Control Systems

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1 Feedback Circuits

Abstract—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.

Download python codes using

svn co https://github.com/gadepall/school/trunk/control/codes

1 FEEDBACK CIRCUITS

1.0.1. The non-inverting op-amp configuration shown in fig.1.0.1:1 provides direct implementation of feedback loop. Assuming operational amplifier has infinite input resistance and zero output resistance. Find the expression for feedback factor. **Solution:** Let the gain of the operational

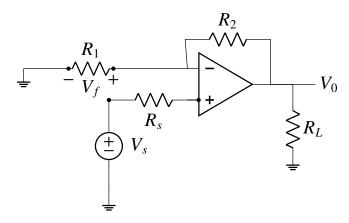


Fig. 1.0.1: 1

amplifier be G. G is the open loop gain of the amplifier. The parameters given are shown in the TABLE.1.0.1:1 The equivalent circuit of the amplifier is in fig.1.0.1:2 From the

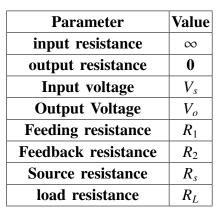


TABLE 1.0.1: 1

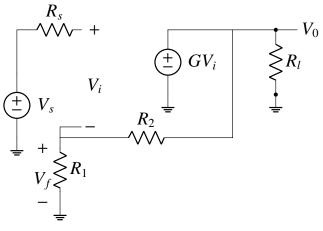


Fig. 1.0.1: 2

equivalent circuit, Applying Ohms law,

$$V_0 = G(V_i) (1.0.1.1)$$

and,
$$V_i = V_+ - V_-$$
 (1.0.1.2)

Now, Applying voltage dividing rule

$$V_{-} = \left[\frac{R_1}{R_1 + R_2}\right] V_0 \tag{1.0.1.3}$$

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Substituting in equ.1.0.1.1

$$V_0 = G(V_+ - \left[\frac{R_1}{R_1 + R_2}\right] V_0) \quad (1.0.1.4)$$

$$\implies V_0 = GV_+ - G\left[\frac{R_1}{R_1 + R_2}\right]V_0 \quad (1.0.1.5)$$

$$G(V_+) = V_0 + G\left[\frac{R_1}{R_1 + R_2}\right] V_0$$
 (1.0.1.6)

But,

$$V_{\rm s} = V_{\perp}$$
 (1.0.1.7)

because, no current flows through resistor.

$$V_0 = G \left[\frac{1}{1 + \frac{GR_1}{R_1 + R_2}} \right] V_s \qquad (1.0.1.8)$$

Gain =
$$\frac{V_0}{V_s} = \left[\frac{G}{1 + \frac{GR_1}{R_1 + R_2}} \right]$$
 (1.0.1.9)

For a negative feedback system,

$$\frac{V_0}{V_i} = \frac{G}{1 + GH} \quad (1.0.1.10)^{1}$$

where.,
$$H = \frac{R_1}{R_1 + R_2}$$
 (1.0.1.11)

The equation.1.0.1.1 looks exactly similar to the Gain of a negative feedback system with

- Open loop gain = G
- Loop gain = P
- Amount of feedback = F
- Feedback factor = f
- closed loop gain = T

Parame- ters	Definition	For given circuit
Open	G	G
loop gain		
Feedback	Н	$\frac{R_1}{R_1+R_2}$
factor		K ₁ + K ₂
Loop gain	GH	$G^{rac{R_1}{R_1+R_2}}$
Amount	1+GH	$1 + \frac{GR_1}{R_1 + R_2}$
of		K ₁ + K ₂
feedback		
Closed	$\frac{G}{1+GH}$	$\frac{G(R_1+R_2)}{R_1+R_2+GR_1}$
loop gain	1+011	$K_1 + K_2 + GK_1$

TABLE 1.0.1: 2

Therefore, This operational amplifier can be modelled as a negative feedback system shown in the fig.1.0.1:3 So, the feedback factor...,

