

# Problem Set 2 (SOLUTIONS)

This problem set will take you through some Stata commands to estimate simple regression equations with dummy variables. You will learn how to interpret the estimated coefficients and test some linear hypotheses. Interpretation of these coefficients will be useful when we do treatment evaluation models later in term 1.

The hypothesis tests discussed in this problem set include standard T-tests and F-tests, which is assumed undergraduate knowledge for this module.

You will need to download the dataset `problemset2.dta`, which is available on Moodle.

## Conditional Expectation Function

Consider the Conditional Expectation Function (CEF),  $E[Y_i|X_i]$ . If  $X$  takes on discrete values:  $X_i \in \{x_1, x_2, \dots, x_m\}$ , then

$$E[Y_i|X_i] = E[Y_i|X_i = x_1] \cdot \mathbf{1}\{X_i = x_1\} + \dots + E[Y_i|X_i = x_m] \cdot \mathbf{1}\{X_i = x_m\}$$

where  $\mathbf{1}\{X_i = x_m\}$  is a dummy variable,  $= 1$  when  $X_i = x_m$ . Since the values of  $X_i$  are mutually exclusive there is no overlap of these dummy variables.

Note, we do not need to assume that  $X$  is a single random variable. It can be a vector of random variables that takes on discrete values.

We can re-arrange this expression using anyone of the values of  $X$ . The natural option is to choose the first, but this is arbitrary.

$$\begin{aligned} E[Y_i|X_i] &= E[Y_i|X_i = x_1] + (E[Y_i|X_i = x_2] - E[Y_i|X_i = x_1]) \cdot \mathbf{1}\{X_i = x_2\} + \dots \\ &\quad + (E[Y_i|X_i = x_m] - E[Y_i|X_i = x_1]) \cdot \mathbf{1}\{X_i = x_m\} \end{aligned}$$

Since,  $E[Y_i|X_i = x_m]$  is a constant ( $X_i$  is set to a specific value), we can express the CEF as a function that is linear in parameters.

$$E[Y_i|X_i] = \beta_1 + \beta_2 D_{i2} + \dots + \beta_m D_{im}$$

where  $D_{im} = \mathbf{1}\{X_i = x_m\}$ .

## Preamble

<IPython.core.display.HTML object>

Create a do-file for this problem set and include a preamble that sets the directory and opens the data. For example,

```
clear
//or, to remove all stored values (including macros, matrices, scalars, etc.)
*clear all

* Replace $rootdir with the relevant path to on your local harddrive.
cd "$rootdir/problem-sets/ps-2"

cap log close
log using problem-set-2-log.txt, replace

use problem-set-2-data.dta
```

## Questions

1. Consider the  $E[\ln(Wage_i)|Gender_i]$ , where  $Gender_i \in \{1\text{"Male"}, 2\text{"Female"}\}$ . Show that this CEF implies a linear model,

$$\ln(Wage_i) = \beta_1 + \beta_2 D_{i2} + \varepsilon_i$$

What do the parameters  $\beta_1$  and  $\beta_2$  imply?

$$\begin{aligned} E[\ln(Wage_i)|G_i] &= E[\ln(Wage_i)|G_i = 1] + (E[\ln(Wage_i)|G_i = 2] - E[\ln(Wage_i)|G_i = 1]) \cdot \mathbf{1}\{G_i = 2\} \\ &= \beta_1 + \beta_2 D_{i2} \end{aligned}$$

$$\Rightarrow E[\varepsilon_i|D_{i2}] = 0.$$

2. Regress `lwage` (log wage) on just a set of binary indicators that will enable you to test the hypothesis that males and females are on average, paid the same wage, *ceteris paribus*. Test this hypothesis.

```
reg lwage female
```

```
* or
```

```
gen male=1-female
```

```
reg lwage male
```

