Neil Gupte
10445674.

CS-810 Midterm.

Pg 1

1 A1) a) Quantum computing is the computing which is done using techniques from the branches of quantum physics and computer science. Qubit is used ~~as~~ for data ~~source~~ manipulations in quantum computing.
The tehniques of superposition and entanglement. are ~~helpful~~ essential for quantum computing.

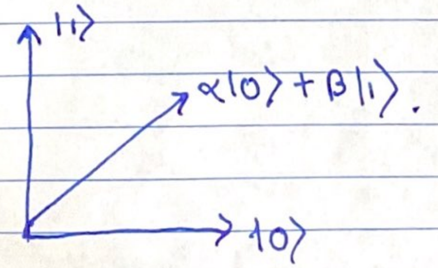
- ★ Beam Splitters can be used to create superposition in Mach Zhender Experiment.
- ★ Young Double slit experiment using beam splitter at the end

Hadamard gates are used in software.

b). Qubit is a two level quantum system. It is the basic data unit of quantum computer. Qubit can also be described as having quantum information of a classical bit. A unit magnitude vector in a complex 2 dimensional vector space can also be defined as qubit.
The general formula →

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle.$$

where $|0\rangle$ and $|1\rangle$ are the computational basis states.



1) b). The difference between a bit and qubit are →.

Bits are used for data manipulations in classical computers while qubit are used in quantum computers.

Bits use electric charge and can be 1 or 0 depending on what charge it has.

Qubits can be created by electrons of hydrogen atoms and the value is calculated by calculating the spins. They can be 1 and/or 0 at the same point of time which a bit cant do.

c). Superposition is a ability the qubit to be present in state $|1\rangle$ and/or state $|0\rangle$ at the same point of time.

Qubit register is analogous to classical program register and are basic storage units to store multiple qubits.

$$|\psi\rangle = c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle$$
$$+ c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle.$$

Neil Gupta
10445674.

2 a). $|0\rangle \xrightarrow{BS1} \frac{|0\rangle + |1\rangle}{\sqrt{2}}$

b) $\frac{|0\rangle + |1\rangle}{\sqrt{2}} \xrightarrow{BS2} \frac{1}{\sqrt{2}}\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} + \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$

$= \frac{1}{\sqrt{2}} \frac{2|0\rangle}{\sqrt{2}} = |0\rangle.$

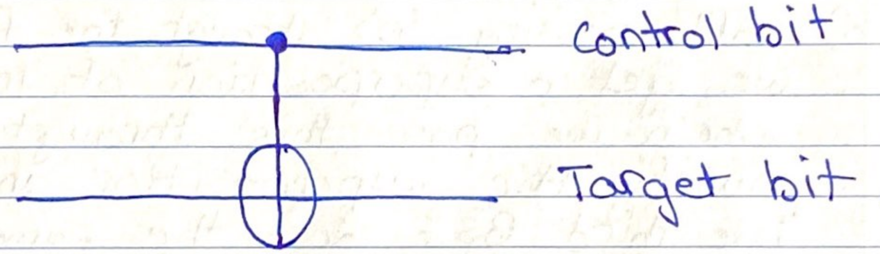
c). $|0\rangle \xrightarrow{BS3} \frac{|0\rangle + |1\rangle}{\sqrt{2}}.$

d). After passing $|0\rangle$ through the first beam splitter we get a superposition of $|0\rangle$ and $|1\rangle$. when we pass this through BS 2 we nullify the superposition and get back $|0\rangle$. The third BS 3 does the same as BS 1 and puts $|0\rangle$ back in superposition.

e). Beam splitter and Hadamard gate are analogous in the result i.e. both put the supplied input into a state of superposition. As with 2 beam splitter ~~is~~, if we apply 2 H gates in a series we get back the original input.

3) a). Quantum C-NOT gate is analoguous to EX-OR of ~~etc~~ boolean gates. It helps to entangle the two Qubits. It is a 2 qubit gate, where one is control bit and other is target bit. If the control bit is 1 the CNOT flips the target bit otherwise it does not.

b). Circuit Representation.



Matrix Representation.

