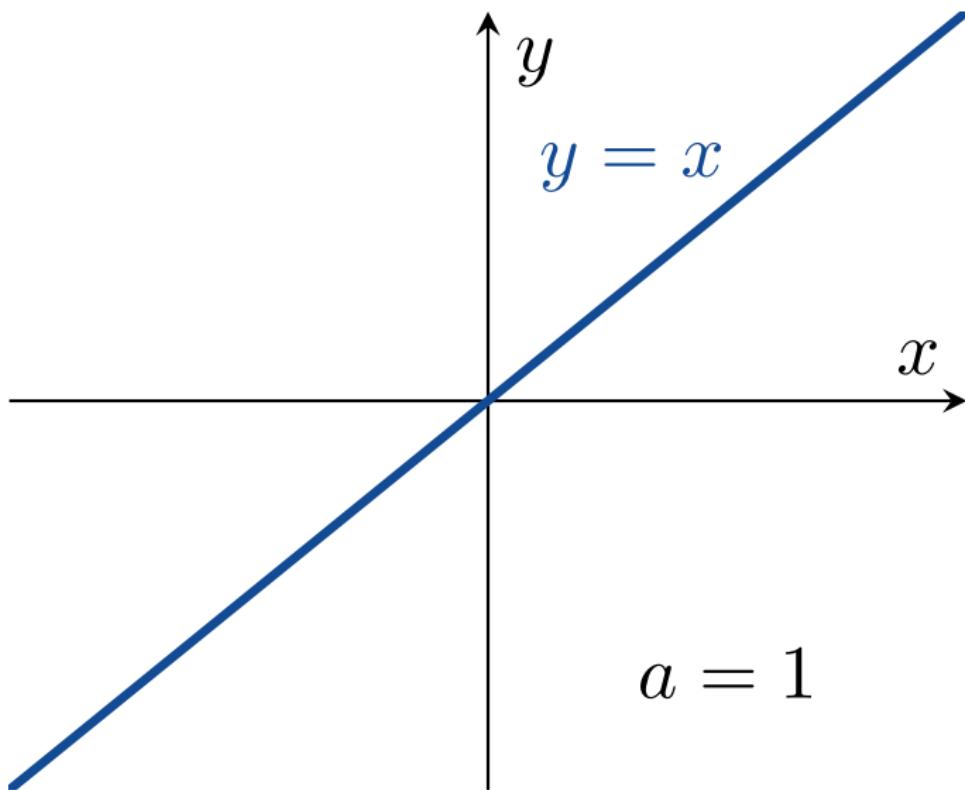
Power Functions

$$f(x) = x^a$$

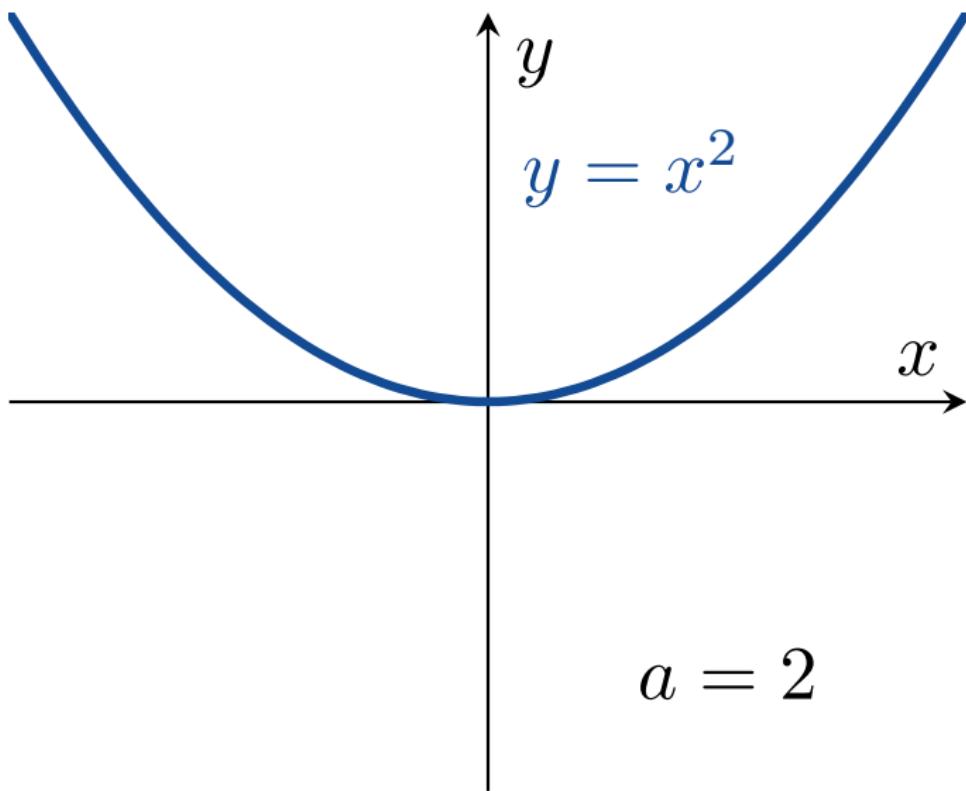
$(a \in \mathbb{R})$

“ x to the power of a ”

