Uber Rocket – Hamster Huey and Calculus Cam

Neil Kale September 21, 2019 Section L

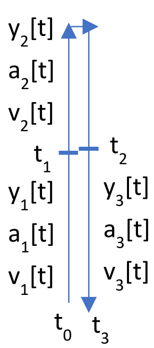
Description

One calm afternoon Calculus Cam decides to launch Hamster Huey into the air using a model rocket. The rocket is launched straight up off the ground, from rest. The rocket engine is designed to burn for specified time while producing non-constant net acceleration given by the equations below. After the engine stops the rocket continues upward in free-fall. A parachute opens after the rocket falls a specified vertical distance from its maximum height. When the parachute opens, assume the rocket instantly stops, and then increases speed to a terminal velocity given by the equation below. Assume the air resistance affects the rocket only during the parachute stage.

Givens

A = -0.9	B = 0	C = 18
$a_y[t]=At^2+Bt+C$		m/s
t_E = 5.1		sec
h = 137		m
v_t = -16 m/s		D = 6
$v_P[t] = v_T(1 - e^{-t/D})$		m/s
t_0 = 0 s		$v_3[t] = v_p[t - t_2]$
$a_2[t] = a_3[t] = -9.8$ m/s^2		$t_1 = t_E$

Diagram



Stage 1: While the rocket burns ($t_0 \rightarrow t_1$)

$a_1[t] = At^2 + Bt + C$
$A = -0.9$
$B = 0$
$C = 18$
$\underline{a_1[t] = -0.9t^2 + 18}$
Find $v_1[t]$ by integrating a[t]:

$v_1[t] = \int a[t]dt$
$v_1[t] = \int (-0.9t^2 + 18)dt$
$v_1[t] = -0.3t^3 + 18t + c$
$v_1[0] = 0$
$\underline{v_1[t] = -0.3t^3 + 18t}$
Find $v_1[t_1]$ via substitution:
$t_1 = t_E = 5.1$ s
$v_1[t_1] = -0.3(5.1)^3 + 18(5.1)$
$v_1[t_1] = -39.7953 + 91.8$
$\underline{v_1[t_1] = 52.005 \text{ m/s}}$
Find $y_1[t_1]$ by integrating v[t] then substituting:
$y_1[t] = \int v[t]dt$
$y_1[t] = \int (-0.3t^3 + 18t)dt$
$y_1[t] = -0.075t^4 + 9t^2 + c$
$y_1[t_0] = y_1[0] = 0$
$\underline{y_1[t] = -0.075t^4 + 9t^2}$
$y_1[t_1] = y[5.1]$
$y_1[5.1] = -0.075(5.1)^4 + 9(5.1)^2$
$y_1[5.1] = -50.739 + 234.09$
$y_1[5.1] = 183.35$
$\underline{y_1[t_1] = 183.35 \text{ m}}$

Stage 2: Engine Stop to Parachute Opening ($t_1 \rightarrow t_2$)

Givens:
$y_2[t_1] = y_1[t_1] = 183.35$ m
$v_2[t_1] = v_1[t_1] = 52.005$ m/s
$\underline{a_2[t] = -9.8 \text{ m/s}^2}$
Vertical fall from y_{max} to parachute opening$(t_2) = h$
$= 137$ m *
Find $v_2[t]$ by integrating $a_2[t]$
$v_2[t] = \int a_2[t]dt$
$v_2[t] = \int -9.8\, dt$
$v_2[t] = -9.8t + c$
$v_2[t_1] = -9.8t_1 + c$
$v_2[t_1] = 52.005$ m/s
$t_1 = 5.1$ s
$52.005 = -9.8 * 5.1 + c$
$c = 52.005 + 9.8 * 5.1$
$c = 52.005 + 49.98$
$c = 101.99$
$\underline{v_2[t] = -9.8t + 101.99}$
Find $y_2[t]$ by integrating v[t]
$y_2[t] = \int v_2[t]dt$

$y_2[t] = \int (-9.8t + 101.99)dt$

$y_2[t] = -4.9t^2 + 101.99t + c$

$y_2[t_1] = -4.9t_1^2 + 101.99t_1 + c$

$y_2[t_1] = 183.35 \text{ m/s}$

$t_1 = 5.1 \text{ s}$

$183.35 = -4.9(5.1)^2 + 101.99(5.1) + c$

$c = 183.35 + 127.449 - 520.58$

$c = -209.35$

$\underline{y_2[t] = -4.9t^2 + 101.99t - 209.35}$

Find t_{max}:

y_{max} occurs at t_{max}; $v_2[t_{max}] = 0$

$-9.8t_{max} + 101.99 = 0$

$t_{max} = \frac{-101.99}{-9.8}$

$\underline{t_{max} = 10.407 \text{ s}}$

Find $y_2[t_{max}]$ by substituting:

