MAMS Mathematica Introduction - p1

The following represents a collection of exercises that you should be able to complete after working your way through the Labs #1-10. These exercises should be completed on your own, with minimal assistance from others. At the completion of the assignment you should add a text cell called **Self Assessment-p1**. In the cell you should evaluate your comfort with these **Mathematica** exercises and acknowledge any assistance that you received. Do not forget to include a header and footer.

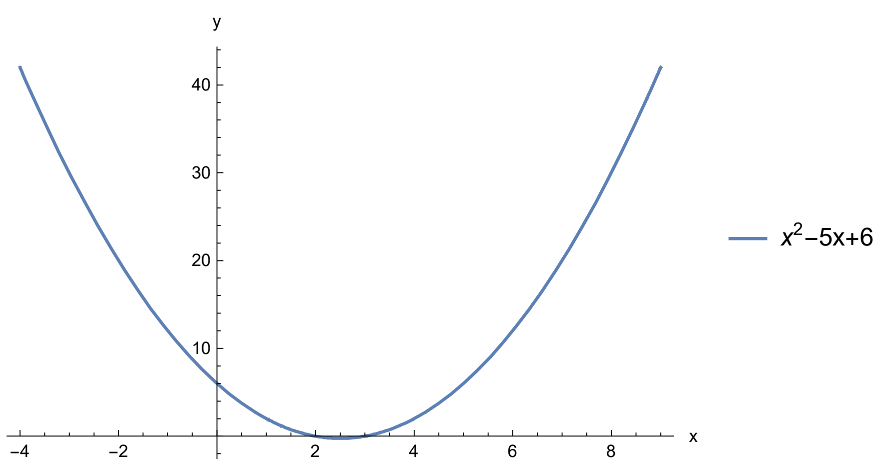
The filename: z_selfassessmentp1_2019.2020_last

#1 Just Do It!

Create a a graph for $\mathbf{y = x^2 - 5\,x + 6}$ in an appropriate window; be sure to label axes.

In[354]:=
```
Plot[x^2 - 5 x + 6, {x, -4, 9},
 AxesLabel → {"x", "y"},
 PlotLegends → LineLegend[{"x²-5x+6"}]]
```

Out[354]=



#2 Just Do It!

Create a graph of $y = x^2 - e^{0.1\,x}$. Make sure to locate and plot all x-intercepts and the turning point in the 1st quadrant. Display the graph along with the ListPlot.

In[390]:=
```
Clear[f, xints, graph1, graph2, turnpt]
f[x_] := x^2 - E^(0.1 x);
xints = Solve[f[x] == 0];
turnpt = Solve[f'[x] == 0];
graph1 = ListPlot[{{-0.953446, 0}, {1.05412, 0}, {89.9951, 0}, {72.84, f[72.84]}},
