

**Exercise 26 (The Laplace Transform).** Find the Laplace Transform of the following functions:

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|------------------------------|---------------------------------|---|
| (a) $f(t) = e^{-2t}$         | (e) $f(t) = \frac{\sinh t}{t}$  | (i) $f(t) = \begin{cases} 2 & 0 < t \leq 3 \\ 0 & t > 3 \end{cases}$                                      |
| (b) $f(t) = 3t^2$            | (f) $f(t) = t^2 \cos 2t$        |   |
| (c) $f(t) = \cos^2 2t$       | (g) $f(t) = \frac{e^{3t}-1}{t}$ | (j) $f(t) = \begin{cases} \sin 2t & \pi \leq t \leq 2\pi \\ 0 & t < \pi \text{ or } t > 2\pi \end{cases}$ |
| (d) $f(t) = t \cos t + te^t$ | (h) $f(t) = te^{-t} \sin^2 t$   |   |

**Exercise 27 (The Inverse Laplace Transform).** Find the inverse Laplace Transform of the following functions:

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|--|---|--|
| (a) $F(s) = \frac{1}{s-2}$                   | (f) $F(s) = \frac{2s+1}{s(s^2+9)}$          | (j) $F(s) = \ln\left(1 + \frac{1}{s^2}\right)$ |
| (b) $F(s) = \frac{1}{s} - \frac{2}{s^{5/2}}$ | (g) $F(s) = \frac{s^3}{(s-4)^4}$            | (k) $F(s) = \arctan\left(\frac{3}{s+2}\right)$ |
| (c) $F(s) = \frac{3s+1}{s^2+4}$              | (h) $F(s) = \frac{s^2-2s}{s^4+5s^2+4}$      | (l) $F(s) = \frac{s}{(s^2+1)^3}$               |
| (d) $F(s) = \frac{2e^{-3s}}{s}$              | (i) $F(s) = \frac{2s^3-s^2}{(4s^2-4s+5)^2}$ | (m) $F(s) = \frac{e^{-s}}{s+2}$                |
| (e) $F(s) = \frac{1}{s(s-3)}$                |   |  |

$f(t)$	$F(s) = \mathcal{L}[f](s)$	
1	$\frac{1}{s}$	$s > 0$
$e^{at}$	$\frac{1}{s-a}$	$s > a$
$t^n \quad (n \in \mathbb{N})$	$\frac{n!}{s^{n+1}}$	$s > 0$
$\sin at$	$\frac{a}{s^2+a^2}$	$s > 0$
$\cos at$	$\frac{s}{s^2+a^2}$	$s > 0$
$\sinh at$	$\frac{a}{s^2-a^2}$	$s >  a $
$\cosh at$	$\frac{s}{s^2-a^2}$	$s >  a $
$e^{at} \sin bt$	$\frac{b}{(s-a)^2+b^2}$	$s > a$
$e^{at} \cos bt$	$\frac{s-a}{(s-a)^2+b^2}$	$s > a$
$t^n e^{at} \quad (n \in \mathbb{N})$	$\frac{n!}{(s-a)^{n+1}}$	$s > a$
$u_c(t)$	$\frac{e^{-cs}}{s}$	$s > 0$
$u_c(t)f(t-c)$	$e^{-cs}F(s)$	
$e^{ct}f(t)$	$F(s-c)$	
$f(ct) \quad (c > 0)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$	
$\int_0^t f(t-\tau)g(\tau)d\tau$	$F(s)G(s)$	
$t^n f(t)$	$(-1)^n F^{(n)}(s)$	