$y_2[t_{max}] = -4.9(10.407)^2 + 101.99(10.407) - 209.35$

$y_2[t_{max}] = -4.9(10.407)^2 + 101.99(10.407) - 209.35$

$y_2[t_{max}] = -530.712 + 1061.41 - 209.35$

$\underline{y_2[t_{max}] = 321.36 \text{ m}}$

Find $y_2[t_2]$:

$y_2[t_2] = y_2[t_{max}] - h$ *

$y_2[t_2] = 321.36 - 137$

$\underline{y_2[t_2] = 184.36 \text{ m}}$ *

Find t_2:

$184.36 = -4.9t_2^2 + 101.99t_2 - 209.35$

$-4.9t_2^2 + 101.99t_2 - 393.71 = 0$

Using solve:

$t_2 = 5.1195 \text{ or } 15.695$

$t_2 > t_{max}$

$t_2 > 10.407$

$\underline{t_2 = 15.695 \text{ s}}$

Stage 3: Parachute Opening to Touchdown ($t_2 \rightarrow t_3$)

Givens:

$v_P[t] = v_T(1 - e^{-t/D})$

$D = 6$; $v_T = -16 \text{ m/s}$

$v_P[t] = -16(1 - e^{-t/6})$

$v_3[t_2] = v_P[0]$

$v_3[t] = v_P[t - t_2]$

$\underline{v_3[t] = -16\left(1 - e^{-\frac{t-t_2}{6}}\right)}$

Find $y_3[t]$ by integrating $v_3[t]$

$y_3[t] = \int v_3[t]dt$

$y_3[t] = -16\int \left(1 - e^{-\frac{t-t_2}{6}}\right)dt$

$y_3[t] = -16\left(t + 6e^{-\frac{t-t_2}{6}} + c\right)$

$y_3[t] = -16t - 96e^{-\frac{t-t_2}{6}} + c$

$y_3[t_2] = y_2[t_2]$

$y_2[t_2] = 184.36 \text{ m}$ *

$\underline{y_3[t_2] = 184.36 \text{ m}}$

$y_3[t_2] = -16t_2 - 96e^{-\frac{t_2-t_2}{6}} + c$

$184.36 = -16t_2 - 96e^{-\frac{t_2-t_2}{6}} + c$

$c = 184.36 + 16t_2 + 96e^{-\frac{0}{6}}$

$t_2 = 15.695 \text{ s}$

$c = 184.36 + 16 * 15.695 + 96$

$c = 531.48$

$\underline{y_3[t] = -16t - 96e^{-\frac{t-t_2}{6}} + 531.48}$

Find t_3, when the object hits the ground:

$y_3[t_3] = 0 \text{ m}$

$0 = -16t_3 - 96e^{-\frac{t_3-t_2}{6}} + 531.48$

Using solve:

$t_3 = 6.8018 \text{ s or } t_3 = 32.875 \text{ s}$

$t_3 > t_2$

$t_3 > 15.695 \text{ s}$

$$\boxed{t_3 = 32.88 \text{ s}}$$

Graphs

Time	Position	Velocity	Acceleration
(s)	(m)	(m/s)	(m/s^2)
Stage 1 ($t_0 \rightarrow t_1$)			
0.00	0.00	0.00	18.00
1.00	8.93	17.70	17.10
2.00	34.80	33.60	14.40
3.00	74.93	45.90	9.90
4.00	124.80	52.80	3.60
5.00	178.13	52.50	-4.50
5.10	183.35	52.00	-5.41
Stage 2 ($t_1 \rightarrow t_2$)			
5.10	183.35	52.01	-9.80
6.00	226.19	43.19	-9.80
7.00	264.48	33.39	-9.80
8.00	292.97	23.59	-9.80
9.00	311.66	13.79	-9.80
10.00	320.55	3.99	-9.80
11.00	319.64	-5.81	-9.80
12.00	308.93	-15.61	-9.80
13.00	288.42	-25.41	-9.80
14.00	258.11	-35.21	-9.80
15.00	218.00	-45.01	-9.80
15.70	184.35	-51.82	-9.80
Stage 3 ($t_2 \rightarrow t_3$)			
15.70	184.36	0.00	-2.67
16.00	184.24	-0.79	-2.53
17.00	182.25	-3.13	-2.15
18.00	178.10	-5.10	-1.82
19.00	172.14	-6.78	-1.54
20.00	164.63	-8.19	-1.30
21.00	155.83	-9.39	-1.10
22.00	145.91	-10.41	-0.93
23.00	135.07	-11.26	-0.79
24.00	123.43	-11.99	-0.67
25.00	111.12	-12.61	-0.57
26.00	98.25	-13.13	-0.48
27.00	84.89	-13.57	-0.41
28.00	71.13	-13.94	-0.34
29.00	57.03	-14.26	-0.29
30.00	42.63	-14.53	-0.25
31.00	27.99	-14.75	-0.21
32.00	13.14	-14.94	-0.18
32.88	0.00	-15.09	-0.15