   PlotStyle → {Magenta, PointSize[0.015]}, AxesLabel → {"x", "y"}];
graph2 = Plot[f[x], {x, -30, 100}, AxesLabel → {"x", "y"}];
Show[{graph1, graph2},
 PlotRange → {-1000, 4000}]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

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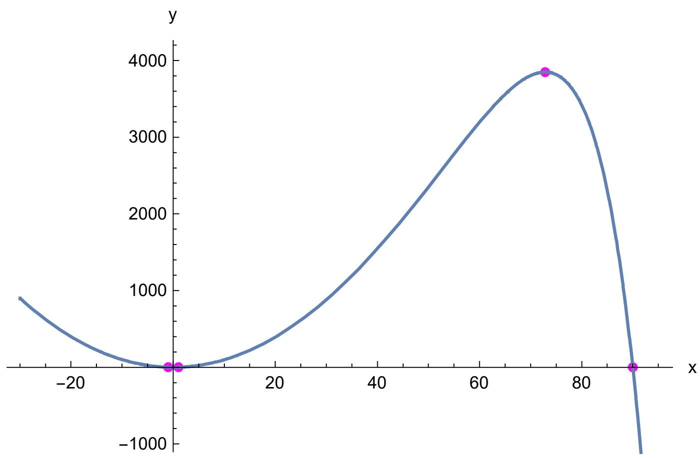
Out[396]=



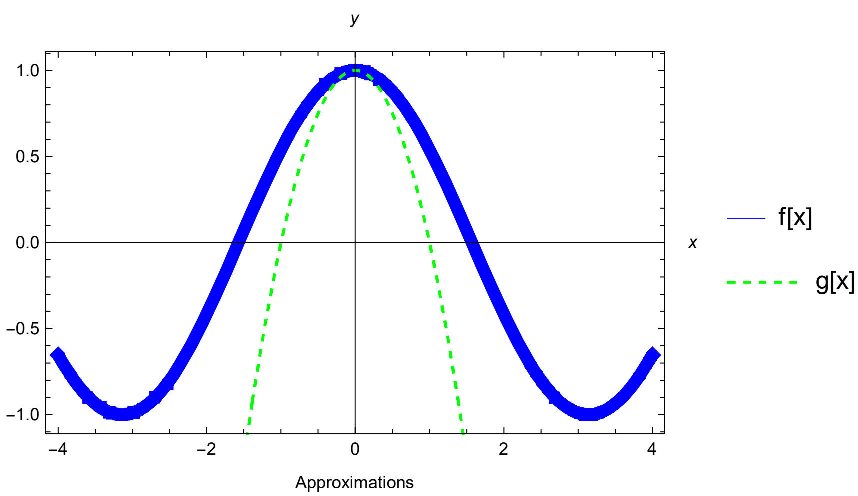
#3 Just Do It!

Let **f[x] = Cos[x]** and **g[x] = 1 –** x^2. Define both of these functions.

Plot them together, making **f[x]** blue and thick while **g[x]** is green with dashes. Put a frame around the plot and label it *Approximations*.

In[250]:=
```
Clear[f, g, grap1, grap2]
f[x_] := Cos[x]
g[x_] := 1 - x^2
grap1 = Plot[f[x], {x, -4, 4},
   PlotStyle → {Blue, Thickness → 0.02}, PlotLegends → LineLegend[{"f[x]"}]];
grap2 = Plot[g[x], {x, -4, 4}, PlotStyle → {Green, Dashed},
   PlotLegends → LineLegend[{"g[x]"}]];
Show[grap1, grap2, Frame → True, FrameLabel → "Approximations"];
Show[%, AxesLabel → {HoldForm[x], HoldForm[y]},
 PlotLabel → None, LabelStyle → {GrayLevel[0]}]
```

Out[256]=



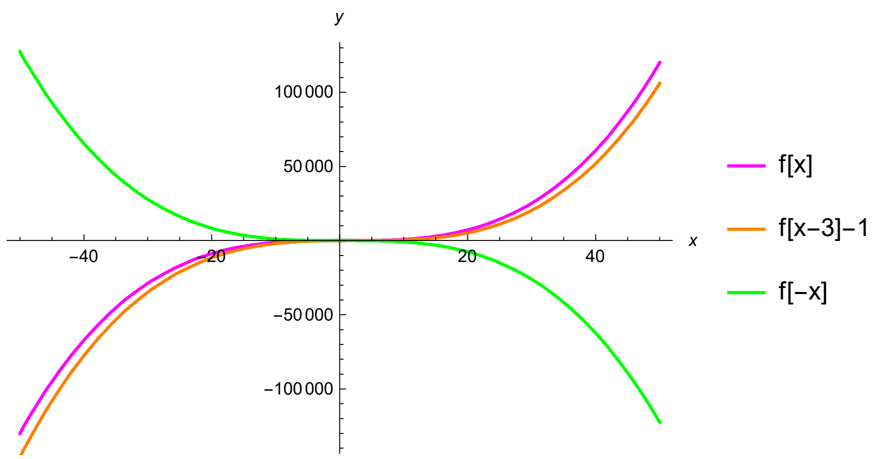
#4 Just Do It!

Let $f(x) = x^3 + x^2 - 2x$. Create the graph below showing $f(x)$, $f(x-3)-1$, and $f(-x)$.