Source	SS	df	MS	Number of obs	=	4,165
				F(1, 4163)	=	491.72
Model	93.6914807	1	93.6914807	Prob > F	=	0.0000
Residual	793.213421	4,163	.190538895	R-squared	=	0.1056
				Adj R-squared	=	0.1054
Total	886.904902	4,164	.212993492	Root MSE	=	.43651

lwage	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
female	-.4744661	.0213967	-22.17	0.000	-.516415	-.4325171
_cons	6.729774	.00718	937.29	0.000	6.715697	6.74385

Source	SS	df	MS	Number of obs	=	4,165
				F(1, 4163)	=	491.72
Model	93.6914807	1	93.6914807	Prob > F	=	0.0000
Residual	793.213421	4,163	.190538895	R-squared	=	0.1056
				Adj R-squared	=	0.1054
Total	886.904902	4,164	.212993492	Root MSE	=	.43651

lwage	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
male	.4744661	.0213967	22.17	0.000	.4325171	.516415
_cons	6.255308	.020156	310.34	0.000	6.215791	6.294824

Alternatively, you could use Stata's factor notation:

```
reg lwage i.female
```

```
//note: defaults to smallest value as base category. This can be changed as follows.
```

```
reg lwage ib1.female
```

Source	SS	df	MS	Number of obs	=	4,165
Model	93.6914807	1	93.6914807	F(1, 4163)	=	491.72
Residual	793.213421	4,163	.190538895	Prob > F	=	0.0000
				R-squared	=	0.1056
				Adj R-squared	=	0.1054
Total	886.904902	4,164	.212993492	Root MSE	=	.43651

lwage	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
1.female	-.4744661	.0213967	-22.17	0.000	-.516415	-.4325171
_cons	6.729774	.00718	937.29	0.000	6.715697	6.74385

Source	SS	df	MS	Number of obs	=	4,165
Model	93.6914807	1	93.6914807	F(1, 4163)	=	491.72
Residual	793.213421	4,163	.190538895	Prob > F	=	0.0000
				R-squared	=	0.1056
				Adj R-squared	=	0.1054
Total	886.904902	4,164	.212993492	Root MSE	=	.43651

lwage	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
0.female	.4744661	.0213967	22.17	0.000	.4325171	.516415
_cons	6.255308	.020156	310.34	0.000	6.215791	6.294824

It is evident from the test p-value that the difference is statistically significant. Note, the standard `reg` command assumes homoskedastic SEs. If we believe that the variance of (log of) wages varies with gender, we should estimate heteroskedastic SEs. However, in this instance it will not make a difference to the conclusion.

```
reg lwage female, r
```

Linear regression

Number of obs = 4,165

F(1, 4163)	=	520.64
Prob > F	=	0.0000
R-squared	=	0.1056
Root MSE	=	.43651

		Robust				
lwage	Coefficient	std. err.	t	P> t	[95% conf. interval]	
female	-.4744661	.0207939	-22.82	0.000	-.5152332	-.4336989
_cons	6.729774	.0072089	933.53	0.000	6.71564	6.743907

3. Extend the specification in (2) that will enable you to test the hypothesis that there is no difference in the wages between the following gender-ethnicity groups. Begin by defining the following dummy variables:

- $\text{female\_black} = \text{female} \times \text{black}$
- $\text{male\_black} = (1 - \text{female}) \times \text{black}$
- $\text{female\_nonblack} = \text{female} \times (1 - \text{black})$
- $\text{male\_nonblack} = (1 - \text{female}) \times (1 - \text{black})$

```
gen female_black=female*black
gen female_nonblack=female*(1-black)
gen male_black=(1-female)*black
gen male_nonblack=(1-female)*(1-black)
```

Then estimate the following regressions:

- lwage on female\_black, female\_nonblack, male\_black, male\_nonblack (without a constant: option nocons)
- lwage on female, black, female\_black
- lwage on female\_black, female\_nonblack, male\_black

For some of these exercises you may be able to use Stata's factor notation. However, in some instances you will need to manually create the above dummy-variable interactions.

