Book 10 Proposition 106

A (straight-line) commensurable (in length) with a (straight-line) which with a rational (area) makes a medial whole is a (straight-line) which with a rational (area) makes a medial whole.

 $\begin{array}{ccccc} A & & B & E \\ \hline C & & D & F \end{array}$

Let AB be a (straight-line) which with a rational (area) makes a medial whole, and (let) CD (be) commensurable (in length) with AB. I say that CD is also a (straight-line) which with a rational (area) makes a medial (whole).

For let BE be an attachment to AB. Thus, AE and EB are (straight-lines which are) incommensurable in square, making the sum of the squares on AE and EB medial, and the (rectangle contained) by them rational [Prop. 10.77]. And let the same construction have been made (as in the previous propositions). So, similarly to the previous (propositions), we can show that CF and FD are in the same ratio as AE and EB, and the sum of the squares on AE and EB is commensurable with the sum of the squares on CF and FD, and the (rectangle contained) by AE and EB with the (rectangle contained) by CF and EB. Hence, EB are also (straight-lines which are) incommensurable in square, making the sum of the squares on EB and EB medial, and the (rectangle contained) by them rational.

CD is thus a (straight-line) which with a rational

(area) makes a medial whole [Prop. 10.77]. (Which is) the very thing it was required to show.