Welcome to

Mathematics for Architects

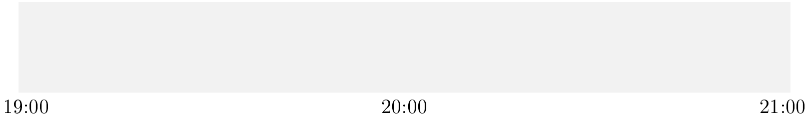
with Dr Neil Course

Lecture 1

- Information about this course
- 1. Sets
- 2. Symbolic Logic
- 3. Numbers

Information about this course

- ≈ 12 classes. Wednesday and Thursday evenings 7pm-9pm.



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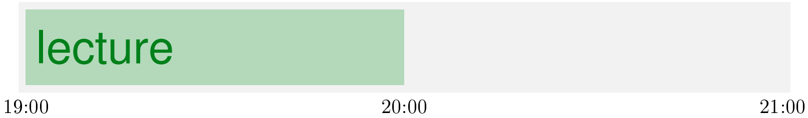
- Each lecture $\approx$ 60 minutes.



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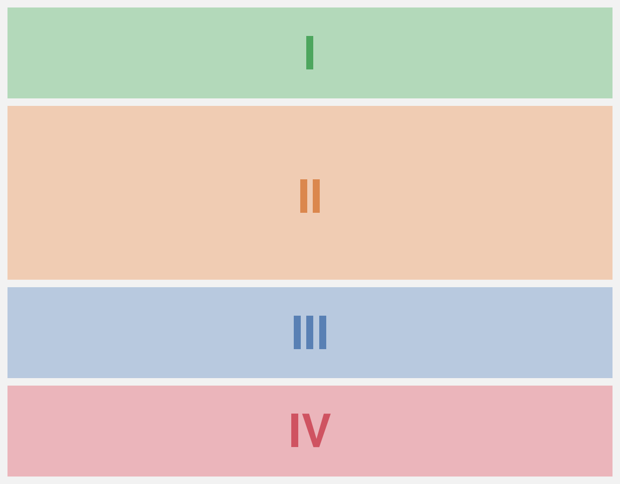
I
II
III
IV

Introduction

Sets; Symbolic Logic; Numbers; Cartesian Coordinates.

II

III

IV

revision?

Introduction

Sets; Symbolic Logic; Numbers; Cartesian Coordinates.

II

III

IV

revision?

Introduction

Sets; Symbolic Logic; Numbers; Cartesian Coordinates.

The Geometry of Space

Polar Coordinates; Conic Sections;
Three Dimensional Cartesian Coordinates;
Vectors; The Dot Product; The Cross Product;
Lines; Planes; Projections.

III

IV

revision?

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Sets; Symbolic Logic; Numbers; Cartesian Coordinates.

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Combinatorics; Probability; Graph Theory.

IV

revision?

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Sets; Symbolic Logic; Numbers; Cartesian Coordinates.

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Combinatorics; Probability; Graph Theory.

Calculus

Limits; Continuity; Differentiation; Integration.

revision?

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$\}$ 2 lectures

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$\}$ 3 lectures

Finite Mathematics
Combinatorics; Probability; Graph Theory.
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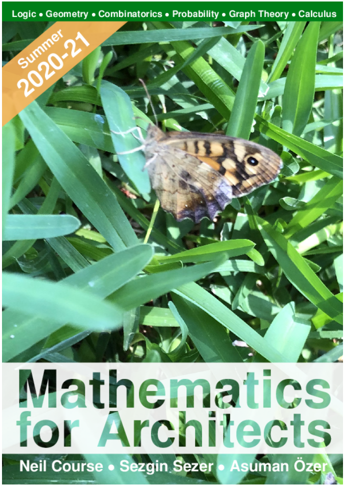
Calculus
Limits; Continuity; Differentiation; Integration.
$\}$ 4 lectures

Lecture Notes



Exams and homework

(This information may change based on the University's decisions)