In each case, identify the base category and write down the parameters of the (implied) model in terms of conditional expectations.

```
* (a)
reg lwage female_black female_nonblack male_black male_nonblack

// note, Stata has dropped one variable due to perfect collinearity

reg lwage female_black female_nonblack male_black male_nonblack, nocons
```

note: male\_black omitted because of collinearity.

Source		SS	df	MS	Number of obs	=	4,165
-----+-----					F(3, 4161)	=	191.17
Model		107.436063	3	35.8120209	Prob > F	=	0.0000
Residual		779.468839	4,161	.187327287	R-squared	=	0.1211
-----+-----					Adj R-squared	=	0.1205
Total		886.904902	4,164	.212993492	Root MSE	=	.43281

lwage		Coefficient	Std. err.	t	P> t	[95% conf. interval]	
-----+-----							
female_black		-.4434409	.0523433	-8.47	0.000	-.5460617	-.34082
female_non~k		-.2101553	.0383456	-5.48	0.000	-.2853332	-.1349774
male_black		0	(omitted)				
male_nonbl~k		.2239593	.0317691	7.05	0.000	.1616749	.2862436
_cons		6.517691	.0309152	210.82	0.000	6.457081	6.578301
-----+-----							

Source		SS	df	MS	Number of obs	=	4,165
-----+-----					F(4, 4161)	>	99999.00
Model		185756.485	4	46439.1213	Prob > F	=	0.0000
Residual		779.468839	4,161	.187327287	R-squared	=	0.9958
-----+-----					Adj R-squared	=	0.9958
Total		186535.954	4,165	44.7865436	Root MSE	=	.43281

lwage		Coefficient	Std. err.	t	P> t	[95% conf. interval]	
-----+-----							
female_black		6.07425	.0422382	143.81	0.000	5.991441	6.15706
female_non~k		6.307536	.0226856	278.04	0.000	6.26306	6.352012
male_black		6.517691	.0309152	210.82	0.000	6.457081	6.578301
male_nonbl~k		6.74165	.0073159	921.51	0.000	6.727307	6.755993
-----+-----							

This model returns four parameter-estimates, each corresponding to the four gender-ethnicity groups. These are essentially conditional mean estimates.

```
* (b)
reg lwage female black female_black

// or, using factor notation:

reg lwage i.female##i.black
```

Source	SS	df	MS	Number of obs	=	4,165
Model	107.436063	3	35.8120209	F(3, 4161)	=	191.17
Residual	779.468839	4,161	.187327287	Prob > F	=	0.0000
				R-squared	=	0.1211
				Adj R-squared	=	0.1205
Total	886.904902	4,164	.212993492	Root MSE	=	.43281

lwage	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
female	-.4341146	.0238361	-18.21	0.000	-.480846	-.3873832
black	-.2239593	.0317691	-7.05	0.000	-.2862436	-.1616749
female_black	-.0093263	.057515	-0.16	0.871	-.1220865	.1034339
_cons	6.74165	.0073159	921.51	0.000	6.727307	6.755993

Source	SS	df	MS	Number of obs	=	4,165
Model	107.436063	3	35.8120209	F(3, 4161)	=	191.17
Residual	779.468839	4,161	.187327287	Prob > F	=	0.0000
				R-squared	=	0.1211
				Adj R-squared	=	0.1205
Total	886.904902	4,164	.212993492	Root MSE	=	.43281

lwage	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
1.female	-.4341146	.0238361	-18.21	0.000	-.480846	-.3873832
1.black	-.2239593	.0317691	-7.05	0.000	-.2862436	-.1616749
female#black						
1 1	-.0093263	.057515	-0.16	0.871	-.1220865	.1034339

_cons		6.74165	.0073159	921.51	0.000	6.727307	6.755993

---

In this model, we have the following:

