Creating Superposition: The Beam Splitter

3.1 ● How is a superposition state created?

While a flipping coin is a simple model of a qubit, it is not very useful for building a quantum computer because it does not exhibit all of the properties of a true quantum superposition. For example, we cannot manipulate the superposition amplitudes. In this section, we will study some real physical examples of quantum particles in a superposition containing two states. These examples include a photon in a beam splitter, an electron in the double-slit experiment, and an electron in a Stern-Gerlach apparatus.

3.2 ● Beam Splitter

In classical optics, a **beam splitter** acts like a partially reflective mirror that splits a beam of light into two. In a 50/50 beam splitter, 50% of the light intensity is transmitted and 50% is reflected, as shown in Figure 3.1.

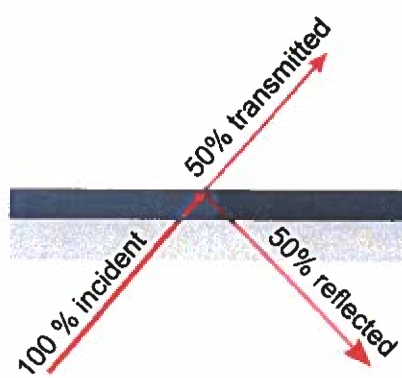

Figure 3.1: A beam splitter reflects 50% of the incident light and transmits 50% of the incident light.

One way to visualize the beam splitter is to imagine a barrier with holes randomly cut out like Swiss cheese, as shown in Figure 3.2. Imagine this barrier is placed in a pond, and a water wave moves toward the barrier. After the wave hits the barrier, we would observe a smaller wave going through the barrier and another would be reflected off the barrier.

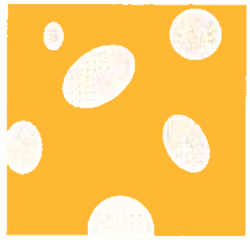

Figure 3.2: A beam splitter reflects 50% of the incident light and transmits 50% of the incident light.

Question 1: What would happen if a classical particle such as a soccer ball is randomly kicked at the barrier? Assume the ball can fit through the holes.

Experiments show that light behaves both like a wave (Young's double-slit experiment) and a particle (photoelectric effect, Compton effect). Classically, light is thought of as a wave consisting of continually oscillating electric and magnetic fields. However, light can also be thought of as a stream of particles called **photons**. Photons have no mass but carry the light's energy from one point to another at the speed of light. A laser beam is comprised of photons. If you turn down the intensity of your laser, you can even send one photon at a time, as shown in Figure 3.3. In practice, setting up a single photon source and detector requires specialized equipment, so we will instead run a simulator to see what happens.

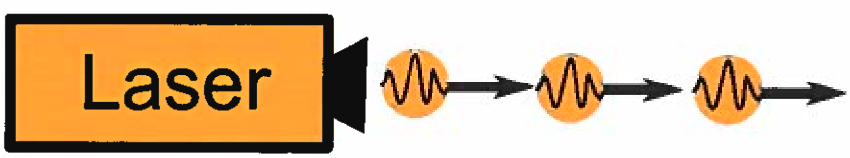

Figure 3.3: Low-intensity light is a stream of single photons.

Question 2: Open the beam splitter simulator[1], go to the Controls screen, and fire a single photon. The setup before the photon hits a beam splitter is shown in Figure 3.2. Which detectors are triggered when the photon passes through the 50/50 beam splitter?

1. Always detector 1

[1] https://www.st-andrews.ac.uk/physics/quvis/simulations_html5/sims/Mach-Zehnder-Interferometer/Mach_Zehnder_Interferometer.html

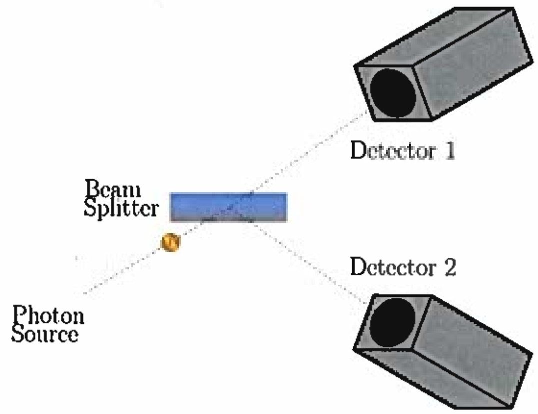

Figure 3.4: A single photon is sent at a beam splitter and the outcome measured with detectors to see whether it transmits or reflects.