In[264]:=

```
Clear[f, grap1, grap2, grap3]
f[x_] := x^3 + x^2 - 2 x
grap1 =
  Plot[f[x - 1], {x, -50, 50}, PlotStyle → {Magenta}, PlotLegends → LineLegend[{"f[x]"}]];
grap2 = Plot[f[x - 3] - 1, {x, -50, 50}, PlotStyle → {Orange},
   PlotLegends → LineLegend[{"f[x-3]-1"}]];
grap3 = Plot[f[-x], {x, -50, 50}, PlotStyle → {Green}, PlotLegends → LineLegend[{"f[-x]"}]];
Show[{grap1, grap2, grap3}];
Show[%, AxesLabel → {HoldForm[x], HoldForm[y]},
 PlotLabel → None, LabelStyle → {GrayLevel[0]}]
```

Out[270]=



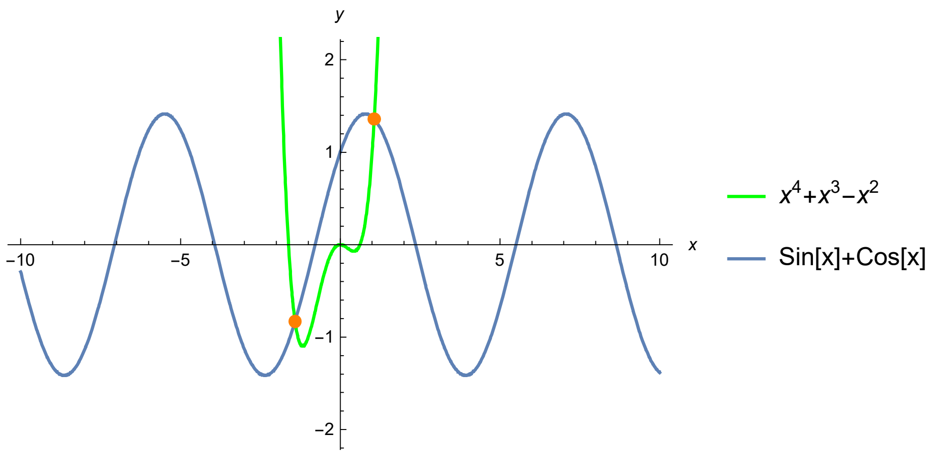
#5 Just Do It!

Determine the intersection points of `x^4+x^3-x^2` and `Sin[x]+Cos[x]`. Display a graph showing the intersection of these two curves.

In[279]:=
```
Clear[f, g, fplot, gplot]
f[x_] := x^4 + x^3 - x^2
g[x_] := Sin[x] + Cos[x]
NSolve[g[x] == f[x], x, Reals]
fplot = Plot[f[x], {x, -10, 10}, PlotStyle → Green, PlotLegends → LineLegend[{"x^4+x^3-x^2"}]];
gplot = Plot[g[x], {x, -10, 10}, PlotLegends → LineLegend[{"Sin[x]+Cos[x]"}]];
ints = ListPlot[{{-1.41328, f[-1.41328]}, {1.06486, f[1.06486]}},
   PlotStyle → {Orange, PointSize → 0.02}];
Show[{fplot, gplot, ints}, PlotRange → {-2, 2}];
Show[%, AxesLabel → {HoldForm[x], HoldForm[y]},
 PlotLabel → None, LabelStyle → {GrayLevel[0]}]
```

Out[282]= {{x → -1.41328}, {x → 1.06486}}

Out[287]=



#6 Just Do It!

Consider two polynomials: **f[x] =** $x^4 - 6x^2 + 2x + 5$ and **g[x] =** $x^3 - 3x + 1$. First, define both functions.

