Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Feedback Circuits

1.0.1. A dc amplifier has an open loop gain of 1000 and two poles, a dominant one at 1kHz and a high frequency one whose location can be controlled. It is required to connect this amplifier in a negative feedback loop that provides a dc closed loop gain of 10 and a maximally flat response.Find the required value of H.

Solution: Given, open loop gain G is dependent on frequency and has two poles.Therefore,G(s) can be written as..,

$$G(s) = \frac{G_0}{(1+\frac{s}{p_1})(1+\frac{s}{p_2})} \tag{1.0.1.1}$$

where,-p1 and -p2 are poles of G(s). Parameters given are shown in Table.1.0.1:1

Parameter	Value
dc open loop gain	1000
dominant pole	-1000Hz
insignificant pole	-p_2
dc closed loop gain	10

TABLE 1.0.1: 1

$$G_0 = 1000 \tag{1.0.1.2}$$

$$\text{Therefore.,}G(s) = \frac{1000}{(1+\frac{s}{p_1})(1+\frac{s}{p_2})} \tag{1.0.1.3}$$

Let, -p1 be the dominant pole.

$$p_1 = 2\pi 10^3 rad/sec \tag{1.0.1.4}$$

Now, we connect the system in a negative feedback of feedback factor H. We know that the closed loop gain of a negative feedback system is.,

$$T(s) = \frac{G(s)}{1+G(s)H} \tag{1.0.1.5}$$

But,also given DC closed loop gain is 10. DC closed loop gain is given in equation.1.0.1.6

$$T(0) = \frac{G_0}{1+G_0H} \tag{1.0.1.6}$$

$$\text{given.,}T(0) = 10 \tag{1.0.1.7}$$

$$\text{and.,}G_0 = 1000 \tag{1.0.1.8}$$

$$\frac{1000}{1+1000H} = 10 \tag{1.0.1.9}$$

$$1+1000H = 100 \tag{1.0.1.10}$$

$$\implies H = \frac{99}{1000} = 0.099 \tag{1.0.1.11}$$

from equation:1.0.1.11, the value of H is 0.099.Therefore, the feedback factor is 0.099.

1.0.2. Find the location at which second ple can be placed.

Solution: Given the negative feedback system should have maximally flat response.from equation.1.0.1.5 and equation.1.0.1.1 the transfer function is,

$$T(s) = \frac{\frac{G_0}{(1+\frac{s}{p_1})(1+\frac{s}{p_2})}}{1+\frac{HG_0}{(1+\frac{s}{p_1})(1+\frac{s}{p_2})}} \tag{1.0.2.1}$$

$$T(s) = \frac{\frac{p_1p_2G_0}{(p_1+s)(p_2+s)}}{1+\frac{p_1p_2HG_0}{(p_1+s)(p_2+s)}} \tag{1.0.2.2}$$

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$$T(s) = \frac{p_1 p_2 G_0}{(p_1 + s)(p_2 + s) + p_1 p_2 H G_0} \quad (1.0.2.3)$$

$$T(s) = \frac{p_1 p_2 G_0}{p_1 p_2 + (p_1 + p_2)s + s^2 + p_1 p_2 H G_0} \quad (1.0.2.4)$$

$$T(s) = \frac{p_1 p_2 G_0}{s^2 + (p_1 + p_2)s + (HG_0 + 1)p_1 p_2} \quad (1.0.2.5)$$

The characteristics equation of above transfer function is.,

$$C.E = s^2 + (p_1 + p_2)s + (HG_0 + 1)p_1 p_2 \quad (1.0.2.6)$$

In general, For a second order amplifier the C.E is.,

$$C.E = s^2 + 2\zeta\omega_n s + \omega_n^2 \quad (1.0.2.7)$$

The quality factor Q of equation.1.0.2.7, is given by

$$Q = \frac{1}{2\zeta} \quad (1.0.2.8)$$

From equation.1.0.2.6,

$$\omega_n = \pm\sqrt{(HG_0 + 1)p_1 p_2} \quad (1.0.2.9)$$

$$\zeta = \pm\frac{2p_1 p_2}{\sqrt{(HG_0 + 1)p_1 p_2}} \quad (1.0.2.10)$$

Therefore,For the given second order amplifier with characteristic equation.1.0.2.6,the Q factor is,

$$Q = \pm\frac{\sqrt{(1 + HG_0)p_1 p_2}}{p_1 + p_2} \quad (1.0.2.11)$$

For a negative feedback amplifier to achieve maximally flat response,

$$Q = 0.7071 \quad (1.0.2.12)$$

$$(1.0.2.13)$$

Therefore, substituting the values of Q and other parameters in equation.1.0.2.11,

$$0.7071 = \pm\frac{\sqrt{(1 + 0.099(1000))(2\pi 10^3)p_2}}{2\pi 10^3 + p_2} \quad (1.0.2.14)$$

