Discrete Mathematics
Peergrade assignment 4

1. Let $A = \{1, 2, 3, 4, 5, 6\}$ and let R be a relation on A defined as follows:

$$m\,R\,n \quad \Leftrightarrow \quad 2 \mid (m+n) \qquad \text{for all } m, n \in A$$

   (a) Draw the directed graph of R.
   (b) Find the equivalence class $[5]_R$.
   (c) How many distinct equivalence classes does the relation R have?

2. (a) Let S be a set of strings over the set $\{a, b\}$ defined recursively as follows:

   I. BASE: $b \in S$.

   II. RECURSION: If $s \in S$, then

   II(a) $sb \in S$

   II(b) $asa \in S$

   III. RESTRICTION: Nothing is in S other than objects defined in I and II above.

   List three strings that are in S and three strings that are not in S.

   (b) Let the sequence $a_0, a_1, a_2, \dots$ be recursively defined as follows:

$$a_n = \begin{cases} 0 & \text{if } n = 0 \\ a_{n-1} + 2n + 3 & \text{if } n > 0 \end{cases}$$

   Using mathematical induction, prove that this sequence satisfies the following equation:

$$a_n = n(n+4) \qquad \text{for all integers } n \geq 0.$$

3. Let R be a relation on $\mathbb{Z}$ defined as follows:

$$m\,R\,n \quad \Leftrightarrow \quad m^2 - n^2 \geq 0 \qquad \text{for all } m, n \in \mathbb{Z}$$

   (a) Is R reflexive?

(b) Is R transitive?

(c) Is R symmetric?

(d) Is R antisymmetric?

Justify each of your answers by providing either a proof or a counterexample. You may use the following property of integers:
For all $a, b, c \in \mathbb{Z}$ if $a \geq b$ and $b \geq c$ then $a \geq c$.