

Black Hole Toaster

I suppose I should be writing essays for the ISSYP¹ application which is due in 3 days, but after spending the last hour deep-diving into Wikipedia to research for this piece, I'm already too far in not to write it.

Suppose we wanted to use a black hole as a toaster. There's a lot of reasons for doing so. For one, it's clean energy - no fossil fuels or greenhouse gases involved. They decompose naturally; in fact, that's the exact quality that will let us use them as toasters - more on that later. Beyond all that, why wouldn't you want to use a black hole as a toaster! It's possibly the coolest way in the universe to go about making breakfast.

The Science of Toast

Toast is the byproduct of a process known as the Maillard Reaction. Essentially, when we heat food to around 300 degrees Fahrenheit, the sugars and amino acids that constitute the meal will interact with one another to form organic compounds that add the tastes and smells that we associate with toasted food. It's a delicate process. If the temperature is too low, then the molecules won't have the activation energy needed to begin reacting with one another. If the temperature is too high, then pyrolysis, the final breakdown of the amino acids and sugars, will begin. In other words - fire in the toaster.²

The Goldilocks Black Hole

Hawking Radiation, named after the late great astrophysicist Stephen Hawking who theorized its existence, refers to black-body radiation emitted by a black hole.³ Black body radiation is released in the form of thermal energy, hence there is a temperature associated with

¹ This place:

<https://perimeterinstitute.ca/outreach/students/programs/international-summer-school-young-physicists>

² https://en.wikipedia.org/wiki/Maillard_reaction

³ I'm still learning more about this, so I'll leave it at that for now.

each black hole, depending on how much radiation it releases. This, in turn, is dependent on the black hole's mass. At the Schwarzschild Radius, more commonly known as the event horizon, a black hole releases radiation whose temperature is given by the following formula, taken from Wikipedia, “

$$T = \frac{\hbar c^3}{8 \pi G k_B M} \approx 1.227 \times 10^{+23} \text{K} \cdot \text{kg} \times \frac{1}{M} = 6.169 \times 10^{-8} \text{K} \times \frac{M_{\odot}}{M}$$

where \hbar is the reduced Planck constant, c is the speed of light, k_B is the Boltzmann constant, G is the gravitational constant, M_{\odot} is the solar mass, and M is the mass of the black hole.”⁴

Given the solar mass, the mass of our sun, is 2×10^{30} kg, and we need the black hole to have a temperature of 300 degrees Fahrenheit, or about 420 K, a quick calculation yields that our black hole must have a mass of about 2.9×10^{20} kg, or about 1% of the mass of our moon. More tangibly, it's the mass of 5.8×10^{21} pop tarts.⁵

Breakfast in... Never

It seems like our black hole toaster is complete! Unfortunately, black holes are finicky things, and relativistic toasting has some relatively unique problems.⁶ As our favorite scientist whose name starts with E⁷ predicted, as we come closer to a source of gravity, time slows down. 1 minute may pass for us, but an observer standing farther away may see us standing there for two minutes! The technical term for this is gravitational time dilation⁸, and it causes us a serious problem. At the Schwarzschild radius of a black hole, time dilation is infinite. A Pop Tart will only take 2 minutes to toast, but from the perspective of an outsider, those two minutes will take forever - literally!

⁴ https://en.wikipedia.org/wiki/Hawking_radiation

⁵ Based on the average mass of the S'Mores Pop-Tarts (n=6) which I ate this week - about 50 grams.

⁶ ba-dum-tshh

⁷ And the runner-ups are... Edison, Euclid, Eratosthenes, and Euler? In no particular order.

