

## Homework 9

### 1) Quantum Teleportation $\rightarrow$

Quantum Teleportation is used to replace the state of one qubit with another qubit which is over a long distance.

We need to keep in mind that the same qubit cannot be cloned this is according to the No cloning theorem.

Suppose A wants to send information to B. Let's assume  $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$  at A.

2) An EPR pair is present between A and B. A has one half and B has other.

3) We create a tensor product of  $|\psi\rangle$  with bell state half which A has.

4) Once we get this first 2 qubits are passed through CNOT gate.

5) After this 1<sup>st</sup> qubit is sent through Hadamard gate and we get a superposition of eight states.

The four possible outcomes upon measuring qubit 1 and 2 result in 2 classical bits  $c_0$  and  $c_1$ . Measurement of this has impact of qubits 3 in B and leaves it in one of the possible 4 states.

6) These  $c_0$  and  $c_1$  are sent to B via classical channel and after doing unitary operation B reproduces original qubit 1.

|   |   |    |
|---|---|----|
| 0 | 0 | I  |
| 0 | 1 | X  |
| 1 | 0 | Z  |
| 1 | 1 | ZX |



## 2) SuperDense Coding $\rightarrow$ .

It can be viewed as the process in which two classical bits of information are transmitted by sending just one quantum bit.

Assume  $s, s_0$  is two bit string Alice wants to send to Bob. There should be an EPR pair which two needs to share.

Alice chooses one of four operation  $U = \{I, X, Y, Z\}$  depend if  $s, s_0$  are  $\in \{00, 01, 10, 11\}$

After this she sends her half of entangled qubit to Bob, Bob combines it with his qubit and applies a CNOT gate on pair  $(q_0, q_1)$ .

After this Hadamard gate is applied on the first qubit of pair which results in unentanglement.

| classical bits | Initial state                                 | Unitary op    | Final state  |
|----------------|---|---------------|--|
| 0 0            | $\frac{1}{\sqrt{2}}( 00\rangle +  11\rangle)$ | $I \otimes I$ | $\frac{1}{\sqrt{2}}( 0\rangle 0\rangle +  1\rangle 1\rangle)$  |
| 0 1            | $\frac{1}{\sqrt{2}}( 00\rangle +  11\rangle)$ | $X \otimes I$ | $\frac{1}{\sqrt{2}}( 10\rangle +  0\rangle 1\rangle)$          |
| 1 0            | $\frac{1}{\sqrt{2}}( 00\rangle +  11\rangle)$ | $Y \otimes I$ | $\frac{1}{\sqrt{2}}( 0\rangle 0\rangle -  1\rangle 1\rangle)$  |
| 1 1            | $\frac{1}{\sqrt{2}}( 00\rangle +  11\rangle)$ | $X \otimes Z$ | $\frac{1}{\sqrt{2}}( 10\rangle 1\rangle -  0\rangle 0\rangle)$ |



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3) Superdense Coding can be viewed as teleportation in reverse.

\* The goal of teleportation is to transfer the unknown state information of the source qubit without measuring or observing to the destination qubit, thereby avoiding the disturbance of first. While in superdense coding the idea is to transmit two classical bits of information by sending a single qubit through the quantum channel. Thus it can be called the reverse of teleportation.

On a high level this is because in one we use classical bits to send the qubit state while in other we send the qubit to get the classical bits. Almost similar unitary transformations are needed in both and shared EPR pair is must between source and destination.