Squaring on both sides and rearranging.,

$$(1 + 1000(0.099))2\pi 10^3 p_2 = 0.7071^2(2\pi 10^3 + p_2)^2 \quad (1.0.2.15)$$

$$(100)2\pi 10^3 = 0.7071^2(2\pi 10^3 + p_2)^2 \quad (1.0.2.16)$$

$$\implies 0.5p_2^2 - 622037.2p_2 + 19733247.6 = 0 \quad (1.0.2.17)$$

Solving above equation.,

$$p_2 = 31.7244 \text{ rad/sec} \quad (1.0.2.18)$$

$$p_2 = 1244042.676 \text{ rad/sec} \quad (1.0.2.19)$$

But, Since p1 is dominating pole,p1 should be close to origin.

$$p_1 <<< p_2 \quad (1.0.2.20)$$

$$p_2 = 1.244 \text{ Mrad/sec} \quad (1.0.2.21)$$

$$(1.0.2.22)$$

$$p_2 = \frac{1.244M}{2\pi}\text{Hz} \quad (1.0.2.23)$$

$$= 197.989 \text{ kHz} \quad (1.0.2.24)$$

$$(1.0.2.25)$$

The second pole frequency is 1.244 Mrad/sec

NOTE:- The poles are at -p1 and -p2,where p1 and p2 are positive numbers.Therefore poles lie on left half of s-plane.So the system is stable.

1.0.3. Verify roots of above equation using python code.

```
codes/ee18btech11005/ee18btech11005_1.py
```

1.0.4. Find the open loop transfer function and closed loop transfer function of the system. **Solution:** Substituting the value of p2 in the equation 1.0.1.1 and 1.0.2.5

$$G(s) = \frac{1000}{(1 + \frac{s}{10^3})(1 + \frac{s}{1.244x10^6})} \quad (1.0.4.1)$$

$$T(s) = \frac{1.244x10^{12}}{s^2 + 1245x10^3 s + 1.244x10^{11}} \quad (1.0.4.2)$$

1.0.5. Verify from the Bode plot of above closed loop transfer function that it has maximally flat response.

Solution: The following code generates the bode plot of the transfer function.

```
codes/ee18btech11005/ee18btech11005_2.py
```

From the magnitude bode plot in figure.1.0.5:1 we can tell that the open loop transfer function

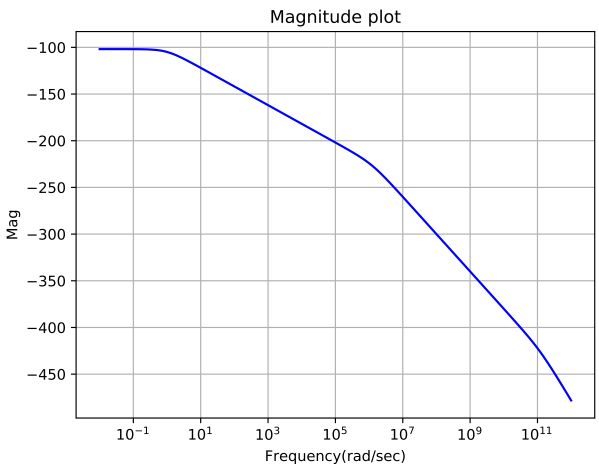

Fig. 1.0.5: 1

after having negative feedback has maximally flat response for the obtained value of p2.

1.0.6. Design a circuit that represents the above transfer function.

Solution: The circuit can be designed using an operational amplifier having negative feedback.Consider the circuit shown in figure.1.0.6:1 For a differential amplifier,