⁸ https://en.wikipedia.org/wiki/Gravitational_time_dilation

In order to get the Pop Tart in any semblance of a reasonable amount of time, we need to step a little further back. The question is, how much further? Well, while we're solving this issue, let's also tackle another one. In order to keep the black hole at the event horizon, it would need to be travelling just under the speed of light, and kept at that speed for all 2 minutes of toasting. In order to stably rotate around the black hole without needing a force to keep it from being sucked past the point of no return, the closest that the Pop Tart can orbit is at the innermost stable circular orbit, which for non-spinning black holes can be found using the following formula, courtesy of Wikipedia, “

$$r_{\text{isco}} = \frac{6GM}{c^2} = 3R_S,$$

where R_S is the Schwarzschild radius of the massive object with mass M .⁹

Thus, by tripling our distance from the black hole, we ought to be able to stably orbit around it and also, hopefully, reduce time dilation enough to have breakfast ready on time.

Turn Up the Heat

However, as we move away from the black hole, the radiation per unit area will decrease. In order to keep the pop-tart sufficiently heated, we'll need to turn up the heat. Assuming that the radiation is spread evenly around the black hole, we can treat it as a form of flux. Let's take a sphere surrounding the black hole at radius r . The surface area of the sphere is $4\pi r^2$. Assuming that the black hole radiates a constant amount of energy E , the radiation flux intensity, or energy per unit area, is equal to $\frac{E}{4\pi r^2}$. Tripling the radius alters the intensity by a factor of $1/r^2$, or $1/9$. In order to keep the radiation flux intensity constant, the energy released by the black hole has to nontuple - increase by a factor of 9. As described earlier, the amount of radiation, or energy, is

⁹ https://en.wikipedia.org/wiki/Innermost_stable_circular_orbit

inversely proportional to the mass of the black hole. Thus, we need to make the black hole 9 times less massive, a mere 3.222E19 kg, which is around the total mass of the rings of Saturn.¹⁰

Suggested Bake Time

While it won't take forever, time dilation will still affect how long the pop-tart will need to bake for. From the pop-tart's perspective, it'll take 2 minutes just as always. However, from our outside perspective, it might seem a little longer. To find out exactly how much, we can use this formula, from Wikipedia, “

$$t_0 = t_f \sqrt{1 - \frac{2GM}{rc^2}} = t_f \sqrt{1 - \frac{r_s}{r}},$$

where t_0 is the time as the pop-tart experiences it, t_f is the time as we observe it at a large distance from the black hole, r_s is the Schwarzschild radius of the black hole, and r is the distance from the pop-tart to the black hole.¹¹

This formula shows why, as we near the Schwarzschild radius, the pop-tart will seem to bake forever. As the $1-r_s/r$ term approaches 0, t_f approaches infinity! Thankfully, we thought this through beforehand. The pop-tart has to bake for 2 minutes, or 120 seconds (t_0), and as we decided earlier, it's at a distance of 3 Schwarzschild radii from the black hole (r_s). A quick calculation yields that, at a distance, we need to wait 147 seconds, or about 2.5 minutes in order to bake the pop tart for 2 minutes.

Closing Remarks

Out in the wilderness of the universe, you may have to make do with whatever black hole you can find. However, by adjusting your distance to account for differing levels of Hawking

¹⁰ [https://en.wikipedia.org/wiki/Orders_of_magnitude_\(mass\)](https://en.wikipedia.org/wiki/Orders_of_magnitude_(mass))

¹¹ https://en.wikipedia.org/wiki/Gravitational_time_dilation

radiation, and cook time to deal with time dilation, you too can achieve the perfect black hole toasted Pop-Tart! Now, the only question is - what flavor?¹²

In all seriousness, I suppose this writing has little to no real world significance. However, I hope that through this writing, I can encourage you, reader, to explore more crazy ideas in your own life, and see where they take you! I've got several years of physics study before I really understand any of what I just talked about, but I don't think that means I shouldn't "waste time" playing with these formulas that I don't understand. Exploring these ideas gets me excited about science! It gets me excited about applying to camps like ISSYP, and excited about cracking open a textbook or watching a Khan Academy video or OCW lecture to better understand all the cool things that science allows us to do. In short, exploring the cool and somewhat crazy side of science empowers me to strive to learn more. If, through my writing, I've shared my excitement with you, then I consider writing this article time well-spent!

¹² Refer to https://www.poptarts.com/en_US/products.html