Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Feedback Circuits

1.0.1. The non-inverting op-amp configuration shown in fig.1.0.1:1 provides direct implementation of feedback loop.Assuming operational amplifier has infinite input resistance and zero output resistance.Find the expression for feedback factor. **Solution:** Let the gain of the operational

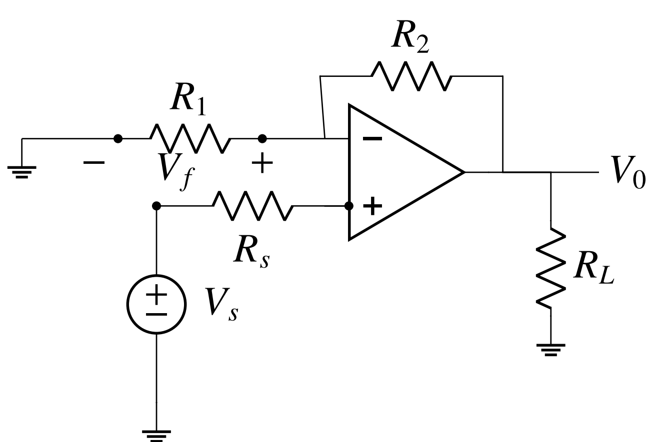

Fig. 1.0.1: 1

amplifier be G. G is the open loop gain of the amplifier. The parameters given are shown in the TABLE.1.0.1:1 The equivalent circuit of the amplifier is in fig.1.0.1:2 From the

Parameter	**Value**
input resistance	∞
output resistance	**0**
Input voltage	V_s
Output Voltage	V_o
Feeding resistance	R_1
Feedback resistance	R_2
Source resistance	R_s
load resistance	R_L

TABLE 1.0.1: 1

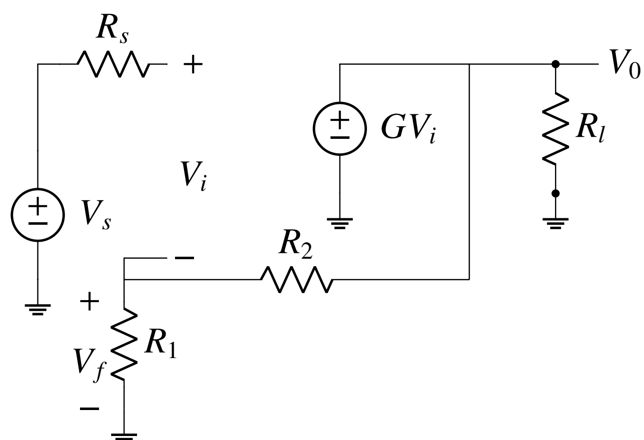

Fig. 1.0.1: 2

equivalent circuit, Applying Ohms law,

$$V_0 = G(V_i) \tag{1.0.1.1}$$

$$\text{and,} V_i = V_+ - V_- \tag{1.0.1.2}$$

Now,Applying voltage dividing rule

$$V_- = \left[\frac{R_1}{R_1 + R_2}\right] V_0 \tag{1.0.1.3}$$

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Substituting in equ.1.0.1.1

$$V_0 = G(V_+ - \left[\frac{R_1}{R_1 + R_2}\right] V_0) \quad (1.0.1.4)$$

$$\implies V_0 = GV_+ - G\left[\frac{R_1}{R_1 + R_2}\right] V_0 \quad (1.0.1.5)$$

$$G(V_+) = V_0 + G\left[\frac{R_1}{R_1 + R_2}\right] V_0 \quad (1.0.1.6)$$

But,

$$V_s = V_+ \quad (1.0.1.7)$$

because, no current flows through resistor.

$$V_0 = G\left[\frac{1}{1 + \frac{GR_1}{R_1+R_2}}\right] V_s \quad (1.0.1.8)$$

$$\text{Gain} = \frac{V_0}{V_s} = \left[\frac{G}{1 + \frac{GR_1}{R_1+R_2}}\right] \quad (1.0.1.9)$$

For a negative feedback system,

$$\frac{V_0}{V_i} = \frac{G}{1 + GH} \quad (1.0.1.10)$$

$$\text{where.,} H = \frac{R_1}{R_1 + R_2} \quad (1.0.1.11)$$

The equation.1.0.1.1 looks exactly similar to the Gain of a negative feedback system with

- Open loop gain = G
- Loop gain = P
- Amount of feedback = F
- Feedback factor = f
- closed loop gain = T

Parameters	Definition	For given circuit
Open loop gain	G	G
Feedback factor	H	$\frac{R_1}{R_1+R_2}$
Loop gain	GH	$G\frac{R_1}{R_1+R_2}$
Amount of feedback	1+GH	$1 + \frac{GR_1}{R_1+R_2}$
Closed loop gain	$\frac{G}{1+GH}$	$\frac{G(R_1+R_2)}{R_1+R_2+GR_1}$