2. Always detector 2
3. Detector 1 OR detector 2
4. Both detector 1 AND detector 2
5. Neither

Question 3: Which detector(s) would trigger if a classical **wave** is sent through the beam splitter?

1. Always detector 1
2. Always detector 2
3. Detector 1 OR detector 2
4. Both detector 1 AND detector 2
5. Neither

Question 4: Which detector(s) would trigger if a classical **particle** is sent through the beam splitter?

1. Always detector 1
2. Always detector 2
3. Detector 1 OR detector 2
4. Both detector 1 AND detector 2
5. Neither

Question 5: What does the photon do at the instance it encounters the 50/50 beam splitter?

1. Splits in half. Half the photon is transmitted and half is reflected
2. The whole photon goes through with 50% probability and reflects with 50% probability
3. The whole photon is both transmitted and reflected, essentially in two places at once

If the photon was split in half, both detectors would be triggered together. As only one detector goes off at a time, the photon could not have split up. In this case, we see that light behaves more like the soccer ball than the water wave.

At this point you may be thinking that the photon was either transmitted or reflected at the beam splitter, and we simply didn't have that information until it hit Detector 1 or 2. Unfortunately, this would be the incorrect interpretation formed by our classical lizard brain. This would be like saying the coin was Heads all along, and all we had to do was look at it to determine its state. Just like how a spinning coin will land on heads 50% of the time and tails 50% of the time, the single photon is in a superposition of both states all the way until the point when it reaches the detectors. This distinction might seem like a matter of semantics, but it will be important once the system becomes more complicated. The experimental setup after the photon hits a beam splitter is shown in Figure 3.5.

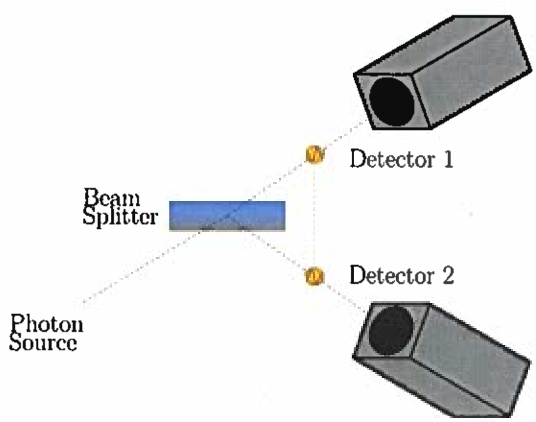

Figure 3.5: The beam splitter puts the photon into a superposition state.

If we let the transmitted path be $|0\rangle$ (detector 1), and the reflected path be $|1\rangle$ (detector 2), then the photon's state after the beam splitter is

$$|\text{photon}\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle. \tag{3.1}$$

Upon measurement, will the superposition collapse into either 0 or 1? Unfortunately, there is no way to predict which detector will be activated at any given time. Quantum mechanics is inherently probabilistic.

The phenomenon of superposition allows quantum computers to perform operations on two bits of information at once with a single qubit. In fact, it is possible to create a general purpose (also called universal) quantum computer using photons as qubits, beam splitters to create superposition, and pieces of glass that slow down the photons along selected paths (phase shifters).[2]

3.3 ● Mach-Zehnder Interferometer

To convince ourselves that the photon really did take two paths at once, let's see what happens when a second beam splitter is added. In reality, this experimental setup is shown in Figure 3.6. The mirrors redirect the photons towards the second

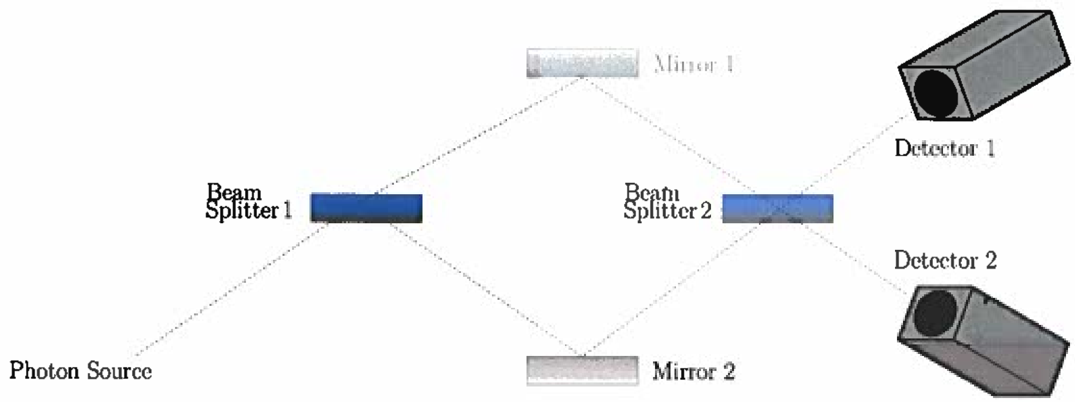

Figure 3.6: Schematic of the Mach-Zehnder interferometer from the beam splitter simulator.

beam splitter. This device configuration is known as a **Mach-Zehnder interferometer**. The set up is very sensitive to the distances between the mirrors and detectors, which essentially have to be the same or differ by an integer number of the photon's wavelength.