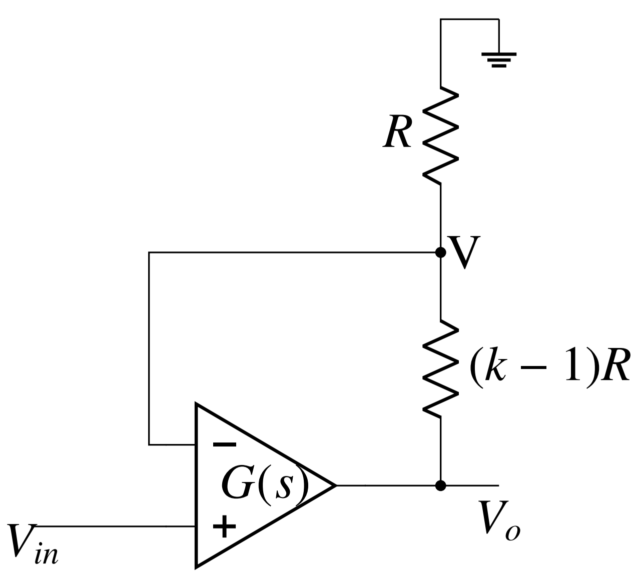

Fig. 1.0.6: 1

$$V_o = G(s)(V_{in} - V) \tag{1.0.6.1}$$

$$\text{where.,}V = \frac{V_o R}{kR} = \frac{V_o}{k} \tag{1.0.6.2}$$

$$V_o = G(s)(V_{in} - \frac{V_o}{k}) \tag{1.0.6.3}$$

$$V_o = G(s)V_{in} - \frac{G(s)V_o}{k} \tag{1.0.6.4}$$

$$V_o = \frac{G(s)V_{in}}{1 + \frac{G(s)}{k}} \tag{1.0.6.5}$$

$$T(s) = \frac{G(s)}{1 + \frac{G(s)}{k}} \tag{1.0.6.6}$$

Where.., G(s) is open loop gain that varies with frequency. Comparing equation.1.0.1.5 and equation.1.0.6.6 We can design the equivalent circuit for the given problem.

$$H = \frac{1}{k} \tag{1.0.6.7}$$

$$\implies k = \frac{1}{H} \tag{1.0.6.8}$$

$$\tag{1.0.6.9}$$

We know.,H = 0.099

$$k = \frac{1}{0.099} = \frac{1000}{99} \tag{1.0.6.10}$$

$$k = 10.101 \tag{1.0.6.11}$$

Choose..,

$$R = 1000\Omega \tag{1.0.6.12}$$

So, the final circuit is shown in figure.1.0.6:2. The above circuit ensures the closed loop transfer function has maximally flat response.

1.0.7. Find the step response of above closed loop system.

Solution: The following code generates the unit step response of closed loop transfer function.

```
codes/ee18btech11005/ee18btech11005_3.py
```

The plot of unit step response is shown in the figure.1.0.7 The steady state value from the plot is 10. The system is stable.

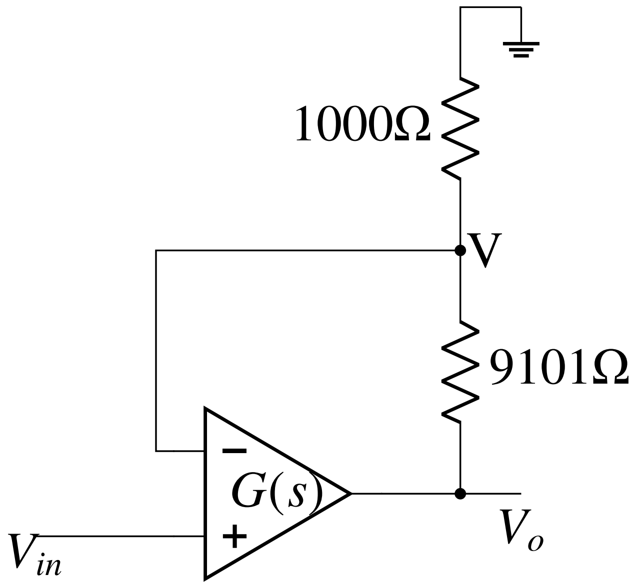

Fig. 1.0.6: 2

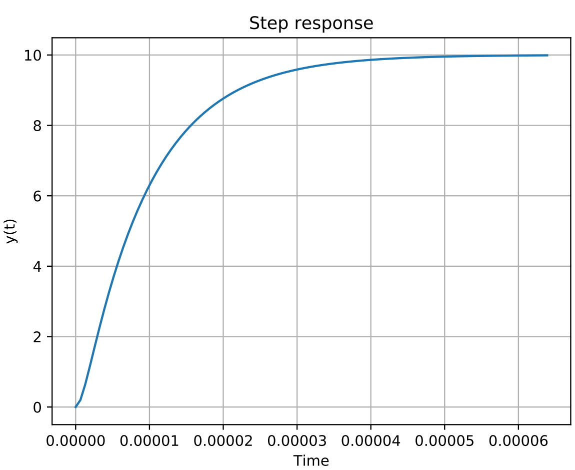

Fig. 1.0.7