- $\beta_1 = E[\ln(Wage_i)|F = 0, B = 0]$
- $\beta_2 = E[\ln(Wage_i)|F = 1, B = 0] - E[\ln(Wage_i)|F = 0, B = 0]$
- $\beta_3 = E[\ln(Wage_i)|F = 0, B = 1] - E[\ln(Wage_i)|F = 0, B = 0]$
- $\beta_4 = (E[\ln(Wage_i)|F = 1, B = 1] - E[\ln(Wage_i)|F = 0, B = 1]) - (E[\ln(Wage_i)|F = 1, B = 0] - E[\ln(Wage_i)|F = 0, B = 0])$

```
* (c)
reg lwage female_black female_nonblack male_black

// note, this one is harder to replicate using factor notation.
```

Source		SS	df	MS	Number of obs	=	4,165
-----+					F(3, 4161)	=	191.17
Model		107.436063	3	35.8120209	Prob > F	=	0.0000
Residual		779.468839	4,161	.187327287	R-squared	=	0.1211
-----+					Adj R-squared	=	0.1205
Total		886.904902	4,164	.212993492	Root MSE	=	.43281
-----							
lwage		Coefficient	Std. err.	t	P> t	[95% conf. interval]	
-----+							
female_black		-.6674001	.0428671	-15.57	0.000	-.7514426	-.5833576
female_non~k		-.4341146	.0238361	-18.21	0.000	-.480846	-.3873832
male_black		-.2239593	.0317691	-7.05	0.000	-.2862436	-.1616749
_cons		6.74165	.0073159	921.51	0.000	6.727307	6.755993

In this model, we have the following:

- $\beta_1 = E[\ln(Wage_i)|F = 0, B = 0]$
- $\beta_2 = E[\ln(Wage_i)|F = 1, B = 1] - E[\ln(Wage_i)|F = 0, B = 0]$
- $\beta_3 = E[\ln(Wage_i)|F = 1, B = 0] - E[\ln(Wage_i)|F = 0, B = 0]$



- $\beta_4 = E[\ln(Wage_i)|F = 0, B = 1] - E[\ln(Wage_i)|F = 0, B = 0]$

4. Compare the estimated coefficients with the sample average values for the lwage for the four subgroups. What do you see?

```
table female black, stat(mean lwage)
```

-----				
		black		
		0	1	Total
-----				
female or male				
0		6.74165	6.517691	6.729774
1		6.307536	6.07425	6.255308
Total		6.700755	6.363002	6.676346
-----				

You can compute the coefficients from each model simply using these averages.

5. In each of the above models, describe the null hypothesis you would test to evaluate whether there is a significant earnings difference between the earnings of black and non-black females.

- $H_0 : \beta_1 = \beta_2$
- $H_0 : \beta_3 + \beta_4 = 0$
- $H_0 : \beta_2 - \beta_3 = 0$

6. Verify your solution to 4 by performing a test using the three set of regression output. You can use the post-estimation `test` command.

```
reg lwage female_black female_nonblack male_black male_nonblack, nocons

test female_black = female_nonblack

reg lwage female black female_black

test female_black + black = 0

reg lwage female_black female_nonblack male_black

test female_black - female_nonblack = 0
```

Source	SS	df	MS	Number of obs	=	4,165
Model	185756.485	4	46439.1213	F(4, 4161)	>	99999.00
Residual	779.468839	4,161	.187327287	Prob > F	=	0.0000
				R-squared	=	0.9958
				Adj R-squared	=	0.9958
Total	186535.954	4,165	44.7865436	Root MSE	=	.43281

lwage	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
female_black	6.07425	.0422382	143.81	0.000	5.991441	6.15706
female_non~k	6.307536	.0226856	278.04	0.000	6.26306	6.352012
male_black	6.517691	.0309152	210.82	0.000	6.457081	6.578301
male_nonbl~k	6.74165	.0073159	921.51	0.000	6.727307	6.755993

