Math 3100 **Spring 2019**

Exam 2
Study Guide and Practice Questions

1. For each of the following series state whether it converges absolutely, converges conditionally, or diverges. Justify your answers.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{2^n}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n}{\sqrt{n^2+1}}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n(2n+1)}{n!}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n}{n^5}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2 4^n}{(2n)!}$ (f) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(\log n)^2}{n}$

2. Prove that if a series converges absolutely, then it is convergent.

3. For what values of p do the following series converge? Justify your answer.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(\log n)^p}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\log n}{n^p}$

4. For which values of x do the following series converge?

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(2x)^n}{2n+1}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^n n}{2^n}$

5. (a) Find a closed form for the power series $\sum_{n=2}^{\infty} x^{2n}$ when $|x| < 1$.

   (b) Find a sequence $\{a_n\}$ so that $\sum_{n=0}^{\infty} a_n x^n = \dfrac{1}{4+x}$ for all $|x| < 4$.

6. Provide counterexamples to the following false statements:

   (a) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

   (b) If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} |b_n|$ converges.

   (c) If $\displaystyle\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$, then $\sum_{n=1}^{\infty} |a_n|$ diverges.

7. Prove that if $\{a_n\}$ is summable, then $\lim_{n\to\infty} a_n = 0$.

8. State and prove the ratio test.

9. Use the ε-δ definition of *continuity* at a point to prove that

$$f(x) = \frac{3+x}{1+x^2}$$

   is continuous at $x_0 = 1$.

10. Prove that if a function $f : \mathbb{R} \to \mathbb{R}$ is *continuous* at x_0, then $\lim_{n\to\infty} f(x_n) = f(x_0)$ for all sequences $\{x_n\}$ with $\lim_{n\to\infty} x_n = x_0$. Use this to show that

$$g(x) = \begin{cases} \cos(x^{-1}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

    is not continuous at $x_0 = 0$.