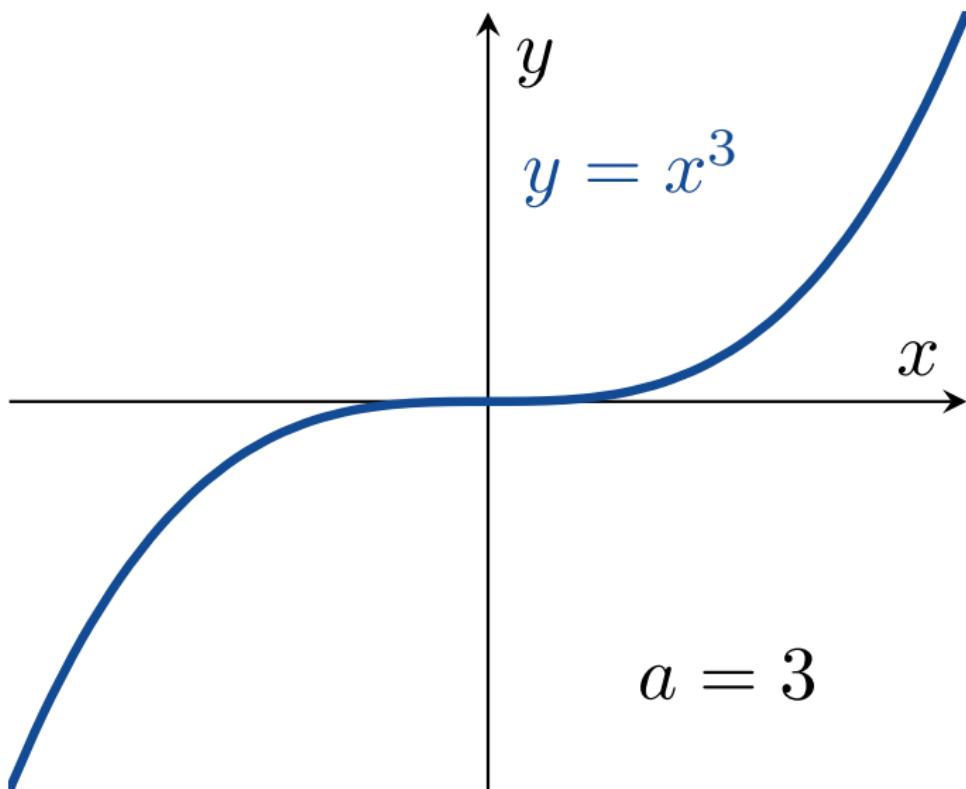
1.1 Functions and Their Graphs



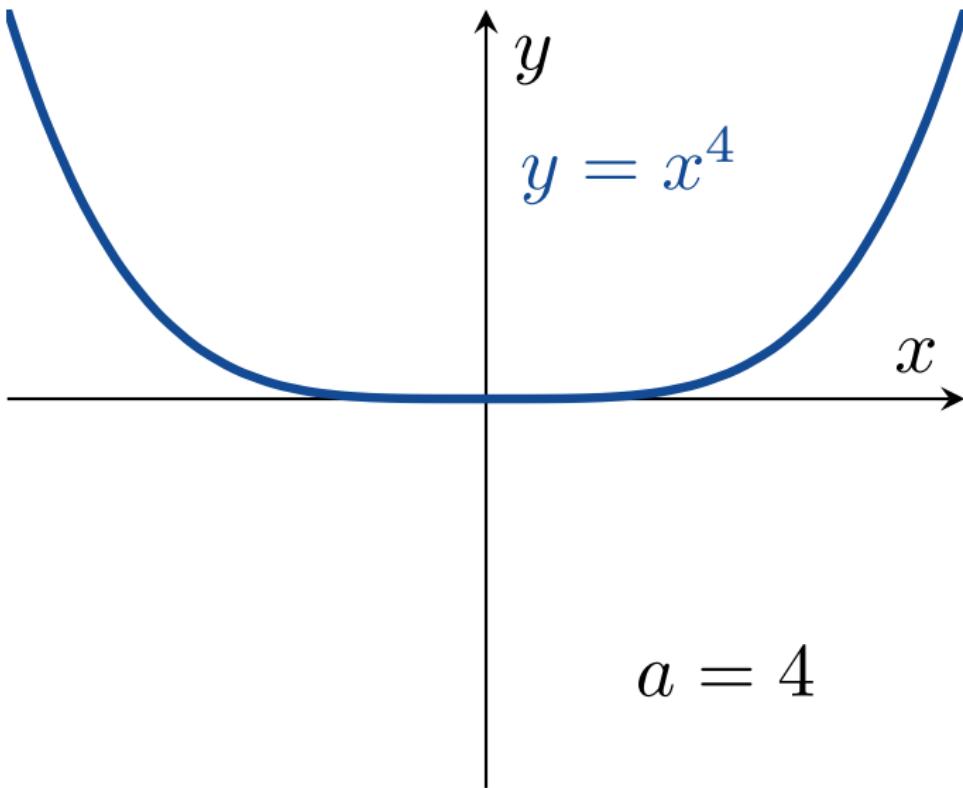
1.1 Functions and Their Graphs



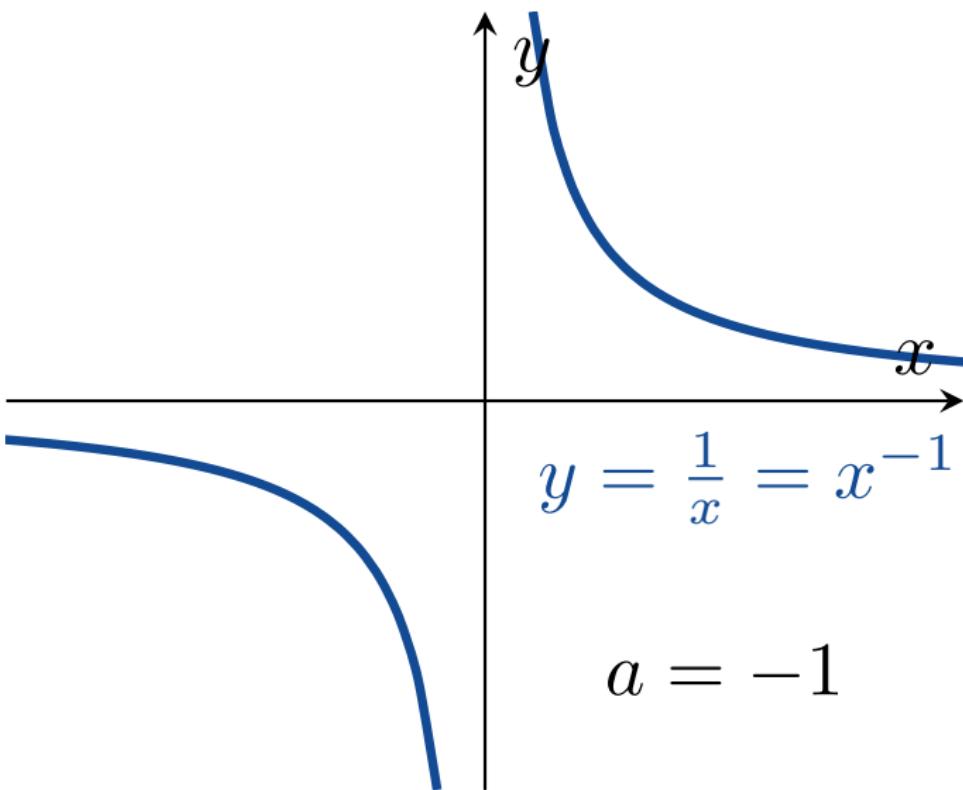
1.1 Functions and Their Graphs



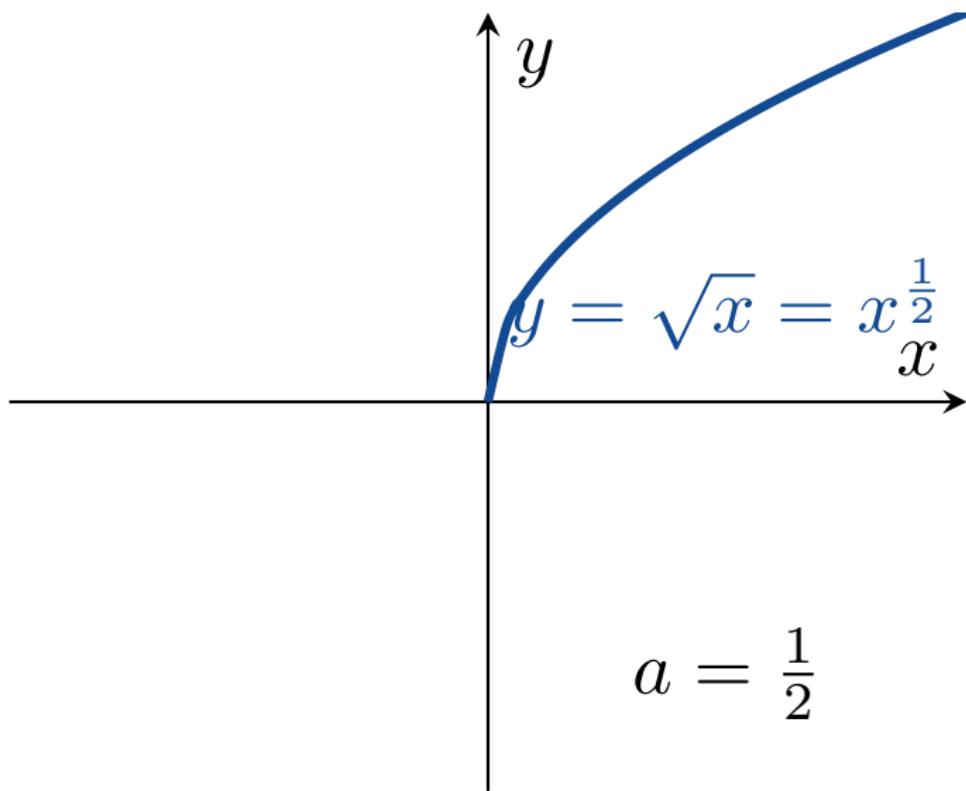
1.1 Functions and Their Graphs



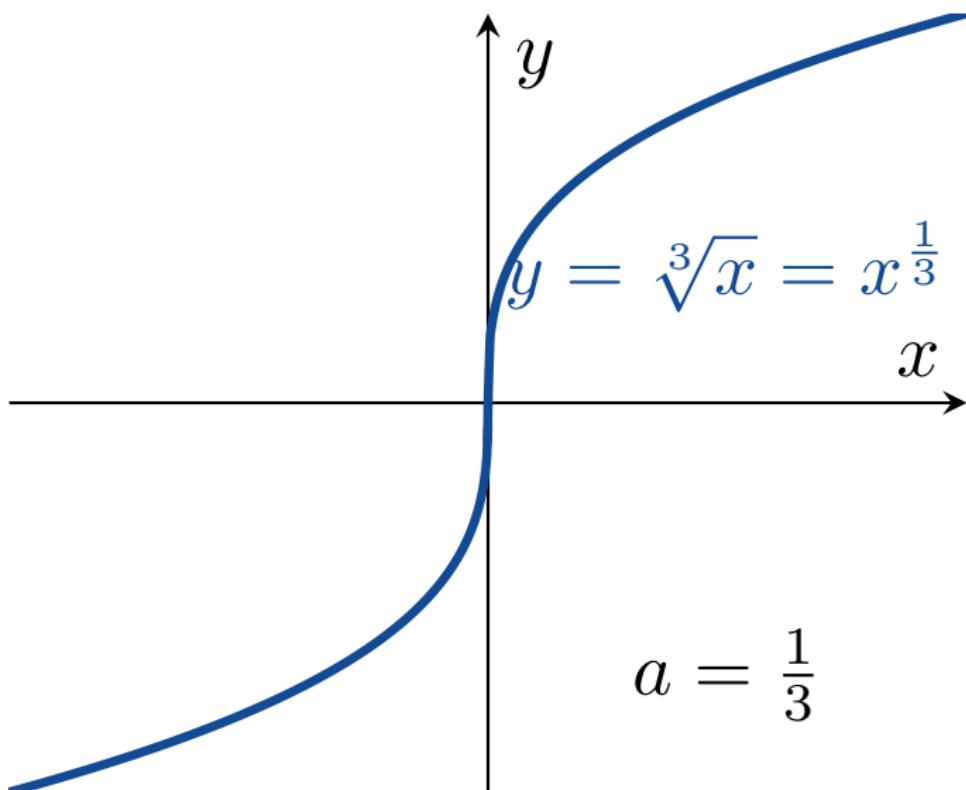
1.1 Functions and Their Graphs



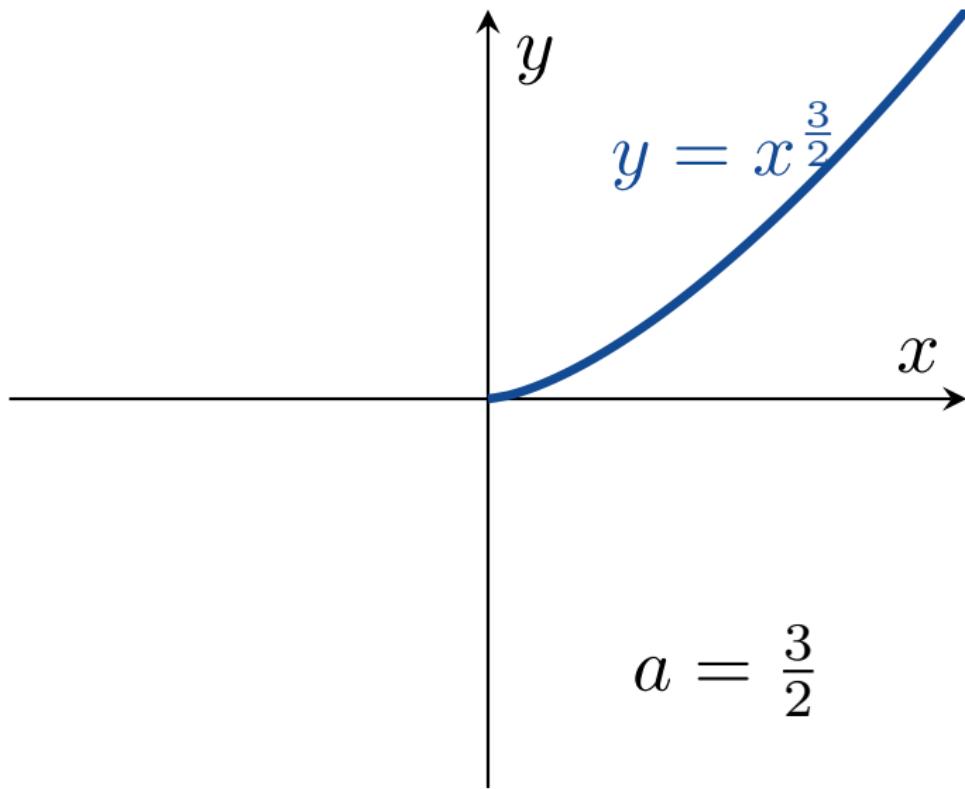
1.1 Functions and Their Graphs



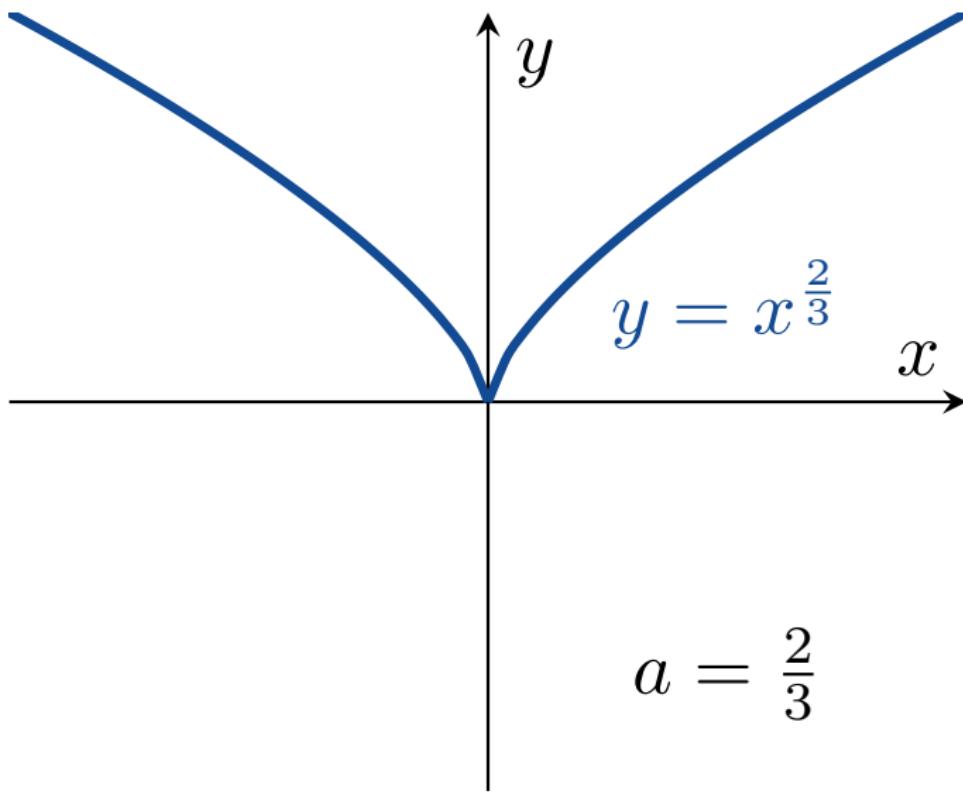
1.1 Functions and Their Graphs



1.1 Functions and Their Graphs



1.1 Functions and Their Graphs





Polynomials

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$
$$(n \in \mathbb{N} \cup \{0\}, a_j \in \mathbb{R}).$$

1.1 Functions and Their Graphs



Polynomials

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$
$$(n \in \mathbb{N} \cup \{0\}, a_j \in \mathbb{R}).$$

The (natural) domain of a polynomial is always $(-\infty, \infty)$. If $n > 0$ and $a_n \neq 0$, then n is called the *degree* of $p(x)$.

Rational Functions

$$f(x) = \frac{p(x)}{q(x)}$$

rational function \nearrow polynomial

A diagram illustrating the definition of a rational function. The expression $f(x) = \frac{p(x)}{q(x)}$ is shown. A vertical line segment points from the term $p(x)$ to the text "rational function". Another line segment points from the term $q(x)$ to the text "polynomial".

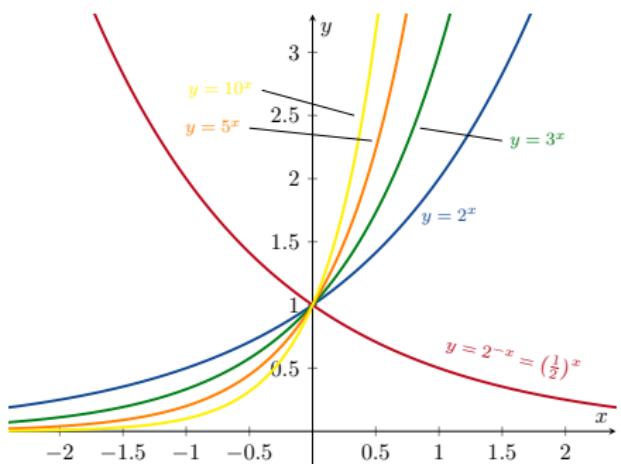
Example

$$f(x) = \frac{2x^3 - 3}{7x + 4}$$

1.1 Functions and Their Graphs

Exponential Functions

$$f(x) = a^x$$
$$(a \in \mathbb{R}, a > 0, a \neq 1)$$



The (natural) domain of an exponential function is $(-\infty, \infty)$.

1.1 Functions and Their Graphs

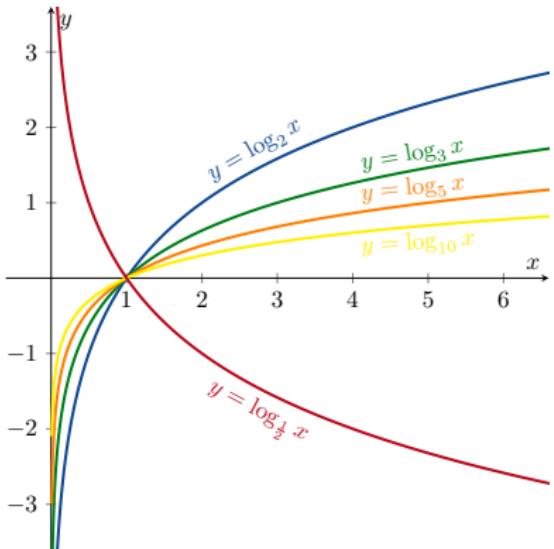


Logarithmic Functions

$$y = \log_a x \iff x = a^y$$

$(a \in \mathbb{R}, a > 0, a \neq 1)$

“log base a of x ”





Break

We will continue at 3pm





Combining Functions; Shifting and Scaling Graphs

1.2 Combining Functions; Shifting and Scaling Graphs



Sums, Differences, Products and Quotients

Consider $f : D(f) \rightarrow \mathbb{R}$ and $g : D(g) \rightarrow \mathbb{R}$.

1.2 Combining Functions; Shifting and Scaling Graphs



Sums, Differences, Products and Quotients

Consider $f : D(f) \rightarrow \mathbb{R}$ and $g : D(g) \rightarrow \mathbb{R}$. We can define 4 new functions:

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x)g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

1.2 Combining Functions; Shifting and Scaling Graphs



Sums, Differences, Products and Quotients

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The domain of $(f + g)$, $(f - g)$ and (fg) is $D(f) \cap D(g)$.

1.2 Combining Functions; Shifting and Scaling Graphs



Sums, Differences, Products and Quotients

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$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

The domain of $(f + g)$, $(f - g)$ and (fg) is $D(f) \cap D(g)$.

The domain of $\left(\frac{f}{g}\right)$ is $D(f) \cap \{x \in D(g) \mid g(x) \neq 0\}$.

1.2 Combining Functions; Shifting and Scaling Graphs



Example

Consider $f : [0, \infty) \rightarrow \mathbb{R}$, $f(x) = \sqrt{x}$ and $g : (-\infty, 1] \rightarrow \mathbb{R}$, $g(x) = \sqrt{1 - x}$.

1.2 Combining Functions; Shifting and Scaling Graphs



Example

Consider $f : [0, \infty) \rightarrow \mathbb{R}$, $f(x) = \sqrt{x}$ and $g : (-\infty, 1] \rightarrow \mathbb{R}$, $g(x) = \sqrt{1 - x}$. Find $(f + g)$, $(f - g)$, (fg) , $\left(\frac{f}{g}\right)$ and $\left(\frac{g}{f}\right)$ and find the domains of these 5 functions.

$$D(f) = [0, \infty), f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1], g(x) = \sqrt{1-x}$$

Function	Formula	Domain
$f + g$		
$f - g$		
fg		
$\frac{f}{g}$		
$\frac{g}{f}$		

$$D(f) = [0, \infty), f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1], g(x) = \sqrt{1-x}$$

Function	Formula	Domain
$f + g$	$\sqrt{x} + \sqrt{1-x}$	
$f - g$		
fg		
$\frac{f}{g}$		
$\frac{g}{f}$		

$$D(f) = [0, \infty), f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1], g(x) = \sqrt{1-x}$$

Function	Formula	Domain
$f + g$	$\sqrt{x} + \sqrt{1-x}$	$[0, 1]$
$f - g$		
fg		
$\frac{f}{g}$		
$\frac{g}{f}$		

$$D(f) = [0, \infty), f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1], g(x) = \sqrt{1-x}$$

Function	Formula	Domain
$f + g$	$\sqrt{x} + \sqrt{1-x}$	$[0, 1]$
$f - g$	$\sqrt{x} - \sqrt{1-x}$	$[0, 1]$
fg	$\sqrt{x}\sqrt{1-x} = \sqrt{x-x^2}$	$[0, 1]$
$\frac{f}{g}$		
$\frac{g}{f}$		

$$D(f) = [0, \infty), f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1], g(x) = \sqrt{1-x}$$

Function	Formula	Domain
$f + g$	$\sqrt{x} + \sqrt{1-x}$	$[0, 1]$
$f - g$	$\sqrt{x} - \sqrt{1-x}$	$[0, 1]$
fg	$\sqrt{x}\sqrt{1-x} = \sqrt{x-x^2}$	$[0, 1]$
$\frac{f}{g}$	$\sqrt{\frac{x}{1-x}}$	
$\frac{g}{f}$	$\sqrt{\frac{1-x}{x}}$	

$$D(f) = [0, \infty), f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1], g(x) = \sqrt{1-x}$$

Function	Formula	Domain
$f + g$	$\sqrt{x} + \sqrt{1-x}$	$[0, 1]$
$f - g$	$\sqrt{x} - \sqrt{1-x}$	$[0, 1]$
fg	$\sqrt{x}\sqrt{1-x} = \sqrt{x-x^2}$	$[0, 1]$
$\frac{f}{g}$	$\sqrt{\frac{x}{1-x}}$	$[0, 1)$
$\frac{g}{f}$	$\sqrt{\frac{1-x}{x}}$	

$$D(f) = [0, \infty), f(x) = \sqrt{x}, \quad D(g) = (-\infty, 1], g(x) = \sqrt{1-x}$$

Function	Formula	Domain
$f + g$	$\sqrt{x} + \sqrt{1-x}$	$[0, 1]$
$f - g$	$\sqrt{x} - \sqrt{1-x}$	$[0, 1]$
fg	$\sqrt{x}\sqrt{1-x} = \sqrt{x-x^2}$	$[0, 1]$
$\frac{f}{g}$	$\sqrt{\frac{x}{1-x}}$	$[0, 1)$
$\frac{g}{f}$	$\sqrt{\frac{1-x}{x}}$	$(0, 1]$

1.2 Combining Functions; Shifting and Scaling Graphs



Composite Functions

$$f \circ g(x) = f(g(x))$$

is the *composite* of f and g .

We read $f \circ g$ as “ f composed with g ”.

1.2 Combining Functions; Shifting and Scaling Graphs



Composite Functions

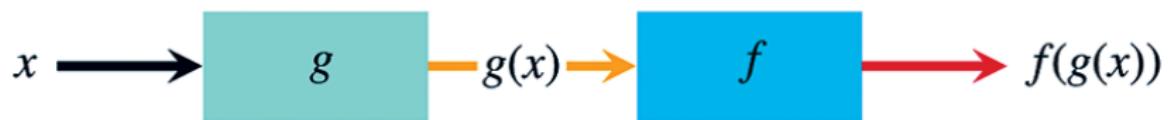
$$f \circ g(x) = f(g(x))$$

is the *composite* of f and g .

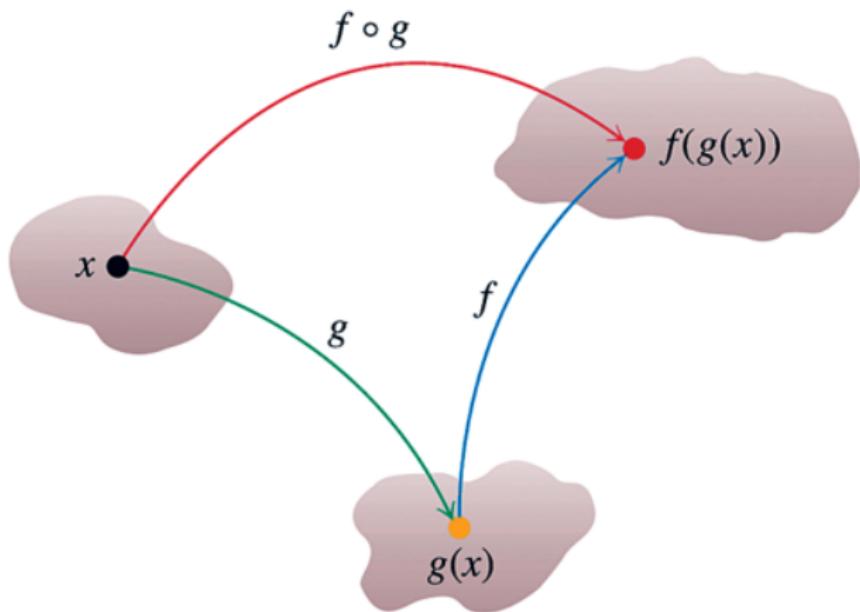
We read $f \circ g$ as “ f composed with g ”.

The domain of $f \circ g$ consists of all numbers x in the domain of g for which the number $g(x)$ lies in the domain of f .

1.2 Combining Functions; Shifting and Scaling Graphs



1.2 Combining Functions; Shifting and Scaling Graphs



1.2 Combining Functions; Shifting and Scaling Graphs



Example

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$, and find the domains of these 4 functions.

Note that the domain of f is $[0, \infty)$ and the domain of g is $(-\infty, \infty)$.

Function	Formula	Domain
$f \circ g$		
$g \circ f$		
$f \circ f$		
$g \circ g$		

1.2 Combining Functions; Shifting and Scaling Graphs



Example

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$, and find the domains of these 4 functions.

Note that the domain of f is $[0, \infty)$ and the domain of g is $(-\infty, \infty)$.

Function	Formula	Domain
$f \circ g$	$f(g(x)) = \sqrt{x+1}$	
$g \circ f$		
$f \circ f$		
$g \circ g$		

1.2 Combining Functions; Shifting and Scaling Graphs



Example

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$, and find the domains of these 4 functions.

Note that the domain of f is $[0, \infty)$ and the domain of g is $(-\infty, \infty)$.

Function	Formula	Domain
$f \circ g$	$f(g(x)) = \sqrt{x+1}$	$[-1, \infty)$
$g \circ f$		
$f \circ f$		
$g \circ g$		

1.2 Combining Functions; Shifting and Scaling Graphs



Example

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$, and find the domains of these 4 functions.

Note that the domain of f is $[0, \infty)$ and the domain of g is $(-\infty, \infty)$.

Function	Formula	Domain
$f \circ g$	$f(g(x)) = \sqrt{x+1}$	$[-1, \infty)$
$g \circ f$	$g(f(x)) = \sqrt{x} + 1$	$[0, \infty)$
$f \circ f$		
$g \circ g$		

1.2 Combining Functions; Shifting and Scaling Graphs



Example

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$, and find the domains of these 4 functions.

Note that the domain of f is $[0, \infty)$ and the domain of g is $(-\infty, \infty)$.

Function	Formula	Domain
$f \circ g$	$f(g(x)) = \sqrt{x+1}$	$[-1, \infty)$
$g \circ f$	$g(f(x)) = \sqrt{x} + 1$	$[0, \infty)$
$f \circ f$	$f(f(x)) = \sqrt{\sqrt{x}} = x^{\frac{1}{4}}$	$[0, \infty)$
$g \circ g$		

1.2 Combining Functions; Shifting and Scaling Graphs



Example

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$, and find the domains of these 4 functions.

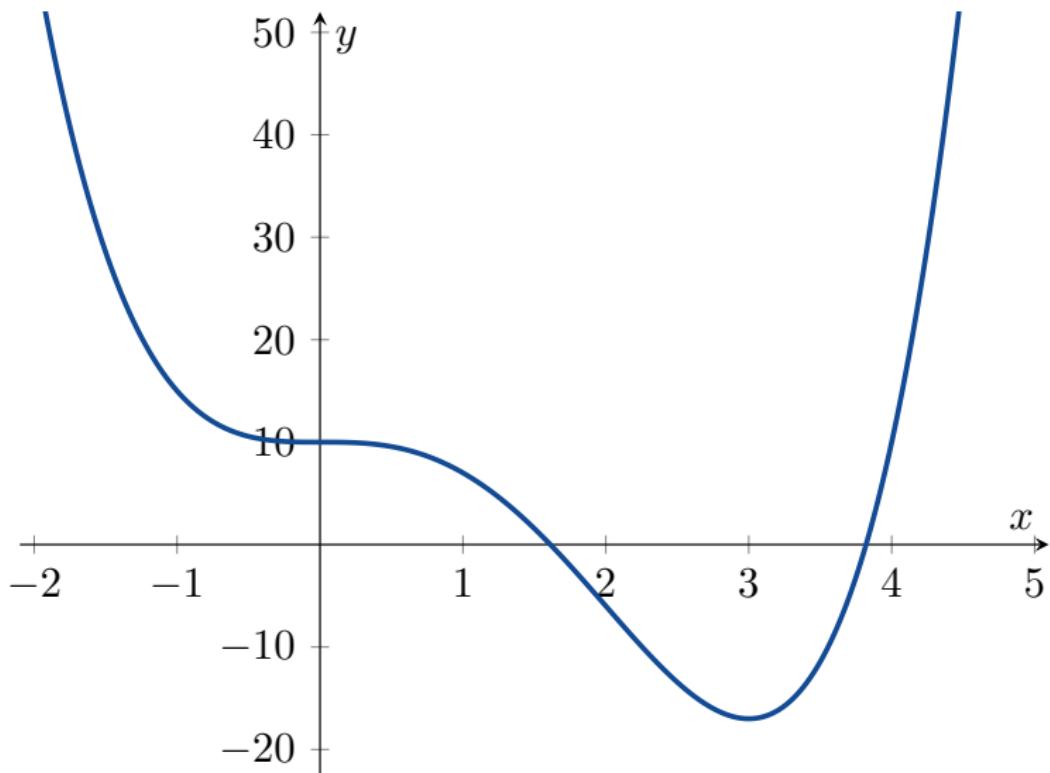
Note that the domain of f is $[0, \infty)$ and the domain of g is $(-\infty, \infty)$.