Question 6: If we assume that the photon was reflected by the first beam splitter, which detectors would be triggered?

1. Always detector 1
2. Always detector 2
3. Detector 1 OR detector 2
4. Both detector 1 AND detector 2

[2]Knill, E.; Laflamme, R.; Milburn, G. J. (2001). "A scheme for efficient quantum computation with linear optics". Nature. Nature Publishing Group. 409 (6816): 46–52.

5. Neither

Question 7: If we assume that the photon was transmitted by the first beam splitter, which detectors would be triggered?

1. Always detector 1
2. Always detector 2
3. Detector 1 OR detector 2
4. Both detector 1 AND detector 2
5. Neither.

Question 8: Construct the Mach-Zehnder interferometer in the beam splitter simulator[3] and fire a single photon. Which detectors are triggered?

1. Always detector 1
2. Always detector 2
3. Detector 1 OR detector 2
4. Both detector 1 AND detector 2
5. Neither

If the photon was either transmitted or reflected by the first beam splitter, it would have a $50/50$ chance of transmission or reflection by the second beam splitter. Thus, both detectors should trigger with equal probability. The experimental results do not agree with this hypothesis, as only one detector is triggered with 100% probability. The results are more intuitively understood from the wave perspective of light.

To understand the operation of the interferometer, it is important to note that the beam splitters have a polarity. The beam splitter consists of a piece of glass coated with a dielectric on one side. When light enters the beam splitter from the dielectric side, the reflected light is **phase shifted** by π. Light entering from the glass side will not experience any phase shift. The phase shift only occurs when the light travels from a low to high index of refraction ($n_{\text{air}} < n_{\text{dielectric}} < n_{\text{glass}}$).

What does it mean for a photon to be phase shifted? In this case, it is more intuitive to think about the wave nature of light. The phase shift would invert the electric and magnetic field oscillations relative to the incoming wave. If a π-shifted wave overlaps with the original wave, destructive interference occurs. This is shown in Figure 3.7.

[3] https://www.st-andrews.ac.uk/physics/quvis/simulations_html5/sims/Mach-Zehnder-Interferometer/Mach_Zehnder_Interferometer.html

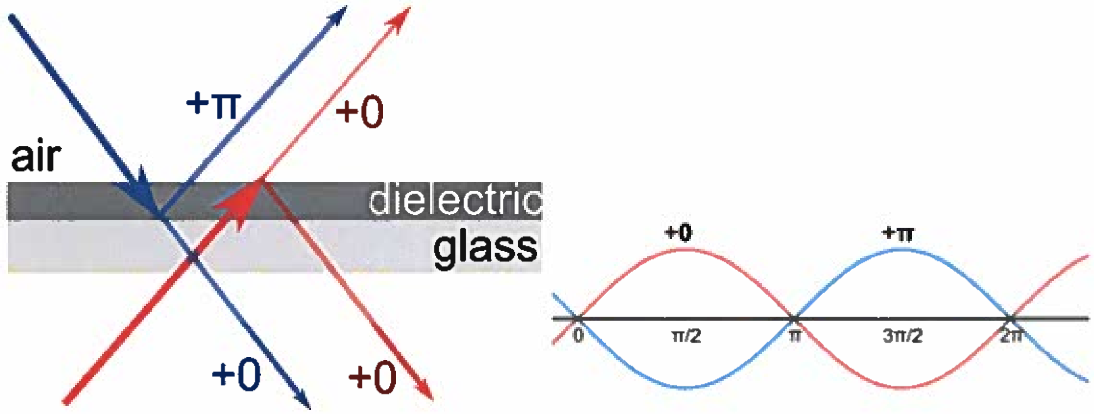

Figure 3.7: The light through a beam splitter is phase shifted if it is reflected from the dielectric side but not phase shifted from if it is reflected from the glass side.

Question 9: If we assume that light is a classical wave exhibiting interference, can you work out which detectors would be triggered? Note that the first beam splitter has the dielectric side on top, while the second has the dielectric on the bottom, as shown in Figure 3.6.