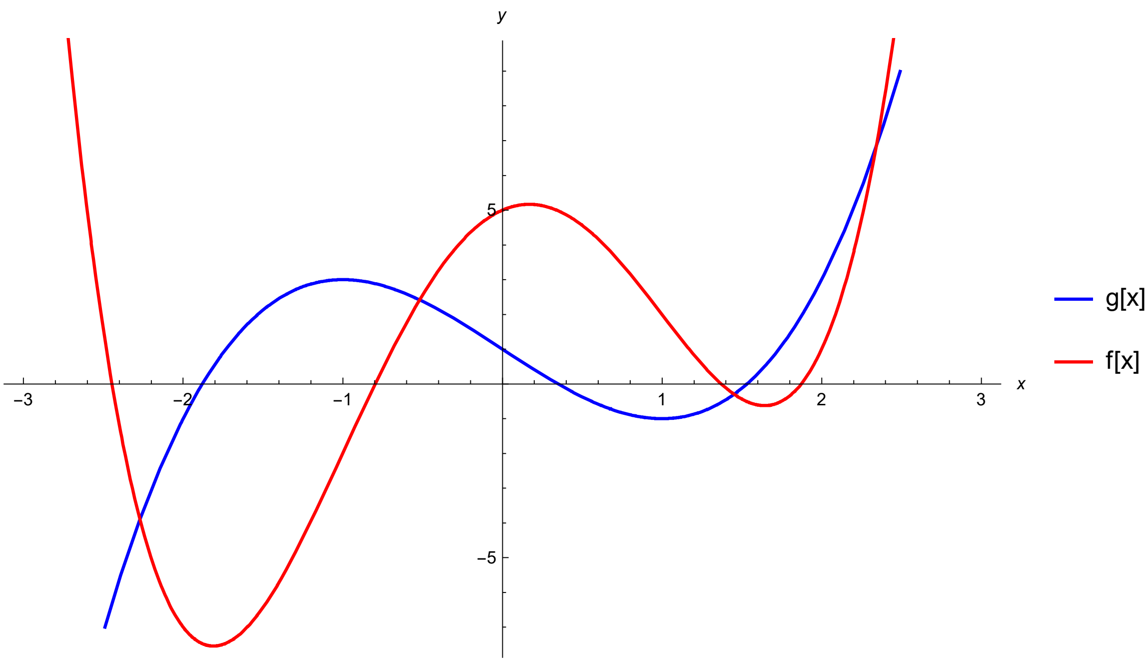
Plot both functions together, finding an appropriate domain. Make the quartic function red and the cubic blue.

Set the two functions equal to each other and solve the equation with the **Solve** command.
Now solve the equation with the **NSolve** command.

Substitute this list of values into either function to find the corresponding y-values. Do the ordered pairs seem to agree with a graphical approximation?

```
Clear[f, g, fplot, gplot, sol]
f[x_] := x^4 - 6 x^2 + 2 x + 5
fplot = Plot[
   f[x],
   {x, -3, 3},
   PlotStyle → Red,
   PlotLegends → LineLegend[{"f[x]"}]];
g[x_] := x^3 - 3 x + 1
gplot = Plot[
   g[x],
   {x, -3, 3},
   PlotStyle → Blue,
   PlotLegends → LineLegend[{"g[x]"}]];
Show[{gplot, fplot}];
Show[%, AxesLabel → {HoldForm[x], HoldForm[y]},
 PlotLabel → None, LabelStyle → {GrayLevel[0]}]
Solve[f[x] == g[x], x]
NSolve[f[x] == g[x], x]
sol = x /. %;
TableForm[Table[{x, f[x]}, {x, sol}], TableHeadings → {None, {x, y}}]
```

Out[294]=



Out[295]= {{x → −2.27…}, {x → −0.519…}, {x → 1.45…}, {x → 2.34…}}

Out[296]= {{x → −2.2706}, {x → −0.51912}, {x → 1.45106}, {x → 2.33866}}

Out[298]//TableForm=

x	y
−2.2706	−3.89453
−0.51912	2.41747
1.45106	−0.297874
2.33866	6.77494

Yes, the values in the table seem to agree with the intersection points seen in the graph generated above.

#7 Just Do It!

a) Execute the following three commands:

In[ ]:=
```
x = 2
```
Out[ ]= 2

In[ ]:=
```
Factor[x^2 - 1]
```
Out[ ]= 3

In[ ]:=
```
Sum[i^2 - Sin[i], {i, 0, x}]
```
Out[ ]= $5 - \mathrm{Sin}[1] - \mathrm{Sin}[2]$

and briefly explain the output that you get.

answer: The first command sets variable x equal to 2. The second command factors x^2-1, however, x is no longer a variable, but rather a placeholder for the value 2, so the command substitutes 2 for x, evaluates the expression, and then executes Factor[3], which is just 3. The third command takes the sum of an expression in terms of i, where i iterates from 0 to x. Again, the command first substitutes 2 for x. The upper bound becomes 2, and thus the iteration stops at i=2.

