Peergrade assignment 1

September 11, 2022

Exercise 1

a.

$\frac{5}{3}x + \frac{2}{5} = 7 \iff$

$\frac{5}{3}x + \frac{2}{5} - \frac{2}{5} = 7 - \frac{2}{5} \iff$ (subtract $\frac{2}{5}$ in both sides)

$\frac{5}{3}x = 7 - \frac{2}{5} \iff$ (simplify)

$\frac{5}{3}x = 7 * (\frac{5}{5}) - \frac{2}{5} \iff$ (multiply 7 with $\frac{5}{5} = 1$)

$\frac{5}{3}x = \frac{35}{5} - \frac{2}{5} \iff$ (calculation)

$\frac{5}{3}x = \frac{33}{5} \iff$ (calculation)

$\frac{3}{5} * \frac{5}{3}x = \frac{3}{5} * \frac{33}{5} \iff$ (multiply both sides with $\frac{3}{5}$)

$x = \frac{3}{5} * \frac{33}{5} \iff$ (simplify)

$x = \frac{99}{25}$ (calculation)

b. Given that $x \neq 0$

$\frac{4}{1+x} + \frac{15}{4} = 5 \iff$

$\frac{4}{1+x} + \frac{15}{4} - \frac{15}{4} = 5 - \frac{15}{4} \iff$ (subtract $\frac{15}{4}$ on both sides)

$\frac{4}{1+x} = 5 - \frac{15}{4} \iff$ (simplify)

$\frac{4}{1+x} = 5 * \frac{4}{4} - \frac{15}{4} \iff$ (multiply 5 with $\frac{4}{4} = 1$)

$\frac{4}{1+x} = \frac{20}{4} - \frac{15}{4} \iff$ (calculation)

$\frac{4}{1+x} = \frac{5}{4} \iff$ (calculation)

$\frac{4}{5} * \frac{4}{1+x} = \frac{5}{4} * \frac{4}{5} \iff$ (multiply both sides with $\frac{4}{5}$)

$\frac{16}{(1+x)*5} = 1 \iff$ (calculation)

$(1 + x) * 5 = 16 * 1 \iff$ (cross-multiplying)

$(1 + x) = \frac{16}{5} \iff$ (cross-multiplying)

$1 + x - 1 = \frac{16}{5} - 1 \iff$ (subtract -1 from both sides)

$x = \frac{16}{5} - 1 \iff$ (simplify)

$x = \frac{16}{5} - \frac{5}{5} \iff$ (calculation)

$x = \frac{11}{5}$ (calculation)

c.

$\frac{|4x|}{5} = (\frac{1}{2} - \frac{1}{3})^{-1} \iff$

$\frac{|4x|}{5} = (\frac{1}{2} * \frac{3}{3} - \frac{1}{3} * \frac{2}{2})^{-1} \iff$ (multiply $\frac{1}{2}$ with $\frac{3}{3} = 1$ and $\frac{1}{3}$ with $\frac{2}{2} = 1$)

$\frac{|4x|}{5} = (\frac{3}{6} - \frac{2}{6})^{-1} \iff$ (calculation)

$\frac{|4x|}{5} = (\frac{1}{6})^{-1} \iff$ (calculation)

$\frac{|4x|}{5} = 6 \iff$ (from Multiplicative inverse rule)

$|4x| = 6 * 5 \iff$ (cross-multiplying)

$|4x| = 30 \iff$ (calculation)

$|x| = \frac{30}{4}$ (calculation)

d.

$\frac{x-3}{3} = 2 * (\frac{2+x}{5} + 1) \iff$

$\frac{x-3}{3} = 2 * (\frac{2+x}{5}) + 2 \iff$ (commutativity, calculation)

$\frac{x-3}{3} - 2 * (\frac{2+x}{5}) = +2 * (\frac{2+x}{5}) - 2 * (\frac{2+x}{5}) + 2 \iff$ (subtract $-2 * (\frac{2+x}{5})$ on both sides)

$\frac{x-3}{3} - 2 * (\frac{2+x}{5}) = 2 \iff$ (simplify)

$\frac{x-3}{3} - \frac{4+2x}{5} = 2 \iff$ (commutativity,calculation)

$\frac{(x-3)*5}{3*5} - \frac{(4+2x)*3}{5*3} = 2 \iff$ (multiply $\frac{(x-3)}{3}$ with $\frac{5}{5} = 1$ and $\frac{(4+2x)}{5}$ with $\frac{3}{3} = 1$)

$\frac{5x-15}{15} - \frac{12+6x}{15} = 2 \iff$ (calculation)

