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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

1 STABILITY

2 ROUTH HURWITZ CRITERION

3 COMPENSATORS

4 NYQUIST PLOT

5 FEEDBACK SYSTEMS

5.0.1. The non-inverting op-amp configuration shown in fig.5.0.1 provides direct implementation of feedback loop. Assuming operational amplifier has infinite input resistance and zero output resistance. Find the expression for feedback factor. **Solution:** Let the gain of the operational

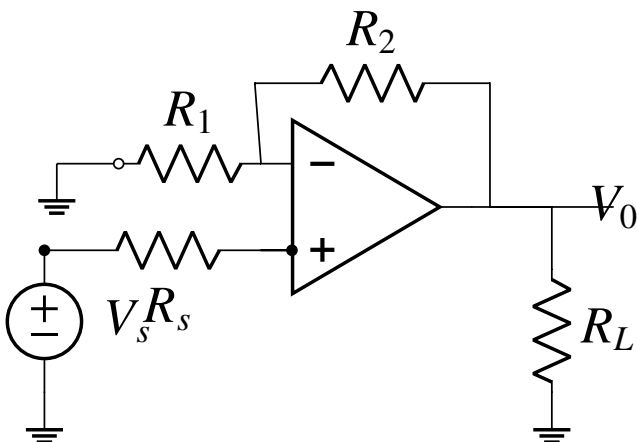


Fig. 5.0.1

amplifier be A . The equivalent circuit of the amplifier is in fig.5.0.1 From the equivalent circuit, Applying Ohms law,

$$V_0 = A(V_+ - V_-) \quad (5.0.1.1)$$

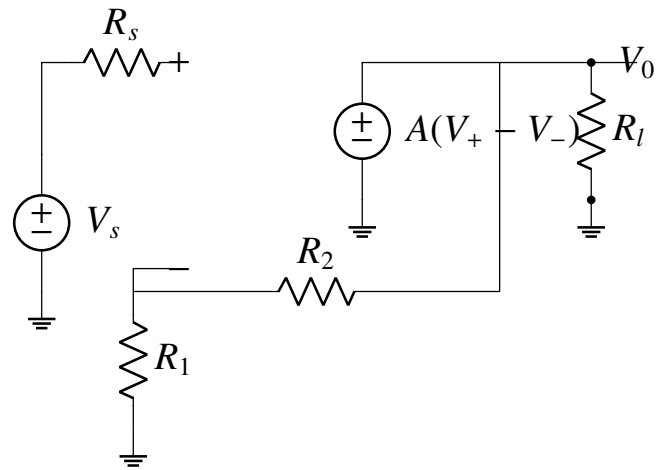


Fig. 5.0.1

Now, Applying voltage dividing rule

$$V_- = \left[\frac{R_1}{R_1 + R_2} \right] V_0 \quad (5.0.1.2)$$

Substituting in equ.5.0.1.1

$$V_0 = A(V_+ - \left[\frac{R_1}{R_1 + R_2} \right] V_0) \quad (5.0.1.3)$$

$$\Rightarrow V_0 = AV_+ - A \left[\frac{R_1}{R_1 + R_2} \right] V_0 \quad (5.0.1.4)$$

$$A(V_+) = V_0 + A \left[\frac{R_1}{R_1 + R_2} \right] V_0 \quad (5.0.1.5)$$

But,

$$V_s = V_+ \quad (5.0.1.6)$$

because, no current flows through resistor, R_s . Since, input resistance is given infinite in fig.5.0.1 The equation can be written as...

$$V_0 = A \left[\frac{1}{1 + \frac{AR_1}{R_1 + R_2}} \right] V_s \quad (5.0.1.7)$$

$$\text{Gain} = \frac{V_0}{V_s} = \left[\frac{A}{1 + \frac{AR_1}{R_1 + R_2}} \right] \quad (5.0.1.8)$$

For a negative feedback system,

$$\frac{V_0}{V_i} = \frac{A}{1 + A\beta} \quad (5.0.1.9)$$

$$\text{where, } \beta = \frac{R_1}{R_1 + R_2} \quad (5.0.1.10)$$

The equation.5.0.1.1 looks exactly similar to the Gain of a negative feedback system with

- Open loop gain = G

- Loop gain = P
- Amount of feedback = F
- Feedback factor = f

where

$$G = A \quad (5.0.1.11)$$

$$P = GH = A \frac{R_1}{R_1 + R_2} \quad (5.0.1.12)$$

$$F = 1 + GH = 1 + \frac{AR_1}{R_1 + R_2} \quad (5.0.1.13)$$

$$f = H = \frac{R_1}{R_1 + R_2} \quad (5.0.1.14)$$

Therefore, This operational amplifier can be modelled as a negative feedback system shown in the fig.5.0.1 So, the feedback factor f.,

