Introduction

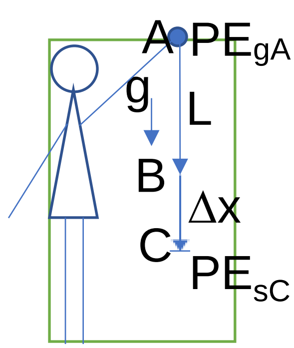
- The purpose of this lab was to design and perform an experiment that analyzed the conservation of energy in a spring-based system.
- Researchable Question:
  - How does the length of a bungee with a fixed mass tied to the end affect the amount that the bungee stretches when the mass is dropped from a fixed height?
- Hypothesis:
  - If the length of the bungee increases, then distance stretched will increase, where distance stretched is proportional to the square-root of bungee length.

Methodology

A wooden dowel was fastened on top of a cabinet with 6 inches extended over the edge. The dowel was normal to the front face of the cabinet. A meterstick was fastened to the front face of the cabinet, normal to the ground, and with the zero end directly beneath the dowel. A rock was fastened to one end of a 1-meter long piece of bungee string with tape. The rock-bungee-tape system was massed.

For each trial, Daanish measured a specified length of bungee string, starting from the point where the rock was fastened to the string. He then pressed the open end of the string against the side of the dowel and held the rock up beside the dowel on the same side on which the string was pressed. Sri started a slow-motion video, holding her phone parallel to the front face of the cabinet, and then told Daanish to drop the rock. Neil then recorded the lowest point that the top of the rock reached.

Diagram



Constants and Equations

m = 0.0422 kg
g = 9.81 m/s^2

$$\Delta x = \frac{mg + \sqrt{m^2 g^2 + 2mkgL}}{k}$$

Data Summary

- Table 1: The table contains measured Δx_{avg} (DV) and experimental k values for each setting of L (IV).

L	k	Δx_{avg}	STDEV	|%RSD|	Δx_T	|%err|
(m)	(kg/s^2)	(m)	(m)	of x_{avg}	(m)	of Δx
10	0.41	10.13	0.77	7.57	5.62	80.22
15	0.12	17.68	0.76	4.27	14.39	22.82
20	0.19	24.25	0.31	1.29	11.80	105.58
30	0.03	34.73	0.84	2.41	42.26	17.83
50	0.02	54.85	4.56	8.31	74.71	26.58
			Avg	4.77	Avg	50.61