TABLE 1.0.1: 2

Therefore,This operational amplifier can be modelled as a negative feedback system shown in the fig.1.0.1:3 So, the feedback factor..,

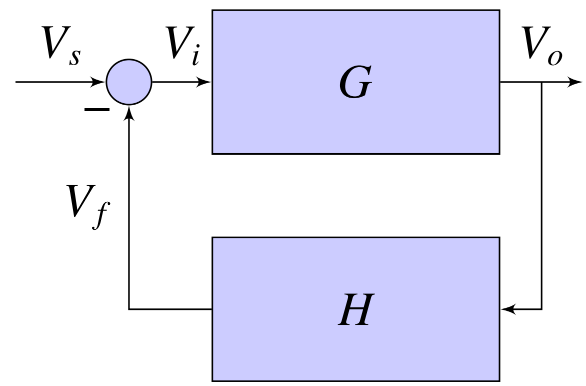

Fig. 1.0.1: 3

$$f = H = \frac{R_1}{R_1 + R_2} \quad (1.0.1.12)$$

1.0.2. Find the condition under which closed loop gain T is almost entirely determined by the feedback network. **Solution:** For T to entirely dependent on feedback network, it should be independent on G(open loop gain) T is given by..,

$$T = \frac{G}{1 + \frac{GR_1}{R_1+R_2}} \quad (1.0.2.1)$$

$$(1.0.2.2)$$

For T to be independent on G..,

$$GH >> 1 \quad (1.0.2.3)$$

$$G\frac{R_1}{R_1 + R_2} >> 1 \quad (1.0.2.4)$$

$$G >> 1 + \frac{R_2}{R_1} \quad (1.0.2.5)$$

Under such condition..,

$$T = \frac{1}{H} \quad (1.0.2.6)$$

$$T = \frac{R_1 + R_2}{R_1} \quad (1.0.2.7)$$

$$T = 1 + \frac{R_2}{R_1} \quad (1.0.2.8)$$

so, the necessary condition for T depend only on feedback network is

$$G >> T \quad (1.0.2.9)$$

1.0.3. If the open loop voltage gain

$$G = 10^4 \tag{1.0.3.1}$$

Find the ratio of R2 and R1 to obtain a closed loop gain of 10. **Solution:** The closed loop gain gain T is given by

$$T = \frac{G}{1+GH} = \frac{G}{1+\frac{GR_1}{R_1+R_2}} = 10 \tag{1.0.3.2}$$

$$\text{where..,} G = 10^4 \tag{1.0.3.3}$$

$$10 = \frac{10^4}{1+\frac{10^4}{1+\frac{R_2}{R_1}}} \tag{1.0.3.4}$$

$$\implies 1+\frac{R_2}{R_1} = \frac{10^4}{\frac{10^4}{10}-1} \tag{1.0.3.5}$$

$$1+\frac{R_2}{R_1} = 10.010 \tag{1.0.3.6}$$

$$\frac{R_2}{R_1} = 9.010 \tag{1.0.3.7}$$

1.0.4. What is the amount of feedback in decibels? **Solution:** The value of F in decibals is given by

$$F(dB) = 20\log(F) \tag{1.0.4.1}$$

$$\text{where..,} F = 1+GH \tag{1.0.4.2}$$

$$F = \frac{G}{T} \tag{1.0.4.3}$$

$$\text{where..,} G = 10^4 \tag{1.0.4.4}$$

$$T = 10 \tag{1.0.4.5}$$

$$F(dB) = 20\log(\frac{10^4}{10}) = 20\log(1000) \tag{1.0.4.6}$$

$$F(dB) = 60dB \tag{1.0.4.7}$$

1.0.5. If G decreases by 20%,what is the corresponding decrease in T? **Solution:** Given

$$G = 10^4 \tag{1.0.5.1}$$

If G decrease by 20% then, the value of G is..,

$$G = (1-0.2)10^4 \tag{1.0.5.2}$$

$$= 8000 \tag{1.0.5.3}$$

For this value of G and ,

$$\frac{R_2}{R_1} = 9.010 \tag{1.0.5.4}$$

The value of T can be solved as follows,

$$T = \frac{G}{1+\frac{G}{1+\frac{R_2}{R_1}}} \tag{1.0.5.5}$$

$$T = \frac{8000}{1+\frac{8000}{1+0.9010}} \tag{1.0.5.6}$$

$$T = 9.99749 \tag{1.0.5.7}$$

The percentage change in T is..,

$$fractionalchange = \frac{10-9.99749}{10} \tag{1.0.5.8}$$

$$= 2.51x10^{-4} \tag{1.0.5.9}$$

$$\%changeinT = 0.00251 \tag{1.0.5.10}$$

Therefore T decreases by 0.0025% when G decreases by 20%

1.0.6. Write a python code that can compute closed loop gain,loop gain,amount of feedback given all input parameters. **Solution:** Code to compute different gains.,

```
codes/ee18btech11005/ee18btech11005.py
```