Quantum Teleportation

One interesting application of entanglement is **quantum teleportation**, which is a technique for transferring an *unknown* quantum state from one place to another. In science fiction, teleportation generally involves a machine scanning a person and another machine reassembling the person on the other end. The original body disintegrates and no longer exists. Similarly, quantum teleportation works by "scanning" the original qubit, sending a recipe, and reconstructing the qubit elsewhere. The original qubit is not physically destroyed in the science fiction sense, but it is no longer in the same state. (Otherwise it would violate the previously mentioned **no-cloning theorem** – which says that a qubit cannot be exactly copied onto another qubit.[1]) The "scanning" part poses a problem though.

Question 1: Create a qubit in the $|1\rangle$ state and pass it through a Hadamard gate. From the measurement histogram, can you tell whether the qubit started as a $|0\rangle$ or $|1\rangle$ initial state?

The measurement histogram should look identical if either of the $|0\rangle$ or $|1\rangle$ states is used initially. Then how can we tell what the initial state was after performing a Hadamard operation? In the beam splitter, we determined where the photon came from by adding a second beam splitter to create interference. The way to measure and distinguish between them is to add a second Hadamard gate.

Question 2: If a qubit is in the unknown state $a|0\rangle + b|1\rangle$, what is the result of a single measurement?

(a) 0

(b) 1

(c) 0 with probability a^2 and 1 with probability b^2

(d) A number between 0 and 1

[1]The no-cloning theorem poses a big problem for correcting errors that happen on quantum computers: https://en.wikipedia.org/wiki/Quantum_error_correction.

Question 3: What is the result of a second measurement after the first from Question 2?

(a) 0 if the first measurement is 0 or 1 if the first measurement is 1

(b) 0 if the first measurement is 1 or 1 if the first measurement is 0

(c) 0 with probability a^2 and 1 with probability b^2

(d) A number between 0 and 1

Given a single qubit, it is not possible to determine how much of a superposition it is in if you only have this single qubit, i.e., you cannot determine the coefficients of $|0\rangle$ and $|1\rangle$ in a general state from one measurement! Note that if the state is known (from measuring many independent qubits that have been prepared identically), then you can just directly send the recipe to prepare this qubit. It is only when the state is unknown and when there is only one qubit that we have to think harder about how to efficiently "scan" the particle.

The way to get around the problem of not being able to measure the qubit (and avoid collapsing the unknown state onto a basis state) is to "scan" the qubit indirectly with the help of entangled particles. This comic[2] illustrates the basic idea. The protocol is as follows:

1. Alice and Bob meet up and make a qubit each (which we will call qubit #2 and #3). At this point, the two qubits are completely independent, i.e., think of the qubits as two different balls that do not contain any information about the other. Then, Alice and Bob decide to entangle their qubits by causing an interaction between the qubits such as application of a two-qubit CNOT gate. Using the previous metaphor, think of entanglement as Alice writing some information on Bob's ball that only she knows how to read, and Bob writing information on Alice's ball that only he knows how to read. For Bob to read Alice's information on his ball, Alice needs to send him a (classical) message with how to understand it, and vice-versa. They do not tell each other how to read the information yet. One possible entangled state (called the Bell-state) that they decide to make is

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle. \tag{8.1}$$

   Alice takes her qubit and walks away, and Bob takes his and walks in a different direction as shown in Figure 8.1.

2. Now Alice obtains a third different qubit in an unknown state (qubit #1) that she wants to transfer to Bob. She can only communicate with him classically by email or phone, and it would take too long to physically bring the qubit to Bob. The current situation is shown in Figure 8.2.

[2] https://www.jpl.nasa.gov/news/news.php?feature=4384

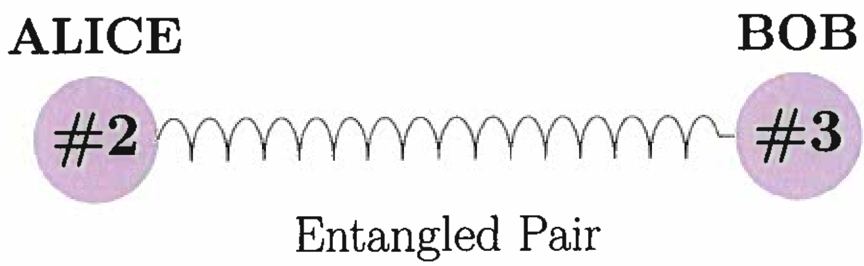

Figure 8.1: Alice and Bob's qubits are entangled.