Calculating k

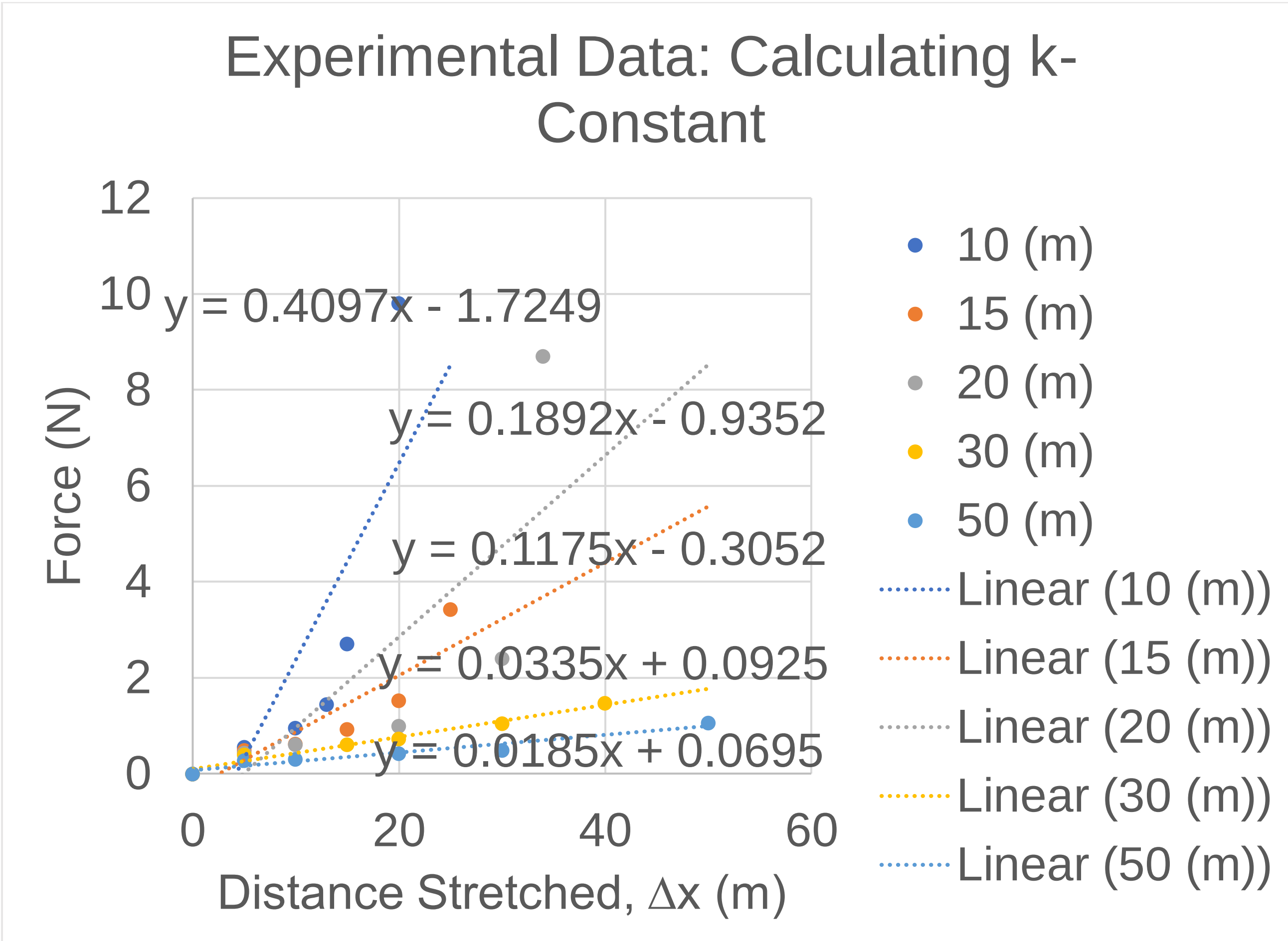


Fig 1: Effect of distance stretched on spring force. The k-constant is the slope of each trendline.

Experimental Data (1)