( 1) female\_black - female\_nonblack = 0

F( 1, 4161) = 23.68  
 Prob > F = 0.0000

Source	SS	df	MS	Number of obs	=	4,165
Model	107.436063	3	35.8120209	F(3, 4161)	=	191.17
Residual	779.468839	4,161	.187327287	Prob > F	=	0.0000
				R-squared	=	0.1211
				Adj R-squared	=	0.1205
Total	886.904902	4,164	.212993492	Root MSE	=	.43281

lwage	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
female	-.4341146	.0238361	-18.21	0.000	-.480846	-.3873832
black	-.2239593	.0317691	-7.05	0.000	-.2862436	-.1616749
female_black	-.0093263	.057515	-0.16	0.871	-.1220865	.1034339
_cons	6.74165	.0073159	921.51	0.000	6.727307	6.755993

( 1) black + female\_black = 0

F( 1, 4161) = 23.68  
 Prob > F = 0.0000

Source	SS	df	MS	Number of obs	=	4,165
				F(3, 4161)	=	191.17
Model	107.436063	3	35.8120209	Prob > F	=	0.0000
Residual	779.468839	4,161	.187327287	R-squared	=	0.1211
				Adj R-squared	=	0.1205
Total	886.904902	4,164	.212993492	Root MSE	=	.43281

lwage	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
female_black	-.6674001	.0428671	-15.57	0.000	-.7514426	-.5833576
female_non~k	-.4341146	.0238361	-18.21	0.000	-.480846	-.3873832
male_black	-.2239593	.0317691	-7.05	0.000	-.2862436	-.1616749
_cons	6.74165	.0073159	921.51	0.000	6.727307	6.755993

( 1) female\_black - female\_nonblack = 0

F( 1, 4161) = 23.68  
 Prob > F = 0.0000

7. In each case, test equality across all four gender-ethnicity groups. Again, you should get the same result.

```
reg lwage female_black female_nonblack male_black male_nonblack, nocons

test female_black = female_nonblack = male_black = male_nonblack

reg lwage female black female_black

test female_black = black = female = 0

reg lwage female_black female_nonblack male_black

test female_black = female_nonblack = male_black = 0
```

Source	SS	df	MS	Number of obs	=	4,165
				F(4, 4161)	>	99999.00
Model	185756.485	4	46439.1213	Prob > F	=	0.0000
Residual	779.468839	4,161	.187327287	R-squared	=	0.9958
				Adj R-squared	=	0.9958

Total | 186535.954      4,165    44.7865436    Root MSE      =      .43281

lwage	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
female_black	6.07425	.0422382	143.81	0.000	5.991441	6.15706
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male_black	6.517691	.0309152	210.82	0.000	6.457081	6.578301
male_nonbl~k	6.74165	.0073159	921.51	0.000	6.727307	6.755993

- ( 1) female\_black - female\_nonblack = 0  
 ( 2) female\_black - male\_black = 0  
 ( 3) female\_black - male\_nonblack = 0

F( 3, 4161) = 191.17  
 Prob > F = 0.0000

Source	SS	df	MS	Number of obs	=	4,165
Model	107.436063	3	35.8120209	F(3, 4161)	=	191.17
Residual	779.468839	4,161	.187327287	Prob > F	=	0.0000
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female_black	-.0093263	.057515	-0.16	0.871	-.1220865	.1034339
_cons	6.74165	.0073159	921.51	0.000	6.727307	6.755993

- ( 1) - black + female\_black = 0  
 ( 2) - female + female\_black = 0  
 ( 3) female\_black = 0

F( 3, 4161) = 191.17  
 Prob > F = 0.0000

Source	SS	df	MS	Number of obs	=	4,165
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Model		107.436063	3	35.8120209	Prob > F	=	0.0000
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male_black	-.2239593	.0317691	-7.05	0.000	-.2862436	-.1616749
_cons	6.74165	.0073159	921.51	0.000	6.727307	6.755993