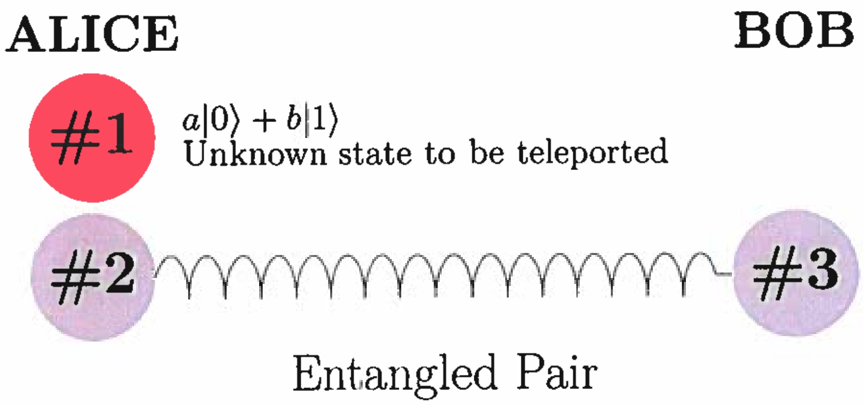

Figure 8.2: Alice has a qubit (#1) in an unknown state she wants to transfer to Bob.

3. Alice interacts her two qubits using a CNOT gate (qubits #1 and #2) and measures the qubit she originally had (qubit #2). She then sends the unknown qubit to be teleported (qubit #1) through a Hadamard gate and afterwards measures the output. Recall that the Hadamard gate is used to create a superposition of states. The current situation is shown in Figure 8.3.

   Because Alice's original qubit (qubit #2) was entangled with Bob's, the CNOT interaction with qubit #1 immediately changes the state of Bob's qubit. By doing the math and drawing the full quantum circuit, one finds that Bob's qubit has changed into one of four possible superposition states. The four possible superposition states that Bob's qubit can be in depends on Alice's original qubit #2 through the initial entanglement in Step 1, as well as depending on the unknown qubit #1 to be teleported from the CNOT gate in Step 3. The reason we need to measure the state of Alice's qubit #2 and qubit #1 is to figure out the way Bob's qubit depends on these two. The current situation is shown in Figure 8.4. Note that Bob has not done anything with his qubit at this stage.

4. Alice sends the two classical bits of information from the measurements to

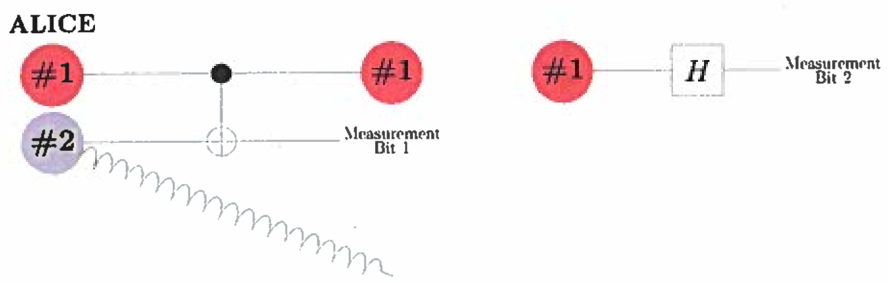

Figure 8.3: Alice passes her two qubits through a CNOT gate.