b) Execute the following three commands:

In[ ]:=
```
Clear[x]
```

In[ ]:=
```
Factor[x^2 - 1]
```
Out[ ]= $(-1 + x)(1 + x)$

In[ ]:=
```
Sum[i^2 - Sin[i], {i, 0, x}]
```
Out[ ]= $\frac{1}{6}\left(x + 3 x^2 + 2 x^3 - 3 \mathrm{Cot}\left[\frac{1}{2}\right] + 3 \mathrm{Cos}[x]\, \mathrm{Cot}\left[\frac{1}{2}\right] - 3 \mathrm{Sin}[x]\right)$

and briefly explain why the outputs of the last two commands are different than they were in Part a).

answer: The first command clears the assignment x=2. Now x isn't a placeholder, it is a variable. The two commands don't substitute 2 for x before evaluating, and instead evaluate in terms of the variable, x.

#8 Just Do It!

A local package delivery service has the following rate structure:

a) Any package weighing less than a pound costs $5.00.

b) A package weighing at least one pound but less than 10 pounds costs 25 cents a pound for each additional pound or part of a pound over 1 pound.

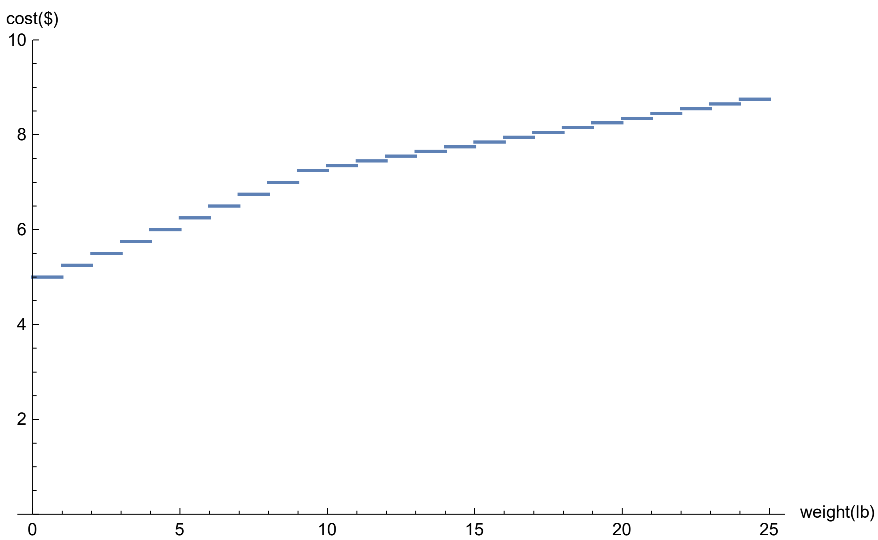
c) A package weighing at least 10 pounds but less than 25 pounds costs 10 cents a pound for each additional pound or part of a pound over 10 pounds.

d) No packages weighing 25 pounds or more are accepted.

Define function **cost[x]** that gives the cost of delivering a package weighing x pounds for $0 < x < 25$. Graph **cost[x]** on the interval $0 < x < 25$.

In[307]:=
```
Clear[cost]
cost[x_] := {        5                        0 < x ≤ 1
               5 + 0.25 * (Ceiling[x - 1])    1 ≤ x < 10
               7.25 + 0.10 (Ceiling[x - 10])  10 ≤ x < 25
Plot[cost[x], {x, 0, 25}, PlotRange → {0, 10}];
Show[%, AxesLabel → {HoldForm[weight[lb]], HoldForm[cost[$]]},
 PlotLabel → None, LabelStyle → {GrayLevel[0]}]
```

Out[310]=



#9 Just Do It!

Recall that a function **f[x]** is **periodic** if there is a positive constant p such that **f[x + p] = f[x]** for all **x**.

The six trigonometric functions are periodic, and so is any constant function. If there is a smallest positive number **p** with the property, it is called the period of **f**.

The built-in Mathematica function **Mod[m,n]** can, among many other things, help us to define periodic functions, different from the standard trigonometric functions.