Function	Formula	Domain
$f \circ g$	$f(g(x)) = \sqrt{x+1}$	$[-1, \infty)$
$g \circ f$	$g(f(x)) = \sqrt{x} + 1$	$[0, \infty)$
$f \circ f$	$f(f(x)) = \sqrt{\sqrt{x}} = x^{\frac{1}{4}}$	$[0, \infty)$
$g \circ g$	$g(g(x)) = (x+1)+1 = x+2$	$(-\infty, \infty)$

1.2 Combining Functions; Shifting and Scaling Graphs



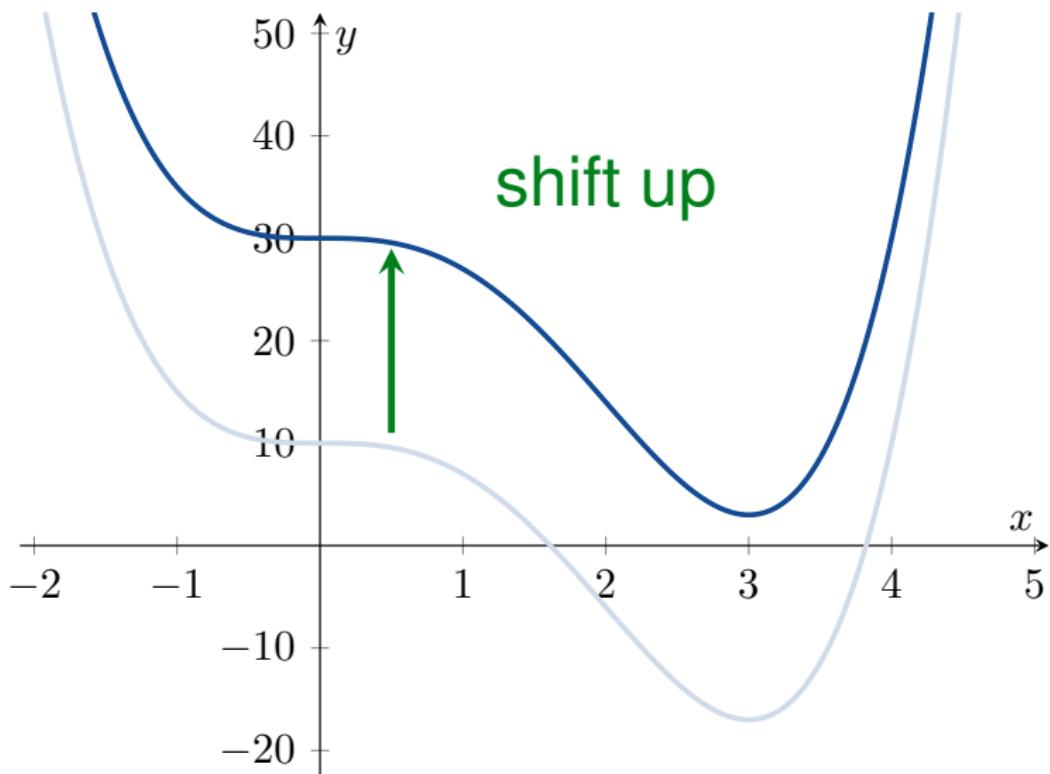
Shifting



1.2 Combining Functions; Shifting and Scaling Graphs



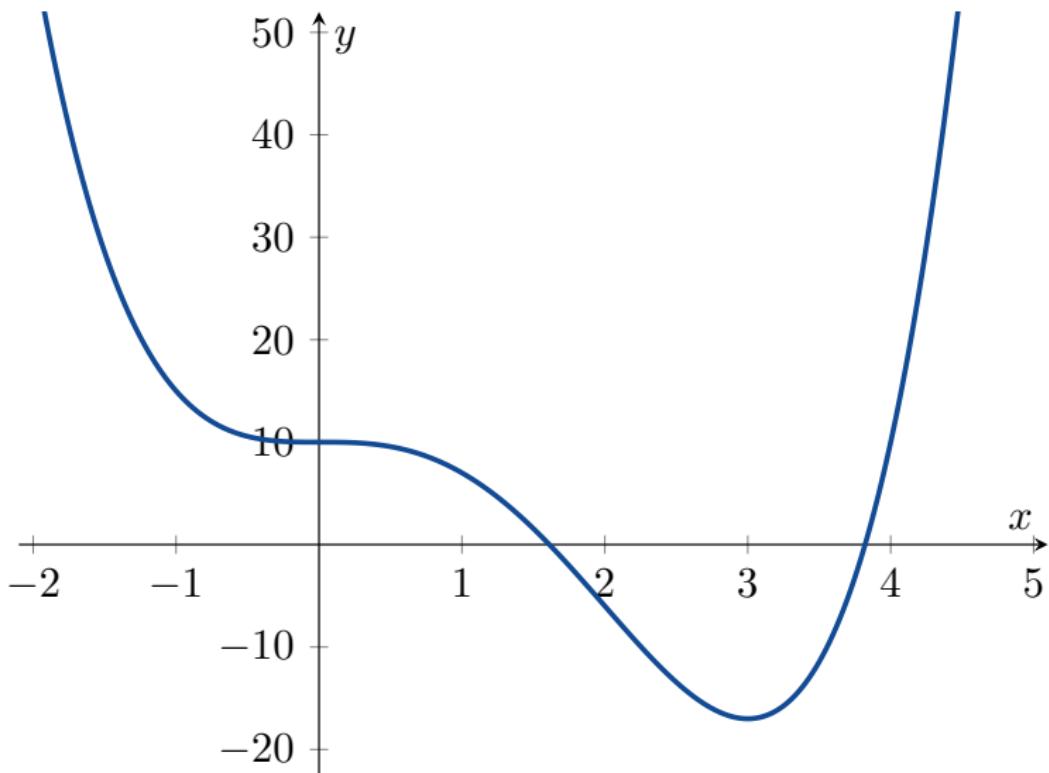
Shifting



1.2 Combining Functions; Shifting and Scaling Graphs



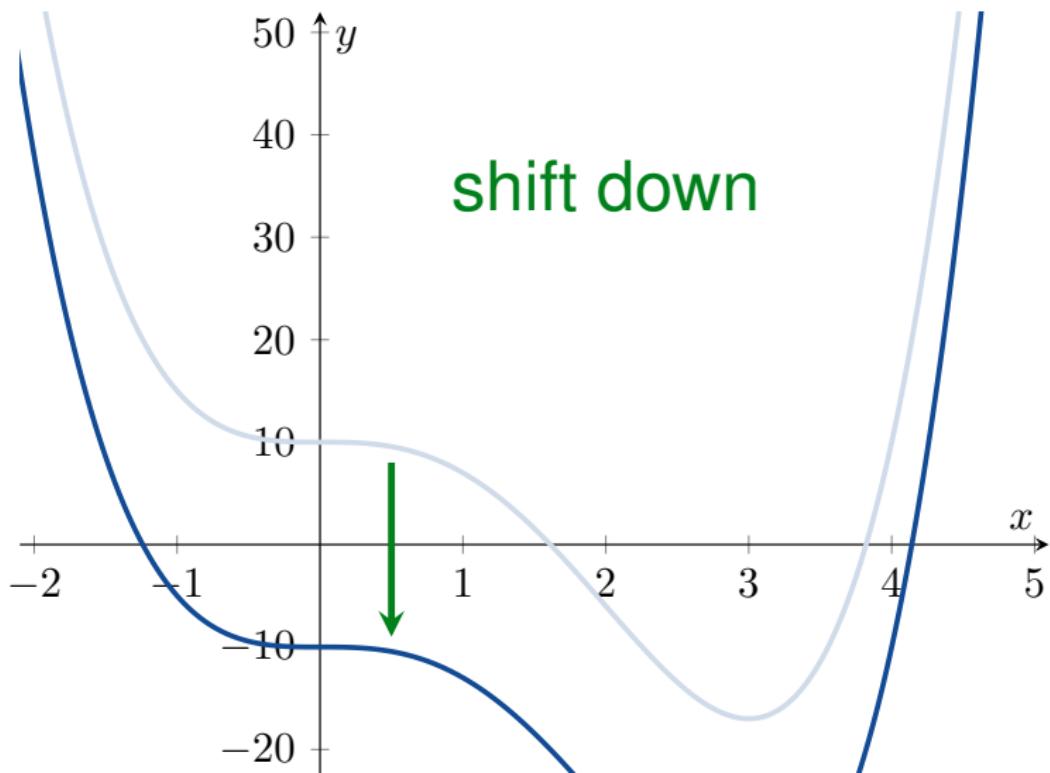
Shifting



1.2 Combining Functions; Shifting and Scaling Graphs



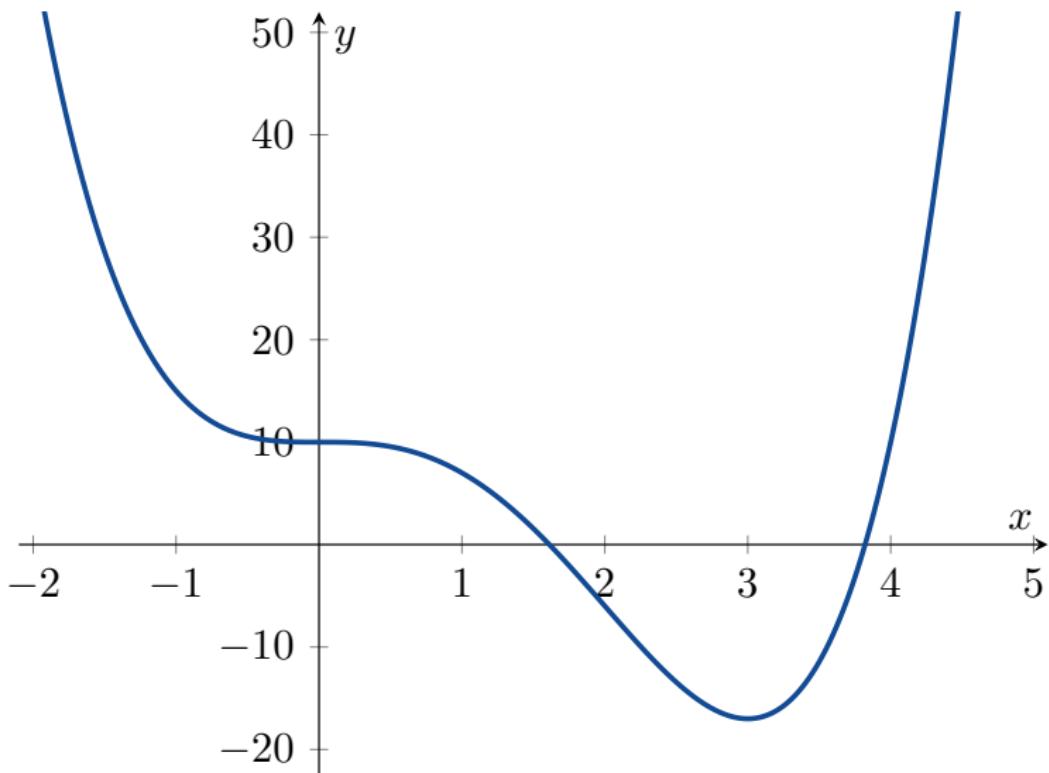
Shifting



1.2 Combining Functions; Shifting and Scaling Graphs



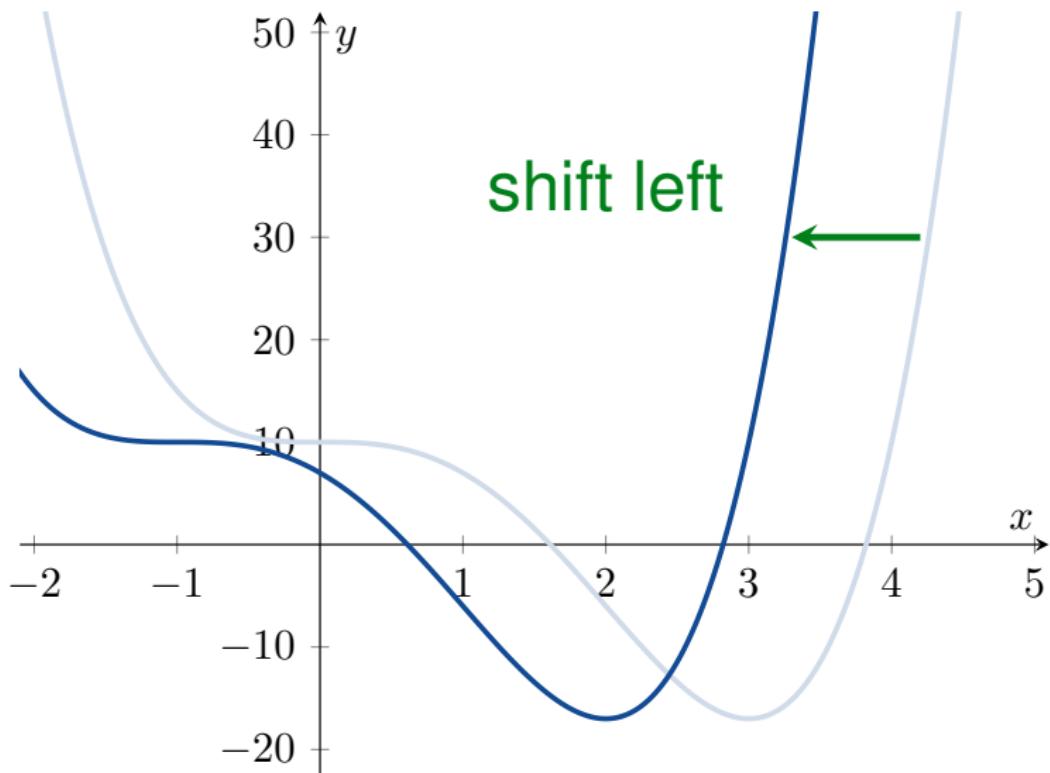
Shifting



1.2 Combining Functions; Shifting and Scaling Graphs



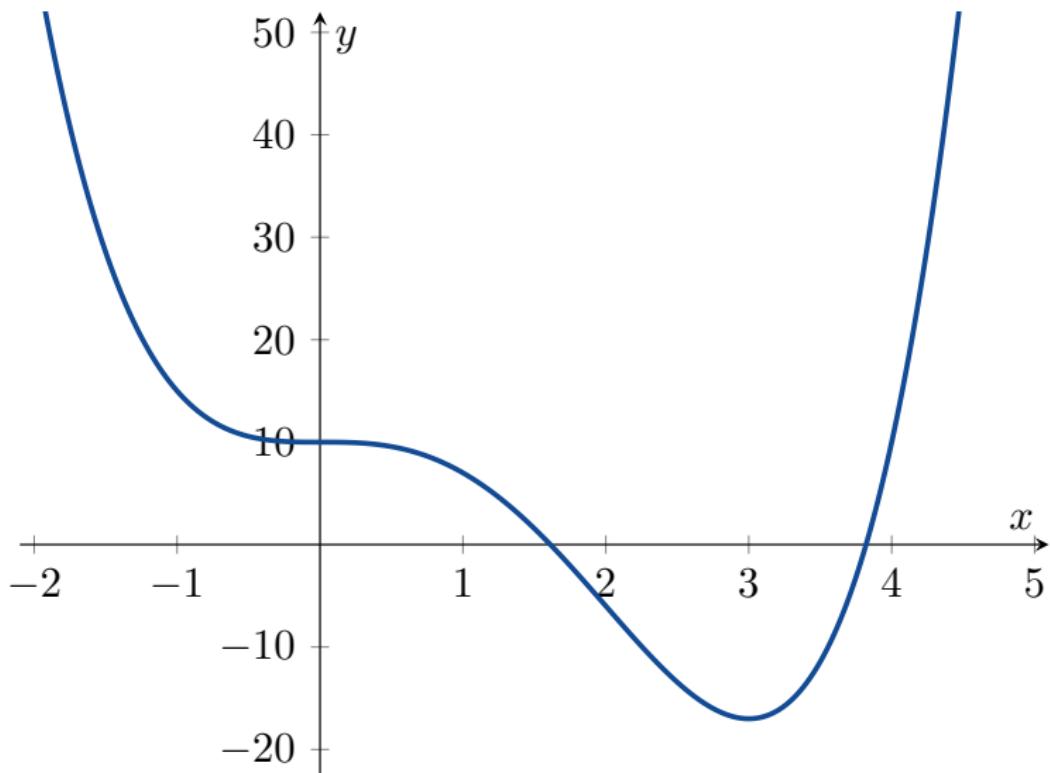
Shifting



1.2 Combining Functions; Shifting and Scaling Graphs



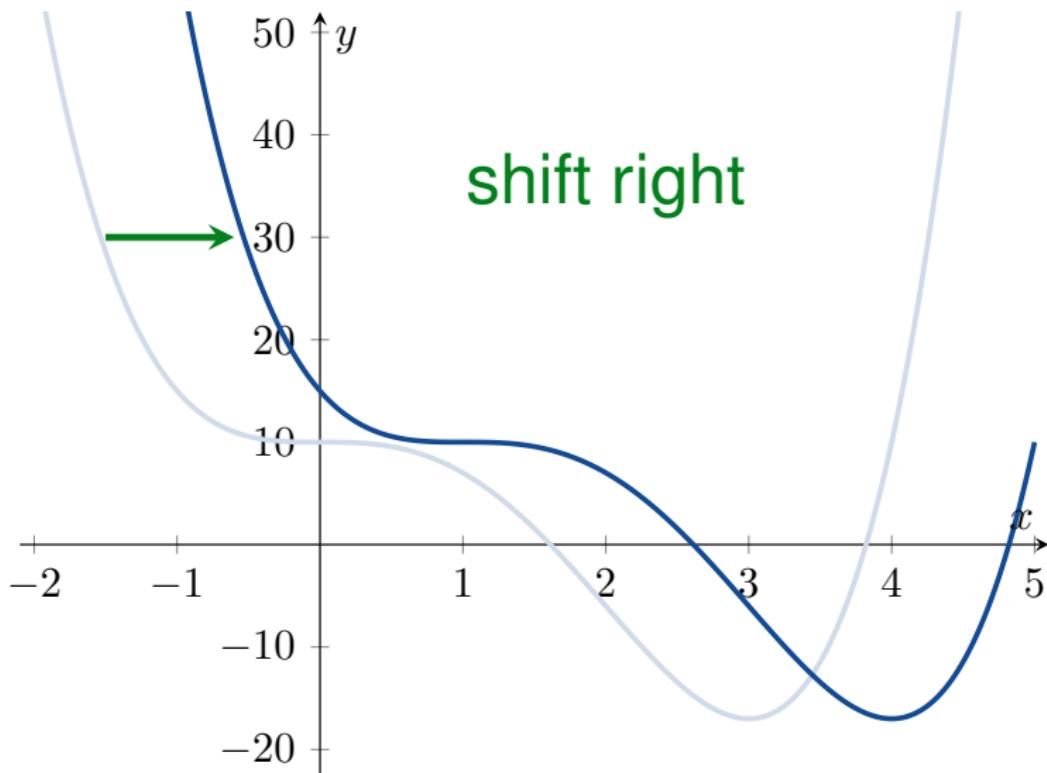
Shifting



1.2 Combining Functions; Shifting and Scaling Graphs



Shifting



1.2 Combining Functions; Shifting and Scaling Graphs



$y = f(x) + k$ shifts the graph up k units.
(or down $|k|$ units if $k < 0$)

