Discrete Mathematics
Peergrade assignment 1

1. Solve for x the following equations. Present the result in the simplest form.

   (a) $\frac{5}{3}\,x + \frac{2}{5} = 7$

   (b) $\frac{4}{1+x} + \frac{15}{4} = 5$

   (c) $|\frac{4\,x}{5}| = (\frac{1}{2} - \frac{1}{3})^{-1}$

   (d) $\frac{x-3}{3} = 2\,(\frac{2+x}{5} + 1)$

   Note: you need to show *how* you obtained the solution. Mark each step with a justification.

2. Show that $\rightarrow$ and $\sim$ form a *functionally complete* set of logical operators, that is that we can express other logical operators using *only* these two.
   To do that, find formulas written using only p, q, $\rightarrow$ and $\sim$, which encode

   (a) disjunction[1] $p \vee q$,

   (b) conjunction $p \wedge q$ and

   (c) biconditional $p \leftrightarrow q$

   In each step, use a truth table to show that your formula is logically equivalent to the operator it encodes.

3. Use the logical equivalences in Figure 1 below to verify the following logical equivalences step by thep. Supply a reason for each step. i.e. state, which of the equivalences from Figure 1 you used.

   (a) $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

   (b) $(p \wedge {\sim}q) \rightarrow r \equiv (p \wedge {\sim}r) \rightarrow q$

   (c) $(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$

   (d) ${\sim}((p \rightarrow q) \wedge {\sim}q) \vee {\sim}p \equiv \mathbf{t}$

[1] Hint: start by looking at Figure 1, particularly the Conditional (12) law.

Given any statement variables p, q, and r, a tautology $\mathbf{t}$ and a contradiction $\mathbf{c}$, the following logical equivalences hold:

1. Commutative laws:	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. Associative laws:	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive laws:	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity laws:	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
5. Negation laws:	$p \vee {\sim}p \equiv \mathbf{t}$	$p \wedge {\sim}p \equiv \mathbf{c}$
6. Double negative law:	${\sim}({\sim}p) \equiv p$	
7. Idempotent laws:	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. Universal bound laws:	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
9. De Morgan's laws:	${\sim}(p \wedge q) \equiv {\sim}p \vee {\sim}q$	${\sim}(p \vee q) \equiv {\sim}p \wedge {\sim}q$
10. Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. Negations of $\mathbf{t}$ and $\mathbf{c}$:	${\sim}\mathbf{t} \equiv \mathbf{c}$	${\sim}\mathbf{c} \equiv \mathbf{t}$
12. Conditional:	${\sim}p \vee q \equiv p \to q$	$p \to q \equiv {\sim}q \to {\sim}p$
13. Biconditional:	$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$	$p \leftrightarrow q \equiv {\sim}p \leftrightarrow {\sim}q$

Figure 1: Logical equivalences.

For example, to verify the equivalence $(p \wedge q) \wedge q \equiv p \wedge q$, you would write:

$$\begin{aligned} & (p \wedge q) \wedge q \\ \equiv\ & p \wedge (q \wedge q) && (2) \\ \equiv\ & p \wedge q && (7) \end{aligned}$$

or

$$\begin{aligned} & (p \wedge q) \wedge q \\ \overset{(2)}{\equiv}\ & p \wedge (q \wedge q) \\ \overset{(7)}{\equiv}\ & p \wedge q \end{aligned}$$