

Control Systems

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CONTENTS

1 Feedback Circuits 1

Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

1 FEEDBACK CIRCUITS

1.0.1. A dc amplifier has an open loop gain of 1000 and two poles, a dominant one at 1kHz and a high frequency one whose location can be controlled. It is required to connect this amplifier in a negative feedback loop that provides a dc closed loop gain of 10 and a maximally flat response. Find the required value of H.

Solution: Given, open loop gain G is dependent on frequency and has two poles. Therefore, $G(s)$ can be written as..,

$$G(s) = \frac{G_0}{(1 - \frac{s}{p_1})(1 - \frac{s}{p_2})} \quad (1.0.1.1)$$

where, p_1 and p_2 are poles of $G(s)$. Parameters given are shown in Table.1.0.1:1

Parameter	Value
dc open loop gain	1000
dominant pole	1000Hz
insignificant pole	p_2
dc closed loop gain	10

TABLE 1.0.1: 1

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$$G_0 = 1000 \quad (1.0.1.2)$$

$$\text{Therefore, } G(s) = \frac{1000}{(1 - \frac{s}{p_1})(1 - \frac{s}{p_2})} \quad (1.0.1.3)$$

Let, p_1 be the dominant pole.

$$p_1 = 2\pi 10^3 \text{ rad/sec} \quad (1.0.1.4)$$

Now, we connect the system in a negative feedback of feedback factor H. We know that the closed loop gain of a negative feedback system is.,

$$T(s) = \frac{G(s)}{1 + G(s)H} \quad (1.0.1.5)$$

But, also given DC closed loop gain is 10. DC closed loop gain is given in equation.1.0.1.6

$$T(0) = \frac{G_0}{1 + G_0H} \quad (1.0.1.6)$$

$$\text{given, } T(0) = 10 \quad (1.0.1.7)$$

$$\text{and, } G_0 = 1000 \quad (1.0.1.8)$$

$$\frac{1000}{1 + 1000H} = 10 \quad (1.0.1.9)$$

$$1 + 1000H = 100 \quad (1.0.1.10)$$

$$\Rightarrow H = \frac{99}{1000} = 0.099 \quad (1.0.1.11)$$

from equation:1.0.1.11, the value of H is 0.099. Therefore, the feedback factor is 0.099.

1.0.2. Find the location at which second pole can be placed.

Solution: Given the negative feedback system should have maximally flat response. from equation.1.0.1.5 and equation.1.0.1.1 the transfer function is,

$$T(s) = \frac{\frac{G_0}{(1 - \frac{s}{p_1})(1 - \frac{s}{p_2})}}{1 + \frac{HG_0}{(1 - \frac{s}{p_1})(1 - \frac{s}{p_2})}} \quad (1.0.2.1)$$

$$T(s) = \frac{\frac{p_1 p_2 G_0}{(p_1 - s)(p_2 - s)}}{1 + \frac{p_1 p_2 H G_0}{(p_1 - s)(p_2 - s)}} \quad (1.0.2.2)$$

$$T(s) = \frac{p_1 p_2 G_0}{(p_1 - s)(p_2 - s) + p_1 p_2 H G_0} \quad (1.0.2.3)$$

$$T(s) = \frac{p_1 p_2 G_0}{p_1 p_2 - (p_1 + p_2)s + s^2 + p_1 p_2 H G_0} \quad (1.0.2.4)$$

$$T(s) = \frac{p_1 p_2 G_0}{s^2 - (p_1 + p_2)s + (H G_0 + 1)p_1 p_2} \quad (1.0.2.5)$$

The characteristics equation of above transfer function is.,

$$C.E = s^2 - (p_1 + p_2)s + (H G_0 + 1)p_1 p_2 \quad (1.0.2.6)$$

In general, For a second order amplifier the C.E is.,

$$C.E = s^2 + 2\zeta\omega_n s + \omega_n^2 \quad (1.0.2.7)$$

The quality factor Q of equation.1.0.2.7, is given by

$$Q = \frac{1}{2\zeta} \quad (1.0.2.8)$$

From equation.1.0.2.6,

$$\omega_n = \pm \sqrt{(H G_0 + 1)p_1 p_2} \quad (1.0.2.9)$$

$$\zeta = \pm \frac{2p_1 p_2}{\sqrt{(H G_0 + 1)p_1 p_2}} \quad (1.0.2.10)$$

Therefore, For the given second order amplifier with characteristic equation.1.0.2.6, the Q factor is,

$$Q = \pm \frac{\sqrt{(1 + H G_0)p_1 p_2}}{p_1 + p_2} \quad (1.0.2.11)$$

For a negative feedback amplifier to achieve maximally flat response,

$$Q = 0.7071 \quad (1.0.2.12)$$

$$(1.0.2.13)$$

Therefore, substituting the values of Q and other parameters in equation.1.0.2.11,

$$0.7071 = \pm \frac{\sqrt{(1 + 0.099(1000))(2\pi 10^3)p_2}}{2\pi 10^3 + p_2} \quad (1.0.2.14)$$

