Information

A graph is bipartite if its vertex set can be partitioned into two parts, such that there are no edges between vertices in the same part.

Question 8

Answer saved

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Which graphs are bipartite? (The names are explained in Question 1)

- [x] a. S_n for $n \geq 2$
- [ ] b. W_n for $n > 2$
- [x] c. C_n for even n
- [ ] d. K_n for even n
- [x] e. P_n for any $n \geq 2$
- [ ] f. I_n for $n \geq 2$ (the independent graph on n vertices, which has no edges)
- [x] g. $K_{n,m}$ for any $n \geq 2$, $m \geq 2$
- [x] h. P_2
- [ ] i. K_3

Question 9

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Recall that an undirected graph is connected if every pair of vertices is connected by a path.

Which claims are true?

- [ ] a. All bipartite graphs are connected
- [ ] b. Bipartite graphs of 3 or more vertices can never be connected (there must be at least two vertices in one part, which cannot share an edge by definition.)
- [x] c. We can determine if a bipartite graph is connected by running Depth-First Search from an arbitrary vertex.
- [x] d. We can determine if a bipartite graph is connected by running Union–Find: union all edge endpoints, then check that there is only one component.
- [ ] e. We can determine if a bipartite graph is connected by checking that every vertex in “left” part has at least one incident edge (which must necessarily go to the “right” part.)
- [ ] f. We can determine if a bipartite graph is connected by counting the edges. (It must have $(n/2)^2$ edges.)
- [x] g. We can determine if a bipartite graph is connected by running Breadth-First Search from an arbitrary vertex.

Question 10

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Let E be the number of edges and V the (total) number of vertices in a bipartite graph.
Let a and b denote the sizes of the two parts.

Select the true statements.

- [x] a. $a + b = V$
- [x] b. Possibly, $E \sim \frac{1}{4}V^2$
- [ ] c. Possibly, $E = \frac{1}{2}V^2$
- [x] d. Possibly, $V < E$
- [x] e. It is guaranteed that $E = O(V^2)$
- [ ] f. It is guaranteed that $V = O(E)$
- [ ] g. It is guaranteed that $V \sim E$
- [ ] h. It must be that $a = b$

Question **11**

Incomplete answer

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Theorem: Let G be a connected, undirected graph. Then G is bipartite if and only if it does not contain a odd cycle.

(An odd cycle is a cycle of odd length. We let $d(v, w)$ denote the distance from vertex v to vertex w.)

Proof.

($\Rightarrow$)

Assume G is bipartite.	Assume G contains no odd cycles.
Let w be a closed walk in G.	Consider any vertex v.
Let P and Q be shortest paths	connecting v to p and q, respectively.

($\Leftarrow$)

Define A = { w | d(v, w) is even }.	with p, q both in A.
Then travel from w to q along Q.	Similarly, define B = { w | d(v, w) is odd }.
with p, q both in B	

We will argue by contradiction.

It remains to show that (A, B) is a bipartition.	Assume for contradiction that {p, q} is an edge,
follow P from p to a vertex w from Q.	Note that |P| and |Q| have the same parity.
In particular, every cycle in G is even.	In particular, the edges of P, Q, and {p, q}
contain an odd cycle.	Thus, |P|+|Q|+1 is odd.
Thus, every closed walk in G has even length.	

(To see this odd cycle, [Let (L, R) be a bipartition of G.] [The walk w must alternate between L and R.] .)

The other case, [(Both are even, or both odd.)] is similar.

Please put an answer in each box.

Question 12

Answer saved

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Start with 4 vertices and no edges.
Now add 3 random edges, independently and at random.

(To be quite precise, the random process is this: Repeatedly pick an edge from the $\binom{4}{2}$ different pairs of distinct vertices.)

What is the chance that the resulting graph is bipartite?

- a. 1
- b. $\frac{1}{9}$
- c. 0
- d. $\frac{1}{4}$
- e. $\frac{2}{3}$
- f. $\frac{1}{2}$
- g. $\frac{1}{6}$
- h. $\frac{2}{7}$

Clear my choice