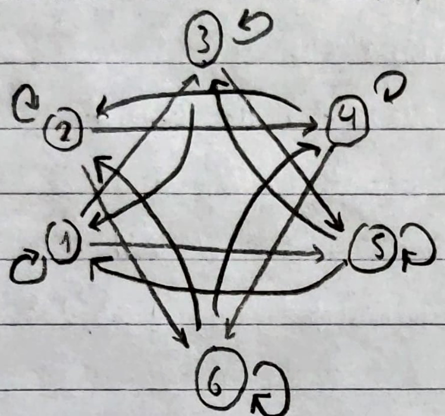
Peergrade Assignment 4

· Question 1.

$A = \{1, 2, 3, 4, 5, 6\}$

$mRn \Leftrightarrow 2 \mid (m+n)$ for all $m,n \in A$

(a) Given the above $R: \{(1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6)\}$



(b) $[5]_R = \{5, 3, 1\}$

(c) $[1]_R = \{1, 3, 5\}$

$[2]_R = \{2, 4, 6\}$

$[3]_R = \{1, 3, 5\}$

$[4]_R = \{2, 4, 6\}$

$[5]_R = \{1, 3, 5\}$

$[6]_R = \{2, 4, 6\}$

There are ~~three~~ two equivalence classes

$[1]_R = \{1, 3, 5\} = [3]_R = [5]_R$

$[2]_R = \{2, 4, 6\} = [2]_R = [6]_R$

· Question 2

S set of strings over the set $\{a,b\}$

I BASE: $b \in S$

II RECURSION: if $s \in S$, then

II(a) $sb \in S$

II(b) $asa \in S$

III Restriction: Nothing in S other than object above

(1) $b \in S$ by I	(2) $a \notin S$ by I
(2) $bb \in S$ by II (a) and (1)	(2) $ab \notin S$ by II(a) and (1)
(3) $aba \in S$ by II (b) and (1)	(3) $abb \notin S$ by II(a) and (2)
(4) $bbb \in S$ by (2) and (2)	(4) $aaa \notin S$ by II(b) and (1)

3 Strings belonging : $\{bb, aba, bbb\} \in S$ | 3 strings not belonging $\{ab, abb, aaa\} \notin S$

(b) $a_0, a_1, a_2, \ldots$ recursively defined as follow

$$a_n = \begin{cases} 0 & \text{if } n=0 \\ a_{n-1}+2n+3 & \text{if } n>0 \end{cases}$$

Using mathematical induction prove: $a_n = n(n+4)$ $\forall$ int $n \geq 0$

Let $a_1, a_2, a_3 \ldots$ be the sequence defined by specifying that $a_0 = 0$ and $a_n = a_{n-1} + 2n + 3$ $\forall$ $n>0$ and let the property $P(n)$ be the equation: $P(n): n(n+4)$

· Show that $P(0)$ is true:

To establish $P(0)$ we must show that

$$a = 0 \cdot (0+4)$$

Now, the left-hand side of $P(0)$ is

$$a_0 = 0 \quad \text{by definition}$$

and the right hand side of $P(1)$ is.

$$0 \cdot (0+4) = 0 \cdot 5 =$$

Thus the two sides of $P(0)$ equal the same quantity, and hence $P(0)$ is TRUE

· Question 2(b) (continuing from previous page)

· Show that for every Integer $n \geq 0$, if $P(n)$ is true then $P(n+1)$ is also true

[Suppose that $P(n)$ is true for a particular but arbitrarily chosen integer $n \geq 0$. That is:]

Suppose that k is any integer with $k \geq 0$ such that

$a_k = k(k+4)$ ← $P(k)$ inductive hypothesis (1)

[We must show that $P(k+1)$ is True. That is:]

$a_{k+1} = (k+1)((k+1)+4)$ ← $P(k+1)$ (2)

But the left side of a_{k+1} is:

$a_{k+1} = a_{(k+1-1)} + 2(k+1) + 3$

$= a_k + 2(k+1) + 3$ by definition of

$= k(k+4) + 2k + 2 + 3$ by (1)

$= k^2 + 4k + 2k + 5$ by basic algebra

$= k^2 + 6k + 5$ by basic algebra

And the left side (2) $P(k+1)$ can be expanded to

$P(k+1) = (k+1)((k+1)+4)$

$= (k+1)(k+5)$ mulpiplying each element

$= k^2 + 5k + k + 5$ by basic algebra

$= k^2 + 6k + 5$ by basic algebra

which equals the right-hand side of $P(k+1)$. [Since the basis and inductive steps, it follows by mathematical induction that the given formula holds for every integer $n \geq 0$]

· Question 3

$$mRn \Leftrightarrow m^2 - n^2 \geq 0 \quad \forall\, m,n \in \mathbb{Z}$$

We can use that for all $a,b,c \in \mathbb{Z}$ if $a \geq b$ and $b \geq c$, then $a \geq c$

(a) Is Reflexive since $\forall\, m \in \mathbb{Z}$ mRm by definition $m^2 - m^2 \geq 0$

which is TRUE as $m^2 - m^2 = 0 \geq 0$

(b) Is Transitive: if $m^2 - n^2$, $n^2 - h^2$ where $m,n,h \in \mathbb{Z}$

since mRn and nRh

By definition $m^2 - n^2 \geq 0$ and $n^2 - h^2 \geq 0$

thus $m^2 \geq n^2$ and $n^2 \geq h^2$

so $m^2 \geq h^2$

Thus for mRn: $m^2 - n^2 \geq 0$

for nRh: $n^2 - h^2 \geq 0$

and mRh = $m^2 - h^2 \geq 0$ is TRUE as $m^2 \geq h^2$

(c) Is NOT symmetric: if $m^2 - n^2 \geq 0$ then $n^2 - m^2 = -(m^2 - n^2) \leq 0$

Counter example: let $m = 1$ and $n = 0$

$1^2 - 0^2 = 1$ but $0^2 - 1^2 = -1 \leq 0$

(d) Is NOT anti-symmetric:

Counter example let $m = 1$ and $n = -1$, then

$m^2 - n^2 = 1^2 - (-1)^2 = 0 \geq 0$ and $n^2 - m^2 = (-1)^2 - 1^2 = 0 \geq 0$

Hence mRn and nRm but $m \neq n$