Control Systems

G V V Sharma*

Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Feedback Circuits

1.0.1. The non-inverting op-amp configuration shown in fig.1.0.1 provides direct implementation of feedback loop.Assuming operational amplifier has infinite input resistance and zero output resistance.Find the expression for feedback factor. **Solution:** Let the gain of the operational amplifier be G. The equivalent circuit of the amplifier is in fig.1.0.1 The parameters given are shown in the table.1.0.1 From the equivalent circuit, Applying Ohms law,

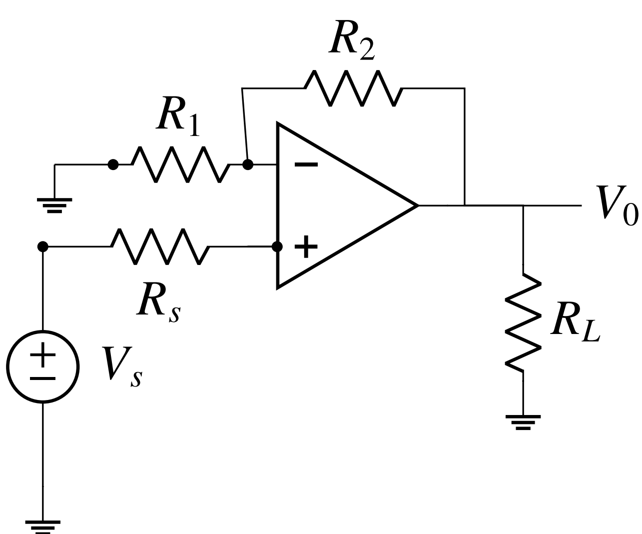

Fig. 1.0.1

Parameter	Value
input resistance	∞
output resistance	**0**
Input voltage	V_s
Output Voltage	V_o
Feedback resistance	R_2
load resistance	R_L

TABLE 1.0.1

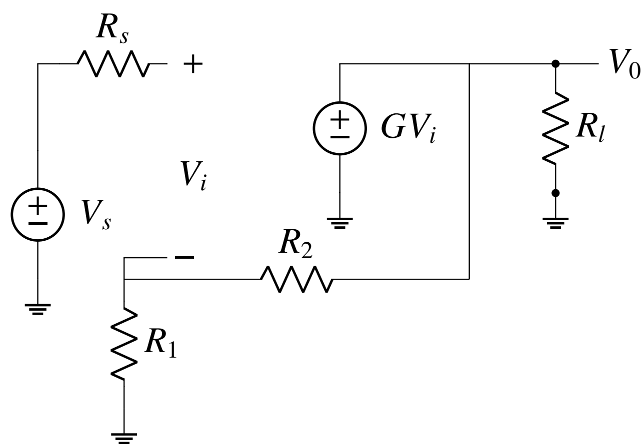

Fig. 1.0.1

$$V_0 = G(V_i) \tag{1.0.1.1}$$

$$\text{and,} V_i = V_+ - V_- \tag{1.0.1.2}$$

Now,Applying voltage dividing rule

$$V_- = \left[\frac{R_1}{R_1 + R_2}\right] V_0 \tag{1.0.1.3}$$

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

Substituting in equ.1.0.1.1

$$V_0 = G(V_+ - \left[\frac{R_1}{R_1+R_2}\right] V_0) \quad (1.0.1.4)$$

$$\implies V_0 = GV_+ - G\left[\frac{R_1}{R_1+R_2}\right] V_0 \quad (1.0.1.5)$$

$$G(V_+) = V_0 + G\left[\frac{R_1}{R_1+R_2}\right] V_0 \quad (1.0.1.6)$$

But,

$$V_s = V_+ \quad (1.0.1.7)$$

because, no current flows through resistor.

$$V_0 = G\left[\frac{1}{1+\frac{GR_1}{R_1+R_2}}\right] V_s \quad (1.0.1.8)$$

$$\text{Gain} = \frac{V_0}{V_s} = \left[\frac{G}{1+\frac{GR_1}{R_1+R_2}}\right] \quad (1.0.1.9)$$

For a negative feedback system,

$$\frac{V_0}{V_i} = \frac{G}{1+GH} \quad (1.0.1.10)$$

$$\text{where.,} H = \frac{R_1}{R_1+R_2} \quad (1.0.1.11)$$

The equation.1.0.1.1 looks exactly similar to the Gain of a negative feedback system with

- Open loop gain = G
- Loop gain = P
- Amount of feedback = F
- Feedback factor = f
- closed loop gain = T

Parameters	Definition	For given circuit
Open loop gain	G	G
Feedback factor	H	$\frac{R_1}{R_1+R_2}$
Loop gain	GH	$G\frac{R_1}{R_1+R_2}$
Amount of feedback	1+GH	$1+\frac{GR_1}{R_1+R_2}$
Closed loop gain	$\frac{G}{1+GH}$	$\frac{G(R_1+R_2)}{R_1+R_2+GR_1}$

