UberPulley – "Jerky" Jerry's Jabberwocky Jumper

Neil Kale October 19, 2019 Section L

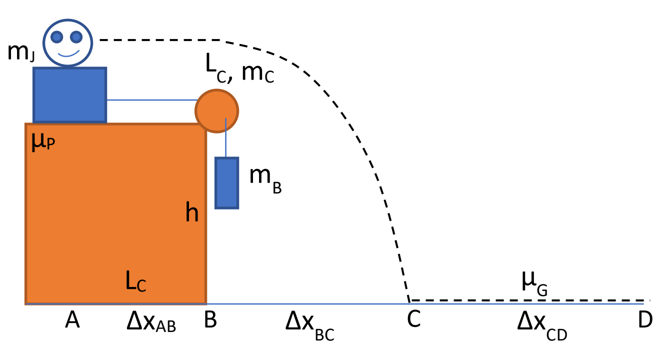
Description

"Jerky" Jerry decided to make a jabberwocky jumper using a pulley system (see diagram). His method was to attach one end of a chain to a barrel of rocks, and the other end to the jumper. He placed the barrel and chain over a massless frictionless pulley, and then walked along a platform away from the pulley to point A (the full length of the chain). When he sat in the jumper he accelerated along the platform to point B and then launched off it while releasing the chain from the jumper and avoiding the pulley. He flew through the air as a projectile to point C, transitioning 75% of his speed into the horizontal direction, and eventually slid to a stop at point D. Note: Ignore any heights of the jumper, pulley, and barrel. Ignore any frictional and normal forces of the chain.

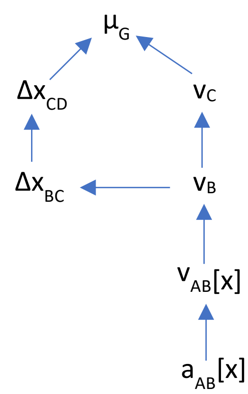
Givens

Total mass of Jerry and Jumper	m_J =	79	(kg)
Total mass of Barrel	m_B =	181	(kg)
Total mass of Chain	m_C =	50	(kg)
Length of Chain (and Δx_{AB})	L_C, Δx_{AB} =	12	(m)
Height of Platform	h =	22	(m)
Coefficient of friction between jumper and platform	μ_P =	0.21	
Total horizontal distance from platform to final location	Δx_{BD} =	67	(m)
Coefficient of friction between jumper and ground	μ_G =	???	

Diagram



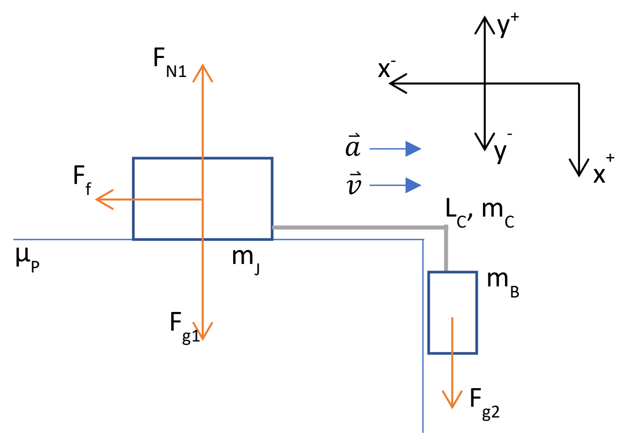
Strategy



The coefficient of friction between Jerry and the ground is dependent on how fast Jerry hits the ground (v_C) and how far he travels from there (Δx_{CD}). The velocity at C and Δx_{BC}, which leads to Δx_{CD} since Δx_{BD} is known, can be found using kinematic equations if the velocity at B is known. In turn, the velocity at B can be derived by determining the acceleration of the block over AB. This is found as shown below using the forces on the system of Jerry, the barrel, and the chain.

Method

1. Find $a_{AB}[x]$



$$\sum F_y : F_{N1} - F_{g1} = m_T * a_y$$

$$a_y = 0$$

$$F_{N1} - F_{g1} = 0$$

$$\underline{F_{N1} = m_J * g}$$

$$\sum F_x : \overrightarrow{F_{g2}} - \overleftarrow{F_f} = m_T * a_x$$

$$\frac{\left(m_B + m_C * \frac{x}{L_C}\right) * g - \mu(m_J * g)}{m_J + m_B + m_C} = a_x$$

$$\frac{\left(181 + 50 * \frac{x}{12}\right) * 9.8 - 0.21(79 * 9.8)}{79 + 181 + 50} = a_x$$

