Math 4900/6900 Additional Problems 1

1. Let $f(x) = |x|$ on the interval $[-\pi, \pi]$.
   (i) Compute the Fourier series of f. (Making use of the fact that f is even should simplify your calculations)
   (ii) How do we know (from Math 4100) that this infinite series converges uniformly to something?

2. Let $P(x) = \sum_{n=-N}^{N} a_n e^{inx}$ be trigonometric polynomial. Compute the Fourier series of P and show that it equals $P(x)$.

3. The function $P_r(x)$, called the **Poisson kernel**, is defined for $x \in [-\pi, \pi]$ and $0 \leq r < 1$ by the absolutely and uniformly convergent series

$$P_r(x) = \sum_{n=-\infty}^{\infty} r^{|n|} e^{inx} = \frac{1 - r^2}{1 - 2r\cos x + r^2}.$$

   Show that the Fourier coefficients $\widehat{P_r}(n)$ equal $r^{|n|}$ for all $n \in \mathbb{Z}$.