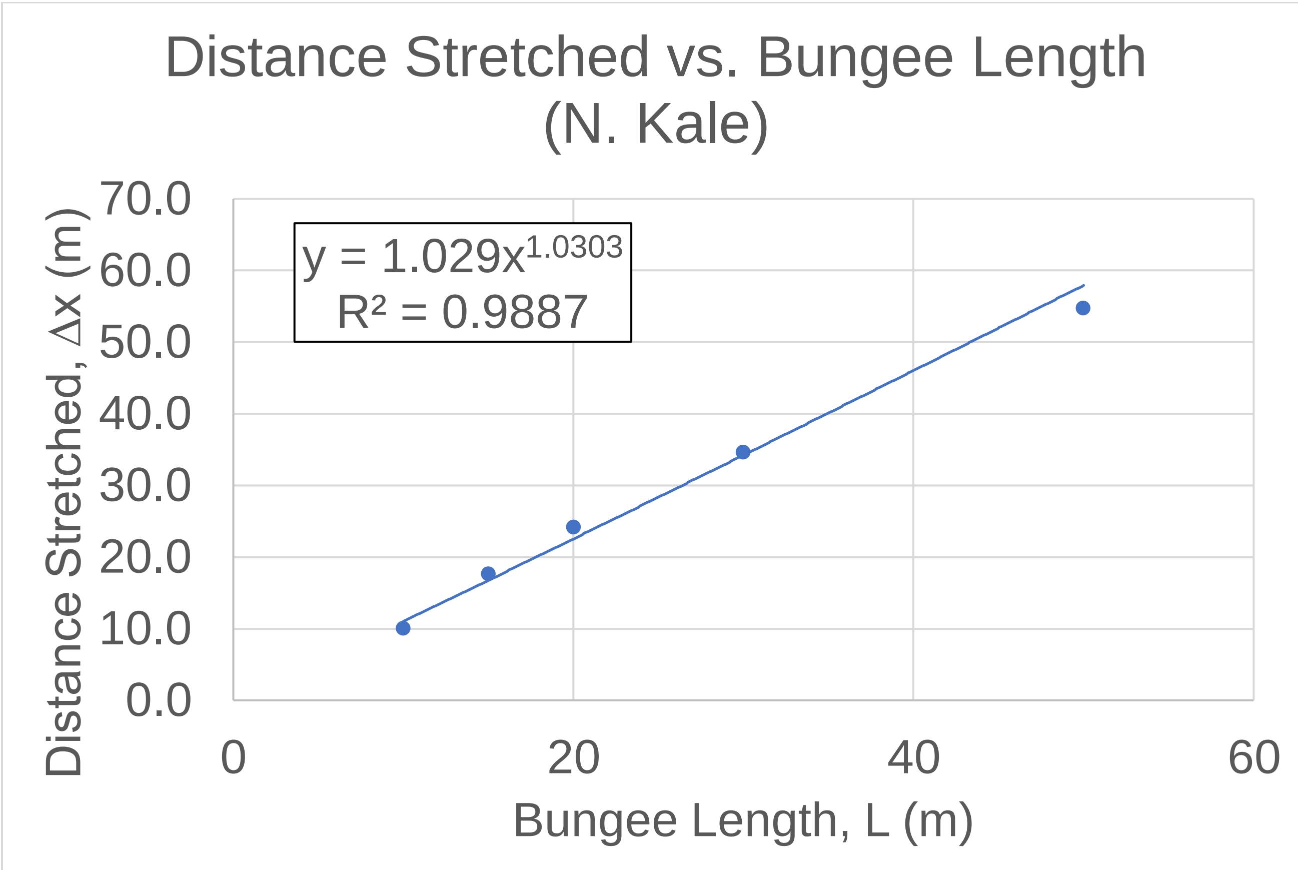


Fig 2: Effect of bungee length on distance stretched.

Experimental Data (2)

