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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

1 STABILITY

2 ROUTH HURWITZ CRITERION

3 COMPENSATORS

4 NYQUIST PLOT

5 FEEDBACK SYSTEMS

5.0.1. The non-inverting op-amp configuration shown in fig.5.0.1 provides direct implementation of feedback loop. Assuming operational amplifier has infinite input resistance and zero output resistance. Find the expression for feedback factor. **Solution:** Let the gain of the operational

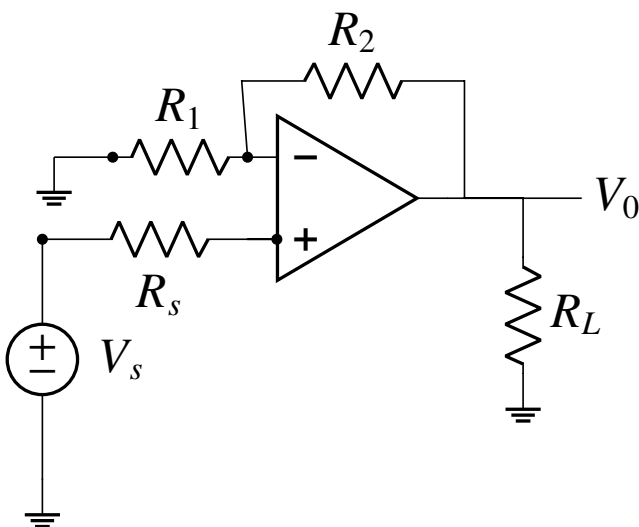


Fig. 5.0.1

amplifier be G . The equivalent circuit of the amplifier is in fig.5.0.1 From the equivalent

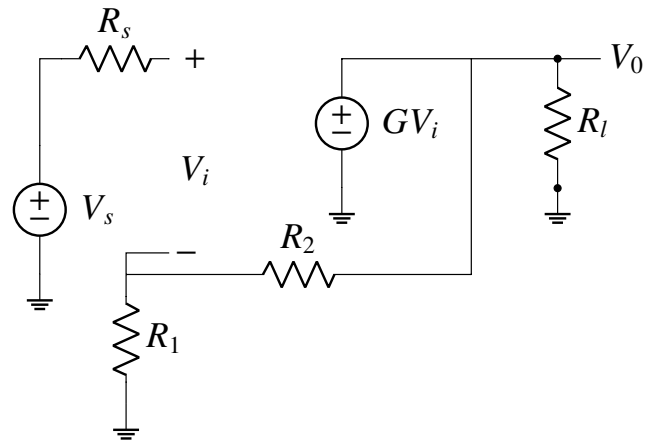


Fig. 5.0.1

circuit, Applying Ohms law,

$$V_0 = G(V_i) \quad (5.0.1.1)$$

$$\text{and, } V_i = V_+ - V_- \quad (5.0.1.2)$$

Now, Applying voltage dividing rule

$$V_- = \left[\frac{R_1}{R_1 + R_2} \right] V_0 \quad (5.0.1.3)$$

Substituting in equ.5.0.1.1

$$V_0 = G(V_+ - \left[\frac{R_1}{R_1 + R_2} \right] V_0) \quad (5.0.1.4)$$

$$\Rightarrow V_0 = GV_+ - G \left[\frac{R_1}{R_1 + R_2} \right] V_0 \quad (5.0.1.5)$$

$$G(V_+) = V_0 + G \left[\frac{R_1}{R_1 + R_2} \right] V_0 \quad (5.0.1.6)$$

But,

$$V_s = V_+ \quad (5.0.1.7)$$

because, no current flows through resistor, R_s . Since, input resistance is given infinite in fig.5.0.1 The equation can be written as...,

$$V_0 = G \left[\frac{1}{1 + \frac{GR_1}{R_1 + R_2}} \right] V_s \quad (5.0.1.8)$$

$$\text{Gain} = \frac{V_0}{V_s} = \left[\frac{G}{1 + \frac{GR_1}{R_1 + R_2}} \right] \quad (5.0.1.9)$$

For a negative feedback system,

$$\frac{V_0}{V_i} = \frac{G}{1 + GH} \quad (5.0.1.10)$$

$$\text{where, } H = \frac{R_1}{R_1 + R_2} \quad (5.0.1.11)$$

The equation.5.0.1.1 looks exactly similar to the Gain of a negative feedback system with

- Open loop gain = G
- Loop gain = P
- Amount of feedback = F
- Feedback factor = f
- closed loop gain = T

where

$$G = G \quad (5.0.1.12)$$

$$P = GH = G \frac{R_1}{R_1 + R_2} \quad (5.0.1.13)$$

$$F = 1 + GH = 1 + \frac{GR_1}{R_1 + R_2} \quad (5.0.1.14)$$

$$f = H = \frac{R_1}{R_1 + R_2} \quad (5.0.1.15)$$

Therefore, This operational amplifier can be modelled as a negative feedback system shown in the fig.5.0.1 So, the feedback factor f .,