$$\frac{1773.8 + 40.83x - 162.58}{310} = a_x$$

$$a_y = 0 \therefore a_x = a_{AB}[x]$$

$$\underline{a_{AB}[x] = 5.1975 + 0.1317x}$$

2. Find v[x]

$$a \equiv \frac{dv}{dt}$$

$$a \equiv \frac{dv}{dt} * \frac{dx}{dx}$$

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$$a\,dx \equiv v\,dv$$

$$\int_{x_0}^{x} (5.1975 + 0.1317x)dx \equiv \int_{v_0}^{v} v\,dv$$

$$0.06585x^2 + 5.1975x = \frac{1}{2}v^2 - \frac{1}{2}v_0^2$$

$$v_0 = 0$$

$$0.1317x^2 + 5.1975x = v^2$$

$$\underline{v = \pm\sqrt{0.1317x^2 + 5.1975x}}$$

3. Find v_B

$$v_B = v[x_B]$$

$$v_B = v[x_0 + \Delta x_{AB}]$$

$$v_B = v[0 + 12]$$

$$v[12] = \pm\sqrt{0.1317(12)^2 + 5.1975(12)}$$

$$v[12] = \pm\sqrt{18.966 + 124.74}$$

$$\underline{v[12] = 11.988\frac{m}{s}}$$

4. Find v_C

From B to C, the only force acting on Jerry is gravity. Acceleration is constant, thus the kinematic EQS hold true.

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$a_{y(BC)} = -g$$

$$v_{Cy}^2 = v_{By}^2 - 2g(y_C - y_B)$$

$$y_B = h$$

$$v_{Cy}^2 = 0^2 + 2(-9.8)(0 - 22)$$

$$v_{Cy}^2 = 431.2$$

$$\underline{v_{Cy} = 20.7654\frac{m}{s}}$$

$$a_{x(BC)} = 0$$

$$\therefore v_{Cx} = v_{Bx}$$

$$\underline{v_{Cx} = 11.988\frac{m}{s}}$$

$$v_C = \sqrt{v_{Cx}^2 + v_{Cy}^2}$$

$$v_C = \sqrt{20.765^2 + 11.988^2}$$

$$v_C = \sqrt{431.2 + 143.706}$$

$$\underline{v_C = 23.977\frac{m}{s}}$$

5. Find Δx_{BC}

$$v = v_0 + at$$

$$v_{Cy} = v_{By} + g * \Delta t_{BC}$$

$$20.7654 = 0 + 9.8 * \Delta t_{BC}$$

$$\underline{\Delta t_{BC} = 2.1189\ \text{s}}$$

Since $a_{x(BC)} = 0$,

$$\Delta x = v_{0x} * t$$

$$\Delta x_{BC} = v_{Bx} * \Delta t_{BC}$$

$$\Delta x_{BC} = 11.988 * 2.1189$$

$$\underline{\Delta x_{BC} = 25.401\ \text{m}}$$

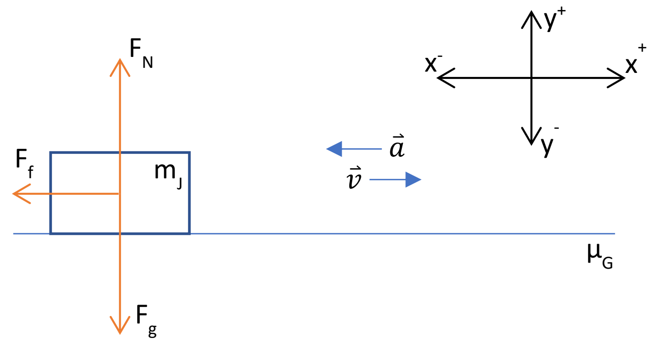
6. Find Δx_{CD}

$$\Delta x_{CD} = \Delta x_{BD} - \Delta x_{BC}$$

$$\Delta x_{CD} = 67 - 25.401$$

$$\underline{\Delta x_{CD} = 41.599\ \text{m}}$$

7. Find μ_G



$$\sum F_y : F_N - F_g = m_J * a_y$$

$$a_y = 0$$

$$F_N = m_J g$$

$$\sum F_x : -\overleftarrow{F_f} = m_J * a_x$$

To find a_x:

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$v_D^2 = v_{C2}^2 + 2a_{CD}(\Delta x_{CD})$$

$$v_D^2 = (0.75v_{C1})^2 + 2a_{CD}(41.599)$$

$$v_D^2 = 0$$

$$a_{CD} = -\frac{(0.75 * 23.977)^2}{2 * 41.599}$$

$$a_{CD} = -\frac{(17.9829)^2}{83.198}$$

$$\underline{a_{CD} = -3.88686\frac{m}{s^2}}$$

Continuing to find F_f:

$$-\overleftarrow{F_f} = m_J * a_x$$

$$-\mu_G * m_J * g = m_J * a_x$$

$$-\mu_G * 9.8 = -3.88686$$

$$\boxed{\mu_G = 0.3966}$$