1.2 Combining Functions; Shifting and Scaling Graphs

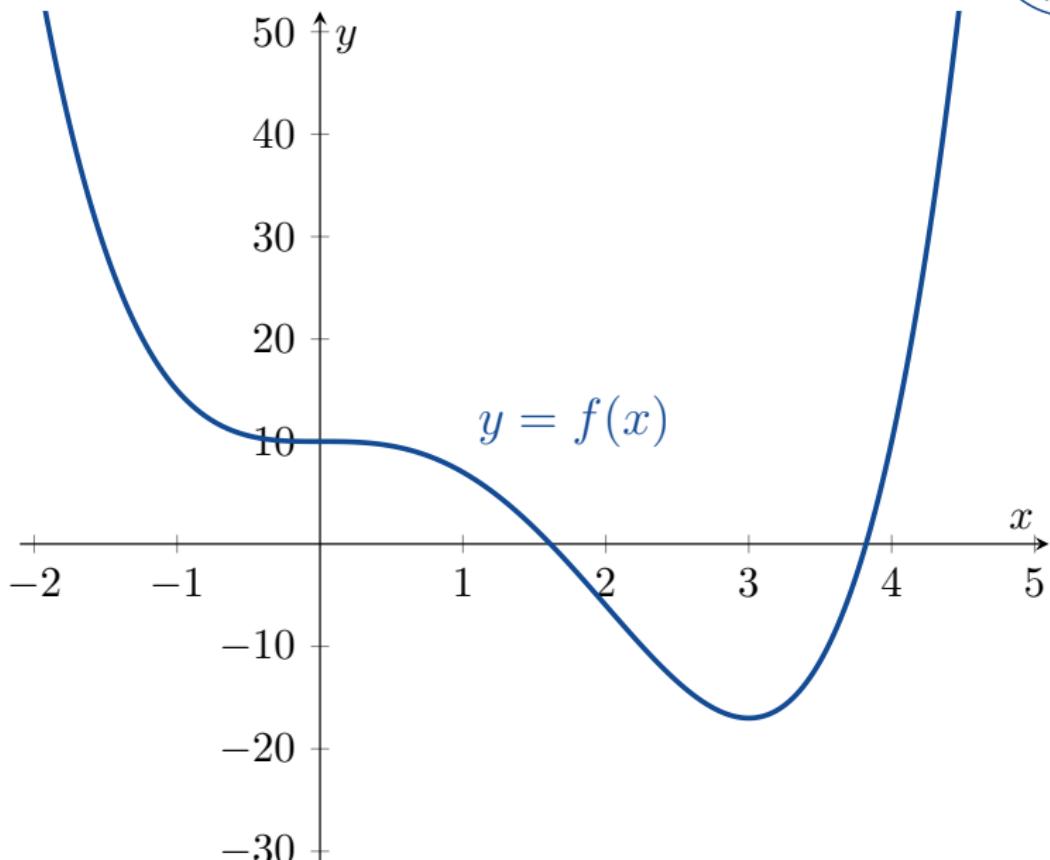


$y = f(x) + k$ shifts the graph up k units.
(or down $|k|$ units if $k < 0$)

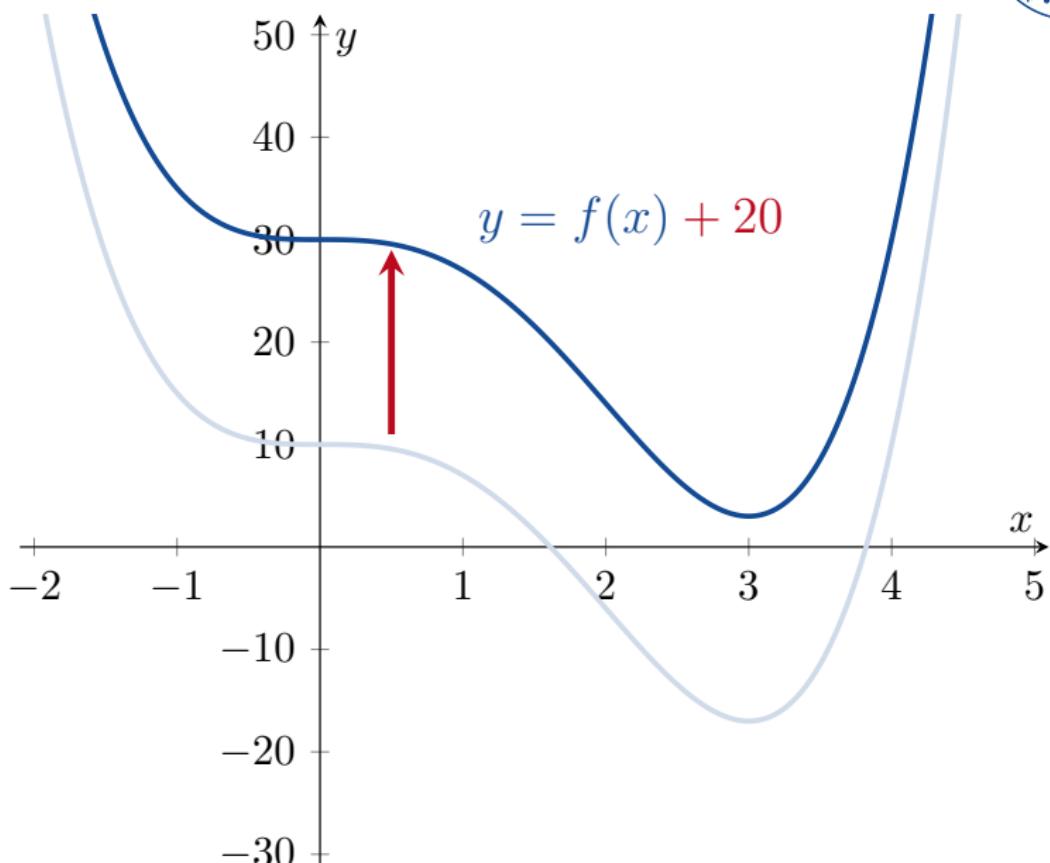


$y = f(x + k)$ shifts the graph left k units.
(or right $|k|$ units if $k < 0$)

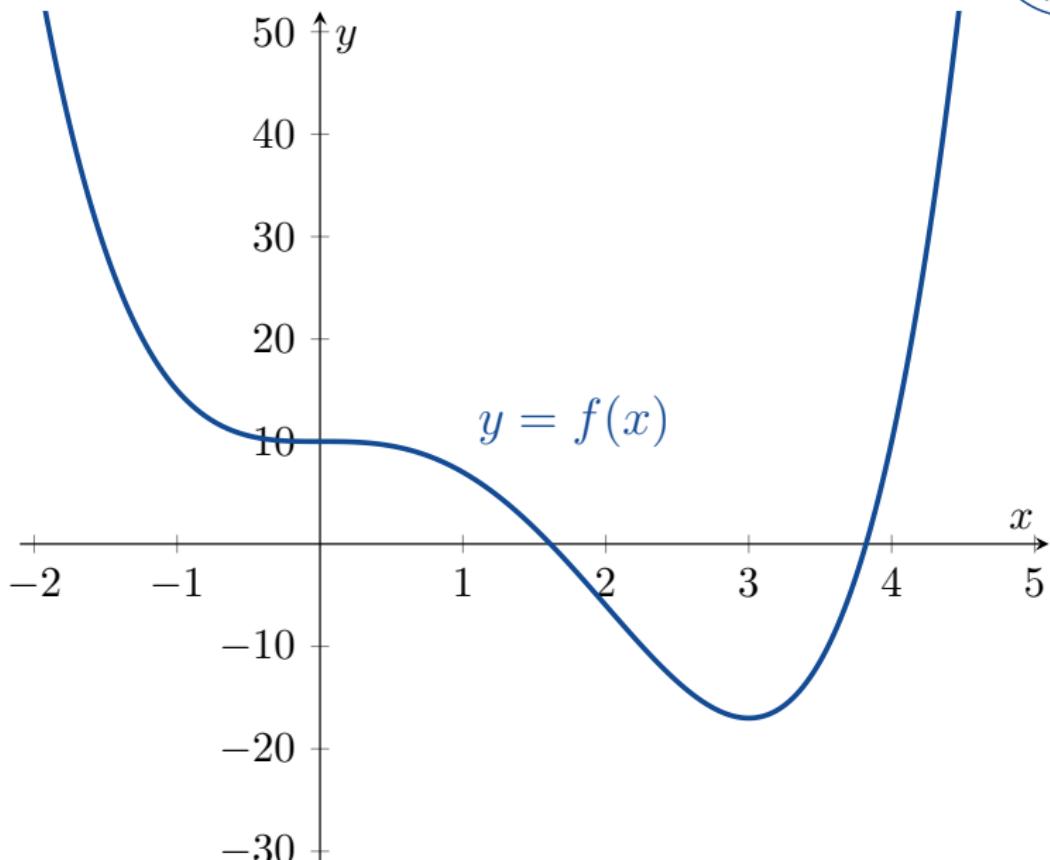
1.2 Combining Functions; Shifting and Scaling Graphs



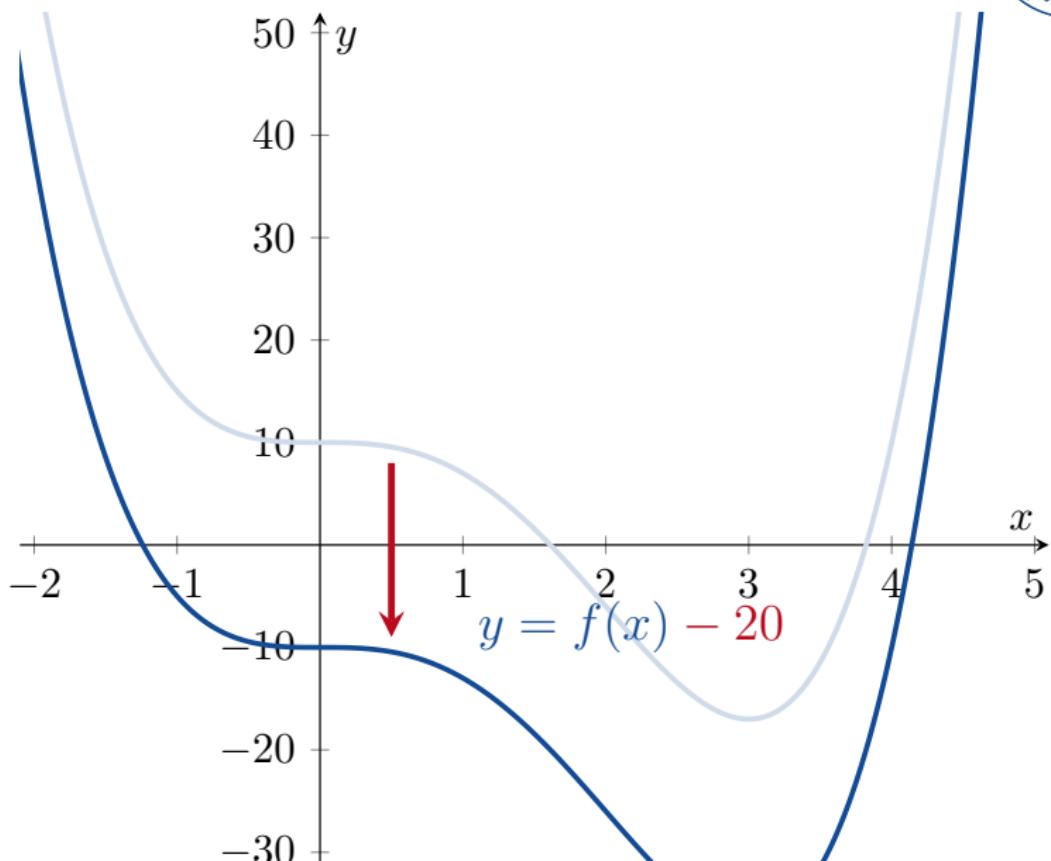
1.2 Combining Functions; Shifting and Scaling Graphs



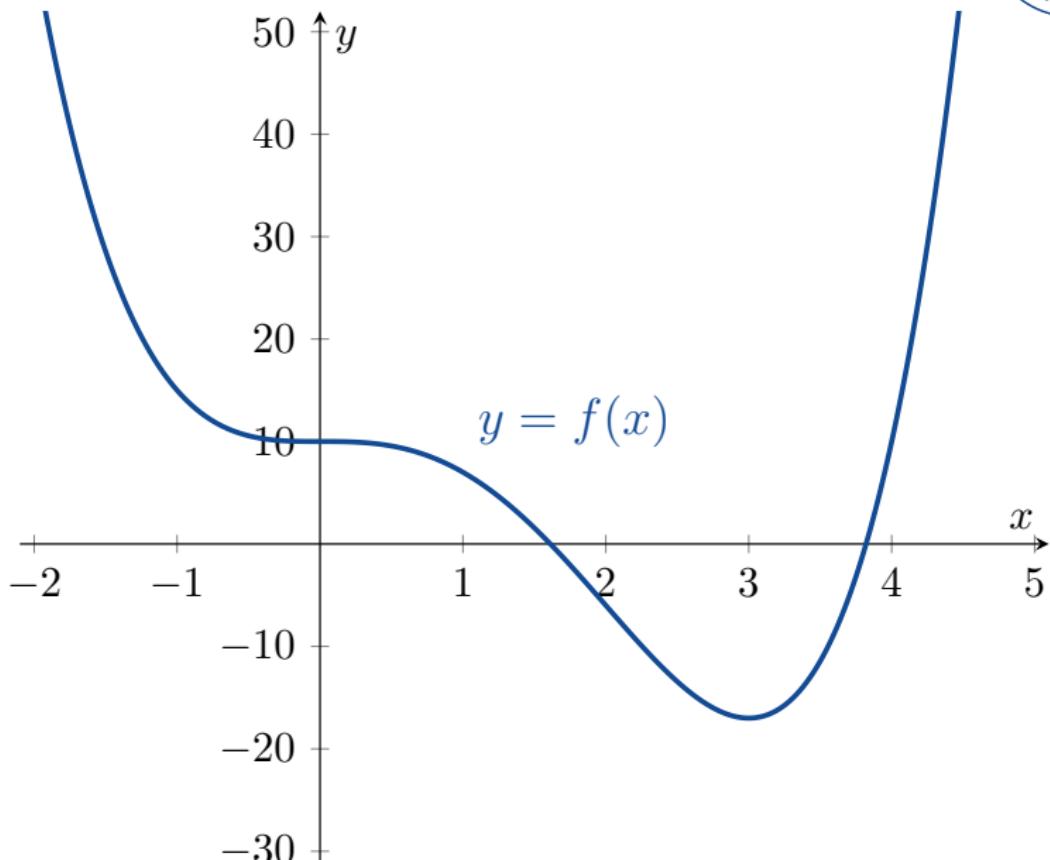
1.2 Combining Functions; Shifting and Scaling Graphs



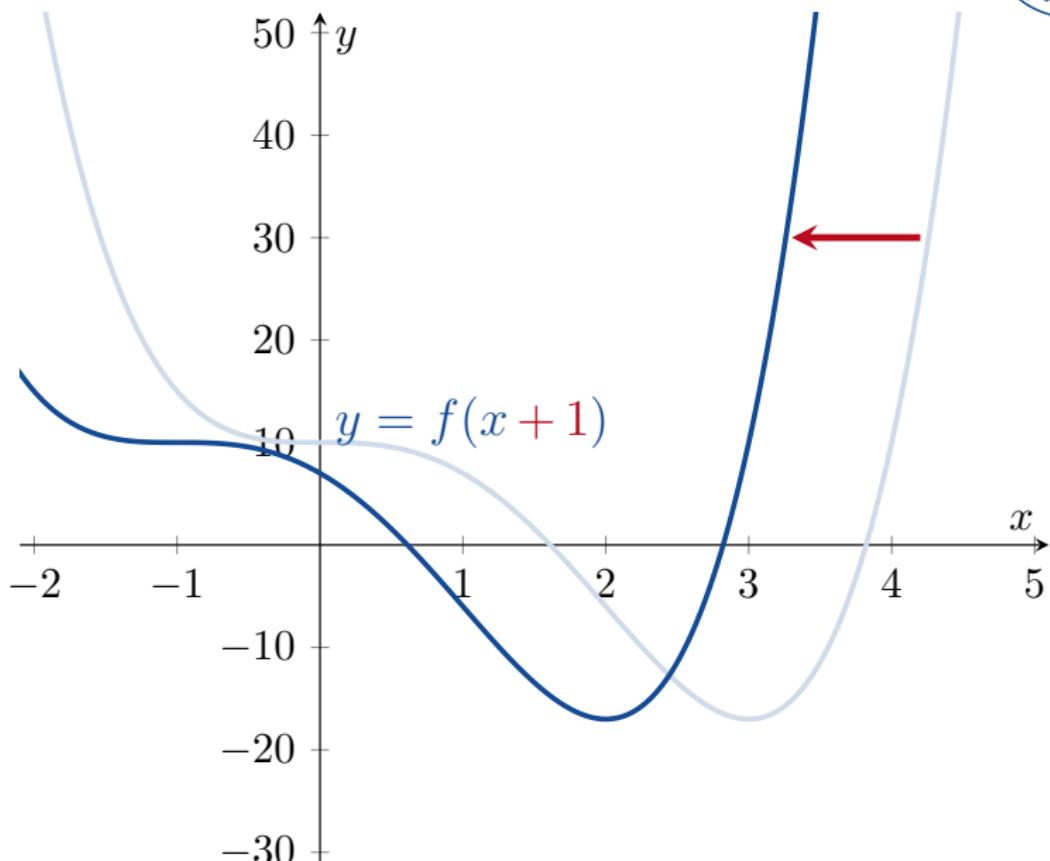
1.2 Combining Functions; Shifting and Scaling Graphs



