36. (a). Let $u = 3x^2 + 5$. Then we have that

$$f'(x) = \frac{d}{dx}u^5 = \left(\frac{d}{du}u^5\right)\left(\frac{du}{dx}\right) = (5u^4)(6x) = 30x(3x^2+5)^4$$

by the Chain Rule.

(b). Now let $u = x^3 - 6x + 10$. It follows that

$$g'(x) = \frac{d}{dx}\cos u = \left(\frac{d}{du}\cos u\right)\left(\frac{du}{dx}\right) = (-\sin u)(3x^2-6) = (6-3x^2)\sin(x^3-6x+10)$$

by the Chain Rule.

(c). *The first version of this homework had a misprint on this question. I will accept correct derivatives of either function. Both solutions are given here.*

Suppose first that $h(x) = \frac{1}{(2x-5)^3}$. Let $u = 2x - 5$. Then we have that

$$h'(x) = \frac{d}{dx}u^{-3} = \left(\frac{d}{du}u^{-3}\right)\left(\frac{d}{dx}2x-5\right) = -3u^{-4}(2) = \frac{-6}{(2x-5)^4}$$

by the Chain Rule.

Now suppose that $h(x) = \frac{x}{(2x-5)^3} = (x)\left(\frac{1}{(2x-5)^3}\right)$. By the product rule, we have that

$$\begin{aligned} h'(x) &= (x)'\left(\frac{1}{(2x-5)^3}\right) + (x)\left(\frac{1}{(2x-5)^3}\right)' \\ &= (1)\left(\frac{1}{(2x-5)^3}\right) + (x)\left(\frac{-6}{(2x-5)^4}\right) \\ &= \frac{1}{(2x-5)^3} - \frac{6x}{(2x-5)^4} \\ &= \frac{-4x-5}{(2x-5)^4}. \end{aligned}$$

Alternately, you could use the quotient rule to calculate this derivative.

37. (a) $\int \left(\frac{x^2}{2} + 4x^3\right)\,dx = \frac{x^3}{6} + x^4 + C.$

(b) $\int \frac{t+1}{t^3}\,dt = \int \left(t^{-2} + t^{-3}\right)\,dt = -t^{-1} - \frac{t^{-2}}{2} + C = C - \frac{1}{t} - \frac{1}{2t^2}.$

(c) $\int \left(\frac{1-\cos 6\theta}{2}\right)\,d\theta = \int \left(\frac{1}{2} - \frac{1}{2}\cos 6\theta\right)\,d\theta = \frac{1}{2}\theta - \frac{1}{12}\sin 6\theta + C.$

38. It is incorrect because $\frac{d}{dx}\left(\cos x + x^3 - x^2 + 7x\right) \neq \sin x + 3x^2 - 2x + 7.$

39. (a) We have

$$\int_{-1}^{1}\left(x^2 - 2x + 3\right)\,dx = \left[\frac{x^3}{3} - x^2 + 3x\right]_{-1}^{1} = \left(\frac{1}{3} - 1 + 3\right) - \left(\frac{-1}{3} - 1 - 3\right) = \frac{20}{3}.$$

(b) We have

$$\begin{aligned} \int_{-\sqrt{3}}^{\sqrt{3}}(t+1)(t^2+4)\,dt &= \int_{-\sqrt{3}}^{\sqrt{3}} t^3 + t^2 + 4t + 4\,dt = \left[\frac{t^4}{4} + \frac{t^3}{3} + 2t^2 + 4t\right]_{-\sqrt{3}}^{\sqrt{3}} \\ &= \left(\frac{9}{4} + \sqrt{3} + 6 + 4\sqrt{3}\right) - \left(\frac{9}{4} - \sqrt{3} + 6 - 4\sqrt{3}\right) = 10\sqrt{3}. \end{aligned}$$

(c) We have

$$\begin{aligned}
\int_0^{\pi} \frac{1}{2}\left(\cos x + |\cos x|\right)\, dx &= \int_0^{\frac{\pi}{2}} \frac{1}{2}\left(\cos x + |\cos x|\right)\, dx + \int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}\left(\cos x + |\cos x|\right)\, dx \\
&= \int_0^{\frac{\pi}{2}} \frac{1}{2}\left(\cos x + \cos x\right)\, dx + \int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}\left(\cos x - \cos x\right)\, dx \\
&= \int_0^{\frac{\pi}{2}} \cos x\, dx + \int_{\frac{\pi}{2}}^{\pi} 0\, dx \\
&= [\sin x]_0^{\frac{\pi}{2}} = \sin\frac{\pi}{2} - \sin 0 = 1 - 0 = 1.
\end{aligned}$$

40. We have that

$$\frac{dy}{dx} = \frac{d}{dx}\int_2^{x^2} \sin(t^3)\, dt = \left(\frac{d}{du}\int_2^{u} \sin t^3\, dt\right)\left(\frac{d}{dx}x^2\right) = \left(\sin u^3\right)(2x) = 2x\sin x^6.$$