# Book 4
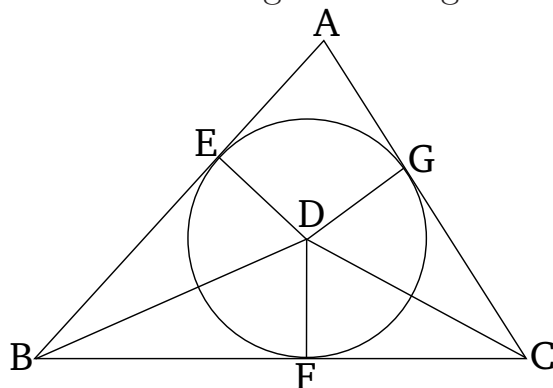# Proposition 4

To inscribe a circle in a given triangle.



Let $ABC$ be the given triangle. So it is required to inscribe a circle in triangle $ABC$.

Let the angles $ABC$ and $ACB$ have been cut in half by the straight-lines $BD$ and $CD$ (respectively) [Prop. 1.9], and let them meet one another at point $D$, and let $DE$, $DF$, and $DG$ have been drawn from point $D$, perpendicular to the straight-lines $AB$, $BC$, and $CA$ (respectively) [Prop. 1.12].

And since angle $ABD$ is equal to $CBD$, and the right-angle $BED$ is also equal to the right-angle $BFD$, $EBD$ and $FBD$ are thus two triangles having two angles equal to two angles, and one side equal to one side—the (one) subtending one of the equal angles (which is) common to the (triangles)—(namely), $BD$. Thus, they will also have the remaining sides equal to the (corresponding) remaining sides [Prop. 1.26]. Thus, $DE$ (is) equal to $DF$. So, for the same (reasons), $DG$ is also equal to $DF$. Thus, the three straight-lines $DE$, $DF$, and $DG$ are

equal to one another. Thus, the circle drawn with center $D$, and radius one of $E$, $F$, or $G$, will also go through the remaining points, and will touch the straight-lines $AB$, $BC$, and $CA$, on account of the angles at $E$, $F$, and $G$ being right-angles. For if it cuts (one of) them then it will be a (straight-line) drawn at right-angles to a diameter of the circle, from its extremity, falling inside the circle. The very thing was shown (to be) absurd [Prop. 3.16]. Thus, the circle drawn with center $D$, and radius one of $E$, $F$, or $G$, does not cut the straight-lines $AB$, $BC$, and $CA$. Thus, it will touch them and will be the circle inscribed in triangle $ABC$. Let it have been (so) inscribed, like $FGE$ (in the figure).

Thus, the circle $EFG$ has been inscribed in the given triangle $ABC$. (Which is) the very thing it was required to do.