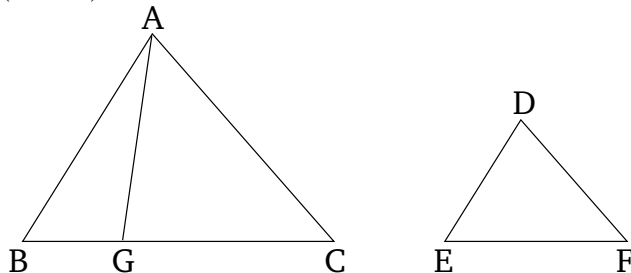# Book 6
# Proposition 19

Similar triangles are to one another in the squared ratio of (their) corresponding sides.



Let $ABC$ and $DEF$ be similar triangles having the angle at $B$ equal to the (angle) at $E$, and $AB$ to $BC$, as $DE$ (is) to $EF$, such that $BC$ corresponds to $EF$. I say that triangle $ABC$ has a squared ratio to triangle $DEF$ with respect to (that side) $BC$ (has) to $EF$.

For let a third (straight-line), $BG$, have been taken (which is) proportional to $BC$ and $EF$, so that as $BC$ (is) to $EF$, so $EF$ (is) to $BG$ [Prop. 6.11]. And let $AG$ have been joined.

Therefore, since as $AB$ is to $BC$, so $DE$ (is) to $EF$, thus, alternately, as $AB$ is to $DE$, so $BC$ (is) to $EF$ [Prop. 5.16]. But, as $BC$ (is) to $EF$, so $EF$ is to $BG$. And, thus, as $AB$ (is) to $DE$, so $EF$ (is) to $BG$. Thus, for triangles $ABG$ and $DEF$, the sides about the equal angles are reciprocally proportional. And those triangles having one (angle) equal to one (angle) for which the sides about the equal angles are reciprocally proportional are equal [Prop. 6.15]. Thus, triangle $ABG$ is equal to triangle $DEF$. And since as $BC$ (is) to $EF$, so $EF$ (is) to $BG$, and if three straight-lines are proportional then

1

the first has a squared ratio to the third with respect to the second [Def. 5.9] , $BC$ thus has a squared ratio to $BG$ with respect to (that) $CB$ (has) to $EF$. And as $CB$ (is) to $BG$, so triangle $ABC$ (is) to triangle $ABG$ [Prop. 6.1] . Thus, triangle $ABC$ also has a squared ratio to (triangle) $ABG$ with respect to (that side) $BC$ (has) to $EF$. And triangle $ABG$ (is) equal to triangle $DEF$. Thus, triangle $ABC$ also has a squared ratio to triangle $DEF$ with respect to (that side) $BC$ (has) to $EF$.

Thus, similar triangles are to one another in the squared ratio of (their) corresponding sides. [(Which is) the very thing it was required to show].

# Corollary

So it is clear, from this, that if three straight-lines are proportional, then as the first is to the third, so the figure (described) on the first (is) to the similar, and similarly described, (figure) on the second. (Which is) the very thing it was required to show.