For two real numbers **m** and **n**, **Mod[m,n]** gives the remainder for the division of the number **m** by the number **n**; that is, if **m = q*n + r** where **r** is between **0** and **n** but not equal to **n** then **Mod[m,n] = r**.

Run the following to check out a few values:

In[ ]:=
```
Mod[3.5, 2]
```

Out[ ]= 1.5

In[ ]:=
```
Mod[3.5, 2.5]
```

Out[ ]= 1.

In[ ]:=
```
Mod[1.5, 2]
```

Out[ ]= 1.5

In[ ]:=
```
Mod[-3.5, 2]
```

Out[ ]= 0.5

In[ ]:=
```
Mod[3.5, -2]
```

Out[ ]= -0.5

Notice that if n is a positive number, then the remainder r is nonnegative and less than n; that is, $n > 0 \implies 0 \leq r < n$.

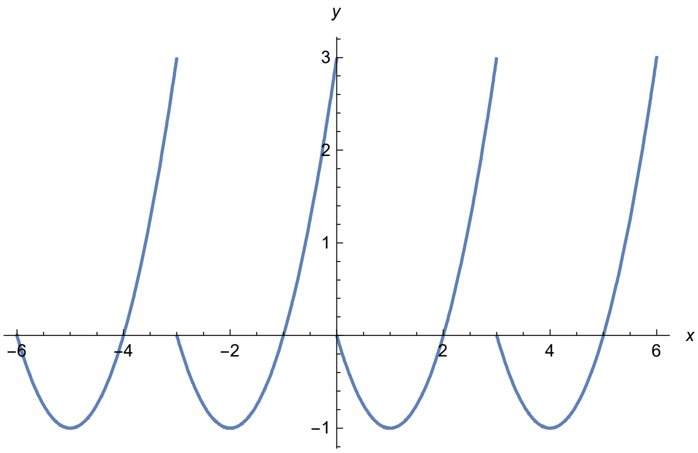
Also, if f[x] is a periodic function of period p, then the value of f at any x is the same as its value at Mod[x,p]; that is, f[x] = f[Mod[x,p]] and 0 ≤ Mod[x,p] < p.

a) Enter the non-periodic function $g[x] = x^2 - 2x$.
b) Use the Mod[m,n] command to define a function f[x] of period 3 that is identical to g[x] on the interval $0 < x < 3$.
c) Graph the function f[x] on the interval $-6 \leq x \leq 6$.

In[349]:=
```
Clear[g, f]
g[x_] := x^2 - 2 x
f[x] = g[Mod[x, 3]]
Plot[f[x], {x, -6, 6}];
Show[%, AxesLabel → {HoldForm[x], HoldForm[y]},
 PlotLabel → None, LabelStyle → {GrayLevel[0]}]
```

Out[351]= $-2\,\text{Mod}[x, 3] + \text{Mod}[x, 3]^2$

Out[353]=



#10 Just Do It!

Plot Cos[3 x] on the interval from -3π to 3π.

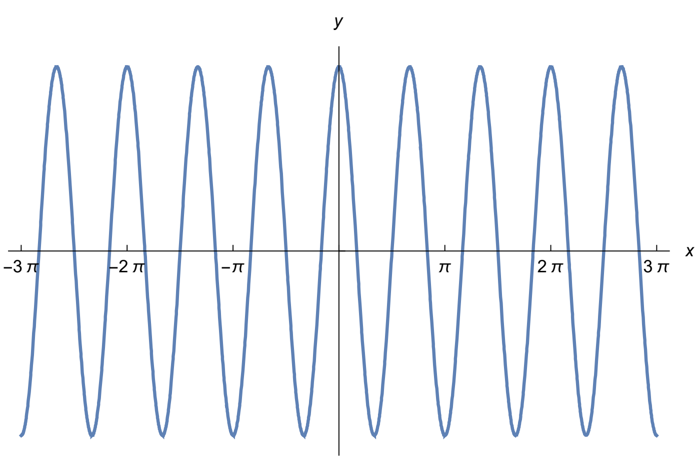
a) Label the axes.

b) Set tick marks on the x-axis from -3π to 3π at intervals of π units apart.