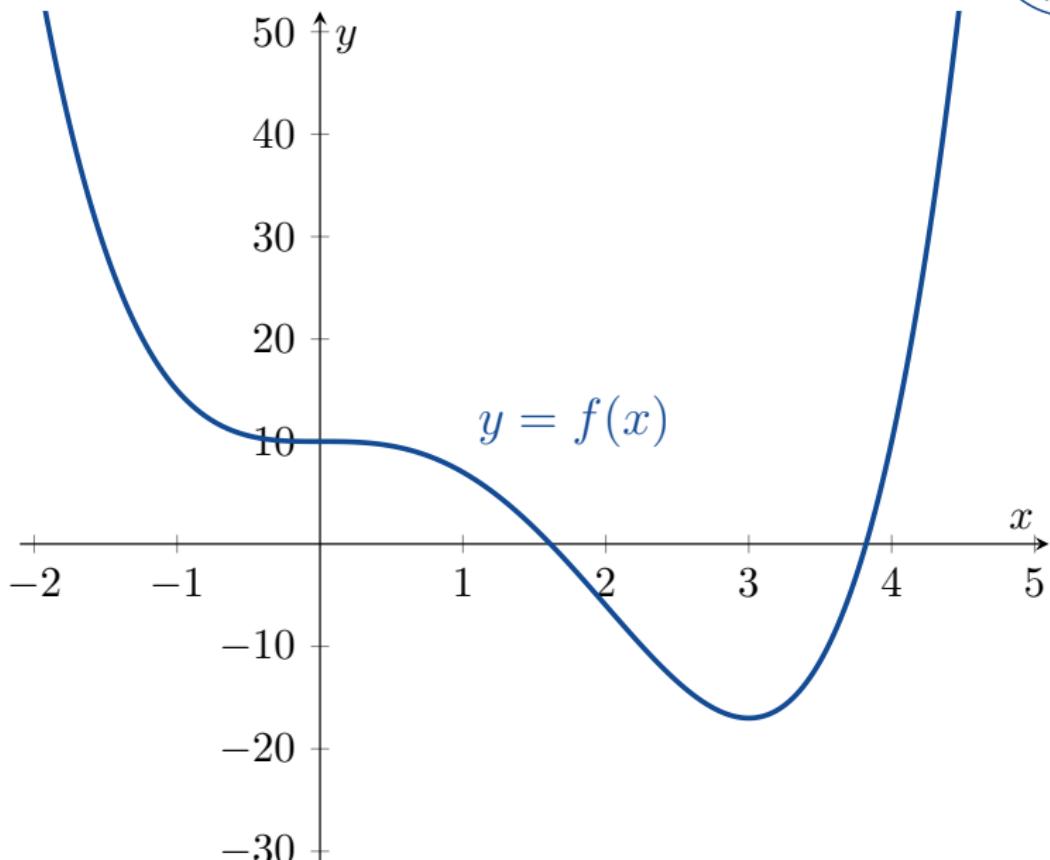
1.2 Combining Functions; Shifting and Scaling Graphs



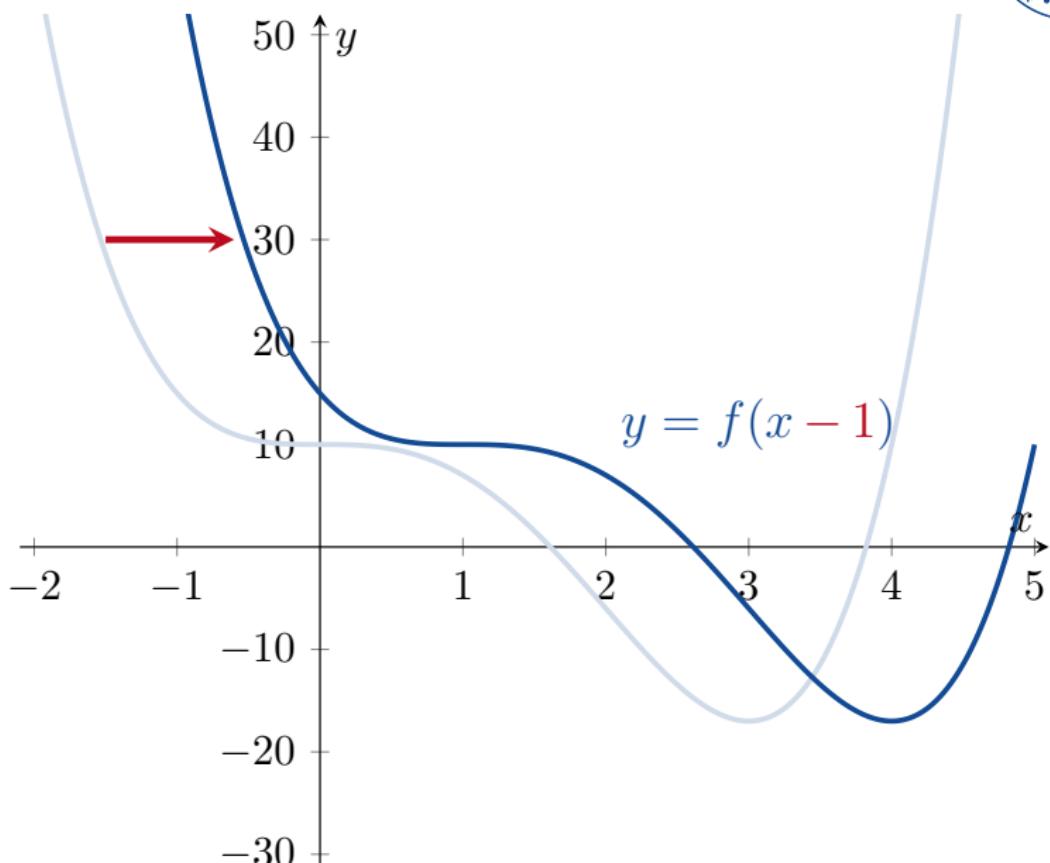
1.2 Combining Functions; Shifting and Scaling Graphs



1.2 Combining Functions; Shifting and Scaling Graphs



1.2 Combining Functions; Shifting and Scaling Graphs

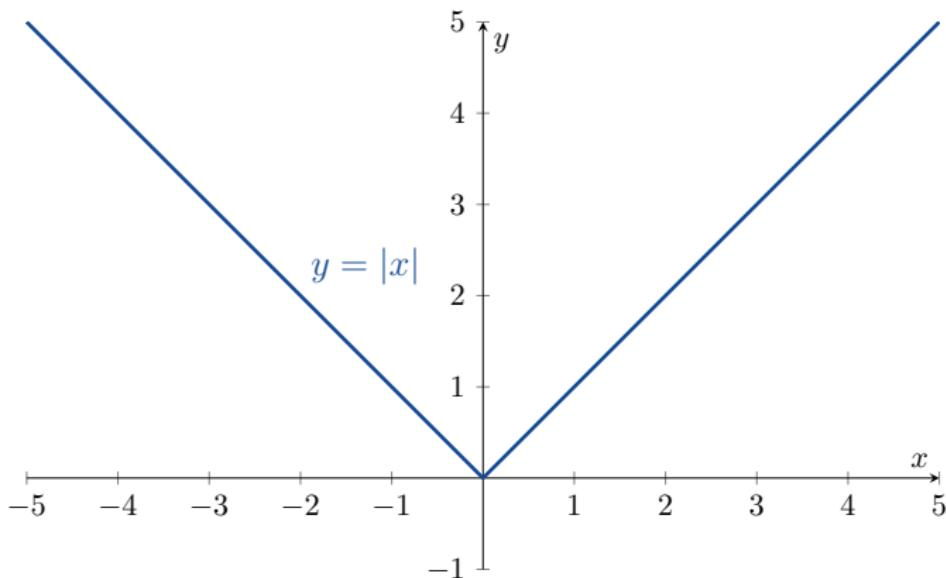


1.2 Combining Functions; Shifting and Scaling Graphs



Example

Shift the function $y = |x|$ by 2 units to the right and 1 unit up.

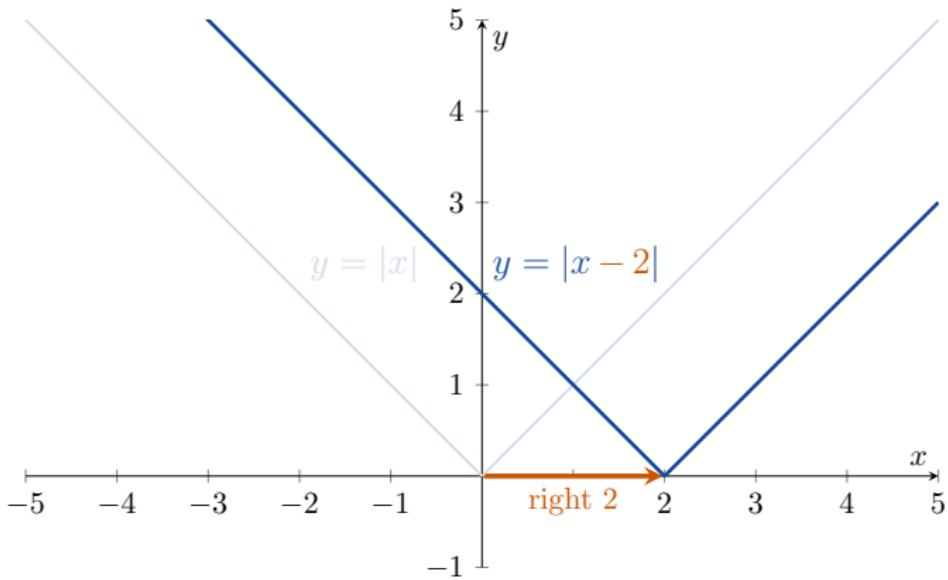


1.2 Combining Functions; Shifting and Scaling Graphs



Example

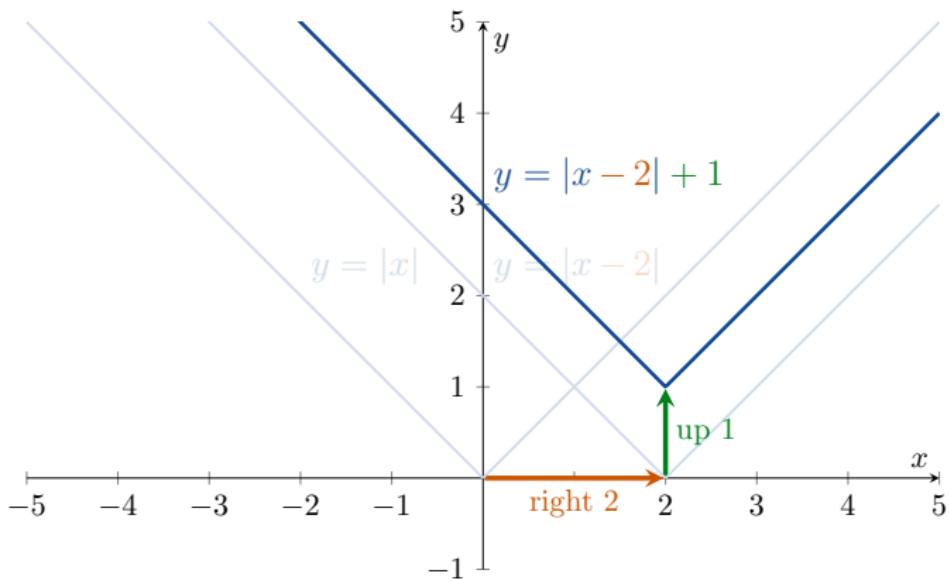
Shift the function $y = |x|$ by 2 units to the right and 1 unit up.



1.2 Combining Functions; Shifting and Scaling Graphs

Example

Shift the function $y = |x|$ by 2 units to the right and 1 unit up.



1.2 Combining Functions; Shifting and Scaling Graphs



Scaling and Reflecting

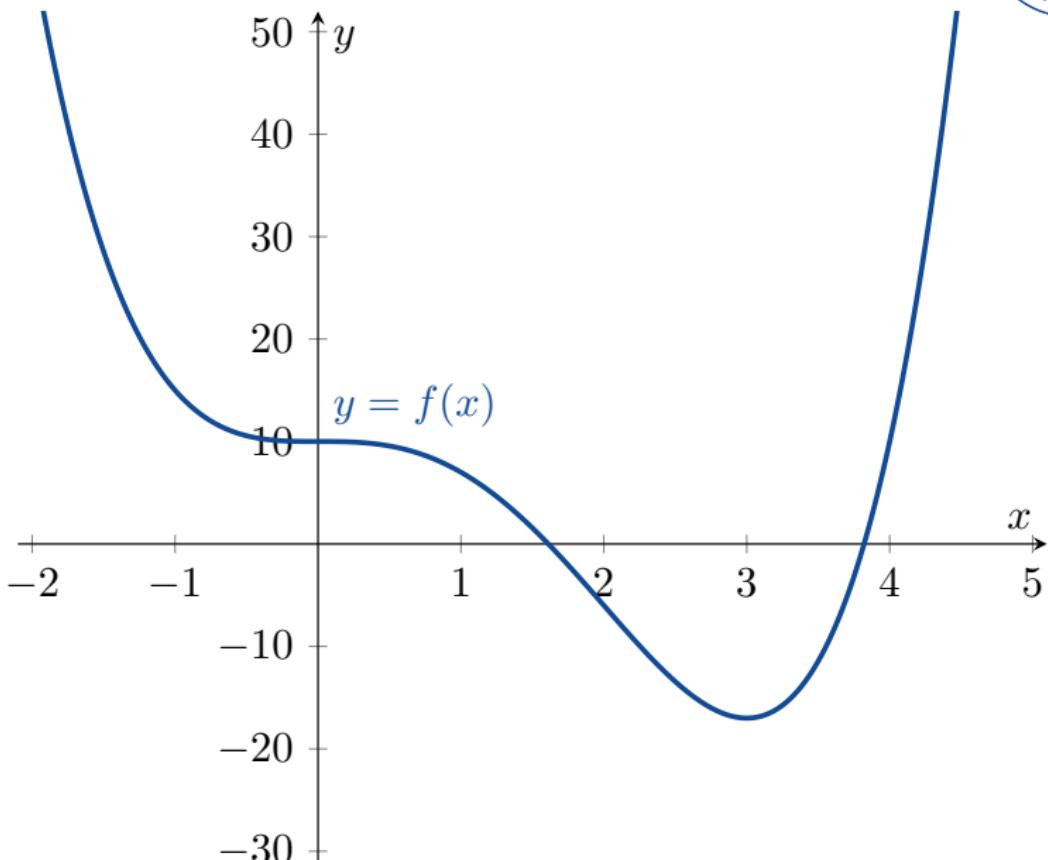
For $c > 1$, the graph is scaled:

- ↑ $y = cf(x)$ stretch vertically by a factor of c
- ↓ $y = \frac{1}{c}f(x)$ squash vertically by a factor of c
- ← $y = f(cx)$ squash horizontally by a factor of c
- ← → $y = f\left(\frac{x}{c}\right)$ stretch horizontally by a factor of c

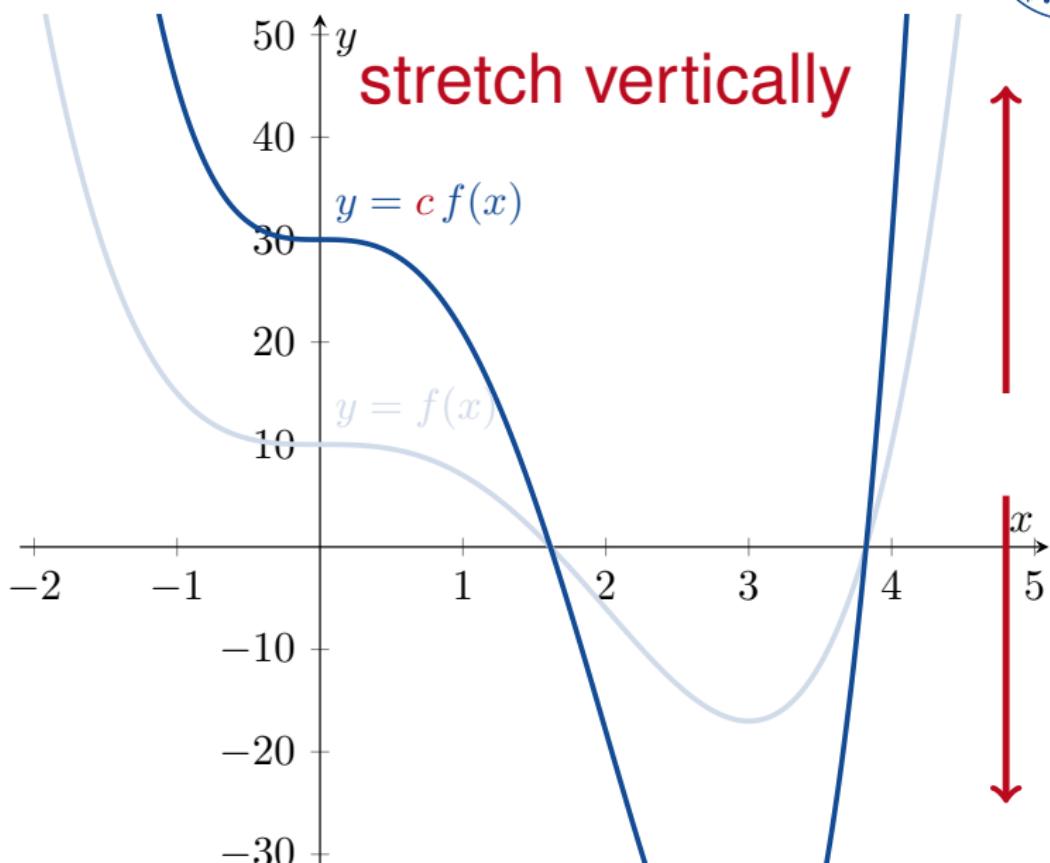
For $c = -1$, the graph is reflected:

- y
mirror $y = -f(x)$ reflect vertically, about the x -axis
- ↑
y
mirror $y = f(-x)$ reflect horizontally, about the y -axis

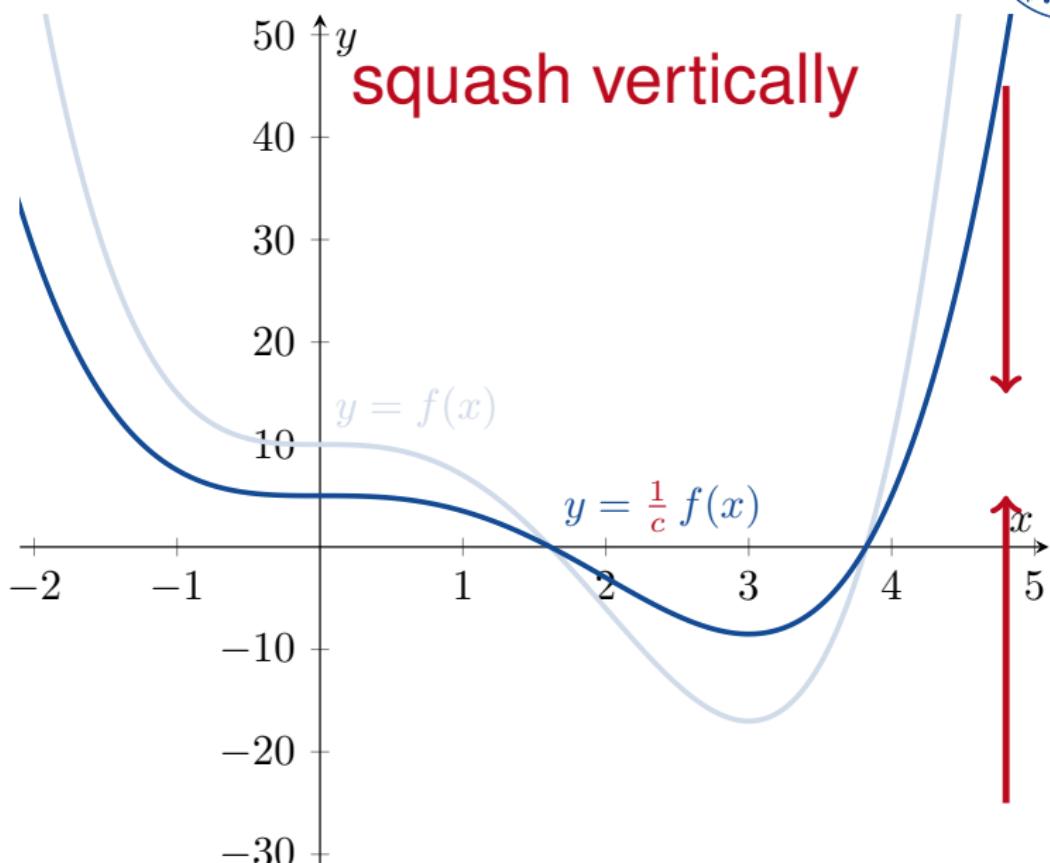
1.2 Combining Functions; Shifting and Scaling Graphs



