



SON TESLİM TARİHİ: Salı 16 Şubat 2016 saat 16:00'e kadar.

**Egzersiz 1 (Symbolic Logic).**

- (a) [20p] Use a truth table to prove that

$$(P \wedge (Q \vee R)) = ((P \wedge Q) \vee (P \wedge R)).$$

e.g.

$P$	$Q$	$R$	$Q \vee R$	$(P \wedge (Q \vee R))$	$P \wedge Q$	$P \wedge R$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

- (b) [20p] Negate the following proposition:

$$(\forall \varepsilon > 0)(\exists N \in \mathbb{N})(\forall n \in \mathbb{N})(n > N \Rightarrow |a_n| < \varepsilon).$$

**Egzersiz 2 (Proof by Induction).** [30p] Use proof by induction to prove that

$$1 + 2 + 3 + 4 + 5 + \dots + n = \frac{n(n+1)}{2}$$

for all  $n \in \mathbb{N}$ .

[Note: It is possible to prove this result as follows:

$$\begin{aligned}
 2(1 + 2 + 3 + 4 + 5 + \dots + n) &= \begin{array}{cccccccc} 1 & + & 2 & + & 3 & + & 4 & + & 5 & + & \dots & + & n \\ + & n & + & (n-1) & + & (n-2) & + & (n-3) & + & (n-4) & + & \dots & + & 1 \end{array} \\
 &= (n+1) + (n+1) + (n+1) + (n+1) + (n+1) + \dots + (n+1) \\
 &= n(n+1).
 \end{aligned}$$

However, I want you to prove it using *proof by induction*.]

**Egzersiz 3 (Proof by Contrapositive).** [30p] Let  $x, y \in \mathbb{R}$ . Use *proof by contrapositive* to prove that

$$xy \notin \mathbb{Q} \implies ((x \notin \mathbb{Q}) \vee (y \notin \mathbb{Q})).$$