$5x - 15 - 12 - 6x = 2 * 15 \iff$ (cross-multiplying, calculation)

$x = -57$ (calculation)

Exercise 2

a.

p	q	$\neg p$	p$\rightarrow q$	$\neg p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Thus $p \rightarrow q \equiv \neg p \vee q$

b.

p	q	$\neg q$	p $\wedge q$	p$\rightarrow \neg q$	$\neg(p \wedge q)$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	F	T	T

Thus $p \rightarrow \neg q \equiv \neg(p \wedge q)$

c.

p	q	p$\rightarrow q$	q $\rightarrow p$	$\neg(q \rightarrow p)$	(p$\rightarrow q) \rightarrow (\neg(q \rightarrow p))$
T	T	T	T	F	F
T	F	F	T	F	T
F	T	T	F	T	T
F	F	T	T	F	F

$\neg((p \rightarrow \neg q) \rightarrow (\neg(q \rightarrow p)))$	p $\longleftrightarrow q$
T	T
F	F
F	F
T	T

Thus $p \longleftrightarrow q \equiv ((p \rightarrow \neg q) \rightarrow (\neg(q \rightarrow p)))$

Exercise 3

a.

$$
\begin{aligned}
&(p \to r) \wedge (q \to r) \\
\overset{(12)}{\equiv} &(\neg p \vee r) \wedge (\neg q \vee r) \\
\overset{(1)}{\equiv} &(r \vee \neg p) \wedge (r \vee \neg q) \\
\overset{(3)}{\equiv} &r \vee (\neg p \wedge \neg q) \\
\overset{(9)}{\equiv} &r \vee \neg(p \vee q) \\
\overset{(1)}{\equiv} &\neg(p \vee q) \vee r \\
\overset{(12)}{\equiv} &(p \vee q) \to r
\end{aligned}
$$

b.

$$
\begin{aligned}
&(p \wedge \neg q) \to r \\
\overset{(12)}{\equiv} &\neg(p \wedge \neg q) \vee r \\
\overset{(1)}{\equiv} &r \vee \neg(p \wedge \neg q) \\
\overset{(9)}{\equiv} &r \vee (\neg p \vee \neg(\neg q)) \\
\overset{(6)}{\equiv} &r \vee (\neg p \vee q) \\
\overset{(2)}{\equiv} &q \vee (\neg p \vee r) \\
\overset{(6)}{\equiv} &q \vee (\neg p \vee \neg(\neg r)) \\
\overset{(9)}{\equiv} &q \vee \neg(p \wedge \neg r) \\
\overset{(1)}{\equiv} &\neg(p \wedge \neg r) \vee q \\
\overset{(12)}{\equiv} &(p \wedge \neg r) \to q
\end{aligned}
$$

c.

$$
\begin{aligned}
&(p \wedge q) \to r \\
\overset{(12)}{\equiv} &\neg(p \wedge q) \vee r \\
\overset{(1)}{\equiv} &r \vee \neg(p \wedge q) \\
\overset{(9)}{\equiv} &r \vee (\neg p \vee \neg q) \\
\overset{(2)}{\equiv} &\neg p \vee (r \vee \neg q) \\
\overset{(1)}{\equiv} &\neg p \vee (\neg q \vee r) \\
\overset{(12)}{\equiv} &p \to (\neg q \vee r) \\
\overset{(12)}{\equiv} &p \to (q \to r)
\end{aligned}
$$

d.

$$
\begin{aligned}
&\neg((p \to q) \land \neg q) \lor \neg p \\
&\overset{(12)}{\equiv} (\neg((\neg p \lor q) \land \neg q)) \lor \neg p \\
&\overset{(9)}{\equiv} \neg(\neg p \lor q) \lor \neg(\neg q) \lor \neg p \\
&\overset{(6)}{\equiv} (\neg(\neg p \lor q) \lor q) \lor \neg p \\
&\overset{(9)}{\equiv} (\neg(\neg p) \land \neg q) \lor q) \lor \neg p \\
&\overset{(6)}{\equiv} (p \land \neg q) \lor q) \lor \neg p \\
&\overset{(1)}{\equiv} (q \lor (p \land \neg q)) \lor \neg p \\
&\overset{(3)}{\equiv} ((q \lor p) \land (q \lor \neg q)) \lor \neg p \\
&\overset{(5)}{\equiv} ((q \lor p) \land t) \lor \neg p \\
&\overset{(4)}{\equiv} (q \lor p) \lor \neg p \\
&\overset{(2)}{\equiv} q \lor (p \lor \neg p) \\
&\overset{(5)}{\equiv} q \lor t \\
&\overset{(5)}{\equiv} t
\end{aligned}
$$