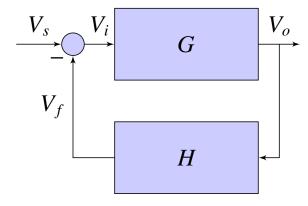


Fig. 1.0.1: 3

$$f = H = \frac{R_1}{R_1 + R_2} \tag{1.0.1.12}$$

(1.0.1.10)
1.0.2. Find the condition under which closed loop gain T is almost entirely determined by the feedback network. **Solution:** For T to entirely dependent on feedback network, it should be independent on G(open loop gain) T is given by...

$$T = \frac{G}{1 + \frac{GR_1}{R_1 + R_2}} \tag{1.0.2.1}$$

(1.0.2.2)

For T to be independent on G.,

$$GH >> 1$$
 (1.0.2.3)

$$G\frac{R_1}{R_1 + R_2} >> 1 \tag{1.0.2.4}$$

$$G >> 1 + \frac{R_2}{R_1} \tag{1.0.2.5}$$

Under such condition...

$$T = \frac{1}{H} \tag{1.0.2.6}$$

$$T = \frac{R_1 + R_2}{R_1} \tag{1.0.2.7}$$

$$T = 1 + \frac{R_2}{R_1} \tag{1.0.2.8}$$

so, the necessary condition for T depend only on feedback network is

$$G >> T$$
 (1.0.2.9)

1.0.3. If the open loop voltage gain

$$G = 10^4 \tag{1.0.3.1}$$

Find the ratio of R2 and R1 to obtain a closed loop gain of 10. **Solution:** The closed loop gain gain T is given by

$$T = \frac{G}{1 + GH} = \frac{G}{1 + \frac{GR_1}{R_1 + R_2}} = 10$$

(1.0.3.2)

where..,
$$G = 10^4$$
 (1.0.3.3)

$$10 = \frac{10^4}{1 + \frac{10^4}{1 + \frac{R_2}{R_1}}} \tag{1.0.3.4}$$

$$\implies 1 + \frac{R_2}{R_1} = \frac{10^4}{\frac{10^4}{10} - 1} \tag{1.0.3.5}$$

$$1 + \frac{R_2}{R_1} = 10.010 \tag{1.0.3.6}$$

$$\frac{R_2}{R_1} = 9.010\tag{1.0.3.7}$$

1.0.4. What is the amount of feedback in decibels? **Solution:** The value of F in decibals is given by

$$F(dB) = 20\log(F)$$
 (1.0.4.1)

where...,
$$F = 1 + GH$$
 (1.0.4.2)

$$F = \frac{G}{T} \tag{1.0.4.3}$$

where..,
$$G = 10^4$$
 (1.0.4.4)

$$T = 10 (1.0.4.5)$$

$$F(dB) = 20\log(\frac{10^4}{10}) = 20\log(1000)$$
(1.0.4.6)

$$F(dB) = 60dB (1.0.4.7)$$

1.0.5. If G decreases by 20%, what is the corresponding decrease in T? **Solution:** Given

$$G = 10^4 \tag{1.0.5.1}$$

If G decrease by 20% then, the value of G is..,

$$G = (1 - 0.2)10^4 \tag{1.0.5.2}$$

$$= 8000$$
 (1.0.5.3)

For this value of G and,

$$\frac{R_2}{R_1} = 9.010 \tag{1.0.5.4}$$

The value of T can be solved as follows,

$$T = \frac{G}{1 + \frac{G}{1 + \frac{R_2}{R_1}}} \tag{1.0.5.5}$$

$$T = \frac{8000}{1 + \frac{8000}{1 + 0.9010}} \tag{1.0.5.6}$$

$$T = 9.99749 \tag{1.0.5.7}$$

The percentage change in T is..,

$$fractional change = \frac{10 - 9.99749}{10}$$
 (1.0.5.8)

$$= 2.51x10^{-4} \qquad (1.0.5.9)$$

$$%changeinT = 0.00251$$
 (1.0.5.10)

Therefore T decreases by 0.0025% when G decreases by 20%

1.0.6. Write a python code that can compute closed loop gain,loop gain,amount of feedback given all input parameters. **Solution:** Code to compute different gains.,

codes/ee18btech11005/ee18btech11005.py