The Inverse Matrix of the Invertible Partitioned Matrix

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1 Lemma

Lemma 1 *If matrix A and D are invertible, the partitioned matrix*

$$T = \begin{pmatrix} A & 0 \\ 0 & D \end{pmatrix}$$

is invertible, and the inverse matrix is

$$T^{-1} = \begin{pmatrix} A^{-1} & 0 \\ 0 & D^{-1} \end{pmatrix}$$

Lemma 2 *If matrix B and C are invertible, the partitioned matrix*

$$T = \begin{pmatrix} 0 & B \\ C & 0 \end{pmatrix}$$

is invertible, and its inverse matrix is

$$T^{-1} = \begin{pmatrix} 0 & C^{-1} \\ B^{-1} & 0 \end{pmatrix}$$

2 Theorem

Theorem 1 *If matrix A and D are invertible, the partitioned matrix*

$$T = \begin{pmatrix} A & B \\ 0 & D \end{pmatrix}$$

is invertible, and its inverse matrix is

$$T^{-1} = \begin{pmatrix} A^{-1} & -A^{-1}BD^{-1} \\ 0 & D^{-1} \end{pmatrix}$$

Theorem 2 *If matrix A and D are invertible, the partitioned matrix*

$$T = \begin{pmatrix} A & 0 \\ C & D \end{pmatrix}$$

is invertible, and its inverse matrix is

$$T^{-1} = \begin{pmatrix} A^{-1} & 0 \\ -D^{-1}CA^{-1} & D^{-1} \end{pmatrix}$$

Theorem 3 *If matrix B and C are invertible, the partitioned matrix*

$$T = \begin{pmatrix} A & B \\ C & 0 \end{pmatrix}$$

is invertible, and its inverse matrix is

$$T^{-1} = \begin{pmatrix} 0 & C^{-1} \\ B^{-1} & -B^{-1}AC^{-1} \end{pmatrix}$$

Theorem 4 *If matrix B and C are invertible, the partitioned matrix*

$$T = \begin{pmatrix} 0 & B \\ C & D \end{pmatrix}$$

is invertible, and its inverse matrix is

$$T^{-1} = \begin{pmatrix} -C^{-1}DB^{-1} & C^{-1} \\ B^{-1} & 0 \end{pmatrix}$$

3 Corollary

Corollary 1 *If matrix*

$$T = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

and the matrix A are invertible, the inverse matrix of T is

$$T^{-1} = \begin{pmatrix} A^{-1} + A^{-1}B(D - CA^{-1}B)^{-1}CA^{-1} & -A^{-1}B(D - CA^{-1}B)^{-1} \\ -(D - CA^{-1}B)^{-1}CA^{-1} & (D - CA^{-1}B)^{-1} \end{pmatrix}$$

Corollary 2 *If matrix*

$$T = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

and the matrix B are invertible, the inverse matrix of T is

$$T^{-1} = \begin{pmatrix} -(C - DB^{-1}A)^{-1}DB^{-1} & (C - DB^{-1}A)^{-1} \\ B^{-1} + B^{-1}A(C - DB^{-1}A)^{-1}DB^{-1} & -B^{-1}A(C - DB^{-1}A)^{-1} \end{pmatrix}$$

Corollary 3 *If matrix*

$$T = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

and the matrix C are invertible, the inverse matrix of T is

$$T^{-1} = \begin{pmatrix} -C^{-1}D(B - AC^{-1}D)^{-1} & C^{-1} + C^{-1}D(B - AC^{-1}D)^{-1}AC^{-1} \\ (B - AC^{-1}D)^{-1} & -(B - AC^{-1}D)^{-1}AC^{-1} \end{pmatrix}$$

Corollary 4 *If matrix*

$$T = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

and the matrix D are invertible, the inverse matrix of T is

$$T^{-1} = \begin{pmatrix} (A - BD^{-1}C)^{-1} & -(A - BD^{-1}C)^{-1}BD^{-1} \\ -D^{-1}C(A - BD^{-1}C)^{-1} & D^{-1} + D^{-1}C(A - BD^{-1}C)^{-1}BD^{-1} \end{pmatrix}$$