In[312]:=
```
Clear[tickspots]
tickspots = Table[Pi * x, {x, -3, 3}]
Plot[
 Cos[3 x],
 {x, -3 * Pi, 3 * Pi},
 AxesLabel → {x, y},
 Ticks → {tickspots}]
```

Out[313]= {-3 π, -2 π, -π, 0, π, 2 π, 3 π}

Out[314]=



#11 Just Do It!

Plot $y = x^2 - 3x + 2$ on a convenient interval.

a) Change the color of the plot to Red.

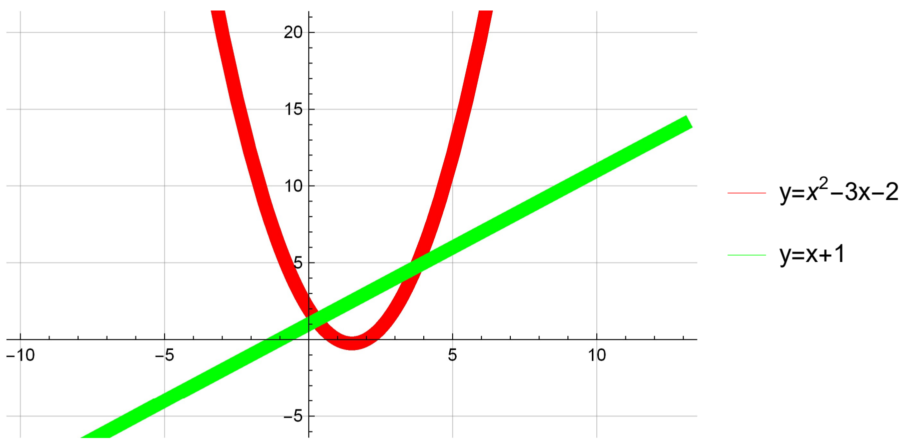
b) Change the Thickness of the plot to 0.02.

c) Include grid lines on your plot.

d) Now make a new graphic that includes the plot of $y = x + 1$ displayed in green with that of $y = x^2 - 3x + 2$ in red.

In[421]:=
```
Clear[grap1, grap2]
grap1 = Plot[
   x^2 - 3 x + 2,
   {x, -10, 13},
   PlotStyle → {Thickness → 0.02, Red},
   GridLines → Automatic,
   PlotLegends → LineLegend[{"y=x^2-3x-2"}]];
grap2 = Plot[
   x + 1,
   {x, -10, 13},
   PlotStyle → {Green, Thickness → 0.02},
   PlotLegends → LineLegend[{"y=x+1"}]];
Show[grap1, grap2, PlotRange → {-5, 20}]
```

Out[424]=



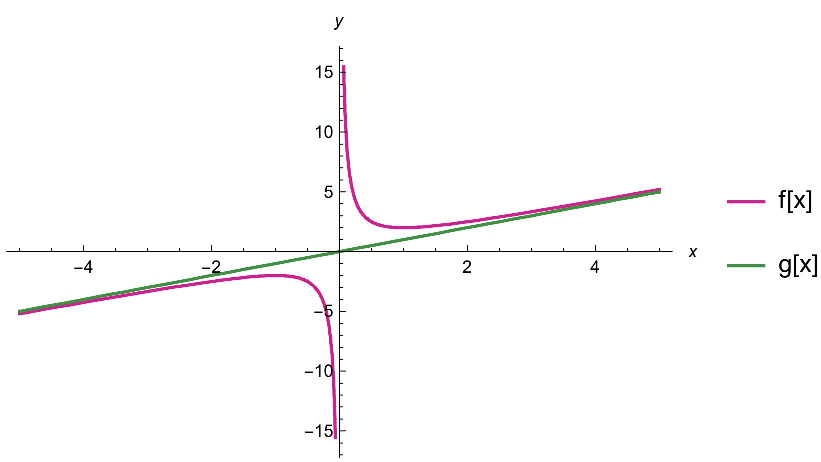
#12 Just Do It!

Plot f[x]= x + $\frac{1}{x}$ and g[x] = x on the interval -5 ≤ x ≤ 5. Then:

a) Label the plot "g[x] is an asymptote of f[x]".
b) Make the graph of f[x] VioletRed and that of g[x] CobaltGreen.
c) Make the label appear in 18-point font.