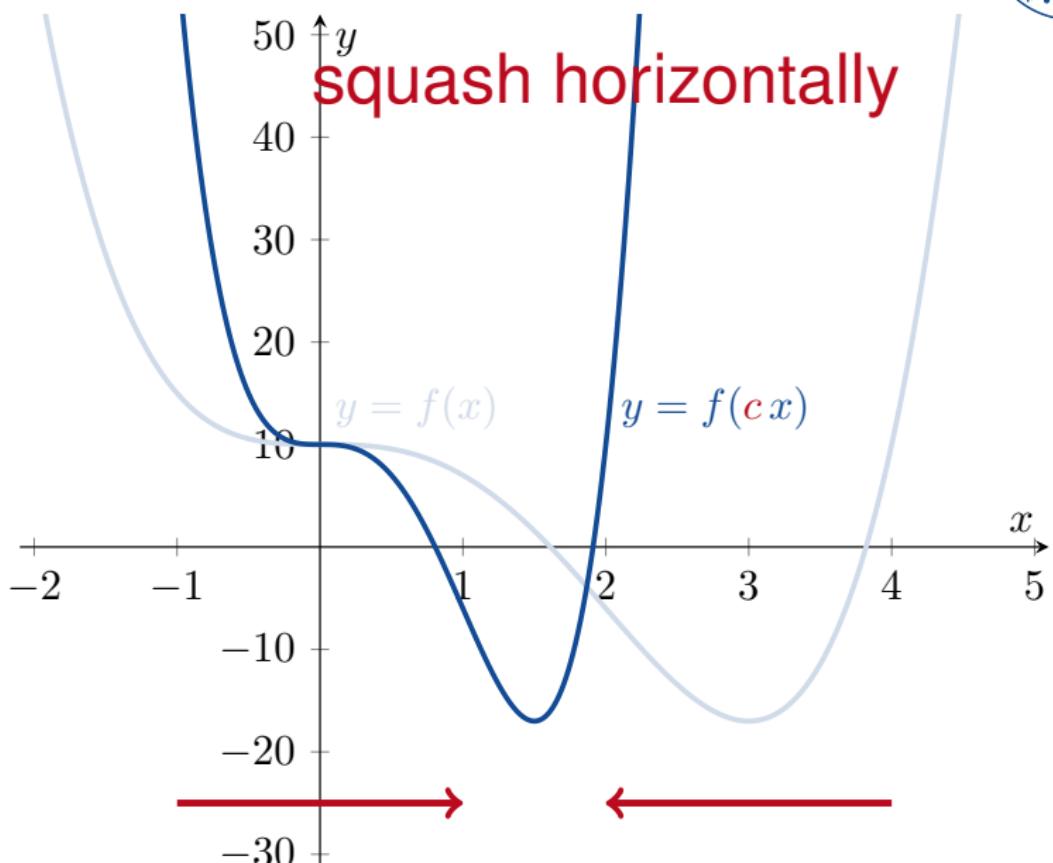
1.2 Combining Functions; Shifting and Scaling Graphs



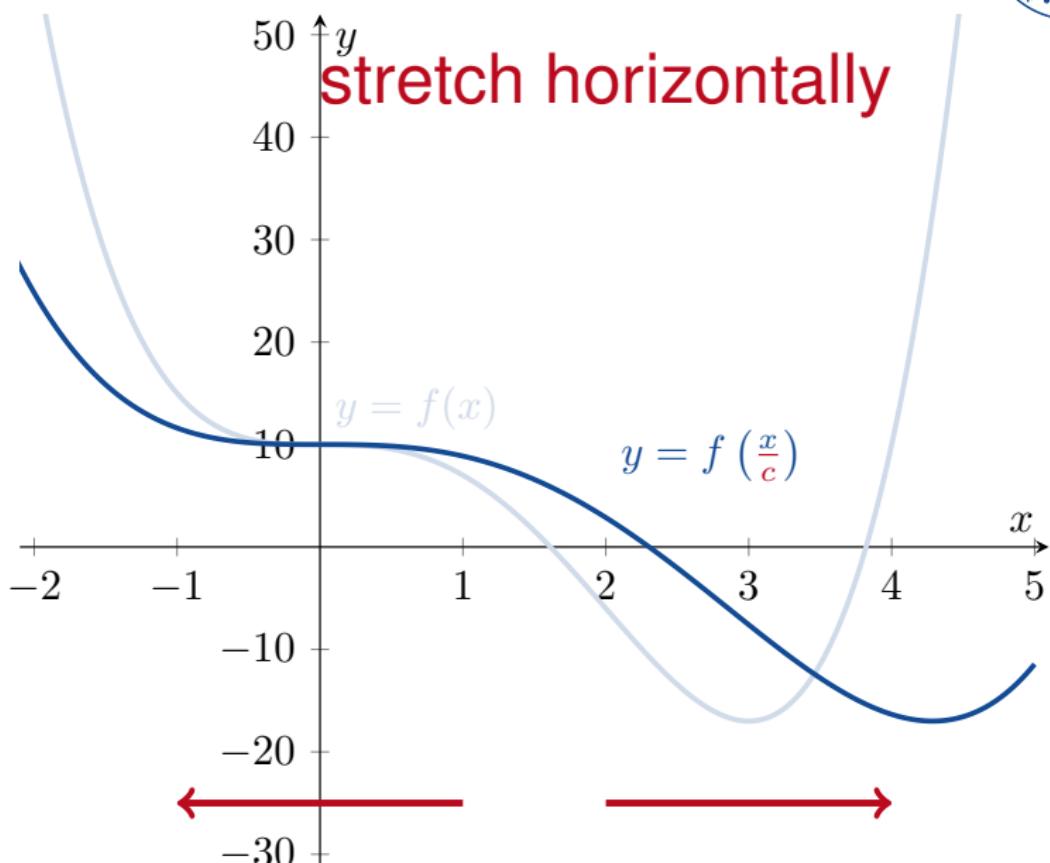
1.2 Combining Functions; Shifting and Scaling Graphs



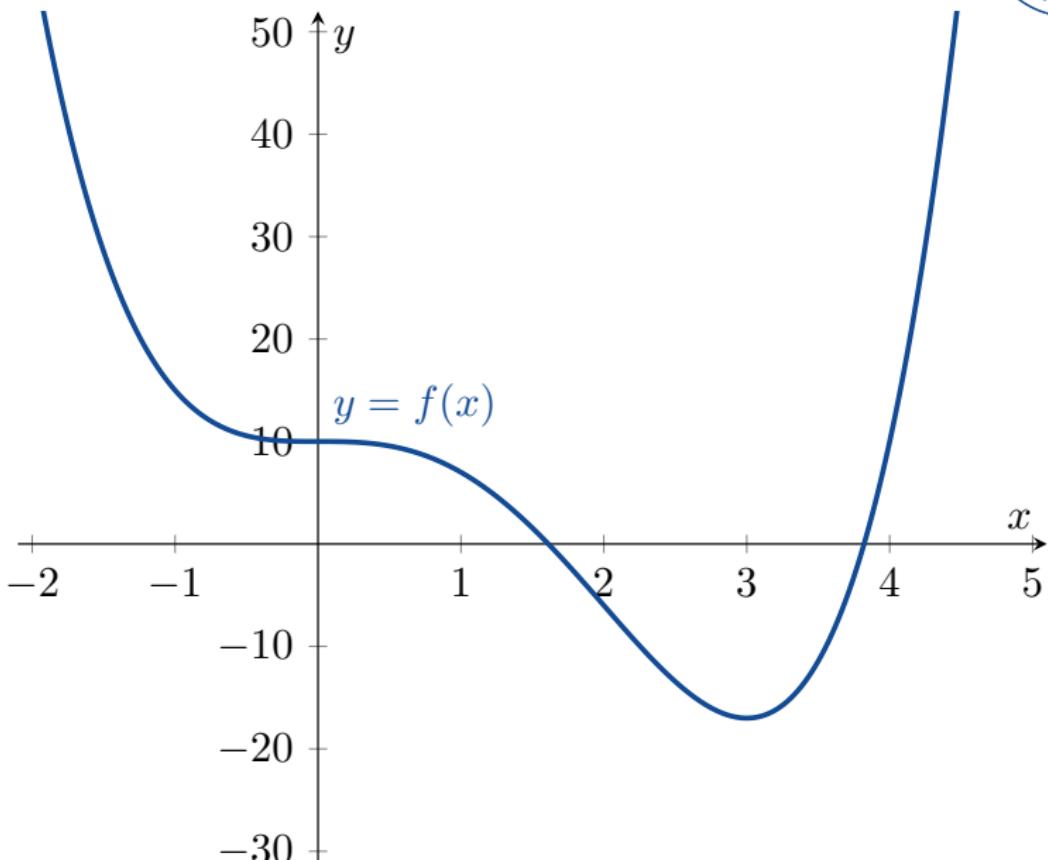
1.2 Combining Functions; Shifting and Scaling Graphs



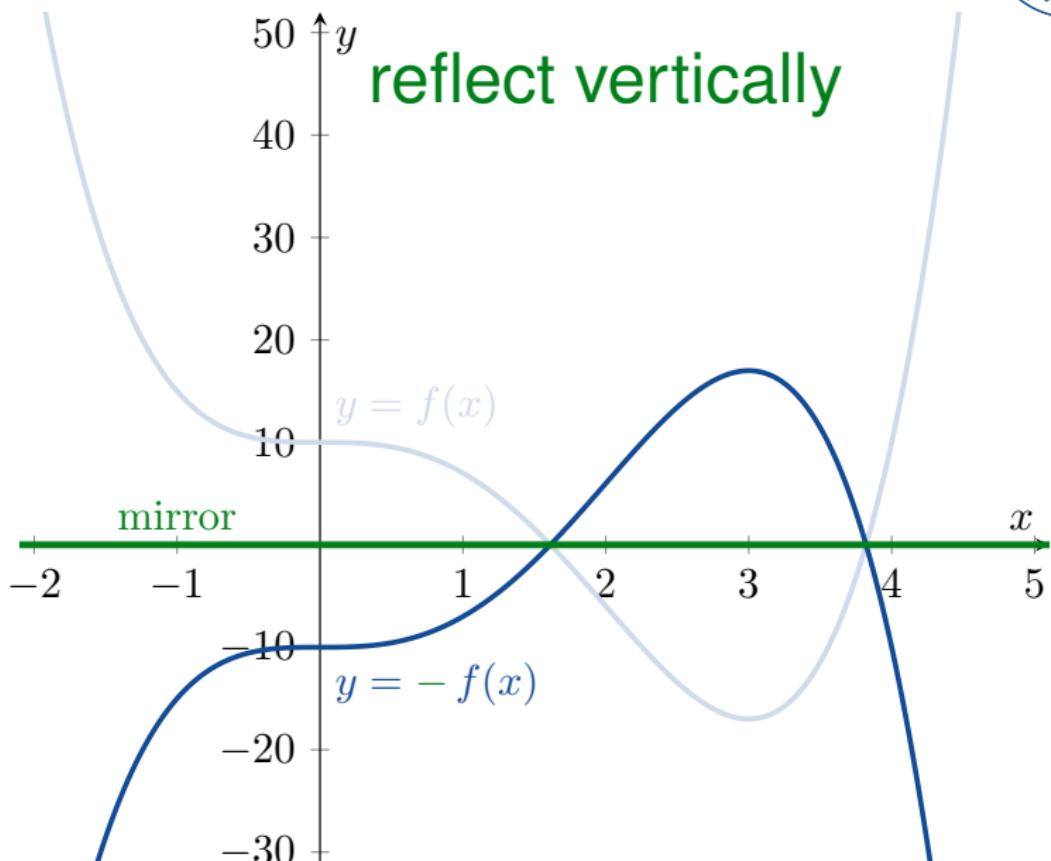
1.2 Combining Functions; Shifting and Scaling Graphs



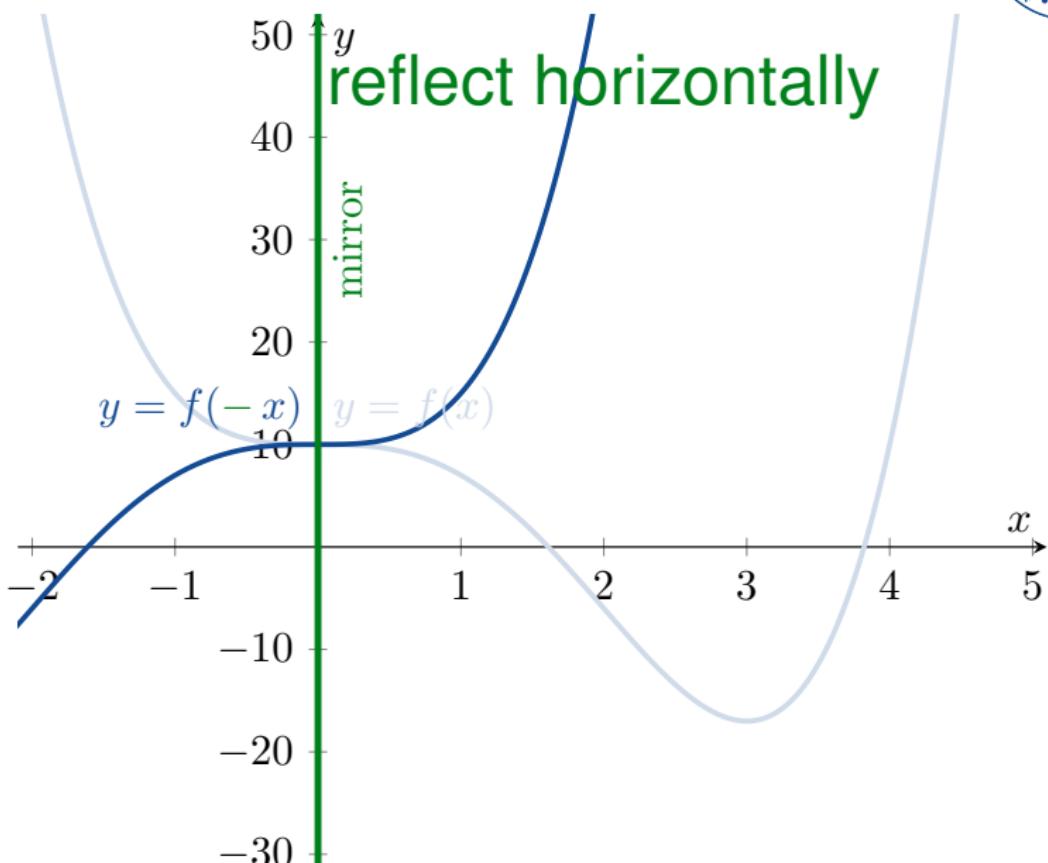
1.2 Combining Functions; Shifting and Scaling Graphs



1.2 Combining Functions; Shifting and Scaling Graphs



1.2 Combining Functions; Shifting and Scaling Graphs



1.2 Combining Functions; Shifting and Scaling Graphs



Please read Example 5 in your textbook.



13 Trigonometric Functions

1.3 Trigonometric Functions



Angles

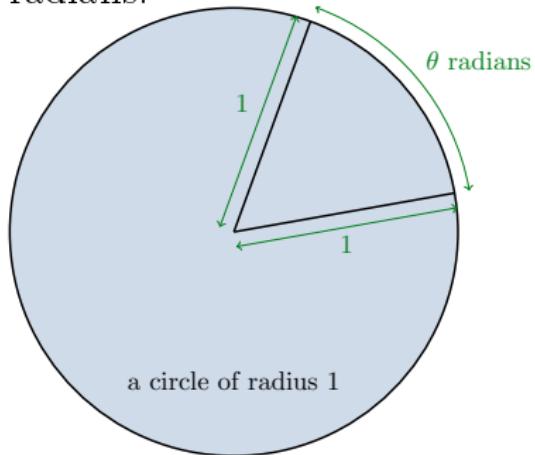
There are two ways to measure angles. Using degrees or using radians.

1.3 Trigonometric Functions



Angles

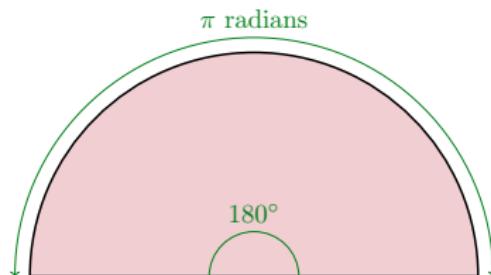
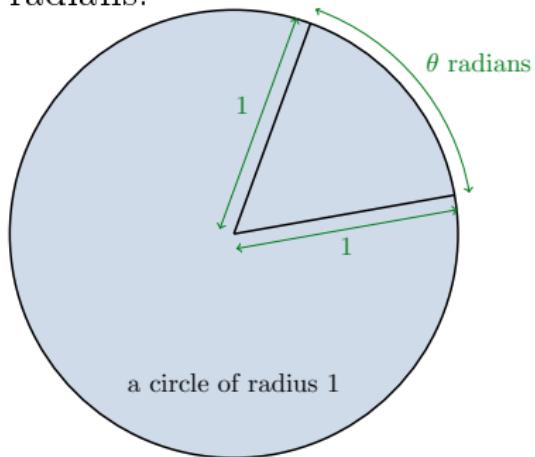
There are two ways to measure angles. Using degrees or using radians.



1.3 Trigonometric Functions

Angles

There are two ways to measure angles. Using degrees or using radians.