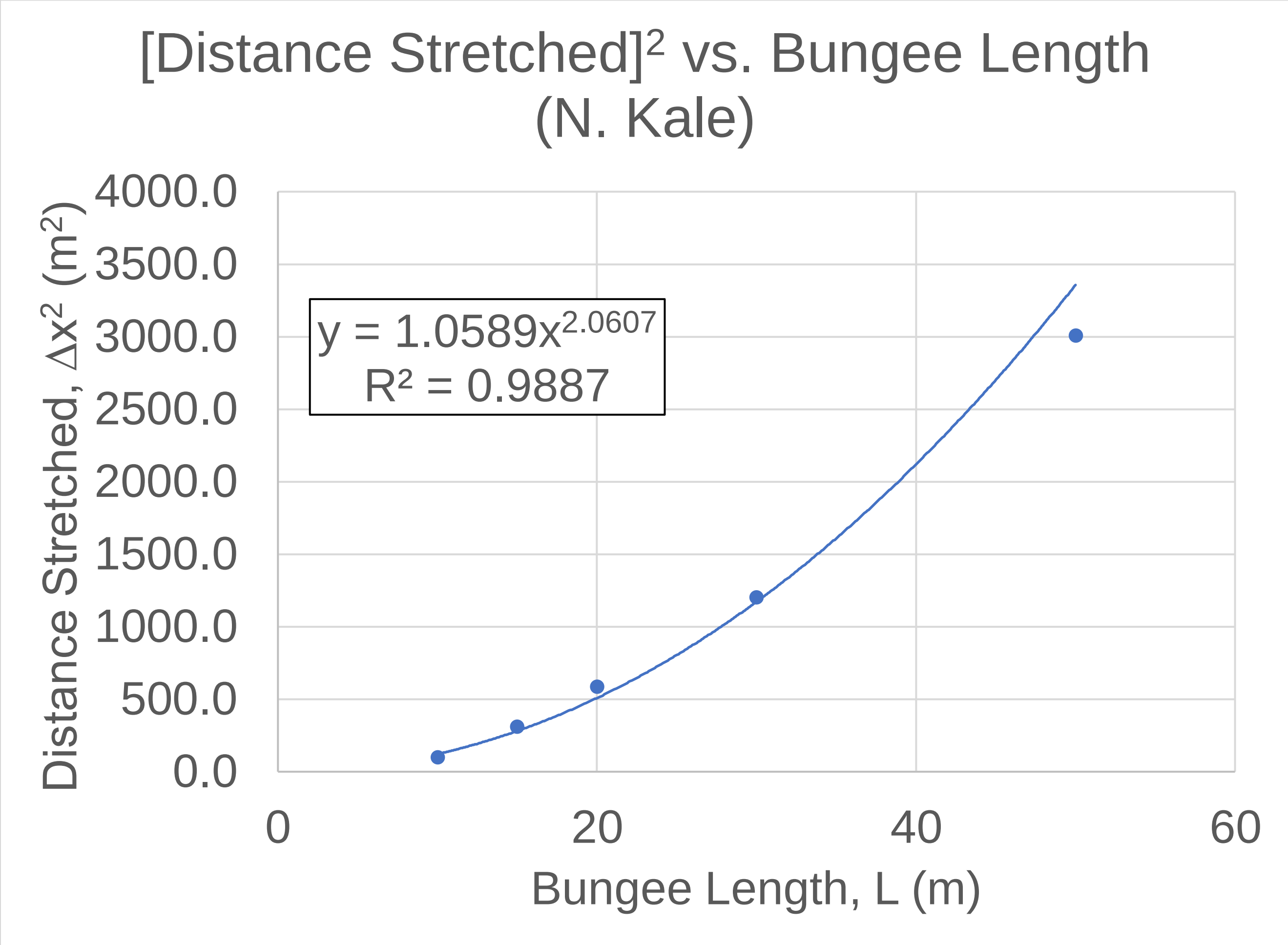


Fig 3: Effect of bungee length on [distance stretched]^2.

Experimental Data (3)