In[464]:=
```
Clear[f, g, fplot, gplot]
f[x_] := x + 1/x
g[x_] := x
fplot = Plot[f[x], {x, -5, 5},
   PlotStyle → ColorData["Legacy", "VioletRed"], PlotLegends → LineLegend[{"f[x]"}]];
gplot = Plot[g[x], {x, -5, 5}, PlotStyle → ColorData["Legacy", "CobaltGreen"],
   PlotLegends → LineLegend[{"g[x]"}]];
Show[{fplot, gplot}];
Show[%, PlotLabel → HoldForm[g[x] is an asymptote of f[x]], LabelStyle → {18, GrayLevel[0]}];
Show[%, AxesLabel → {HoldForm[x], HoldForm[y]},
 PlotLabel → None, LabelStyle → {GrayLevel[0]}]
```

Out[471]=



#13 Just Do It!

Create a plot of $3^{2x} - 1$

a) Store the plot into a variable called **threeplot**. Include a semicolon so that the plot does not show.

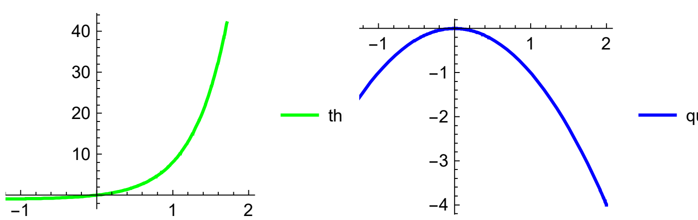
b) Create, but do not display, a plot named **quadplot** of $-x^2$. Also suppress this output.

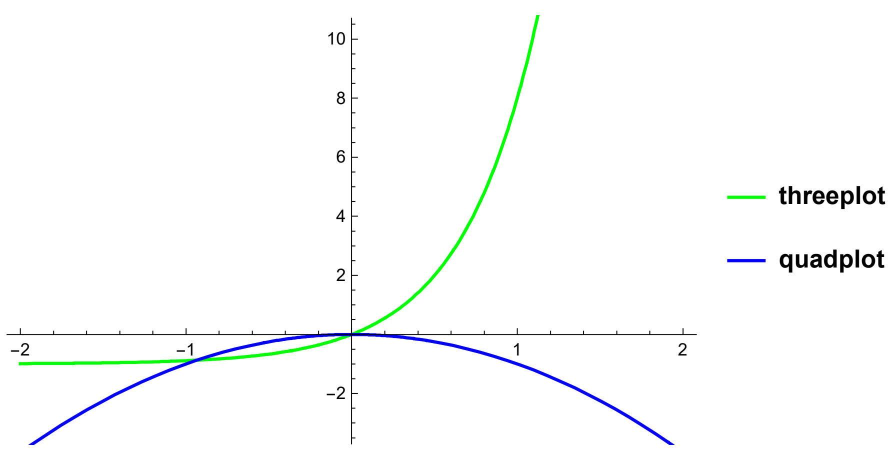
c) Show both plots together side-by-side.

d) Now show both of the plots together on the same axes.

In[492]:=
```
Clear[threeplot, quadplot]
threeplot = Plot[
   3^(2 x) - 1,
   {x, -2, 2},
   PlotStyle → Green,
   PlotLegends → LineLegend[{"threeplot"}]];
quadplot = Plot[
   -x^2,
   {x, -2, 2},
   PlotStyle → Blue,
   PlotLegends → LineLegend[{"quadplot"}]];
Show[GraphicsRow[{threeplot, quadplot}]]
Show[{threeplot, quadplot}, PlotRange → {-3, 10}]
```

Out[495]=





Self Assessment-p1

I am fairly comfortable with the aspects of functions and ‘solve’ that were covered in the Labs and this self-assessment. I understand what the Mod function does, and some of its uses. I can make Plots and List-Plots and do basic alterations to their designs, however, I’m still clumsy with the graphics tools. I also have some trouble with formatting Show commands. My main assistance during this project came from the Language Documentation Center. This was mainly to reference syntax. I also used Google to troubleshoot from time to time. For example, the command for using special colors like Cobalt Green is different now than it was when the labs were written, and I had to use Google to find out what was the appropriate command.