Control Systems

G V V Sharma*

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3.1. Plot the Bode magnitude and phase plots for the following system

$$G(s) = \frac{50(s+3)(s+5)}{s(s+2)(s+4)(s+6)} \tag{3.1.1}$$

Solution: The magnitude and phase plot are as follows: Fig3.1
The python code to obtain the graphs and results:

codes/ee18btech11031.py

3.2. *Gain* and *Phase* of Transfer Function

$$G(j\omega) = \frac{50(j\omega+3)(j\omega+5)}{j\omega(j\omega+2)(j\omega+4)(j\omega+6)} \tag{3.2.1}$$

Gain:

$$\frac{100\sqrt{(\omega)^2+9}\sqrt{(\omega)^2+25}}{\omega\sqrt{(\omega)^2+4}\sqrt{(\omega)^2+16}\sqrt{(\omega)^2+36}} \tag{3.2.2}$$

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

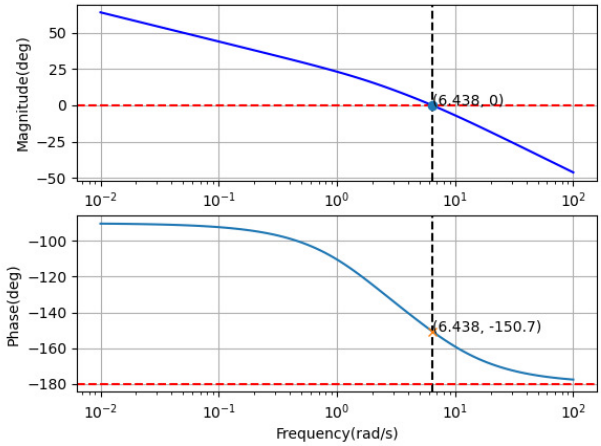

Fig. 3.1: Graphs

Phase:

$$\tan^{-1}(0) + \tan^{-1}\left(\frac{\omega}{3}\right) + \tan^{-1}\left(\frac{\omega}{5}\right) - \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{6}\right) \tag{3.2.3}$$

3.3. Find the Phase Margin(PM) and verify using the same code

$$PM = \angle G(j\omega_{gc}) + 180^\circ \tag{3.3.1}$$

$$\omega_{gc} = \text{Gain Crossover Frequency} \tag{3.3.2}$$

$$\text{At } \omega_{gc} \; |G(s)| = 1 \tag{3.3.3}$$

Solution:

$$\frac{100\sqrt{\left(\omega_{gc}\right)^2+9}\sqrt{\left(\omega_{gc}\right)^2+25}}{\omega gc\sqrt{\left(\omega_{gc}\right)^2+4}\sqrt{\left(\omega_{gc}\right)^2+16}\sqrt{\left(\omega_{gc}\right)^2+36}} = 1 \tag{3.3.4}$$

Solving Eq. (3.3.4) *or* from Fig 3.1 :

$$\implies \omega_{gc} = 6.438 \tag{3.3.5}$$

$$\angle G\left(j\omega_{gc}\right) = -150.725 \tag{3.3.6}$$

$$\implies PM = 29.275 \tag{3.3.7}$$

3.4. Find the Gain Margin (GM) and verify using the same code.

$$GM = 0 - G(j\omega_{pc})db \quad (3.4.1)$$

$$\omega_{pc} = \text{Phase Crossover Frequency} \quad (3.4.2)$$

$$\text{At } \omega_{pc}, \angle G(s) = -180^\circ \quad (3.4.3)$$

Solution: From Fig 3.1 ,we can say that phase never crosses -180° . So , the gain margin is *infinite* and from the equation: 3.4.3, ω_{pc} is non-existent.