2D-Pose Graph Optimization Simultaneous Localization and Mapping

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Abstract

Building a good map of an environment has been a long prevalent problem in the domain of robotics. Especially in areas where there is no access to external referencing systems such as GPS sensors. This problem is popularly known as Simultaneous localization and mapping problem.

In this project, I have attempted to study, code and demonstrate the working of 2D-pose Graph Simultaneous localization and mapping Optimization on the data sets that are provided by Luca Carlone, Department of Aeronautics and Astronautics, MIT which is taken from the website [4] and which is the part of research work [2]. Moreover, I have not used any external exclusive Graph Slam library for implementation. Given that pose graph SLAM can be divided into two functional modules namely the front-end which comprises converting raw data into graph nodes and edges and the back-end which takes the nodes and edges as input to find the optimized poses, this project will only address the back-end optimization function.

1 Introduction

Having a map of the environment is very essential to carry out tasks such as search and rescue, transportation, surveillance in mining, or nuclear reactors. Good accurate maps facilitate the robot to perform complex actions, only based on their sensors without having to depend on external positioning reference systems like the GPS. The task of building maps when the robot pose is uncertain is commonly referred to as the simultaneous localization and mapping (SLAM) problem. There are several methods proposed to solve the SLAM problem, especially the methods that are classified under two board categories namely filtering and smoothing

[11]. The filtering constitutes techniques such as Kalman filter, Extended Kalman filter, Information filter [3], particle filter [10] Filtering approaches are also called as online approaches as the estimates are made accurate as new observations are obtained. On the other hand, smoothening approaches only work after a full set of observations are obtained and therefore they are also termed as offline SLAM methods. [22] [5]. Least square error minimization techniques are popular concerning smoothening approaches.

A Graph SLAM approach is a smoothening based approach in which we use a graph formulation for representing poses in the form of graph vertices and constraints between poses in the form of edges. The constraints can be contradictory as the observations can be affected by sensor noise or imperfect data associations. The crux of the Graph SLAM problem is to find poses that are most consistent with the measurements obtained. Thus we convert the SLAM problem to an error minimization problem. Although the graph-based SLAM was proposed in 1997 [17], it is only recently, the improvements in sparse linear algebra has led to techniques which can reduce the computationally expensive operations involved in graph SLAM. Later on, in the text, we will determine that to solve the error minimization problem we have to invert a squared error matrix which is a computationally expensive process and efficient solvers such as Cholesky factorization [21], QR factorization [6], Iterative methods like conjugate gradients (subgraph) and many others essentially exploit the sparsity of this matrix to efficiently solve the non-linear least square minimization problem.

2 Problem Formulation

The complete SLAM problem is to estimate the position and orientation of the robot or trajectory as it travels in the unknown environment and simultaneously building the map of the environment. As the measurements made by the sensors are inherently noisy in nature the SLAM problem is usually formulated in a probabilistic manner.

A sequence of random variables define a robots trajectory in the unknown environment $x_{1:T} = x_1, ...x_T$ .Given the set of odometric measurements that were applied to the robot $u_{1:T} = \{u_1, u_2, u_3, ...u_t\}$ and the set of observations of the environment performed by the robot $z_{1:T} = \{z_1, z_2, z_3, ...z_t\}$ we estimate the most likely map of the environment and the posterior probability of the robot's pose. This is called the full SLAM problem.

$$p(x_{1:T}, m|z_{1:T}, u_{1:T}, x_0)$$

The x_0 is the initial position in the map which can be chosen without loss of generality.

In graph-based SLAM, the nodes constitute the poses of the robot in the environment and the spatial constraints between the poses and the observations z_t from the odometer become the part of the graph edges. These spatial constraints are a probability distribution over the relative transformation between poses. The transformations include sequential odometry measurements that come from the robot's sequential positions and also from matching the observations that are obtained at two robot locations. The graph SLAM problem is essentially divided into two parts namely graph construction and graph optimization [10]. In the graph construction problem, we take the raw measurements as the input and construct the graph with nodes and edges, whereas for the graph optimization problem we take in as input the edges of the graph that encapsulates the constraints and determine the most likely configuration of poses. The graph construction is also commonly known as the front end [11] and it is heavily dependant on the sensors, whereas the graph optimization is the back end part which is generally sensor independent.

In this project, we take as input the constructed graph dataset as provided by Luca Carlone , Department of Aeronautics and Astronautics, MIT [4] and do the graph optimization using the algorithm mentioned in this research work [11].

