Peergrade Assignment 3

- Question 1

(a) As $f: \mathbb{Z} \to \mathbb{Z}$ $\forall$ int n and 0, 1, 2 are int

$f(0) = 2 \cdot 0 - 5 = -5$

$f(1) = 2 \cdot 1 - 5 = -3$

$f(2) = 2 \cdot 2 - 5 = 4 - 5 = -1$

(b) In order to be one-to-one

if $f(n_1) = f(n_2)$ then $n_1 = n_2$

So by substituting $f(n) = 2n - 5$ with n_1, n_2

$2 \cdot n_1 - 5 = 2n_2 - 5$

$2n_1 - 5 + 5 = 2n_2 - 5 + 5$ (add +5 in both sides)

$2n_1 = 2n_2$ (calculation)

$\frac{2n_1}{2} = \frac{2n_2}{2}$ (divide both sides by 2)

$n_1 = n_2$

Thus, f is one-to-one

(c) f is onto if and only if for all element y in Y, there is an element x in X such that $f(x) = y$

Consider $-4 \in \mathbb{Z}$

We claim that $-4 \neq f(n)$, for any integer n, because if there were an integer n such that $-4 = f(n)$, then by definition of f

$-4 = 2n - 5$

$-4 + 5 = 2n - 5 + 5$ (add +5 in both sides)

$1 = 2n$ (calculation)

$\frac{1}{2} = \frac{2n}{2}$ (divide both sides by 2)

$\frac{1}{2} = n$, which is not an integer

Thus, f is NOT onto

(d) By substituting where n with $2n-5$

$f \circ f = 2(2n-5)-5$

$= 4n - 10 - 5$

$= 4n - 15$

· Question 2

(a) $\sum_{k=m}^{n} a_k - 3 \sum_{k=m}^{n} b_k$

$= \sum_{k=m}^{n} (2k-5) - 3 \cdot \sum_{k=m}^{n} (2-k)$ · (by substitution)

$= \sum_{k=m}^{n} (2k-5) - 3 \sum_{k=m}^{n} (2-k)$ (by alg. calculation)

$= \sum_{k=m}^{n} (2k-5) - \sum_{k=m}^{n} 3 \cdot (2-k)$ (by Theorem 5.11 (2))

$= \sum_{k=m}^{n} (2k-5) - \sum_{k=m}^{n} (6-3k)$ (by alg calculation)

$= \sum_{k=m}^{n} (2k-5) - (6-3k)$ (by Theorem 5.11 (1))

$= \sum_{k=m}^{n} 2k - 5 - 6 + 3k$ (remove parenthesis)

$= \sum_{k=m}^{n} 5k - 11$ (by algebraic calculation)

(b) $\prod_{k=m}^{n} a_k \cdot \prod_{k=m}^{n} b_k$

$= \prod_{k=m}^{n} (2k-5) \cdot \prod_{k=m}^{n} (2-k)$ (by substitution)

$= \prod_{k=m}^{n} (2k-5)(2-k)$ (by Theorem 5.11 (3))

$= \prod_{k=m}^{n} (4k-2k^2-10+5k)$ (by algebraic simplification)

$= \prod_{k=m}^{n} (-2k^2+9k-10)$ (by algebraic simplification)

• Question 3

$P(n) = 3 \mid n^3+5n-6$ $\forall$ integer $n \geq 0$

[Basic Step P(0)]

$P(0) = 0^3+5\cdot 0-6$ and $-\frac{6}{3} = -2$ so P(0) is TRUE

[Inductive Step]

Suppose k integer and ≥ 0. If P(k) is TRUE, then P(k+1) should be TRUE

$P(k) = k^3+5k-6$ suppose it is TRUE

Since 3 divides k^3+5k-6, it follows that

$k^3+5k-6 = 3\cdot q$ for some integer q.

$P(k+1) = (k+1)^3+5(k+1)-6$

$= k^3+3k^2+3k+1+5k+5-6$ (by using the binomial exp.s and alg. simplification

$= k^3+3k^2+8k$ (alg calculation)

$= k^3+5k-6+3k^2+6$

$= 3\cdot q + (3k^2+3k+6)$

We brake this up to 2 caes, to consider odd and even values of k

[Case 1]

k is an even integer, so $k = 2 \cdot a$ for some int a. Thus:

$P(k+1) = 3 \cdot q + 3k^2 + 3k + 6$

$= 3 \cdot q + 3(2 \cdot a)^2 + 3(2a) + 6$

$= 3 \cdot q + 3(4a^2) + 6a + 6$

$= 3q + 12a^2 + 6a + 6$

$= 3(q + 4a^2 + 2a + 2)$

Let $m = q + 4a^2 + 2$ is an integer as a product from integers, thus $P(k+1) = 3 \cdot m$, meaning that is divisible by 3

[Case 2]

k is odd, so $k = 2b + 1$ for some int b. Thus:

$P(k+1) = 3q + 3k^2 + 3k + 6$

$= 3 \cdot q + 3(2b+1)^2 + 3(2b+1) + 6$

$= 3 \cdot q + 3(4b^2 + 4b + 1) + 6b + 3 + 6$

$= 3 \cdot q + 12b^2 + 12b + 3 + 6b + 3 + 6$

$= 3(q + 4b^2 + 4b + 1 + 2b + 3)$

$= 3(q + 4b^2 + 6b + 3)$

Let $z = q + 4b^2 + 3b + 3$ is an integer as a product of integers thus $P(k+1) = 3 \cdot z$ meaning that is divisable by 3.