Peergrade Assignment 5

- Question 1

$D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$C = \{\text{yellow}, \text{red}, \text{blue}, \text{green}\}$

(a) We want 3 elements of the same colour out of 4 available colours

According to Generalised Pigeon Principle

Let X (number of draws) be finite of size n and C finite of size m

Let $f: X \to C$ be a function

If $k \cdot m < n$ then there is $a \in C$ that a is the image of at least $k+1$ elements of X

- We want $k+1 = 3$ draws of same colour, hence $k = 2$

We look for the smallest n such that

$2 \cdot 4 < n$

The smallest n is 9

Thus we have to draw 9 times

(b) Since we have 10 digits and 4 colours there are 40 denominations, assuming the drawings are elements of a set X and distinct denominations are elements of a set Y, there have to be more elements in X than in Y, so $f: X \to Y$ cannot be one-to-one (theorem 9.4.1)

According to Generalised pigeon principles Pigeon Principle

Let X (number of draws) be finite of size n and Y (distinct denominations) finite size of m, so $f: X \to C$ be a function

- We want $k+1 = 2$, Thus $k = 1$ and the smallest n such that

$1 \cdot 40 < n$, so $n = 41$

We have to draw 41 times

Question 2

Student Council of 21 members

11 women 10 men

Comittee of 8 members with preisely 4 women

(a) Groups of 8 with precisely 4 women will have precisely four men

$$\left[\begin{matrix}\text{4 out}\\\text{of}\\\text{11 women}\end{matrix}\right]\binom{11}{4}\times\left[\begin{matrix}\text{4 out}\\\text{of}\\\text{10 men}\end{matrix}\right]\binom{10}{4} = \frac{11!}{4!(11-4)!}\cdot\frac{10!}{4!(10-4)!} = \frac{11!}{4!\,7!}\cdot\frac{10!}{4!\,6!}$$

$$= \frac{11\times10\times9\times8}{4\times3\times2\times1}\cdot\frac{10\times9\times8\times7}{4\times3\times2\times1} = \frac{7920}{24}\times\frac{5040}{24}$$

$$= 330\times210 = 69.300 \text{ ways}$$

(b) Let A = male student 1 and B = male student 2 (That they dislike each other)

$$\left[\begin{matrix}\text{4 out}\\\text{of}\\\text{11 women}\end{matrix}\right]\binom{11}{4}\times\left(\left[\begin{matrix}\text{number of}\\\text{comittes with}\\\text{A and 2 others}\\\text{(none of them B)}\end{matrix}\right]\binom{8}{3} + \left[\begin{matrix}\text{number of}\\\text{comittes with}\\\text{B and 2 other}\\\text{(none of them A)}\end{matrix}\right]\binom{8}{3} + \left[\begin{matrix}\text{number of}\\\text{comittes with}\\\text{neither}\\\text{A or B}\end{matrix}\right]\binom{8}{4}\right)$$

$$= \frac{11!}{4!(11-4)!}\times\left(\frac{8!}{3!(8-3)!}+\frac{8!}{3!(8-3)!}+\frac{8!}{4!(8-4)!}\right) = \frac{11!}{4!\,7!}\times\left(\frac{8!}{3!\,5!}+\frac{8!}{3!\,5!}+\frac{8!}{4!\,4!}\right)$$

$$= \frac{11\times10\times9\times8}{4\times3\times2\times1}\times\left(\frac{8\times7\times6}{3\times2\times1}+\frac{8\times7\times6}{3\times2\times1}+\frac{8\times7\times6\times5}{4\times3\times2\times1}\right) = \frac{7920}{24}\times\left(\frac{336}{6}+\frac{336}{6}+\frac{1680}{24}\right)$$

$$= 330\times(56+56+70) = 330\times182$$

$$= 60060 \text{ ways}$$

(c) Let A be female student 1 and B = female student 2

that either they are together in a committee or they don't

$$\begin{bmatrix}\text{4 out of}\\ \text{10 men}\end{bmatrix}\binom{10}{4}\times\left(\begin{bmatrix}\text{numbers of}\\ \text{committees}\\ \text{with neither}\\ \text{A and B}\end{bmatrix}\binom{9}{2}+\begin{bmatrix}\text{number of}\\ \text{committees}\\ \text{with}\\ \text{A and B}\end{bmatrix}\binom{9}{4}\right)$$

