Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Feedback Circuits

1.0.1. Three identical inverting amplifier stages each characterized by a low frequency gain K and a single pole response with

$$f_{3dB} = 100kHz \tag{1.0.1.1}$$

are connected in a feedback loop with H = 1. What is the minimum value of K at which the circuit oscillates? What would the frequency of oscillation be?
Solution: The given data is shown in the table.1.0.1

Parameter	Value
pole frequency	100kHz
feedback factor	1

TABLE 1.0.1: INPUT TABLE

The open loop gain of amplifier stages is given by..,

$$G_1(s) = G_2(s) = G_3(s) = \frac{-K}{1 + \frac{s}{2\pi 10^5}} \tag{1.0.1.2}$$

The three gains are cascaded.The net open loop gain is given by.,

$$G(s) = G_1(s)G_2(s)G_3(s) \tag{1.0.1.3}$$

$$G(s) = \left[\frac{-K}{1 + \frac{s}{2\pi 10^5}}\right]^3 \tag{1.0.1.4}$$

$$G(s) = -\left[\frac{K}{1 + \frac{s}{2\pi 10^5}}\right]^3 \tag{1.0.1.5}$$

1.0.2. Draw the control block diagram of above question.
Solution: The control system block is shown in the figure.1.0.2

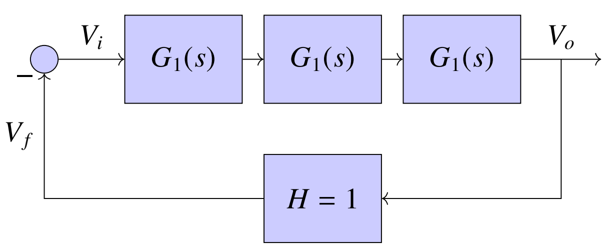

Fig. 1.0.2: CONTROL SYSTEM BLOCK

1.0.3. Find the loop gain of the system.
Solution: The loop gain is given by

$$LG(s) = G(s)H \tag{1.0.3.1}$$

$$LG(s) = -\left[\frac{K}{1 + \frac{s}{2\pi 10^5}}\right]^3 \tag{1.0.3.2}$$

$$\text{Substitute..,} s = j\omega \tag{1.0.3.3}$$

$$LG(j\omega) = -\left[\frac{K}{1 + \frac{j\omega}{2\pi 10^5}}\right]^3 \tag{1.0.3.4}$$

1.0.4. Find the frequency of oscillation of above circuit.
Solution: According to BarkHausen's criteria..,

- The magnitude of loop gain of the system at the oscillating frequency should be greater than or equal to 1.
- The phase angle of loop gain at the oscillating frequency must be equal to -180 degrees.

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Let the frequency of oscillation is fs.,

$$\angle LG(j\omega) = -180^\circ \quad (1.0.4.1)$$

$$-180^\circ - 3tan^{-1}\left[\frac{\omega_s}{2\pi 10^5}\right] = -180^\circ \quad (1.0.4.2)$$

$$-3tan^{-1}\left[\frac{\omega_s}{2\pi 10^5}\right] = -360^\circ \quad (1.0.4.3)$$

$$\frac{\omega_s}{2\pi 10^5} = \tan(120^\circ) \quad (1.0.4.4)$$

$$\implies \omega_s = 1.088 \times 10^6 \text{ rad/sec} \quad (1.0.4.5)$$

Thus,the frequency of oscillation is 1.088M rad/sec.

1.0.5. Find the minimum value of K.

Solution: We know the magnitude of loop gain is greater than or equal to 1.The range of K can be found as follows,

$$|LG(j\omega_s)| \geq 1 \quad (1.0.5.1)$$

$$\frac{K^3}{\left|1 + \frac{j\omega_s}{2\pi 10^5}\right|^3} \geq 1 \quad (1.0.5.2)$$

$$K^3 \geq \left|1 + \frac{j1.088 \times 10^6}{2\pi 10^5}\right|^3 \quad (1.0.5.3)$$

$$K^3 \geq |1 + j1.73|^3 \quad (1.0.5.4)$$

$$K^3 \geq 8 \quad (1.0.5.5)$$

$$K \geq 2 \quad (1.0.5.6)$$

Therefore, The minimum value of K is 2.

1.0.6. Design the three stage oscillator for the above given question.

Solution: Consider the circuit shown in the figure.1.0.6.

Considering the first stage.,

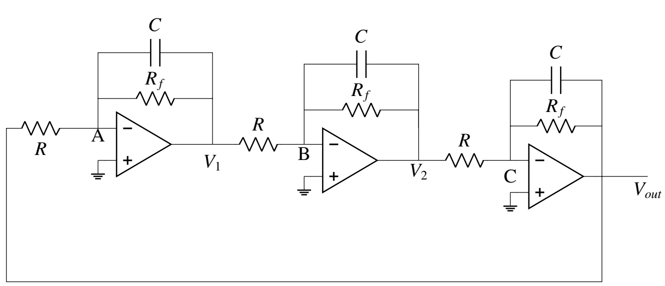

Fig. 1.0.6: 1

$$\frac{V_1}{V_{out}} = -\frac{R_f \| \frac{1}{sC}}{R} \quad (1.0.6.1)$$

$$\frac{V_1}{V_{out}} = -\frac{\frac{R_f}{R}}{1 + sCR_f} \quad (1.0.6.2)$$

Similarly for second stage.,

$$\frac{V_2}{V_1} = -\frac{\frac{R_f}{R}}{1 + sCR_f} \quad (1.0.6.3)$$

Similarly For third stage.,

$$\frac{V_{out}}{V_2} = -\frac{\frac{R_f}{R}}{1 + sCR_f} \quad (1.0.6.4)$$

Loop gain of the oscillator is computed as follows.,

$$LG(s) = \frac{V_1}{V_{out}} \times \frac{V_2}{V_1} \times \frac{V_{out}}{V_2} \quad (1.0.6.5)$$

$$LG(s) = \left[-\frac{\frac{R_f}{R}}{1 + sCR_f}\right]^3 \quad (1.0.6.6)$$

Comparing with the equation.1.0.3.4

$$\frac{R_f}{R} = K \quad (1.0.6.7)$$

$$CR_f = \frac{1}{2\pi 10^5} \quad (1.0.6.8)$$

For the circuit to oscillate,

$$K \geq 2 \quad (1.0.6.9)$$

$$R_f \geq 2R \quad (1.0.6.10)$$

Choose

$$R = 1000\Omega \quad (1.0.6.11)$$

$$\implies R_f = 2000\Omega \quad (1.0.6.12)$$

$$\implies C = 7.9577 \times 10^{-10}F \quad (1.0.6.13)$$

Parameter	**Value**
R	1000Ω
R_f	2000Ω
C	7.9577×10^{-10}

TABLE 1.0.6