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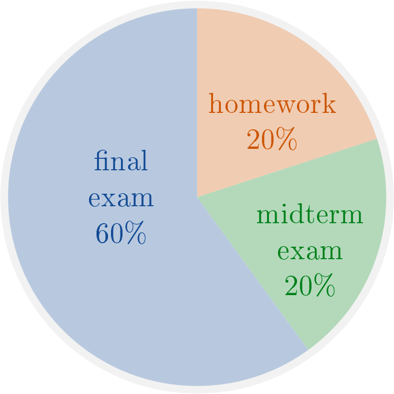

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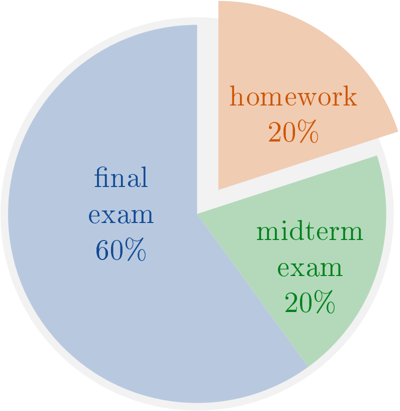

10 multiple choice tests on O'Learn.

Exams and homework

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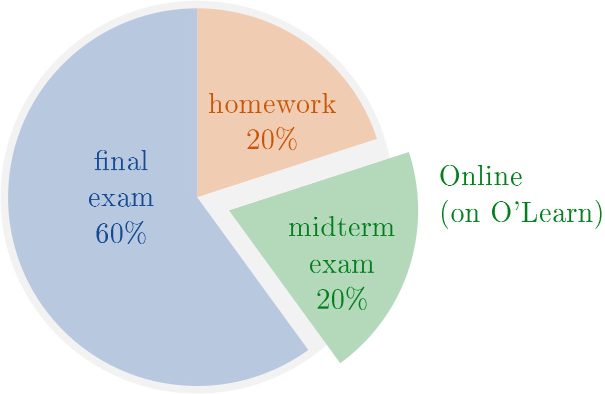

Exams and homework

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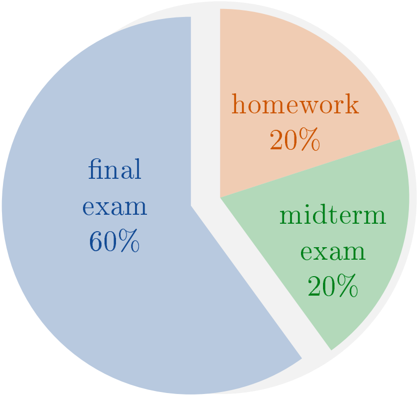
Online.



Expectations

If this was a classroom course, the expectation is that for each hour of lectures, you would study an extra 1-2 hours outside of class.

classroom course	lectures (8 hours)	other study (8-16 hours)

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For an online course, you are still expected to study a total of 16-24 hours each week.

online course	class (4 hours)	other study (12-20 hours)

Your other study may include:

- Do the online homework tests each week;

⋮

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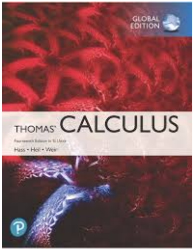
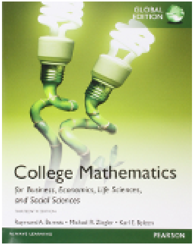
- Solve the problems in the lecture notes;
- Use the O'Learn Discussion Board;
- Read books;
- Watch online videos (e.g. blackpenredpen on YouTube has good Calculus videos);

$\vdots$

Two good books



George B. Thomas Jr., Maurice D. Weir and Joel Hass,
Thomas' Calculus,
Pearson.



Raymond A. Barnett, Michael R. Ziegler and Karl E. Byleen,
College Mathematics for Business, Economics, Life Sciences, and Social Sciences,
Pearson.

slide number

current chapter

1 Sets

Definition

A set is a collection of objects, specified in such a way that we can tell whether any given object is or is not in the collection.

Example

For example

$$A = \{1, 2, 3, 4, 5\},$$
$$B = \{\text{apple}, \text{banana}, \text{cherry}\}$$

and

$$C = \{n, e, i, l\}$$

are sets.

Definition

The symbol $\in$ means "is in the set".

Example

If

$$B = \{\text{apple}, \text{banana}, \text{cherry}\}$$

then

$$\text{banana} \in B$$

and

$$\text{date} \notin B$$

Definition

Each object in a set is called an *element* of the set.