1. Always detector 1
2. Always detector 2
3. Detector 1 OR detector 2
4. Both detector 1 AND detector 2
5. Neither

Particle Explanation

The behavior of the interferometer can also be viewed from the particle perspective, though it may be less intuitive. Recall from the single beam splitter experiment that the photon did not split up or clone itself. It was in a superposition state, essentially taking both paths. The second beam splitter treats the photon as if it came in from both top and bottom simultaneously. The bottom photon is phase shifted relative to the top photon, resulting in destructive interference at Detector 2. Since there is no phase shift at Detector 1, there is no cancellation and it triggers with 100% probability, as shown in Figure 3.8.

Question 10: If the photon is sent into the Mach-Zehnder interferometer from the upper left instead of the bottom left, which detector(s) would be triggered and with what probability?

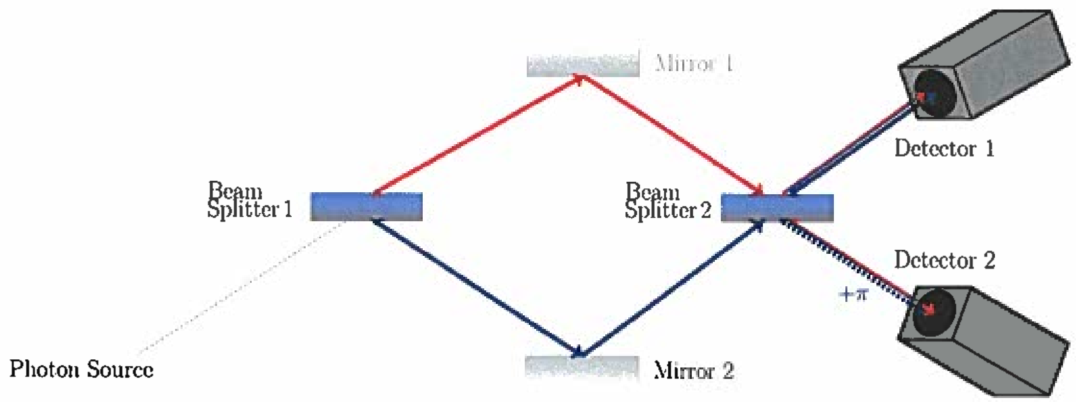

Figure 3.8: The blue path shows the photon's path if it is reflected by the first beam splitter. The red path shows the path if the photon is transmitted. Red and blue interfere constructively at Detector 1 while destructively at Detector 2.

Even though the output of the first beam splitter is 50/50, the second beam splitter can distinguish whether the laser was fired from the top of the bottom. The first beam splitter creates a superposition state, but adding a second one undoes the superposition and recovers the original state. This is a non-classical operation. It would be like starting with the coin heads up, flipping it, flipping it again while it is still in the air, and then always getting heads when it lands! This is highlighted in Figure 3.9.

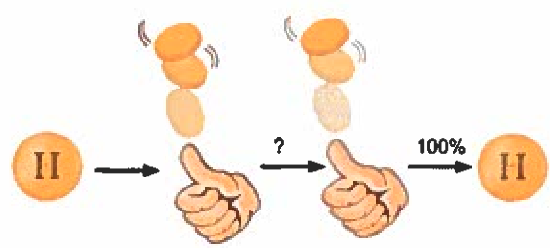

Figure 3.9: Coin analogy for the interferometer. Sending a photon through one beam splitter puts it in superposition, but adding a second beam splitter undoes the superposition and recovers the original state.

There is hidden information in the superposition state. In the Mach-Zehnder photon qubit, the information is encoded in the form of the phase shift. In the experiment shown in Figure 3.8, we choose the phase shift to have a value of π. However, we could have just as easily chosen the phase shift to have any value between 0 and 2π (the angles of a circle). Each separate choice of phase shift would produce a different type of superposition state that would still produce the same measurable 50/50 outcome.[4] This phase shift information is present in the

[4] A complex amplitude $e^{i\phi}$ with infinite possible phase angles ϕ does not affect the probability since $|e^{i\phi}|^2 = 1$.

amplitudes but not the square of the amplitudes (and hence hidden from us in the Mach-Zehnder experiment-though we could make an other experiment to try to determine this information). Here are two simple examples of distinct states that can be created in the Mach-Zehnder experiment which still have the same 50/50 probability:

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \quad \text{or} \quad \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle. \tag{3.2}$$

In these two states the minus sign represents the phase shift, which is what allows certain states to cancel out when adding them together. As you can see, quantum superposition is inextricably linked to wave-particle duality.