( 1) female\_black - female\_nonblack = 0

( 2) female\_black - male\_black = 0

( 3) female\_black = 0

F( 3, 4161) = 191.17

Prob > F = 0.0000

8. Try to replicate the F-statistic for one of the above models. Hint, the F-stat for these models is the same as that of the whole model.

```
reg lwage female black female_black
ereturn list
scalar fstat = (e(r2)*e(df_r))/((1-e(r2))*e(df_m))
scalar list fstat
```

Source		SS	df	MS	Number of obs	=	4,165
-----+-----					F(3, 4161)	=	191.17
Model		107.436063	3	35.8120209	Prob > F	=	0.0000
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---

scalars:

```

e(N) = 4165
e(df_m) = 3
e(df_r) = 4161
e(F) = 191.1735422330181
e(r2) = .1211359442526956
e(rmse) = .4328132235793649
e(mss) = 107.4360627542734
e(rss) = 779.4688391479763
e(r2_a) = .1205023003768864
e(ll) = -2419.902951629166
e(ll_0) = -2688.805870567022
e(rank) = 4

```

macros:

```

e(cmdline) : "regress lwage female black female_black"
e(title) : "Linear regression"
e(marginsok) : "XB default"
e(vce) : "ols"
e(depvar) : "lwage"
e(cmd) : "regress"
e(properties) : "b V"
e(predict) : "regres_p"
e(model) : "ols"
e(estat_cmd) : "regress_estat"

```

matrices:

```

e(b) : 1 x 4
e(V) : 4 x 4
e(beta) : 1 x 3

```

functions:

```

e(sample)
fstat = 191.17354

```

In the case where the F-test corresponds to the test of the entire model, you can write the F-statistic in terms of  $R^2$ .

9. Estimate the following model:

$$lwage = \beta_1 + \beta_2 F + \beta_3 B + \beta_4 F \times B + \beta_5 exp + \beta_6 exp^2 + \beta_7 educ + \varepsilon$$

- Interpret the estimated coefficients  $\hat{\beta}_7$ .
- Interpret the effect of experience variable **exp**. Use the median level of experience to make your calculation.

```
sum educ, det
reg lwage i.female##i.black exper expsq educ

di (exp(_b[educ])-1)*100
```

years of education						
Percentiles		Smallest				
1%	6	4				
5%	8	4				
10%	9	4	Obs		4,165	
25%	12	4	Sum of wgt.		4,165	
50%	12		Mean		12.84538	
		Largest	Std. dev.		2.787995	
75%	16	17				
90%	17	17	Variance		7.772916	
95%	17	17	Skewness		-.2581161	
99%	17	17	Kurtosis		2.71273	
Source	SS	df	MS	Number of obs	=	4,165
				F(6, 4158)	=	411.81
Model	330.586	6	55.0976667	Prob > F	=	0.0000
Residual	556.318902	4,158	.13379483	R-squared	=	0.3727
				Adj R-squared	=	0.3718
Total	886.904902	4,164	.212993492	Root MSE	=	.36578
lwage	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
1.female	-.4032047	.0203208	-19.84	0.000	-.4430444	-.363365
1.black	-.1551546	.0269249	-5.76	0.000	-.2079417	-.1023674
female#black						

1 1		-.002071	.0488405	-0.04	0.966	-.0978246	.0936826
exper		.0427346	.0022404	19.07	0.000	.0383422	.047127
expsq		-.0006982	.0000494	-14.14	0.000	-.0007951	-.0006014
educ		.0731837	.0020983	34.88	0.000	.0690698	.0772975
_cons		5.303667	.0362462	146.32	0.000	5.232606	5.374729

7.5928119

- A one unit increase in years of educ is associated with an increase of 7.59% in expected wages, holding other regressors fixed.

```
sum exper, det
return list
di (exp(_b[exper]+2*r(p50)*_b[expsq])-1)*100
```