1.3 Trigonometric Functions



We have that

$$\pi \text{ radians} = 180 \text{ degrees}$$

1.3 Trigonometric Functions



We have that

$$\pi \text{ radians} = 180 \text{ degrees}$$

$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$

$$1 \text{ degree} = \frac{\pi}{180} \text{ radians.}$$

1.3 Trigonometric Functions

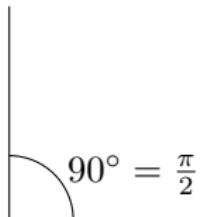
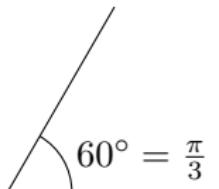
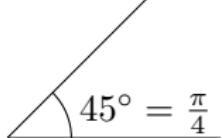


We have that

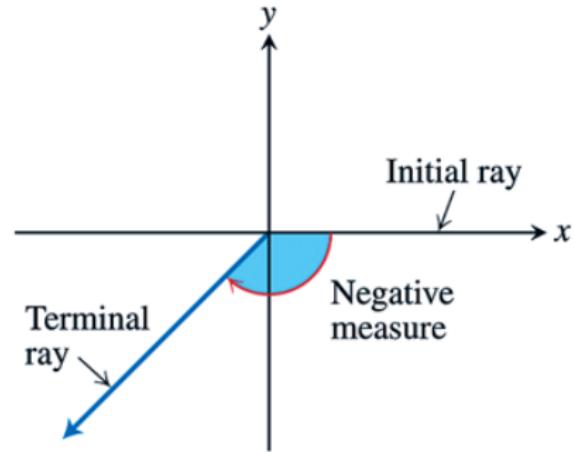
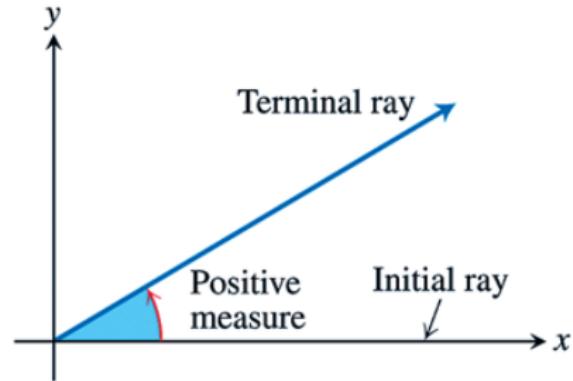
$$\pi \text{ radians} = 180 \text{ degrees}$$

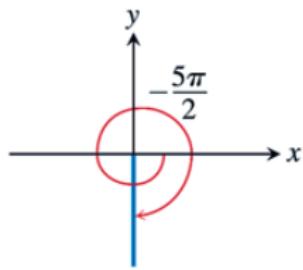
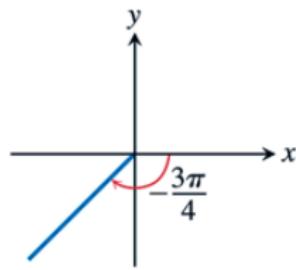
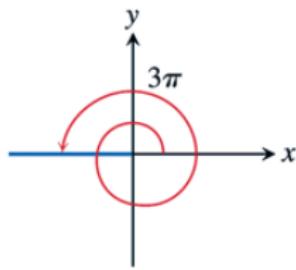
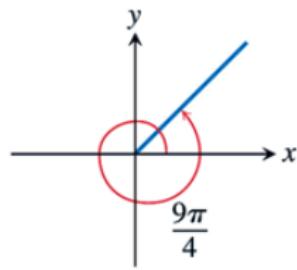
$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$

$$1 \text{ degree} = \frac{\pi}{180} \text{ radians.}$$



Degrees	-180	-135	-90	-45	0	30	45	60	90	120	135	150	180	270	360
θ (radians)	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π





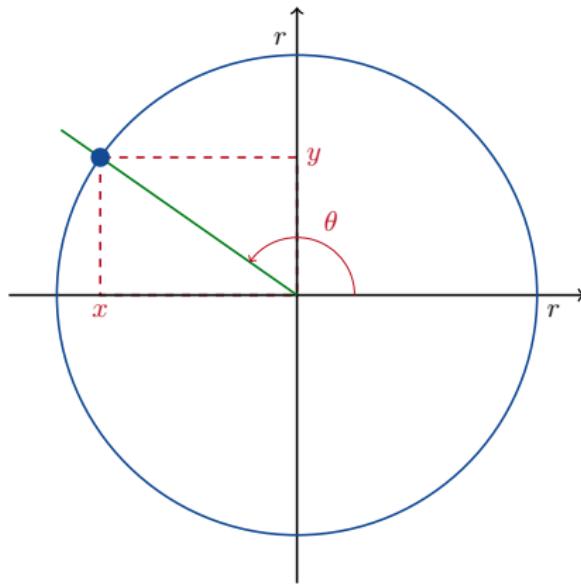
1.3 Trigonometric Functions



Remark

In Calculus, we use radians!!!! If you see an angle in this course, it will be in radians. Calculus doesn't work with degrees!!

6 Trigonometric Functions



sine

cosine

tangent

secant

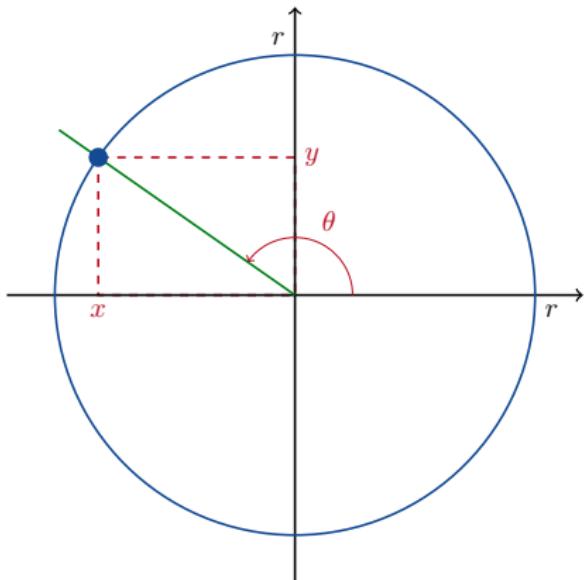
cosecant

cotangent

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

6 Trigonometric Functions



sine

$$\sin \theta = \frac{y}{r}$$

cosine

$$\cos \theta = \frac{x}{r}$$

tangent

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

secant

$$\sec \theta = \frac{1}{\cos \theta}$$

cosecant

$$\operatorname{cosec} \theta = \csc \theta = \frac{1}{\sin \theta}$$

cotangent

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

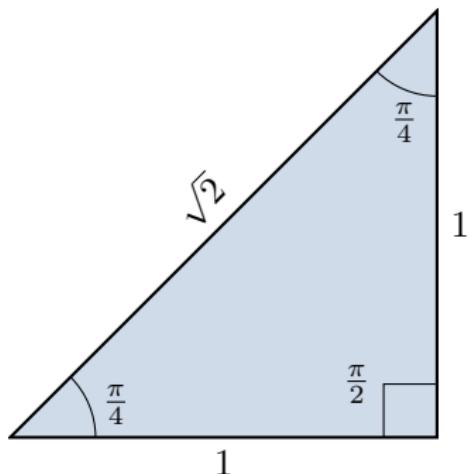
1.3 Trigonometric Functions



Remark

Note that $\tan \theta$ and $\sec \theta$ are only defined if $\cos \theta \neq 0$; and $\operatorname{cosec} \theta$ and $\cot \theta$ are only defined if $\sin \theta \neq 0$.

1.3 Trigonometric Functions



$$\sin 45^\circ = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

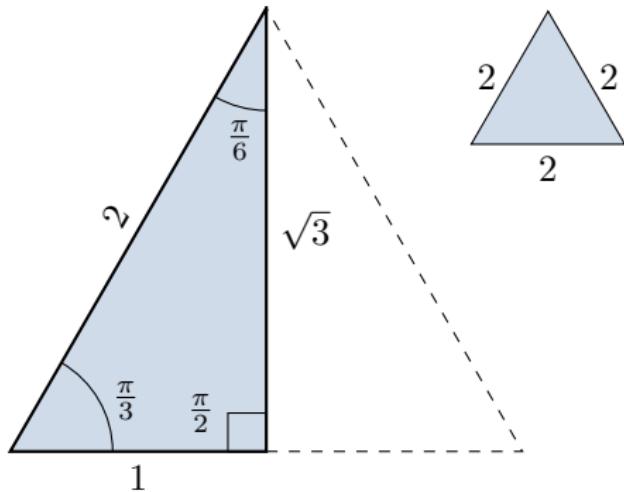
$$\tan 45^\circ = \tan \frac{\pi}{4} = 1$$

$$\sec 45^\circ = \sec \frac{\pi}{4} = \sqrt{2}$$

$$\operatorname{cosec} 45^\circ = \operatorname{cosec} \frac{\pi}{4} = \sqrt{2}$$

$$\cot 45^\circ = \cot \frac{\pi}{4} = 1$$

1.3 Trigonometric Functions



$$\sin 60^\circ = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \cos \frac{\pi}{3} = \frac{1}{2}$$

$$\tan 60^\circ = \tan \frac{\pi}{3} = \sqrt{3}$$

$$\sec 60^\circ = \sec \frac{\pi}{3} = 2$$

$$\operatorname{cosec} 60^\circ = \operatorname{cosec} \frac{\pi}{3} = \frac{2}{\sqrt{3}}$$

$$\cot 60^\circ = \cot \frac{\pi}{3} = \frac{1}{\sqrt{3}}$$

Degrees	-180	-135	-90	-45	0	30	45	60	90	120	135	150	180	270	360
θ (radians)	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
$\sin \theta$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\cos \theta$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	1
$\tan \theta$	0	1		-1	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$		$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0		0

1.3 Trigonometric Functions



Periodicity

Definition

A function $f(x)$ is called *periodic* if there exists a positive number p such that

$$f(x + p) = f(x)$$

for all x .

1.3 Trigonometric Functions



Periodicity

Definition

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The smallest such $p > 0$ is called the *period* of f .

1.3 Trigonometric Functions



Example

The period of $\tan x$ is π since

$$\tan(x + \pi) = \tan x$$

for all x , and this is the smallest $p > 0$ which works.

1.3 Trigonometric Functions

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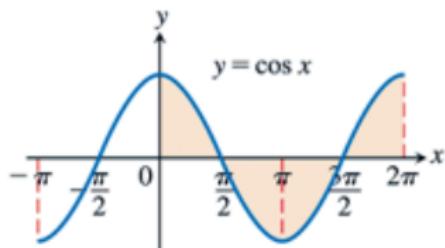
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Example

The period of $\sin x$ is 2π since

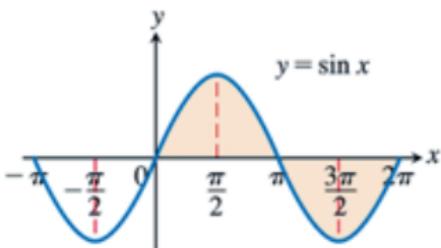
$$\sin(x + 2\pi) = \sin x$$

for all x , and this is the smallest $p > 0$ which works.



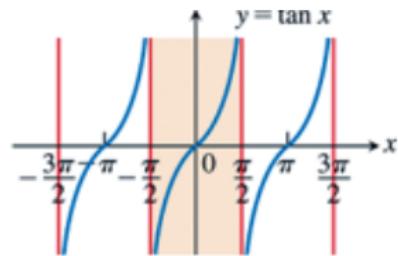
Domain: $-\infty < x < \infty$
 Range: $-1 \leq y \leq 1$
 Period: 2π

(a)



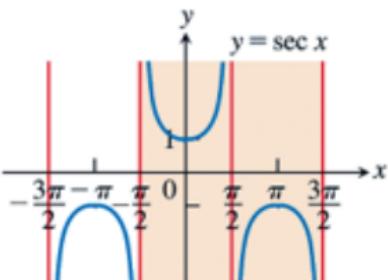
Domain: $-\infty < x < \infty$
 Range: $-1 \leq y \leq 1$
 Period: 2π

(b)



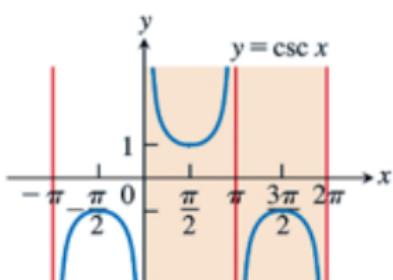
Domain: $x \neq \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \dots$
 Range: $-\infty < y < \infty$
 Period: π

(c)



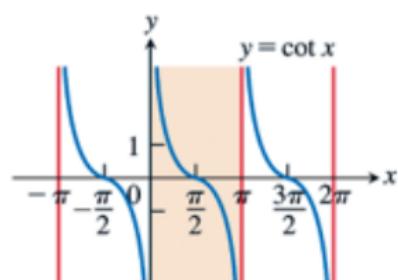
Domain: $x \neq \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \dots$
 Range: $y \leq -1$ or $y \geq 1$
 Period: 2π

(d)



Domain: $x \neq 0, \pm \pi, \pm 2\pi, \dots$
 Range: $y \leq -1$ or $y \geq 1$
 Period: 2π

(e)



Domain: $x \neq 0, \pm \pi, \pm 2\pi, \dots$
 Range: $-\infty < y < \infty$
 Period: π

(f)

Trigonometric Identities

$$\cos \theta = \sin \left(\frac{\pi}{2} - \theta \right)$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

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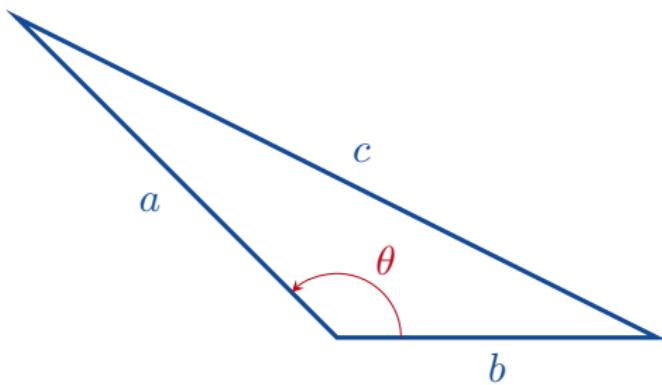
$$\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$$

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1.3 Trigonometric Functions

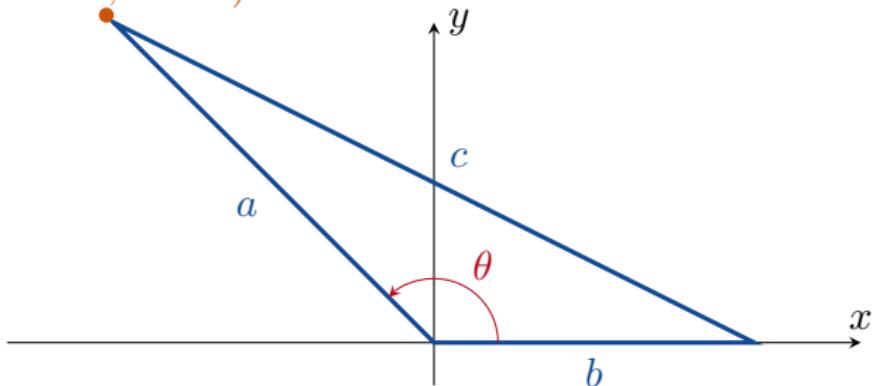


The Law of Cosines



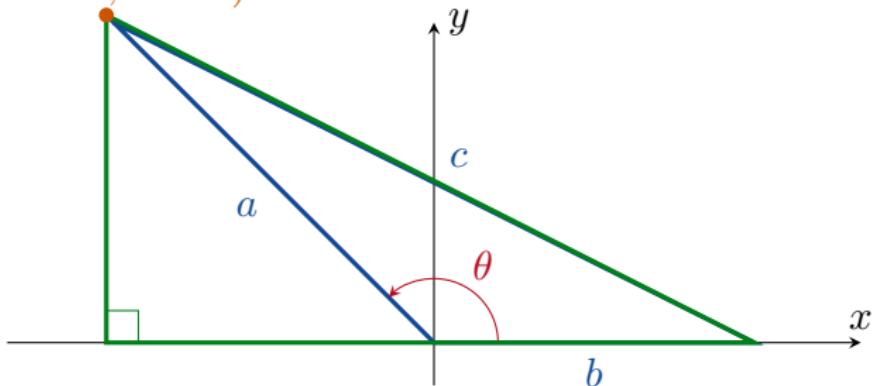
The Law of Cosines

$$(a \cos \theta, a \sin \theta)$$

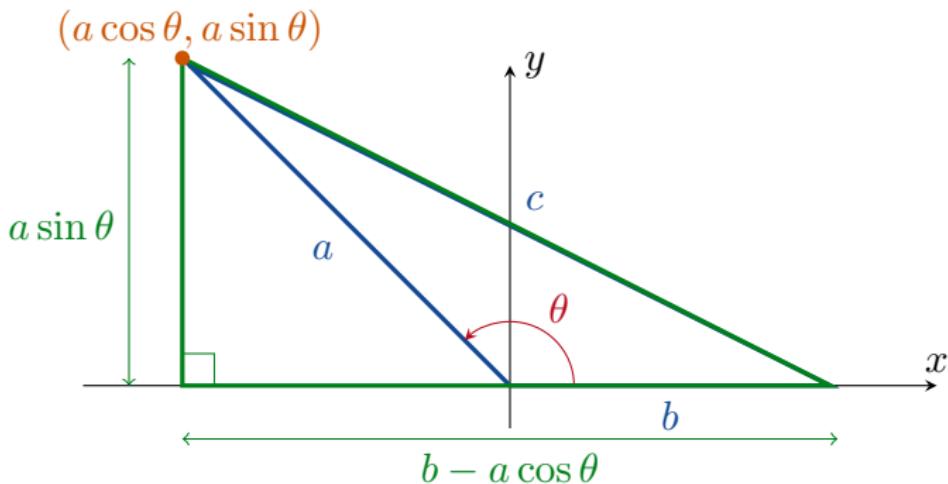


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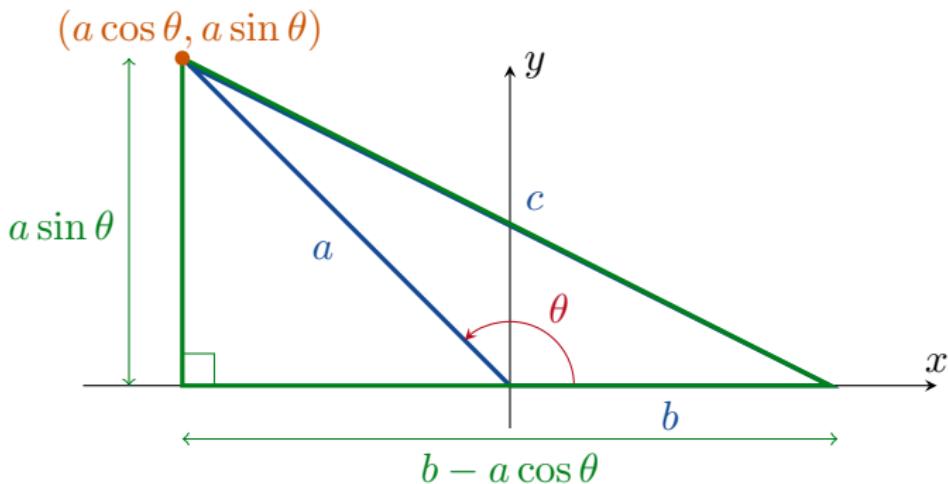
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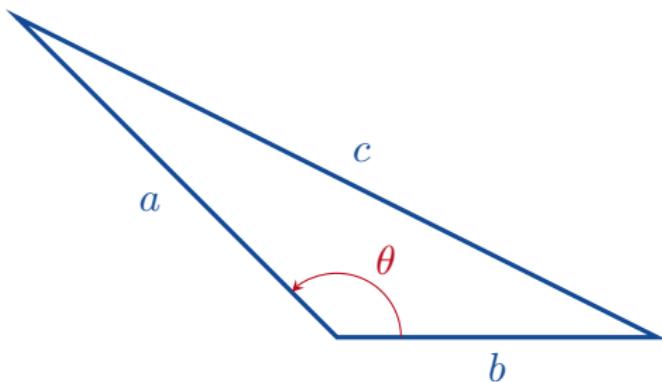
The Law of Cosines