Furthermore, in the Mach-Zehnder experiment we created a superposition, performed a phase shift and then observed wave interference. These experimental operations are equivalent to mathematically applying (matrix/gate) operations on a qubit, as we shall see later. As such, the Mach-Zehnder is an example of how we can technologically implement qubits (the photon) and operations (superposition/phase shift, etc) to build a quantum computer.[5] In quantum computing, people talk about the superposition of states rather than the wave behavior. Yet, as we have seen, both frameworks lead to the same understanding of the Mach-Zehnder interferometer. Later we will use the interferometer to implement a quantum algorithm.

3.4 Check Your Understanding

1. ● Your friend who is explaining superposition to you says that:

   "A particle in the state $(1/\sqrt{2})|0\rangle + (1/\sqrt{2})|1\rangle$ represents a lack of knowledge of the system. Over time, the particle is changing back and forth between the state $|0\rangle$ and $|1\rangle$. The superposition state says that overall, the particle is in each of the two states for half of the time."

   What parts of this statement do you agree with and what do you not agree with?

2. ■ Only one detector is triggered if a single photon is sent through the beam splitter experiment shown in Figure 3.6. If the laser outputs two photons at the same time, what is the probability that both detectors will be triggered simultaneously? Now how about three photons? Ten photons? Note that this is why a higher power beam of light appears to reach both detectors simultaneously.

3. ◆ In practice, it is difficult to put the detectors the exact same distance from the beam splitter. The difference in distance is measured using the time delay

[5] It should be noted that the technology has progressed so that most qubits are at present implemented using superconducting transmons and not using a Mach-Zehnder.

δt between photons. The experiment is shown in Figure 3.10 and the data in Figure 3.11.

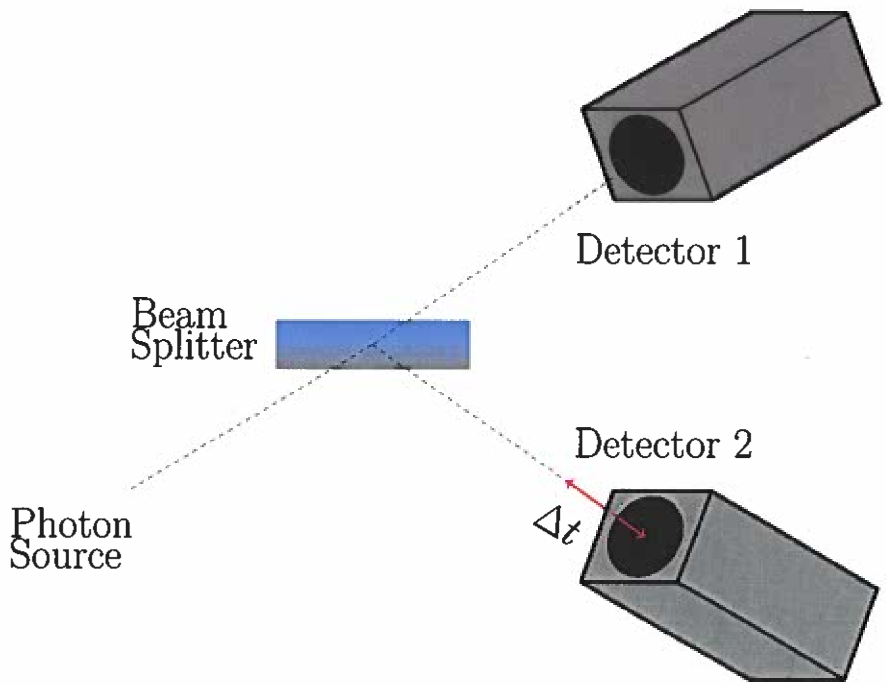

Figure 3.10: The experiment varies the position of Detector 2 and records the number of coincidences, i.e., the number of times both detectors are triggered simultaneously.

3.a. Does the data shown in Figure 3.11 at $\delta t = 0$ support that light is a particle or a wave?

3.b. Why are there large coincidence counts when $\delta t \neq 0$? (Hint: Look at the spacing between the peaks.)

4. ◆ Using matrices given in Figure 3.12, show how the superposition state is created by applying the beam splitter matrix transformation to the initial photon vector state.

5. ◆ Construct the matrix representation for a $30/70$ beam splitter.

6. ■ Unsettled by the Mach-Zehnder interferometer, you decide to determine once and for all which path the photon takes after the first beam splitter. You place another detector (indicated by the eyeball) on the upper path as shown in Figure 3.13. If the eyeball sees a photon, what would be seen at Detectors 1 and 2?