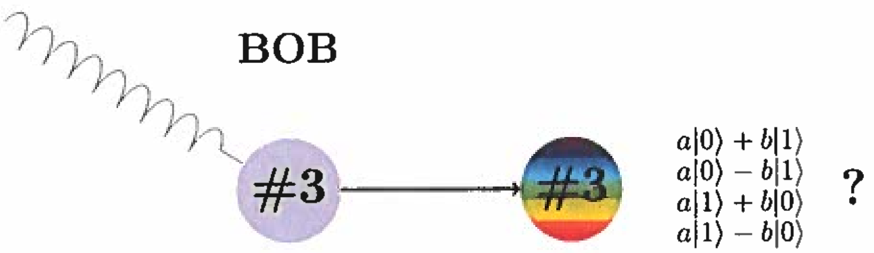

Figure 8.4: Four possible superposition states of Bob's qubit.

Bob by email or phone.

5. Bob uses the two classical bits as the recipe for turning his qubit (now in an unknown state) into the correct state identical to qubit #1. Depending on the values of the classical bits, Bob will know which of the four possibilities he has and he can then change it into the correct state using Z and/or X gates. If he has the correct state already, he does nothing. The result of this is shown in Figure 8.5.

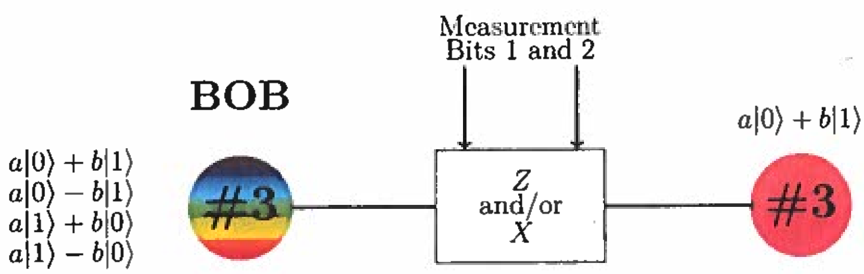

Figure 8.5: Final situation of entanglement between Bob and Alice.

It is important to understand that neither Alice nor Bob know what qubit #1's coefficients a or b are at any point in the process. All they know at the end is that qubit #1 has been teleported from Alice to Bob.

Why is this protocol interesting? To answer this, imagine Alice and Bob met a long time ago and each took one qubit of the entangled pair. Bob is now traveling around the world and can only communicate with Alice by phone or email. If Alice wanted to transfer quantum data to Bob without quantum teleportation, she would have to meet Bob and physically give Bob her qubit. Quantum teleportation allows Alice to send *quantum* information using a *classical* communications channel. All she has to do is make some measurements and email Bob the values. Bob can then apply the correct recipe to his qubit to get the data. Quantum teleportation is a useful way of causing interaction between different parts of a quantum computer (by teleporting a qubit to a different part of the quantum computer you want to interact with) as well as quantum cryptography (to prevent eavesdropping when sending information).

8.1 Check Your Understanding

1. ● Could quantum teleportation be used to teleport a physical object from one place to another? Why or why not?
2. ● What would lead someone to think quantum teleportation can transmit information faster than the speed of light? Explain why this is not possible.
3. ■ By the no-cloning theorem, it is not possible to make a copy of an unknown qubit. At what point in the teleportation protocol does the unknown qubit collapse into a definite state?
4. ■ In the original protocol, Alice applies the CNOT and then measures Bit 1 (see Figure 8.3). After this, Alice then applies the Hadamard to qubit #1 and then measures Bit 2 (see Figure 8.3). What happens if she decides to reverse the procedure by measuring Bit 2 first, before applying the two-qubit CNOT gate?
5. ● If Bob knows that his qubit is in the $b|0\rangle + a|1\rangle$ state, which gate(s) would he need to use to change it back into the original needed $a|0\rangle + b|1\rangle$ state?
   (a) X
   (b) Z
   (c) X then Z
6. ● If Bob knows that his qubit is in the $a|0\rangle - b|1\rangle$ state, which gate(s) would he need to use to change it back into the original needed $a|0\rangle + b|1\rangle$ state?
   (a) X
   (b) Z
   (c) X then Z

7. ● If Bob knows that his qubit is in the $a|1\rangle - b|0\rangle$ state, which gate(s) would he need to use to change it back into the original needed $a|0\rangle + b|1\rangle$ state?

   (a) X
   (b) Z
   (c) X then Z