Squaring on both sides and rearranging.,

$$(1 + 1000(0.099))2\pi 10^3 p_2 = 0.7071^2(2\pi 10^3 + p_2)^2 \quad (1.0.2.15)$$

$$(100)2\pi 10^3 = 0.7071^2(2\pi 10^3 + p_2)^2 \quad (1.0.2.16)$$

$$\Rightarrow 0.5p_2^2 - 622037.2p_2 + 19733247.6 = 0 \quad (1.0.2.17)$$

Solving above equation.,

$$p_2 = 31.7244 \text{ rad/sec} \quad (1.0.2.18)$$

$$p_2 = 1244042.676 \text{ rad/sec} \quad (1.0.2.19)$$

But, Since p1 is dominating pole, p1 should be close to origin.

$$p_1 \ll p_2 \quad (1.0.2.20)$$

$$p_2 = 1.244 \text{ Mrad/sec} \quad (1.0.2.21)$$

$$(1.0.2.22)$$

$$p_2 = \frac{1.244M}{2\pi} \text{ Hz} = 192.989 \text{ kHz} \quad (1.0.2.23)$$

$$(1.0.2.24)$$

1.0.3. Verify roots of above equation using python code.

codes/ee18btech11005/ee18btech11005_1.py

1.0.4. Now, Find the open loop transfer function and closed loop transfer function of the system.

Solution: Substituting the value of p2 in the equation 1.0.1.1 and 1.0.2.5

$$G(s) = \frac{1000}{(1 - \frac{s}{10^3})(1 - \frac{s}{192.989 \times 10^3})} \quad (1.0.4.1)$$

$$T(s) = \frac{192.989 \times 10^9}{s^2 - 193 \times 10^3 s + 192.989 \times 10^{10}} \quad (1.0.4.2)$$

1.0.5. Verify from the Bode plot of above closed loop transfer function that it has maximally flat response.

Solution: The following code generates the bode plot of the transfer function.

codes/ee18btech11005/ee18btech11005_2.py

From the magnitude bode plot in figure.1.0.5:1 we can tell that the open loop transfer function after having negative feedback has maximally flat response for the obtained value of p2.

1.0.6. Design a circuit that represents the above transfer function.

Solution: The circuit can be designed using an operational amplifier having negative feed-