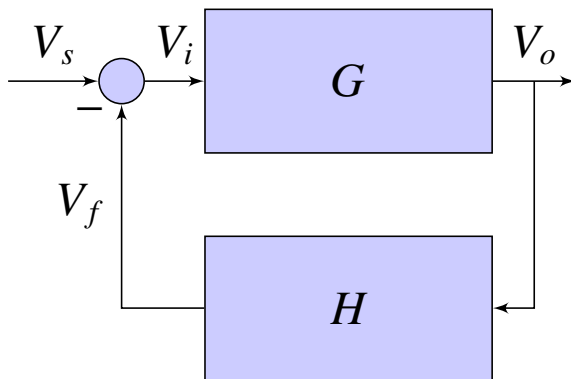


Fig. 5.0.1

$$f = H = \frac{R_1}{R_1 + R_2} \quad (5.0.1.16)$$

5.0.2. Find the condition under which closed loop gain T is almost entirely determined by the feedback network. **Solution:** For T to be entirely dependent on feedback network, it should be independent on G (open loop gain). T is given by.,

$$T = \frac{G}{1 + \frac{GR_1}{R_1 + R_2}} \quad (5.0.2.1)$$

$$(5.0.2.2)$$

For T to be independent on G .,

$$GH \gg 1 \quad (5.0.2.3)$$

$$G \frac{R_1}{R_1 + R_2} \gg 1 \quad (5.0.2.4)$$

$$G \gg 1 + \frac{R_2}{R_1} \quad (5.0.2.5)$$

Under such condition.,

$$T = \frac{1}{H} \quad (5.0.2.6)$$

$$T = \frac{R_1 + R_2}{R_1} \quad (5.0.2.7)$$

$$T = 1 + \frac{R_2}{R_1} \quad (5.0.2.8)$$

so, the necessary condition for A_f depends only on feedback network is

$$G \gg T \quad (5.0.2.9)$$

5.0.3. If the open loop voltage gain

$$G = 10^4 \quad (5.0.3.1)$$

Find the ratio of R_2 and R_1 to obtain a closed loop gain of 10. **Solution:** The closed loop gain T is given by

$$T = \frac{G}{1 + GH} = \frac{G}{1 + \frac{GR_1}{R_1 + R_2}} = 10 \quad (5.0.3.2)$$

$$\text{where.., } G = 10^4 \quad (5.0.3.3)$$

$$10 = \frac{10^4}{1 + \frac{10^4 R_1}{1 + \frac{R_2}{R_1}}} \quad (5.0.3.4)$$

$$\Rightarrow 1 + \frac{R_2}{R_1} = \frac{10^4}{\frac{10^4}{10} - 1} \quad (5.0.3.5)$$

$$1 + \frac{R_2}{R_1} = 10.010 \quad (5.0.3.6)$$

$$\frac{R_2}{R_1} = 9.010 \quad (5.0.3.7)$$

5.0.4. What is the amount of feedback in decibels?

Solution: The value of F in decibels is given by

$$F(dB) = 20 \log(F) \quad (5.0.4.1)$$

$$\text{where...}, F = 1 + GH \quad (5.0.4.2)$$

$$F = \frac{G}{T} \quad (5.0.4.3)$$

$$\text{where...}, G = 10^4 \quad (5.0.4.4)$$

$$T = 10 \quad (5.0.4.5)$$

$$F(dB) = 20 \log\left(\frac{10^4}{10}\right) = 20 \log(1000) \quad (5.0.4.6)$$

$$F(dB) = 60dB \quad (5.0.4.7)$$

5.0.5. If G decreases by 20%, what is the corresponding decrease in T? **Solution:** Given

$$G = 10^4 \quad (5.0.5.1)$$

If G decrease by 20% then, the value of G is...

$$G = (1 - 0.2)10^4 \quad (5.0.5.2)$$

$$= 8000 \quad (5.0.5.3)$$

For this value of G and ,

$$\frac{R_2}{R_1} = 9.010 \quad (5.0.5.4)$$

The value of T can be solved as follows,

$$T = \frac{G}{1 + \frac{G}{1 + \frac{R_2}{R_1}}} \quad (5.0.5.5)$$

$$T = \frac{8000}{1 + \frac{8000}{1 + 0.9010}} \quad (5.0.5.6)$$

$$T = 9.99749 \quad (5.0.5.7)$$

The percentage change in T is...

$$\text{fractional change} = \frac{10 - 9.99749}{10} \quad (5.0.5.8)$$

$$= 2.51 \times 10^{-4} \quad (5.0.5.9)$$

$$\% \text{change in } T = 0.00251 \quad (5.0.5.10)$$

Therefore T decreases by 0.0025% when G decreases by 20%