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

c). After H. gate we get →.

$$|0\rangle \rightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}}.$$

then we combine it with $|0\rangle$.

we get $\dfrac{|00\rangle + |10\rangle}{\sqrt{2}} \xrightarrow{CNOT} \dfrac{|00\rangle + |11\rangle}{\sqrt{2}}$

~~after applying CNOT, a = |00⟩ + |11⟩~~

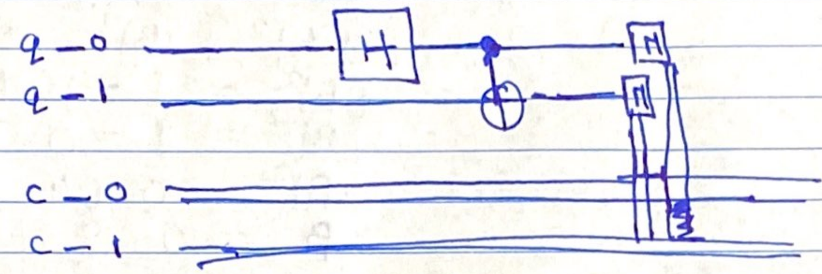
so the final state is $\dfrac{|00\rangle + |11\rangle}{\sqrt{2}}$.

Neil Gupta
10445674

3) d) .1) Lines of code :

```
Circuit. h(0)
Circuit. cx(0, 1)
```

2).



The output we get is $a = |00\rangle$ & $b = |11\rangle$. This is because once we apply the hadamard gate. we get a superposition of $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$.

and once we get a tensor product of that wit $|0\rangle$ and then apply CNOT to it we get $|00\rangle$ and $|11\rangle$ as the CNOT. flips the bits of b because the control bit is 1 in b's case.

4) To find →.

$|\langle 0|\psi\rangle|^2$

As $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$.

$|\langle 0|\psi\rangle|^2$

$= |\langle 0|\alpha|0\rangle + \beta|1\rangle|^2$

$= |\alpha\langle 0|0\rangle + \beta\langle 1|0\rangle|^2$

As $\langle 0.0\rangle = 1$ and $\langle 1|0\rangle = 0$.

$= |\alpha|^2$. is the final probability.

Q4 b). $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

Given $\alpha = 0.6$.

we know the $|\alpha|^2 + |\beta|^2 = 1$.

$\therefore \quad |0.6|^2 + |\beta|^2 = 1.$

$|\beta|^2 = 1 - |0.6|^2$

$|\beta|^2 = 1 - 0.36.$

$|\beta|^2 = 0.64$

$\beta = 0.8$.

Q5 a). $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

(i). $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$.

ii) $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle.$

$= \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

$X|\psi\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix} = \alpha|1\rangle + \beta|0\rangle.$

Neil Gupta
10445674.
pg ④

6 a). $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$H|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

$= \frac{|0\rangle + |1\rangle}{\sqrt{2}}$.

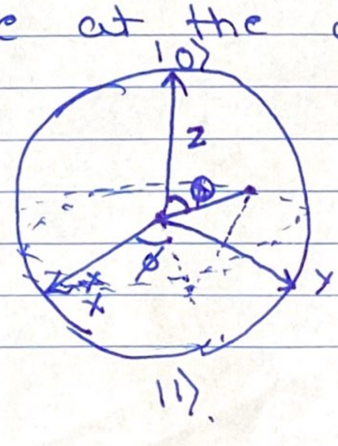
$H|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$= \frac{|0\rangle - |1\rangle}{\sqrt{2}}$.

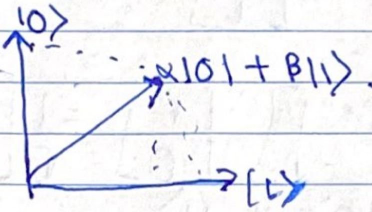
7) a) Bloch sphere is used for visualizing a qubit. The qubit is shown as vector inside the bloch sphere. General state vector is shown by formula =.

$|\psi\rangle = e \cos\frac{\theta}{2} + e^{i\phi}\sin\frac{\theta}{2}$.

It has 3 axes X, Y, Z. $|0\rangle$ and $|1\rangle$ are at the opposite ends.



7) 2). So a qubit can be represented as: ~~single~~ unit magnitude vector of 2 complex vectors in 2d space.



Same is true for any vector shown inside the bloch sphere.
It can be represented by combination of two vectors in either x, y, z axes.