Definition

A set without any elements is called the *empty set* and is denoted by $\varnothing$.

1. Sets

Definition

The symbol | means “such that”.

Example

$$\{x \mid x \text{ is a weekend day}\} = \{\text{Saturday}, \text{ Sunday}\}$$
$$\{x \mid x^2 = 4\} = \{-2, 2\}$$
$$\{\text{all the people who are} > 5\text{m tall}\} = \varnothing.$$

Definition

If every element of a set A is also in a set B, then we say that A is a *subset* of B, and we write $A \subseteq B$.

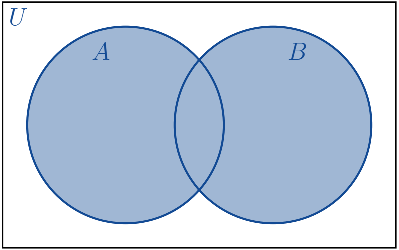
Example

$$\begin{aligned} \{1, 2, 3\} &\subseteq \{1, 2, 3, 4\}, \\ \{\text{banana}\} &\subseteq \{\text{apple}, \text{banana}, \text{cherry}\}, \\ \{\text{Neil},\ \text{Sezgin}\} &\subseteq \{\text{Neil},\ \text{Sezgin}\}. \end{aligned}$$

Definition

The *universal set* is the set of all elements under consideration. We call this set U.

Suppose that A and B are subsets of U.

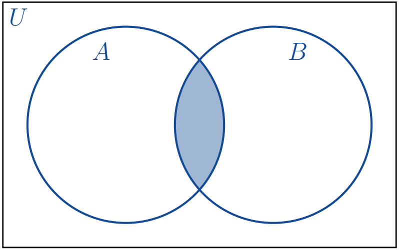


$A \cup B$

Definition

The *union* of A and B is

$$A \cup B = \{e \in U \mid e \in A \text{ or } e \in B\}.$$



$A \cap B$

Definition

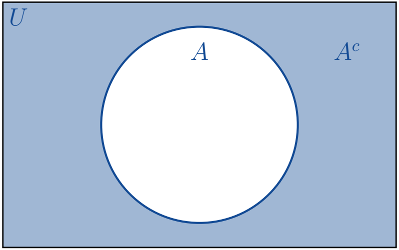
The *intersection* of A and B is

$$A \cap B = \{e \in U \mid e \in A \text{ and } e \in B\}.$$

Example

$$\{a, b, c\} \cup \{b, c, d\} = \{a, b, c, d\}$$

$$\{a, b, c\} \cap \{b, c, d\} = \{b, c\}$$



A^c

Definition

The *complement* of a subset A of U is

$$A^c = \{e \in U \mid e \notin A\}.$$

Example

If U is the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = \{1, 3, 5, 7, 9\}$, then

$$A^c = \{2, 4, 6, 8, 10\}.$$

Symbolic Logic

Definition

A *proposition* is a statement which is either *true* or *false* (but not both).

Example

- “Grass is green” (true)
- “2+5=5” (false)
- “My name is Neil” (true)

are propositions, but

- “Close the door”
- “Is it cold today?”
- “1”

are not propositions.

2. Symbolic Logic

Notation

The symbol for *or* (veya) is $\vee$.

Example

If P is the proposition "It is snowing today" and Q is the proposition "It is raining today", then $P \vee Q$ is the proposition "It is snowing or raining today".

Example

If $M = (x \in A)$ and $N = (x \in B)$, then $M \vee N = (x \in A \cup B)$

Truth Table

(T = true, F = false)

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Notation

The symbol for *and* (ve) is $\wedge$.

Example

If $P =$ "I am hungry" and $Q =$ "I am sleepy", then $P \wedge Q =$ "I am hungry and sleepy".

Example

If $M = (x \in A)$ and $N = (x \in B)$, then $M \wedge N = (x \in A \cap B)$

Truth Table

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Notation

The symbol for *not* (değil) is $\neg$.

Example

If $P =$ “Sizin hocanız kahve seviyor”, then $\neg P =$ “Sizin hocanız kahve sevmiyor”.