By Pythagoras we have that

$$c^2 = (b - a \cos \theta)^2 + (a \sin \theta)^2$$

1.3 Trigonometric Functions

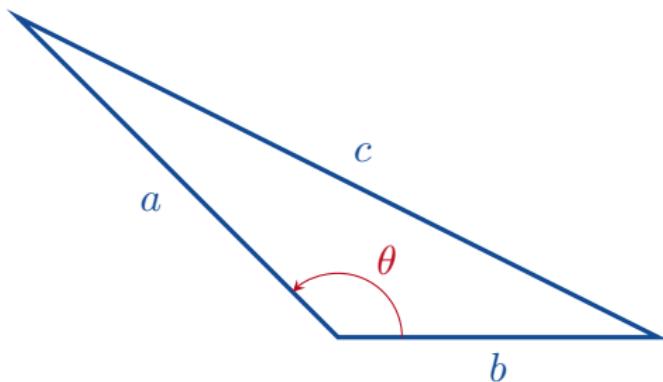


$$c^2 = (b - a \cos \theta)^2 + (a \sin \theta)^2$$

=

=

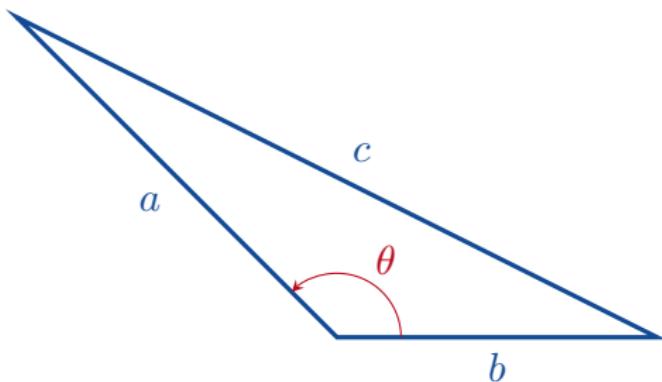
1.3 Trigonometric Functions



$$\begin{aligned}c^2 &= (b - a \cos \theta)^2 + (a \sin \theta)^2 \\&= b^2 - 2a \cos \theta + a^2 \cos^2 \theta + a^2 \sin^2 \theta \\&= b^2 - 2a \cos \theta + a^2\end{aligned}$$

=

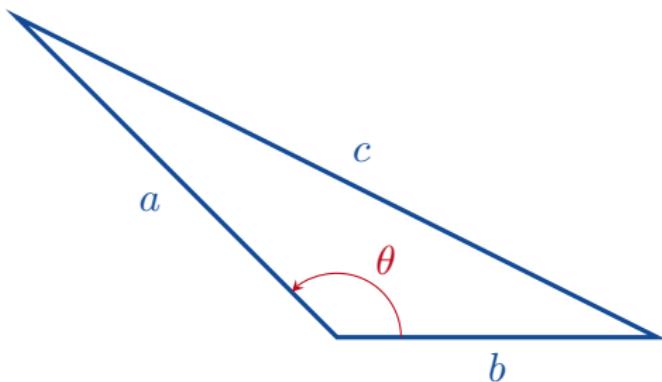
1.3 Trigonometric Functions



$$\begin{aligned}c^2 &= (b - a \cos \theta)^2 + (a \sin \theta)^2 \\&= b^2 - 2a \cos \theta + a^2 \cos^2 \theta + a^2 \sin^2 \theta \\&= b^2 - 2a \cos \theta + a^2 (\cos^2 \theta + \sin^2 \theta) \\&= b^2 - 2a \cos \theta + a^2\end{aligned}$$

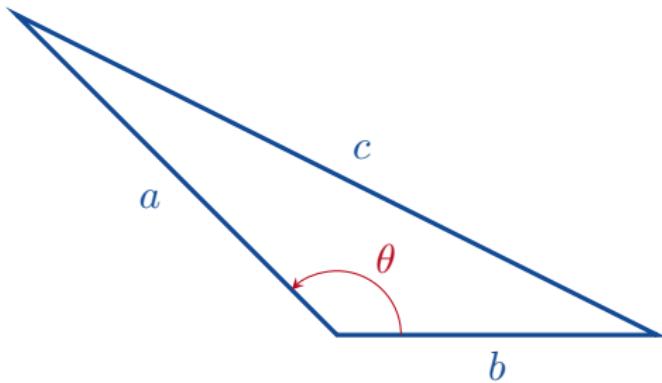
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1.3 Trigonometric Functions



$$\begin{aligned}c^2 &= (b - a \cos \theta)^2 + (a \sin \theta)^2 \\&= b^2 - 2a \cos \theta + a^2 \cos^2 \theta + a^2 \sin^2 \theta \\&= a^2 + b^2 - 2a \cos \theta\end{aligned}$$

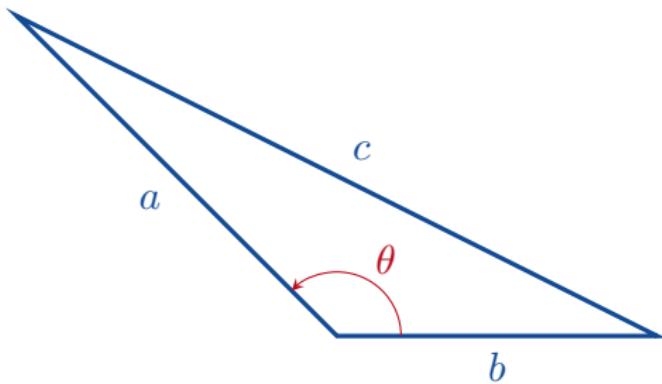
1.3 Trigonometric Functions



Theorem (The Law of Cosines)

$$c^2 = a^2 + b^2 - 2ab \cos \theta$$

1.3 Trigonometric Functions



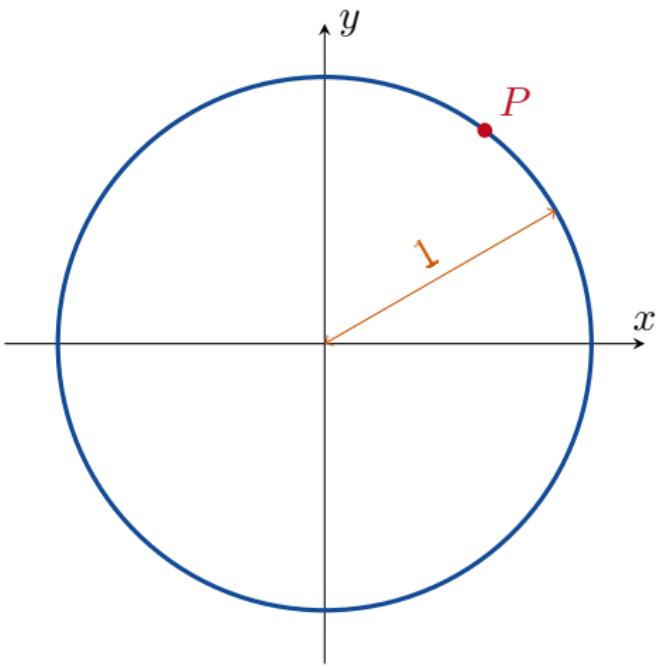
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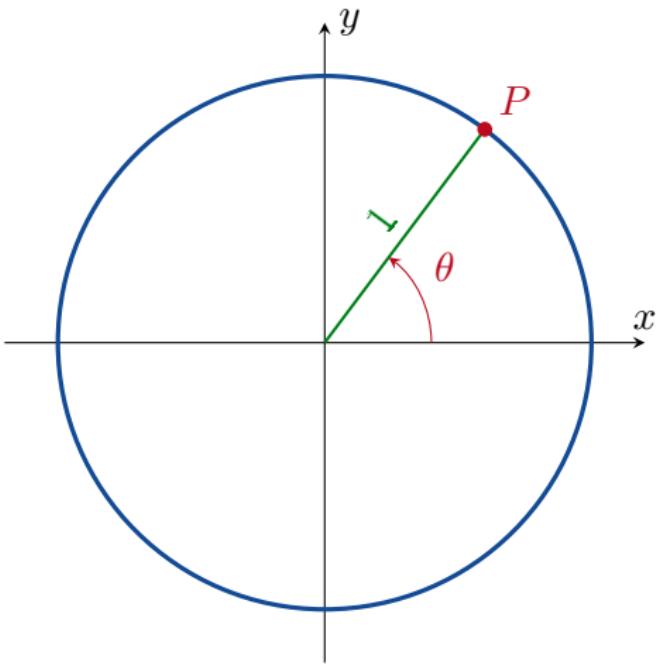
Remark

If $\theta = \frac{\pi}{2}$, then we just get $c^2 = a^2 + b^2$.

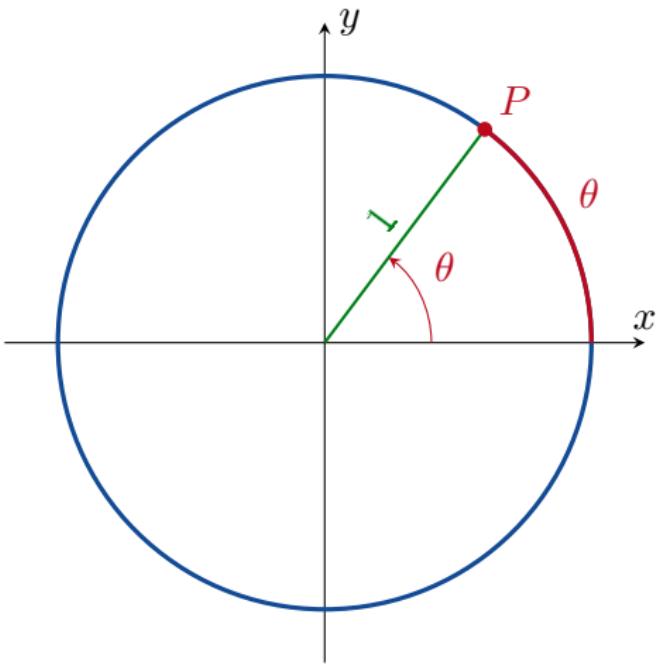
2 Special Inequalities



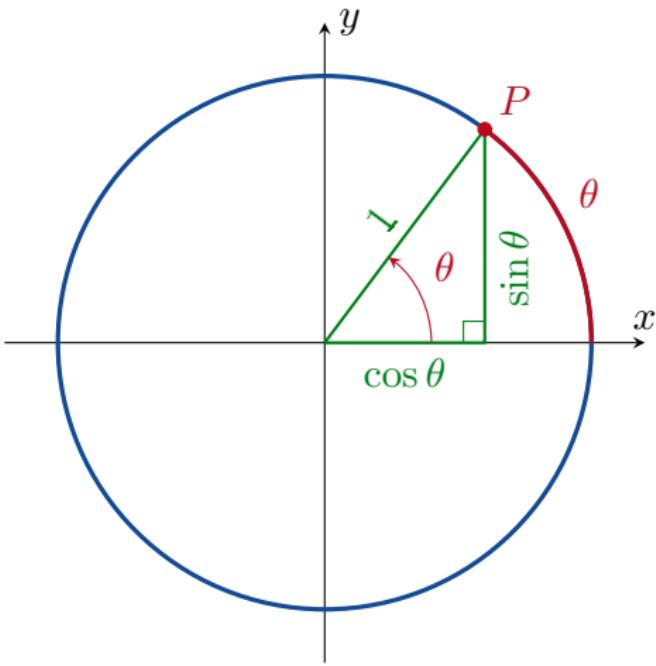
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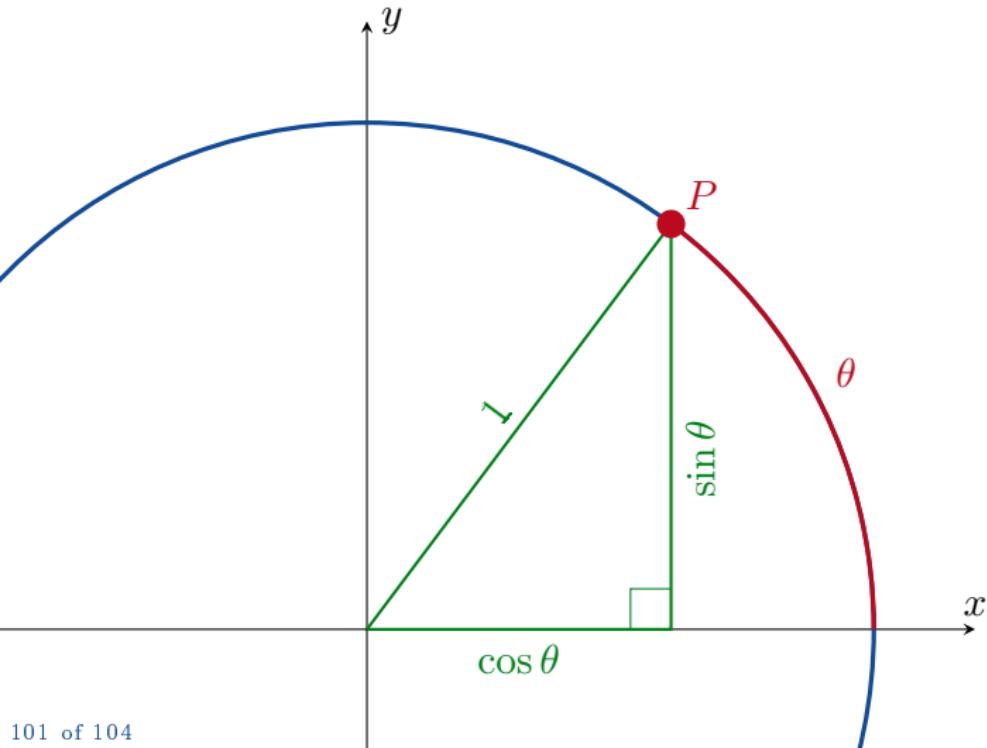
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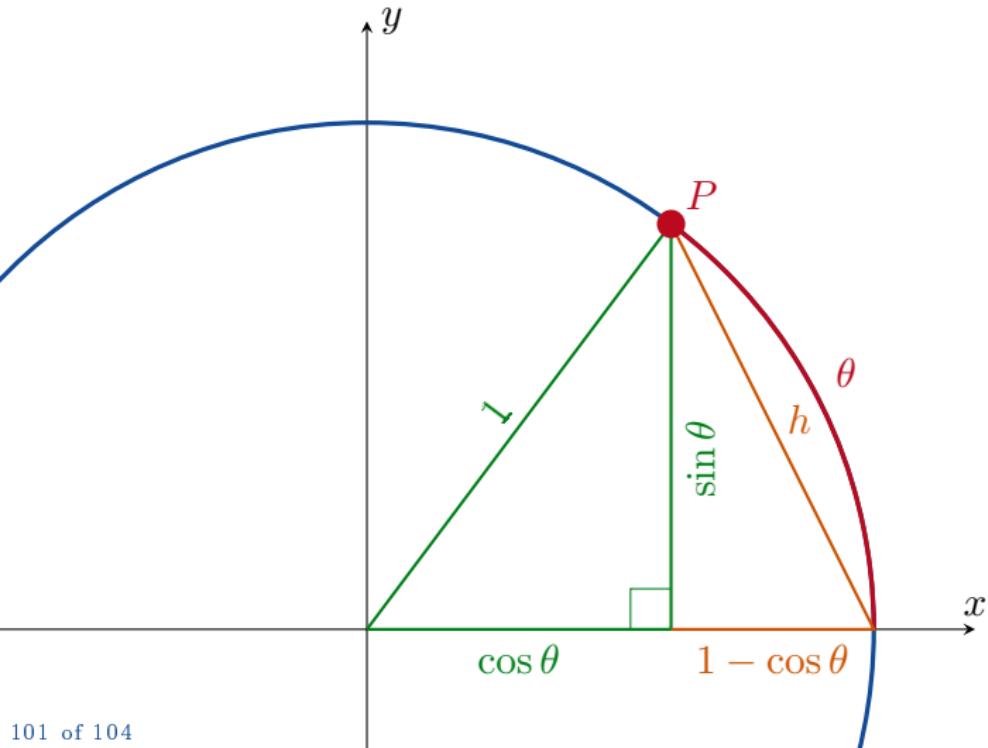
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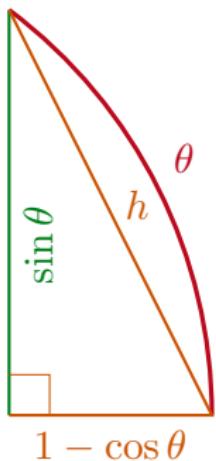
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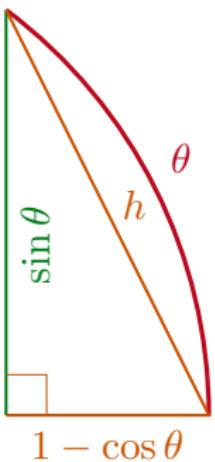
2 Special Inequalities



Since

$$h \leq \theta$$

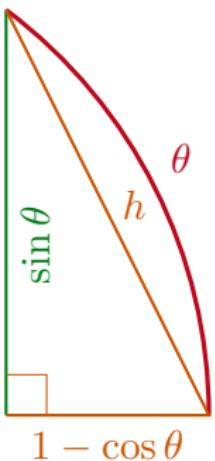
2 Special Inequalities



Since

$$h \leq \theta$$
$$h^2 \leq \theta^2$$

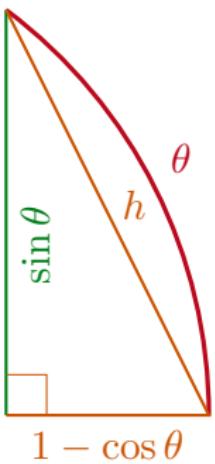
2 Special Inequalities



Since

$$\begin{aligned} h &\leq \theta \\ h^2 &\leq \theta^2 \\ (1 - \cos \theta)^2 + \sin^2 \theta &\leq \theta^2 \end{aligned}$$

2 Special Inequalities



Since

$$h \leq \theta$$

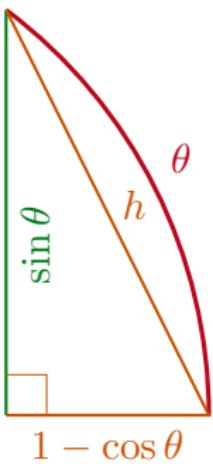
$$h^2 \leq \theta^2$$

$$(1 - \cos \theta)^2 + \sin^2 \theta \leq \theta^2$$

we have that

$$(1 - \cos \theta)^2 \leq \theta^2 \quad \text{and} \quad \sin^2 \theta \leq \theta^2.$$

2 Special Inequalities



Since

$$h \leq \theta$$

$$h^2 \leq \theta^2$$

$$(1 - \cos \theta)^2 + \sin^2 \theta \leq \theta^2$$

we have that

$$(1 - \cos \theta)^2 \leq \theta^2 \quad \text{and} \quad \sin^2 \theta \leq \theta^2.$$

It follows that

$- \theta \leq \sin \theta \leq \theta $	and	$- \theta \leq 1 - \cos \theta \leq \theta .$
--	--------------	---

1.3 Trigonometric Functions



Remark

We will need

$$-|\theta| \leq \sin \theta \leq |\theta| \quad \text{and} \quad -|\theta| \leq 1 - \cos \theta \leq |\theta|$$

next week.

Next Time

- 2.1 Rates of Change and Tangents to Curves
- 2.2 Limit of a Function and Limit Laws
- 2.3 The Precise Definition of a Limit