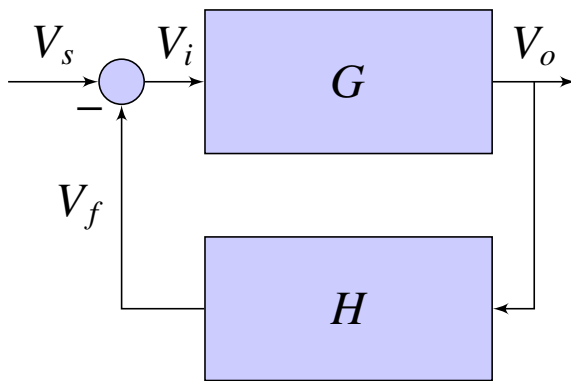


Fig. 5.0.1

$$f = H = \frac{R_1}{R_1 + R_2} \quad (5.0.1.15)$$

5.0.2. Find the condition under which closed loop gain A_f is almost entirely determined by the feedback network. **Solution:** For A_f to be entirely dependent on feedback network, it should be independent on A (open loop gain). A_f is given by...

$$A_f = \frac{A}{1 + \frac{AR_1}{R_1 + R_2}} \quad (5.0.2.1)$$

$$(5.0.2.2)$$

For A_f to be independent on A ...

$$A\beta \gg 1 \quad (5.0.2.3)$$

$$A \frac{R_1}{R_1 + R_2} \gg 1 \quad (5.0.2.4)$$

$$A \gg 1 + \frac{R_2}{R_1} \quad (5.0.2.5)$$

Under such condition...

$$A_f = \frac{1}{\beta} \quad (5.0.2.6)$$

$$A_f = \frac{R_1 + R_2}{R_1} \quad (5.0.2.7)$$

$$A_f = 1 + \frac{R_2}{R_1} \quad (5.0.2.8)$$

so, the necessary condition for A_f depend only on feedback network is

$$A \gg A_f \quad (5.0.2.9)$$

5.0.3. If the open loop voltage gain

$$A = 10^4 \quad (5.0.3.1)$$

Find the ratio of R_2 and R_1 to obtain a closed loop gain of 10. **Solution:** The closed loop gain A_f is given by

$$A_f = \frac{A}{1 + A\beta} = \frac{A}{1 + \frac{AR_1}{R_1 + R_2}} = 10 \quad (5.0.3.2)$$

$$\text{where...} A = 10^4 \quad (5.0.3.3)$$

$$10 = \frac{10^4}{1 + \frac{10^4 R_1}{R_1 + R_2}} \quad (5.0.3.4)$$

$$\Rightarrow 1 + \frac{R_2}{R_1} = \frac{10^4}{\frac{10^4}{10} - 1} \quad (5.0.3.5)$$

$$1 + \frac{R_2}{R_1} = 10.010 \quad (5.0.3.6)$$

$$\frac{R_2}{R_1} = 9.010 \quad (5.0.3.7)$$

5.0.4. What is the amount of feedback in decibels?

Solution: The value of F in decibels is given by

$$F(\text{dB}) = 20 \log(F) \quad (5.0.4.1)$$

$$\text{where...} F = 1 + A\beta \quad (5.0.4.2)$$

$$F = \frac{A}{A_f} \quad (5.0.4.3)$$

$$\text{where...} A = 10^4 \quad (5.0.4.4)$$

$$A_f = 10 \quad (5.0.4.5)$$

$$F(\text{dB}) = 20 \log\left(\frac{10^4}{10}\right) = 20 \log(1000) \quad (5.0.4.6)$$

$$F(\text{dB}) = 60 \text{ dB} \quad (5.0.4.7)$$

5.0.5. If A decreases by 20%, what is the corresponding decrease in Af? **Solution:** Given

$$A = 10^4 \quad (5.0.5.1)$$

If A decrease by 20% then, the value of A is..,

$$A = (1 - 0.2)10^4 \quad (5.0.5.2)$$

$$= 8000 \quad (5.0.5.3)$$

For this value of A and ,

$$\frac{R_2}{R_1} = 9.010 \quad (5.0.5.4)$$

The value of Af can be solved as follows,

$$A_f = \frac{A}{1 + \frac{A}{1 + \frac{R_2}{R_1}}} \quad (5.0.5.5)$$

$$A_f = \frac{8000}{1 + \frac{8000}{1 + 0.9010}} \quad (5.0.5.6)$$

$$A_f = 9.99749 \quad (5.0.5.7)$$

The percentage change in Af is..,

$$fractionalchange = \frac{10 - 9.99749}{10} \quad (5.0.5.8)$$

$$= 2.51 \times 10^{-4} \quad (5.0.5.9)$$

$$\%changeinA_f = 0.00251 \quad (5.0.5.10)$$

Therefore Af decreases by 0.0025% when A decreases by 20%