Example

If $M = (x \geq 7)$, then $\neg M = (x < 7)$

Truth Table

P	$\neg P$
T	F
F	T

Notation

The symbol for *if and only if* (iff/ancak ve ancak) is $\iff$.

Truth Table

P	Q	$P \iff Q$
T	T	T
T	F	F
F	T	F
F	F	T

Notation

The symbol for *implies* (ise) is $\implies$.

Example

Let $P =$ "I am in London" and $Q =$ "I am in the UK." Then $P \implies Q$.

Truth Table

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

Remark

We must only write "$P \implies Q$" if both P and Q are propositions. I don't want to see nonsense like

$$\int_0^1 3x^2 \, dx = \left[x^3\right]_0^1 \implies 1$$

in your work. Yes, "$\int_0^1 3x^2 \, dx = \left[x^3\right]_0^1$" is a proposition. In fact, it is a *true* proposition. But "1" is not a proposition.
If you mean "$=$", then write "$=$".

Remark

If P and Q are propositions, then $(P \vee Q)$, $(P \wedge Q)$, $(\neg P)$, $(P \implies Q)$ and $(P \iff Q)$ are also propositions.

Definition

The *converse* (zıt) of $(P \implies Q)$ is $(Q \implies P)$.

Definition

The *contapositive* (devrik) of $(P \implies Q)$ is $(\neg Q \implies \neg P)$.

Example

$P =$ "It is raining"
$Q =$ "I get wet"
$(P \implies Q) =$ "If it is raining, then I get wet"
converse: $(Q \implies P) =$ "If I get wet, then it is raining"
contrapositive: $(\neg Q \implies \neg P) =$ "If I do not get wet, then it is not raining"

The 22 Identities

1. $(P \vee P) = P$
2. $(P \wedge P) = P$
3. $(P \vee Q) = (Q \vee P)$
4. $(P \wedge Q) = (Q \wedge P)$
5. $((P \vee Q) \vee R) = (P \vee (Q \vee R))$
6. $((P \wedge Q) \wedge R) = (P \wedge (Q \wedge R))$
7. $\neg(P \vee Q) = (\neg P \wedge \neg Q)$
8. $\neg(P \wedge Q) = (\neg P \vee \neg Q)$
9. $(P \wedge (Q \vee R)) = ((P \wedge Q) \vee (P \wedge R))$
10. $(P \vee (Q \wedge R)) = ((P \vee Q) \wedge (P \vee R))$
11. $(P \vee \text{true}) = \text{true}$

12. $(P \wedge \text{false}) = \text{false}$
13. $(P \vee \text{false}) = P$
14. $(P \wedge \text{true}) = P$
15. $(P \vee \neg P) = \text{true}$
16. $(P \wedge \neg P) = \text{false}$
17. $\neg(\neg P) = P$
18. $(P \implies Q) = (\neg P \vee Q)$
19. $(P \iff Q) = ((P \implies Q) \wedge (Q \implies P))$
20. $((P \wedge Q) \implies R) = (P \implies (Q \implies R))$
21. $((P \implies Q) \wedge (P \implies \neg Q)) = \neg P$
22. $(P \implies Q) = (\neg Q \implies \neg P)$

Proof of Identity 18.

P	Q	$P \implies Q$	$\neg P$	Q	$\neg P \vee Q$
T	T	T	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	F	T

Note that the 3rd and 6th columns are the same:

$$T, F, T, T.$$

Therefore $(P \implies Q) = (\neg P \vee Q)$. $\square$

Proof of Identity 22.

P	Q	$P \implies Q$	$\neg Q$	$\neg P$	$\neg Q \implies \neg P$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Therefore $(P \implies Q) = (\neg Q \implies \neg P)$. $\square$

Notation

The symbol for *for all* (her) is $\forall$.

Notation

The symbol for *there exists* (vardır) is $\exists$.

