Peergrade Assignment 2

· Question 1.

In order to define $B = \{x \in \mathbb{N} \mid (x \mid 28)\}$.

since $x \in \mathbb{N}$ such as x divides 28

by definition of $\mathbb{N} = \{0,1,2,3,4,...\}$ and $U = \mathbb{N}$ for B

and Because of Theorem 4.4.1 for positive integers

"For all integers a and b; if a and b are positive integers and a divides b, then $a \leq b$", then $x \leq 28$

Given the above we can define B as

$B = \{1,2,4,7,14,28\}$

(a) $(A \cup C) \cap B$

(i) $A \cup C$

$= \{2,4\} \cup \{1,3,5\}$

$= \{1,2,3,4,5\}$

(ii) $(A \cup C) \cap B$

$= \{1,2,3,4,5\} \cap \{1,2,4,7,14,28\}$

$= \{1,2,4\}$ Thus $(A \cup C) \cap B = \{1,2,4\}$

(b) $A \cup (C - B)$

(i) $(C - B) = \{1,3,5\} - \{1,2,4,7,14,28\}$

$= \{3,5\}$

(ii) $A \cup (C - B) = \{2,4\} \cup \{3,5\}$

$= \{2,3,4,5\}$ Thus $A \cup (C - B) = \{2,3,4,5\}$

(c) $C \times A$

$C \times A = \{1,3,5\} \times \{2,4\}$

$= \{(1,2),(1,4),(3,2),(3,4),(5,2),(5,4)\}$

Thus $C \times A = \{(1,2),(1,4),(3,2),(3,4),(5,2),(5,4)\}$

(a) $P(A)$

$$P(A) = P(\{2,4\})$$
$$= \{\emptyset, \{2\}, \{4\}, \{2,4\}\}$$

Thus $P(A) = \{\emptyset, \{2\}, \{4\}, \{2,4\}\}$

(e) Since $B = \{x \in \mathbb{N} \mid x \in (x \mid 128)\}$

then $B^c = \{x \in \mathbb{N} \mid x \in (x \nmid 128)\}$

thus $B^c = \{0, 3, 5, 6, 8, 9, 10, 11, 12, 13, 15, \ldots, 27, 29, \ldots\}$

$$B^c \cap C = \{0, 3, 5, 6, 8, 9, 10, 11, 12, 13, 15, \ldots, 27, 29, \ldots\} \cap \{1, 3, 5\}$$

$$B^c \cap C = \{3, 5\}$$

· Question 2

gcd (6256, 2346)

(i) Divide 2346 by 6256

$$\begin{array}{r} 2 \leftarrow \text{quotient} \\ 2346 \overline{)6256} \\ \underline{4692} \\ 1564 \leftarrow \text{remainder} \end{array}$$

Thus $6256 = 2 \cdot 4692 + 1564$ and hence $\gcd(6256, 2346) = \gcd(2346, 1564)$ by Lemma 4.10.2

(ii) Divide 1564 by 2346

$$\begin{array}{r} 1 \leftarrow \text{quotient} \\ 1564 \overline{)2346} \\ \underline{1564} \\ 732 \leftarrow \text{remainder} \end{array}$$

Thus $2346 = 1 \cdot 1564 + 732$ and hence $\gcd(2346, 1564) = \gcd(1564, 732)$ by Lemma 4.10.2

(iii) Divide 732 by 1564

$$\begin{array}{r} 2 \leftarrow \text{quotient} \\ 732 \overline{)1564} \\ \underline{1564} \\ 0 \leftarrow \text{remainder} \end{array}$$

Thus $1564 = 2 \cdot 732 + 0$, hence $\gcd(1564, 732) = \gcd(732, 0)$ by Lemma 4.10.2

Putting all equations together.

$$\begin{aligned} \gcd(6256, 2346) &= \gcd(2346, 1564) \\ &= \gcd(1564, 732) \\ &= \gcd(732, 0) \\ &= 732 \end{aligned}$$

Therefore $\gcd(6256, 2346) = 732$

· Question 3

(a) As a is odd, by given facts, there is an integer k such that $a = 2k+1$
As b is odd, by given facts, there is an integer l such that $b = 2l+1$

Then by replacing $a+b$

$a+b = (2k+1) + (2l+1)$

$a+b = 2k+2l+2$

$a+b = 2(k+l+1)$

Let $m = k+l+1$. Then m is an integer as $k, l, 1$ are integers and sum and products of integers are integers.

Thus, by subsituting

$a+b = 2 \cdot m$, m being an integer, so $a+b$ is even by definition

(b) This question is answered by the Method of Proof by Contradiction

so, the contrapositive is that:

Assuming a, b, c are integers. if $a|b$ then there is $k \in \mathbb{Z}$ that $b = a \cdot k$

By the statement 3(b) if $a \nmid bc$

then by substituting $b = a \cdot k$

then $a \nmid a \cdot k \cdot c$ which is false

· Let $m = k \cdot c$, which is an integer as a product of integers k, c

By definition of divisibility a can divide $a \cdot m$, if m is an integer

So, $a \nmid a(k \cdot c)$ is false

Thus $a \nmid b$ by contradiction