

# Discrete Mathematics

## Peergrade assignment 5

1. Let  $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$  be a set of digits and  $C = \{yellow, red, blue, green\}$  be a set of available colours.

Answer the following questions and justify your answer using the (generalized) pigeonhole principle.

- (a) You draw elements from the set  $D \times C$  at random. What is the minimum number of draws before you are sure to have at least three elements with the same colour?
  - (b) You draw elements from the set  $D \times C$  at random. What is the minimum number of draws before you are sure to have the same digit in the same colour at least twice?
2. A student council of a Mathematics and Computer Science Department consists of 21 members, 11 of which are women. A new committee of 8 members is to be selected, with precisely 4 women.
- (a) How many different ways are there to select a committee?
  - (b) Two of the male students in the council strongly dislike each other, and should not be in on the committee together. How many different ways are there to select a committee with that restriction?
  - (c) Two of the female students work very well as a team within a team, and declare that they're either both on this committee, or neither of them is. How many different ways are there to select a committee with that restriction? (ignoring the restriction in (b))
  - (d) Of the 11 women, 8 study maths and 3 computer science. How many different ways are there to select a committee, so that at least one woman from CS is included? (ignoring restrictions in (b) and (c))
3. A novel infectious disease screening test is ordered to be used on the population of a city, which currently has 4% of the population infected. Suppose that the false positive rate is 3% and the false negative rate is 5%. Thus, a person who has the disease, tests positive 95% of the

time, and a person who does not have it, tests negative 97% of the time.

- (a) What is the probability that a randomly chosen person who tests positive is actually infected?
- (b) What is the probability that a randomly chosen person who tests negative is actually not infected?