TABLE 1.0.1

Therefore,This operational amplifier can be modelled as a negative feedback system shown in the fig.1.0.1 So, the feedback factor ..,

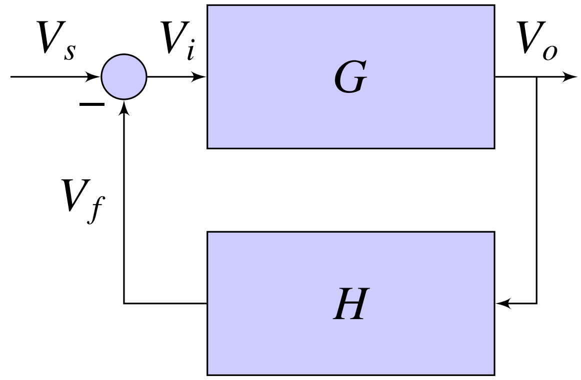

Fig. 1.0.1

$$f = H = \frac{R_1}{R_1+R_2} \quad (1.0.1.12)$$

1.0.2. Find the condition under which closed loop gain T is almost entirely determined by the feedback network. **Solution:** For T to entirely dependent on feedback network, it should be independent on G(open loop gain) T is given by..,

$$T = \frac{G}{1+\frac{GR_1}{R_1+R_2}} \quad (1.0.2.1)$$

$$(1.0.2.2)$$

For T to be independent on G..,

$$GH >> 1 \quad (1.0.2.3)$$

$$G\frac{R_1}{R_1+R_2} >> 1 \quad (1.0.2.4)$$

$$G >> 1 + \frac{R_2}{R_1} \quad (1.0.2.5)$$

Under such condition..,

$$T = \frac{1}{H} \quad (1.0.2.6)$$

$$T = \frac{R_1+R_2}{R_1} \quad (1.0.2.7)$$

$$T = 1 + \frac{R_2}{R_1} \quad (1.0.2.8)$$

so, the necessary condition for T depend only on feedback network is

$$G >> T \quad (1.0.2.9)$$

1.0.3. If the open loop voltage gain

$$G = 10^4 \tag{1.0.3.1}$$

Find the ratio of R2 and R1 to obtain a closed loop gain of 10. **Solution:** The closed loop gain gain T is given by

$$T = \frac{G}{1+GH} = \frac{G}{1+\frac{GR_1}{R_1+R_2}} = 10 \tag{1.0.3.2}$$

$$\text{where..,}G = 10^4 \tag{1.0.3.3}$$

$$10 = \frac{10^4}{1+\frac{10^4}{1+\frac{R_2}{R_1}}} \tag{1.0.3.4}$$

$$\implies 1+\frac{R_2}{R_1} = \frac{10^4}{\frac{10^4}{10}-1} \tag{1.0.3.5}$$

$$1+\frac{R_2}{R_1} = 10.010 \tag{1.0.3.6}$$

$$\frac{R_2}{R_1} = 9.010 \tag{1.0.3.7}$$

1.0.4. What is the amount of feedback in decibels? **Solution:** The value of F in decibals is given by

$$F(dB) = 20\log(F) \tag{1.0.4.1}$$

$$\text{where..,}F = 1+GH \tag{1.0.4.2}$$

$$F = \frac{G}{T} \tag{1.0.4.3}$$

$$\text{where..,}G = 10^4 \tag{1.0.4.4}$$

$$T = 10 \tag{1.0.4.5}$$

$$F(dB) = 20\log(\frac{10^4}{10}) = 20\log(1000) \tag{1.0.4.6}$$

$$F(dB) = 60dB \tag{1.0.4.7}$$

1.0.5. If G decreases by 20%,what is the corresponding decrease in T? **Solution:** Given

$$G = 10^4 \tag{1.0.5.1}$$

If G decrease by 20% then, the value of G is..,

$$G = (1-0.2)10^4 \tag{1.0.5.2}$$

$$= 8000 \tag{1.0.5.3}$$

For this value of G and ,

$$\frac{R_2}{R_1} = 9.010 \tag{1.0.5.4}$$

The value of T can be solved as follows,

$$T = \frac{G}{1+\frac{G}{1+\frac{R_2}{R_1}}} \tag{1.0.5.5}$$

$$T = \frac{8000}{1+\frac{8000}{1+0.9010}} \tag{1.0.5.6}$$

$$T = 9.99749 \tag{1.0.5.7}$$

The percentage change in T is..,

$$fractionalchange = \frac{10-9.99749}{10} \tag{1.0.5.8}$$

$$= 2.51x10^{-4} \tag{1.0.5.9}$$

$$\%changeinT = 0.00251 \tag{1.0.5.10}$$

Therefore T decreases by 0.0025% when G decreases by 20%

1.0.6. Write a python code that can compute closed loop gain,loop gain,amount of feedback given all input parameters. **Solution:** Code to compute different gains.,

```
codes/ee18btech11005/ee18btech11005.py
```