$$= \frac{10!}{4!(10-4)!}\times\left(\frac{9!}{2!(9-2)!}+\frac{9!}{4!(9-4)!}\right)=\frac{10!}{4!\,6!}\times\left(\frac{9!}{2!\,7!}+\frac{9!}{4!\,5!}\right)$$

$$= \frac{10\times9\times8\times7}{4\times3\times2\times1}\times\left(\frac{9\times8}{2\times1}+\frac{9\times8\times7\times6}{4\times3\times2\times1}\right)=\frac{5040}{24}\times\left(\frac{72}{2}+\frac{3024}{24}\right)$$

$$= 210\times(36+126) = 210\times162$$

$= 34.020$ ways

(d) $$\begin{bmatrix}\text{4 out of}\\ \text{10 men}\end{bmatrix}\binom{10}{4}\times\left(\begin{bmatrix}\text{3 women}\\ \text{that}\\ \text{study}\\ \text{MATH}\end{bmatrix}\binom{8}{3}\times\begin{bmatrix}\text{1 woman}\\ \text{from}\\ \text{COMP.SCIENCE}\end{bmatrix}\binom{3}{1}+\begin{bmatrix}\text{2 women}\\ \text{that}\\ \text{study}\\ \text{MATH}\end{bmatrix}\binom{8}{2}\times\begin{bmatrix}\text{2 women}\\ \text{from}\\ \text{COMP.}\\ \text{SCIENCE}\end{bmatrix}\binom{3}{2}\right.$$

$$\left.+\begin{bmatrix}\text{1 woman}\\ \text{that}\\ \text{study}\\ \text{MATH}\end{bmatrix}\binom{8}{1}\times\begin{bmatrix}\text{3 women}\\ \text{from}\\ \text{COMP.}\\ \text{SCIENCE}\end{bmatrix}\binom{3}{3}\right)$$

$$= 210^{*}\times\left(\frac{8!}{3!(8-3)!}\times3+\frac{8!}{2!(8-2)!}\cdot\frac{3!}{2!\times1!}+8\times1\right)$$

$$= 210\times\left(\frac{8!}{3!5!}\times3+\frac{8!}{2!6!}\times3+8\right)=210\times\left(\frac{8\times7\times6}{3\times2\times1}\times3+\frac{8\times7}{2\times1}\times3+8\right)$$

$$= 210\times\left(\frac{336}{6}\times3+\frac{56}{2}\times3+8\right)$$

$$= 210\times(56\times3+28\times3+8) = 210\times(168+84+8) = 210\times260$$

$= 54.600$ ways

* from previous question

• Question 3

Let P be the event that the person has been tested positive for the disease, A the event that the person actually has the disease and B the event that the person does not have the disease. Then:

$P(P|A) = 0,95$ $\quad$ $P(P^c|A) = 0,05$

$P(P|B) = 0,03$ $\quad$ $P(P^c|B) = 0,97$

Also, because 4% of the population have been infected

$P(A) = 0,04$ and $P(B) = 0,96$.

(a) By Baye's theorem:

$$P(A|P) = \frac{P(P|A)\,P(A)}{P(P|A)\,P(A) + P(P|B)\,P(B)}$$

$$= \frac{0,95 \times 0,04}{0,95\times 0,04 + 0,03\times 0,96} = \frac{0,0388}{0,0388 + 0,0288} = \frac{0,0388}{0,0677}$$

$$\cong 0,57311 \cong 57,311\%$$

Thus the probability that a person with a positive test result actually has the disease is approximately 57,311%

(b) By Baye's Theorem:

$$P(B|P^c) = \frac{P(P^c|B)\,P(B)}{P(P^c|B)P(B) \times P(P^c|A)P(A)}$$

$$= \frac{0,97 \times 0,96}{0,97\times 0,96 + 0,05\times 0,04} = \frac{0,9312}{0,9312 + 0,002} = \frac{0,9312}{0,9332}$$

$$\simeq 0,99785 \simeq 99,785\%$$

Thus the probability that a person with a negative does not have the disease is approximately 99,785%