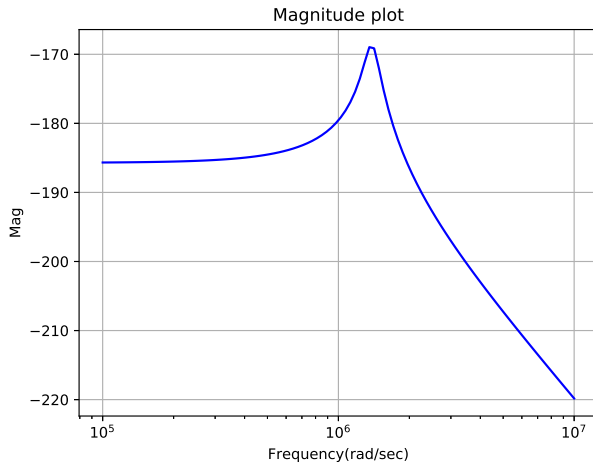


Fig. 1.0.5: 1

back. Consider the circuit shown in figure.1.0.6
For a differential amplifier,

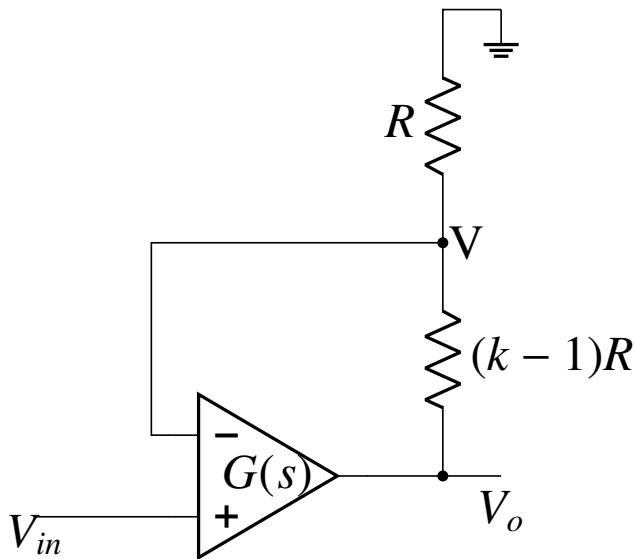


Fig. 1.0.6

$$V_o = G(s)(V_{in} - V) \quad (1.0.6.1)$$

$$\text{where, } V = \frac{V_o R}{kR} = \frac{V_o}{k} \quad (1.0.6.2)$$

$$V_o = G(s)\left(V_{in} - \frac{V_o}{k}\right) \quad (1.0.6.3)$$

$$V_o = G(s)V_{in} - \frac{G(s)V_o}{k} \quad (1.0.6.4)$$

$$V_o = \frac{G(s)V_{in}}{1 + \frac{G(s)}{k}} \quad (1.0.6.5)$$

$$T(s) = \frac{G(s)}{1 + \frac{G(s)}{k}} \quad (1.0.6.6)$$

Where., $G(s)$ is open loop gain that varies with frequency. Comparing equation.1.0.1.5 and equation.1.0.6.6 We can design the equivalent circuit for the given problem.

$$H = \frac{1}{k} \quad (1.0.6.7)$$

$$\Rightarrow k = \frac{1}{H} \quad (1.0.6.8)$$

$$(1.0.6.9)$$

We know., $H = 0.099$

$$k = \frac{1}{0.099} = \frac{1000}{99} \quad (1.0.6.10)$$

$$k = 10.101 \quad (1.0.6.11)$$

Choose.,

$$R = 1000\Omega \quad (1.0.6.12)$$

So, the final circuit is shown in figure.1.0.6.
The above circuit ensures the closed loop trans-

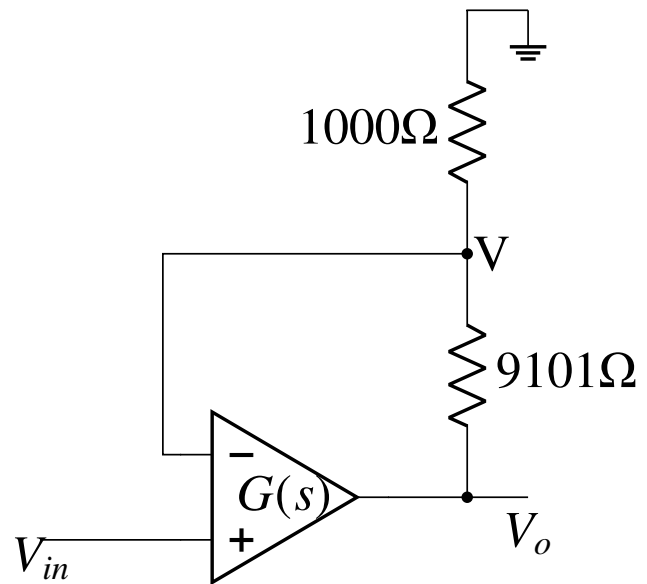


Fig. 1.0.6

fer function has maximally flat response.

1.0.7. Find the step response of above closed loop system.

Solution: The following code generates the unit step response of closed loop transfer function.

codes/ee18btech11005/ee18btech11005_3.py

The plot of unit step response is shown in the figure.1.0.7

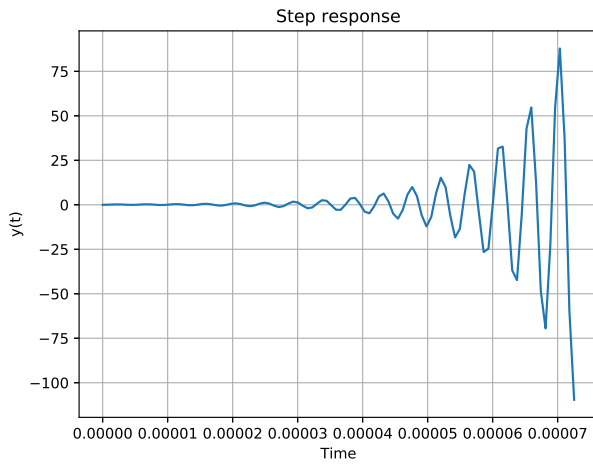


Fig. 1.0.7