2015–16 **MAT234 Matematik IV – Ödev 5** N. Course

SON TESLİM TARİHİ: Salı 5 Nisan 2016 saat 16:00'e kadar.

NEW RULE: Poor quality photos of answers sent by email will no longer be accepted.

I prefer to receive your answers on paper. If you must email your answers, then you must either

(i) prepare them with LaTeX;

(ii) use a word processor;

(iii) write them on paper, then use a proper flatbed scanner to scan them; or

(iv) write them on paper, then use a "scanner" app on your mobile phone to scan them.

Make sure that your name and student number are clearly visible on every page that you send.

Egzersiz 10 (Sequences). Let $a_1 = 2$ and $a_{n+1} = \frac{1}{70}\left(1200 + a_n^2\right)$ for all $n \in \mathbb{N}$.

(a) [5p] Use a calculator or computer to calculate a_2, a_3, a_4, a_5 and a_6.

(b) [15p] Show that $0 \leq a_n \leq 30$ for all $n \in \mathbb{N}$.
[HINT: Use proof by induction.]

(c) [15p] Is (a_n) increasing or decreasing? Prove your answer.

(d) [10p] Show that (a_n) is convergent.

(e) [10p] Calculate $\lim_{n\to\infty} a_n$.

Definition. A sequence (a_n) of real numbers is called a *Cauchy sequence* iff, $\forall\, \varepsilon > 0$ $\exists\, N = N(\varepsilon) \in \mathbb{N}$ such that

$$n, m > N \implies |a_n - a_m| < \varepsilon.$$

Egzersiz 11 (Subsequences and Cauchy Sequences).

(a) [25p] Suppose that

- (x_n) is a sequence of real numbers;
- $x_n \to -\infty$ as $n \to \infty$;
- (x_{n_k}) is a subsequence of (x_n).

Show that $x_{n_k} \to -\infty$ as $k \to \infty$.

(b) [20p] Let $y_n = \frac{25n^2-2}{n^2}$. *Use the definition* to show that (y_n) is a Cauchy sequence.

Ödev 4'ün çözümleri

8. (a)$|z_n| = \left|\left(\frac{2n}{1+n}\right)\cos(n\pi)\right| = \left|\left(\frac{2n}{1+n}\right)(-1)^n\right| = \frac{2n}{1+n} < \frac{2n}{n} = 2$ for all $n \in \mathbb{N}$. Therefore (z_n) is bounded.
(b) Clearly (g_n) is an increasing sequence (it is not possible to have $g_{n+1} < g_n$ since $g_{n+1} := \max\{g_n, \cos(n+1)\}$) and clearly (g_n) is bounded above by 1 (since $\cos x \leq 1$), so this sequence converges by Theorem 5.1.

9. (a) Since $\frac{n}{2^n} \to 0$ as $n \to \infty$, and since cos is a continuous function, it follows that $a_n \to 1$ as $n \to \infty$.
(b) $b_n \to 1$ as $n \to \infty$. We could, for example, use the Sandwich Rule to prove this using the inequality $(n^2) \leq (n^2+1) \leq (n+1)^2$. Details left to you.
(c) $c_n = n^{\frac{2}{n}} = n^{\frac{1}{n}} \times n^{\frac{1}{n}} \to 1 \times 1 = 1$ as $n \to \infty$.
(d) $d_n = \frac{\log n}{n} \to 0$ as $n \to \infty$. Proof left to you.
(e) (e_n) does not have a limit. Proof left to you.
(f) (f_n) does not have a limit. Proof left to you.