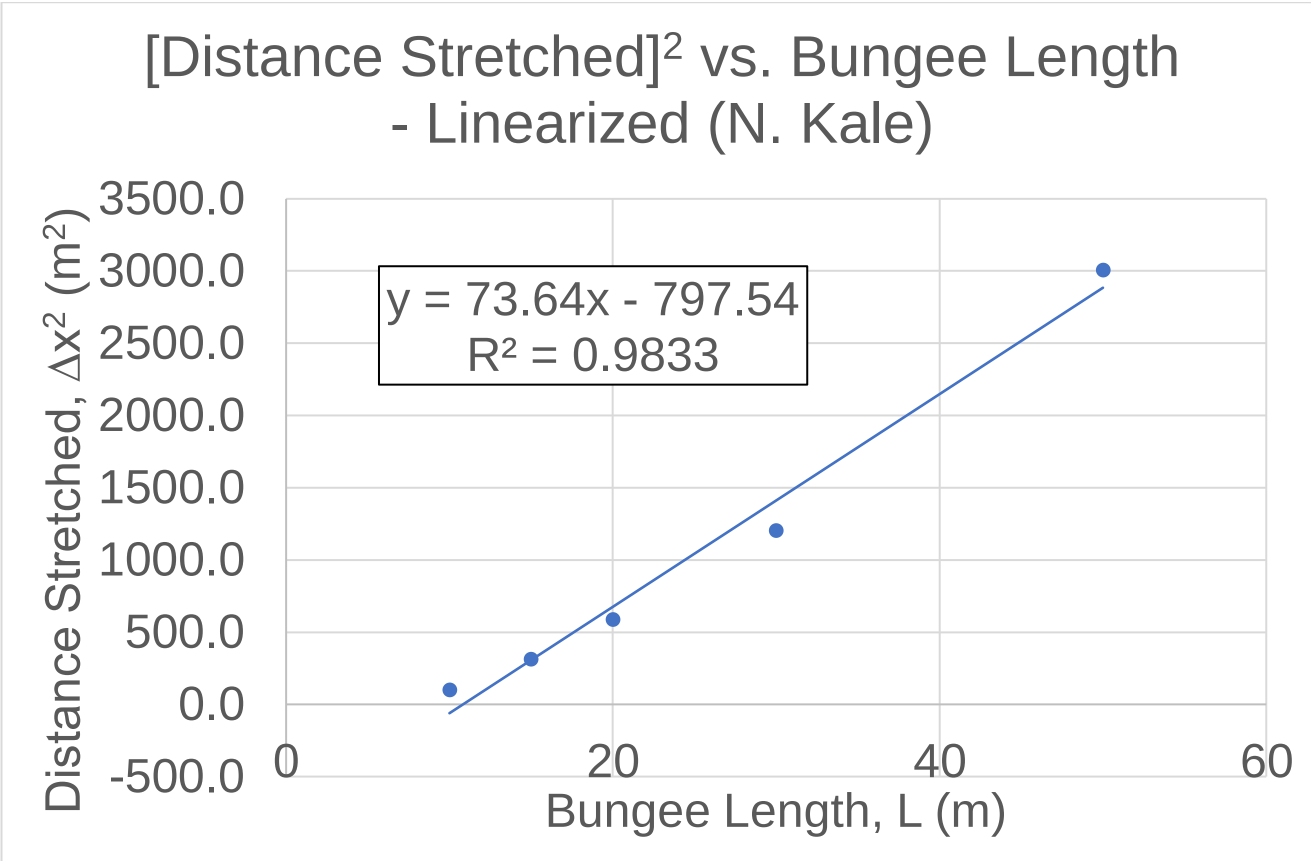


Fig 4: Effect of bungee length on [distance stretched]^2 with linear trendline (Compare to Fig. 3)

Photograph

Fig. 5: Collecting data for the Spring Energy lab.

Discussion

- The slope of the curve (m/m) remains almost constant as L increases, suggesting that the difference in drop time per unit height is constant at all values of L.
- The curve has no limit.
- The linearized fit of Δx^2 vs. L has a lower R^2 value than the linearized fit of Δx vs. L. This suggests that Δx is directly proportional to L, not $L^{1/2}$.

Statistical Analysis

- The data has high precision; the average |%RSD| is 4.77%.
- The average % error is 50.67%, hence the accuracy is very low.
- The R^2 is 0.9887, so the mathematical model is strong.
- The curve of best fit is $\Delta x_{avg} = 1.029L^{1.0303}$

Sources of Error

- The calculated k values were inaccurate.
  - Vernier Force sensor may have been pulled in the opposite direction instead of being held still
    - Increased F recorded → increased k → Δx measured is less than the true value
- Bungee cord deteriorated while testing.
  - k-constant would decrease during the experiment (Less force required for the same stretch)
  - Δx is proportional to $k^{-1/2}$ → Δx measured is greater than the true value
- Drop height was inconsistent. No regulation on whether the top or bottom of the rock was held to the dowel height before dropping

Summary of Outcomes

- The hypothesis was not supported by this experiment. As described in the Statistical Analysis, the linearized fit of Δx^2 vs. L performed better than the linearized fit of Δx vs. L.
- This suggests that Δx is not directly proportional to $L^{1/2}$ as was initially proposed.

Future Extensions

- Redo the experiment with cords that have a defined k value.
  - This will help to verify the results and reduce the possibilities of error.
- Conduct the same testing; however, measure k before and afterward.
- Measure the effect of bungee length on the height to which the mass rebounds after being dropped.
- Determine whether a similar relationship exists for smaller and larger masses.

DV Formula Derivation

$$PE_{gA} = PE_{SB}$$
$$mgh_A = \frac{1}{2}k(\Delta x)^2$$
$$mg(L + \Delta x) = \frac{1}{2}k(\Delta x)^2$$
$$mgL + mg\Delta x = \frac{1}{2}k(\Delta x)^2$$
$$\frac{1}{2}k(\Delta x)^2 - mg\Delta x - mgL -= 0$$

Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$a = \frac{1}{2}k \,; b = -mg \,; c = -mgL$$
$$\Delta x = \frac{-(-mg) \pm \sqrt{(mg)^2 - 4\left(\frac{k}{2}\right)(-mgL)}}{2\left(\frac{k}{2}\right)}$$
$$\Delta x = \frac{mg \pm \sqrt{m^2g^2 + 2mkgL}}{k}$$

Δx must be greater than zero. Therefore, the final equation is

$$\Delta x = \frac{mg + \sqrt{m^2g^2 + 2mkgL}}{k}$$