3 Numbers

3. Numbers

The set

$$\mathbb{N} = \{1, 2, 3, 4, 5, 6, \dots\}$$

is called the set of *natural numbers*. These are the first numbers that children learn. For example

$$
\begin{aligned}
2 &\in \mathbb{N} \quad \text{means “2 is a natural number”} \\
7 &\in \mathbb{N} \quad \text{means “7 is a natural number”} \\
\tfrac{1}{2} &\notin \mathbb{N} \quad \text{means “}\tfrac{1}{2}\text{ is \textbf{not} a natural number”} \\
0 &\notin \mathbb{N} \quad \text{means “0 is \textbf{not} a natural number”} \\
-5 &\notin \mathbb{N} \quad \text{means “}-5\text{ is \textbf{not} a natural number”}
\end{aligned}
$$

In the natural numbers, we can do "+" and "×"

$$2 + 7 = 9 \in \mathbb{N}, \qquad\qquad 2 \times 7 = 14 \in \mathbb{N}.$$

However we can not do "−" because

$$2 - 7 \notin \mathbb{N}.$$

So we invent new numbers!

3. Numbers

The set

$$\mathbb{Z} = \{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$$

is called the set of *integers*. We use a $\mathbb{Z}$ for the German word 'zahlen' (numbers). In $\mathbb{Z}$, we can do "+", "−" and "×" but we can not do "÷". For example $3 \in \mathbb{Z}$, $4 \in \mathbb{Z}$, $-5 \in \mathbb{Z}$ and

$$3 + 4 \in \mathbb{Z}, \qquad 3 - 4 \in \mathbb{Z}, \qquad 3 \times 4 \in \mathbb{Z}, \qquad 3 \div 4 \notin \mathbb{Z},$$

$$3 + (-5) \in \mathbb{Z}, \quad 3 - (-5) \in \mathbb{Z}, \quad 3 \times (-5) \in \mathbb{Z}, \quad 3 \div (-5) \notin \mathbb{Z}.$$

So we invent new numbers!

The set

$$\mathbb{Q} = \{\text{all fractions}\} = \left\{ \frac{a}{b} \;\middle|\; a, b \in \mathbb{Z} \text{ and } b \neq 0 \right\}$$

is called the set of *rational numbers*. We use a $\mathbb{Q}$ for the word 'quotient'. For example

$$0 = \frac{0}{1} \in \mathbb{Q} \qquad \frac{100}{13} \in \mathbb{Q}$$

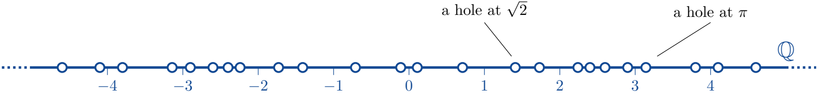
$$1 = \frac{1}{1} \in \mathbb{Q} \qquad \sqrt{2} \notin \mathbb{Q}$$

$$\frac{3}{4} \in \mathbb{Q} \qquad -4 = \frac{8}{-2} \in \mathbb{Q}$$

$$\pi \notin \mathbb{Q} \qquad 0.12345 = \frac{12345}{100000} \in \mathbb{Q}.$$

In $\mathbb{Q}$ we can do "$+$", "$-$", "$\times$" and "$\div$(by a number $\neq 0$)".

Are we happy now? No!
Why? Because if we draw all the rational numbers in a line, then the line has lots of holes in it. In fact, $\mathbb{Q}$ has ∞ many holes in it.



So we invent new numbers!

The set

$$\mathbb{R} = \{\text{all numbers which can be written as a decimal}\}$$

is called the set of *real numbers*. For example

$$\begin{aligned} 0 = 0.0 &\in \mathbb{R} & \qquad \frac{100}{13} &= 7.692307\ldots \in \mathbb{R} \\ \frac{23}{99} = 0.232323\ldots &\in \mathbb{R} & \sqrt{2} &= 1.414213\ldots \in \mathbb{R} \\ \frac{3}{4} = 0.75 &\in \mathbb{R} & \frac{123}{999} &= 0.123123\ldots \in \mathbb{R} \\ \pi = 3.141592\ldots &\in \mathbb{R} & \frac{12345}{100000} &= 0.12345 \in \mathbb{R}. \end{aligned}$$

3. Numbers

The real numbers are complete – this means that if we draw all the real numbers in a line, then there are no holes in the line.

Are we happy now?

Are we happy now?

Yes!

Next Time

- 4. Intervals
- 5. Cartesian Coordinates
- 6. Functions
- 7. Sigma Notation