Book 5 Proposition 5

If a magnitude is the same multiple of a magnitude that a (part) taken away (is) of a (part) taken away (respectively) then the remainder will also be the same multiple of the remainder as that which the whole (is) of the whole (respectively).

For let the magnitude AB be the same multiple of the magnitude CD that the (part) taken away AE (is) of the (part) taken away CF (respectively). I say that the remainder EB will also be the same multiple of the remainder FD as that which the whole AB (is) of the whole CD (respectively).

For as many times as AE is (divisible) by CF, so many times let EB also have been made (divisible) by CG.

And since AE and EB are equal multiples of CF and GC (respectively), AE and AB are thus equal multiples of CF and GF (respectively) [Prop. 5.1]. And AE and AB are assumed (to be) equal multiples of CF and CD (respectively). Thus, AB is an equal multiple of each of GF and CD. Thus, GF (is) equal to CD. Let CF have been subtracted from both. Thus, the remainder GC is equal to the remainder FD. And since AE and EB are equal multiples of CF and GC (respectively), and GC (is) equal to DF, AE and EB are thus equal multiples of CF and FD (respectively). And FD are

assumed (to be) equal multiples of CF and CD (respectively). Thus, EB and AB are equal multiples of FD and CD (respectively). Thus, the remainder EB will also be the same multiple of the remainder FD as that which the whole AB (is) of the whole CD (respectively).

Thus, if a magnitude is the same multiple of a magnitude that a (part) taken away (is) of a (part) taken away (respectively) then the remainder will also be the same multiple of the remainder as that which the whole (is) of the whole (respectively). (Which is) the very thing it was required to show.