



Exercise 19 (Homogeneous Second Order Linear ODEs with constant coefficients). Find the general solution of the following ODEs:

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|--------------------------|---------------------------|----------------------------|-----------------------------|
| (a) $y'' - 2y' + 2y = 0$ | (e) $y'' + 6y' + 13y = 0$ | (i) $4y'' + 12y' + 9y = 0$ | (m) $4y'' + 17y' + 4y = 0$ |
| (b) $y'' + 2y' + 2y = 0$ | (f) $9y'' + 16y = 0$ | (j) $4y'' - 4y' - 3y = 0$ | (n) $4y'' + 20y' + 25y = 0$ |
| (c) $y'' + 2y' - 8y = 0$ | (g) $y'' - 2y' + y = 0$ | (k) $y'' - 2y' + 10y = 0$ | (o) $25y'' - 20y' + 4y = 0$ |
| (d) $y'' - 2y' + 6y = 0$ | (h) $9y'' + 6y' + y = 0$ | (l) $y'' - 6y' + 9y = 0$ | (p) $2y'' + 2y' + y = 0$ |

Solve the following IVPs:

(q)
$$\begin{cases} 9y'' + 6y' + 82y = 0 \\ y(0) = -1 \\ y'(0) = 2 \end{cases}$$

(r)
$$\begin{cases} y'' - 6y' + 9y = 0 \\ y(0) = 0 \\ y'(0) = 2 \end{cases}$$

Exercise 20 (Reduction of Order). In each of the following problems:

- (i) Check that y_1 solves the ODE;
- (ii) Use the method of reduction of order to find a second, linearly independent solution, y_2
[HINT: Start with $y_2(t) = v(t)y_1(t)$.];
- (iii) Check that your y_2 solves the ODE; and
- (iv) Calculate the Wronskian of y_1 and y_2 .

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| (a) $t^2y'' + 2ty' - 2y = 0, \quad t > 0; \quad y_1(t) = t$ | (d) $t^2y'' - t(t+2)y' + (t+2)y = 0, \quad t > 0; \quad y_1(t) = t$ |
| (b) $t^2y'' - 4ty' + 6y = 0, \quad t > 0; \quad y_1(t) = t^2$ | (e) $xy'' - y' + 4x^3y = 0, \quad x > 0; \quad y_1(x) = \sin x^2$ |
| (c) $t^2y'' + 3ty' + y = 0, \quad t > 0; \quad y_1(t) = t^{-1}$ | (f) $(x-1)y'' - xy' + y = 0, \quad x > 1; \quad y_1(x) = e^x$ |