#### years of full-time work experience

Percentiles		Smallest		
1%	3	1		
5%	5	1		
10%	7	1	Obs	4,165
25%	11	1	Sum of wgt.	4,165
50%	18		Mean	19.85378
		Largest	Std. dev.	10.96637
75%	29	50		
90%	36	50	Variance	120.2613
95%	39	50	Skewness	.4000014
99%	44	51	Kurtosis	2.072064

scalars:

```

r(N) = 4165
r(sum_w) = 4165
r(mean) = 19.85378151260504
r(Var) = 120.2612759224727
r(sd) = 10.96637022548814
r(skewness) = .4000013893781186
r(kurtosis) = 2.072064120792274
r(sum) = 82691
r(min) = 1
```



```

r(max) = 51
r(p1) = 3
r(p5) = 5
r(p10) = 7
r(p25) = 11
r(p50) = 18
r(p75) = 29
r(p90) = 36
r(p95) = 39
r(p99) = 44

```

1.7754427

- A one unit increase in years of experience is associated with an increase of 1.78% in expected wages, holding other regressors fixed.

**10.** Theoretically, how would you test the following restrictions for the model below?

- $\beta_2 = \beta_3$
- $\beta_4 + \beta_5 = 1$
- $\beta_2 = \beta_3$  **and**  $\beta_4 + \beta_5 = 1$

$$Y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \varepsilon$$

- Restrictions 1:  $H_0 : \beta_2 = \beta_3 \Rightarrow \beta_2 - \beta_3 = 0$ . This can be written as a simple T-test (or F-tests),

$$\text{T-stat} = \frac{\hat{\beta}_2 - \hat{\beta}_3}{\sqrt{\widehat{\text{Var}}(\hat{\beta}_2 - \hat{\beta}_3)}}$$

where,

$$\text{Var}(\hat{\beta}_2 - \hat{\beta}_3) = \text{Var}(\hat{\beta}_2) + \text{Var}(\hat{\beta}_3) - 2 \cdot \text{Cov}(\hat{\beta}_2, \hat{\beta}_3)$$

Alternatively, rewrite the model, adding and subtracting  $\beta_3 X_2$  (or  $\beta_2 X_3$ ):

$$Y = \beta_1 + (\beta_2 - \beta_3)X_2 + \beta_3(X_2 + X_3) + \beta_4 X_4 + \beta_5 X_5 + \varepsilon$$

Then test the hypothesis that the coefficient on  $X_2$  is  $= 0$ .

- Restrictions 1:  $H_0 : \beta_4 + \beta_5 = 1$ . This can be written as a simple T-test,

$$\text{T-stat} = \frac{\hat{\beta}_4 + \hat{\beta}_5 - 1}{\sqrt{\widehat{Var}(\hat{\beta}_4 + \hat{\beta}_5)}}$$

where,

$$\text{Var}(\hat{\beta}_4 + \hat{\beta}_5) = \text{Var}(\hat{\beta}_4) + \text{Var}(\hat{\beta}_5) + 2 \cdot \text{Cov}(\hat{\beta}_4, \hat{\beta}_5)$$

Alternatively, rewrite the model, adding and subtracting  $\beta_5 X_4 - X_5$  (or  $\beta_2 X_3$ ):

$$Y - X_4 = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + (\beta_4 + \beta_5 - 1)X_4 + (\beta_5)(X_5 - X_4) + \varepsilon$$

Then test the hypothesis that the coefficient on  $X_4$  is  $= 0$ .

c. To test both of the linear restrictions simultaneously, we would use an F-test.

Step 1: estimate the unrestricted model

$$Y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \varepsilon$$

Store the  $SSR_U$

Step 2: estimate the restricted model

$$(Y - X_4) = \gamma_1 + \gamma_2(X_2 + X_3) + \gamma_5(X_5 - X_4) + \varepsilon$$

Store the  $SSR_R$ .

Step 3: Compute the F-statistic

$$\text{F-stat} = \frac{(SSR_R - SSR_U)/(df_R - df_U)}{SSR_U/df_U}$$

**Postamble**

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