3 Related Work

SLAM, in general, is a popular problem in robotics with a variety of approaches, In this section, the focus will be limited to only graph-based approaches. In early works, Lu and Milos[17] introduced the technique that established relative spatial constraints as the robot explores the environment and devised a maximum likelihood criterion to optimally combine the spatial relations that led to global optimization of the system of equations for error minimization. Gutmann and Konolige[12] presented a technique Local registration and global correlation (LRGC) for detecting loop closures and constructing better maps. Howard et al. [13] proposes relaxation to solve a constraint mesh or network with respect to a global frame of reference for localization and mapping. Frese et al.[8] proposes multilevel relaxation approach for SLAM problem.Dellaert and Kaess [22] attempted to exploit the sparseness of the matrix to present an efficient solution to the offline SLAM problem. Kaess et al.[14], on the other hand, introduces an online version known as iSAM that proposes incremental updates interleaved with periodic reordering, which results in good computational time for sparse factorization. Konolige et al.[15] proposes Sparse Pose Adjustment (SPA) method for solving pose graph optimization problem. Olson et al [18] approach the non-linear optimization problem using a variant of stochastic gradient descent on an alternative state-space representation for good computational properties. Grisetti et al. [9] presents an extension to Olson's algorithm, considering SLAM for 3d pose graphs. The implementation is also faster as tree parametrization of nodes decreases computational time. GraphSLAM [11] shows how to reduce the dimensionality of the SLAM problem to ease optimization. Hierarchical SLAM [7] technique uses independent local maps and proposes a loop closure method that has consistency at the global level with computational cost linear to the size of the loop. The process

of converting raw measurements into relative constraints which is to say graph nodes(poses for example (x,y, yaw) in the case of 2d or (x,y,z, yaw, pitch, roll) in case of 3d) and edges(relative measurements) with information matrices which suggest the confidence of measuring instruments, is also known as front-end graph SLAM. χ^2 test of a statistic is often used to make data association in front-end SLAM systems.Olson[19] presented a front-end SLAM using spectral clustering. The back-end algorithm used in this project can be applied to the front-end obtained by Olson and also to any other variant as long as the input requirements to the algorithm are satisfied.

4 Method

As mentioned earlier, we assume that we have the front end part already which is to convert the raw measurements into nodes and edges with constraints (or information matrix) as the input to our graph SLAM optimization algorithm. The algorithm that is used for Graph SLAM optimization which results in the solution to non-linear least squares optimization problem which is also an approximation of the function that we estimate which reduces the error between true poses and estimated poses.The algorithm that is implemented in this project is given below

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Algorithm 1 calculating the optimized poses $\bar{X}^*_{1:T}$ and information matrix H^* for given graph nodes and edges

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Require: $\bar{X}_{1:T}$ list of initial poses and intial contrainsts C = $(e_{ij}\ ,\Omega_{ij})$
Ensure: $\bar{X}^*_{1:T}$ is the new solution and H^* new information matrix
  while $\neg$ converged **do**
    Set $b \leftarrow 0$ and $H \leftarrow 0$
    for all $(e_{ij}, \Omega_{ij}) \in C$ **do**
      $A_{ij} \leftarrow \frac{\partial e_{ij}}{\partial x_i}\Big|_{x=\bar{x}}$ $B_{ij} \leftarrow \frac{\partial e_{ij}}{\partial x_j}\Big|_{x=\bar{x}}$
      $H_[ii]+ = A^T_{ij}\Omega_{ij}A_{ij}$ $H_[ij]+ = A^T_{ij}\Omega_{ij}A_{ij}$
      $H_[ji]+ = B^T_{ij}\Omega_{ij}A_{ij}$ $H_[jj]+ = B^T_{ij}\Omega_{ij}A_{ij}$
      $b_i+ = A^T_{ij}\Omega_{ij}e_{ij}$ $b_j+ = B^T_{ij}\Omega_{ij}e_{ij}$
    end for
    $H_[11]+ = I$
    $\Delta X \leftarrow (H\Delta X = -b)$
    $\bar{x}+ = \Delta x$
  end while
    $x^* \leftarrow \bar{x}$
    $H^* \leftarrow \bar{H}$
    $H^*_{11}- = I$
    return $< x^*, H^* >$

The above algorithm is taken from the research work [11]. Sparse matrix can be formed by several data structures such as linked-list, list of lists ,dictionary of keys, compressed sparse row or compressed sparse column. These data structures help reducing auxiliary space complexity, and also help reduce the theoretical time complexity or number of operations. In this project, compressed sparse row data structure was used, which is essentially three arrays that store row, column and indices of non-zero elements in the matrix. The reason to use sparse matrix is because in most of matrix operations involved in the algorithm are sparse, meaning most of the values in the matrix are zero. After the optimization is applied for set of iterations, we plot the x-pose versus y-pose of the optimized nodes to obtain the plots.

5 Experiments

The results for the datasets INTEL, MIT_B and M3500 that are used in the project are given below.

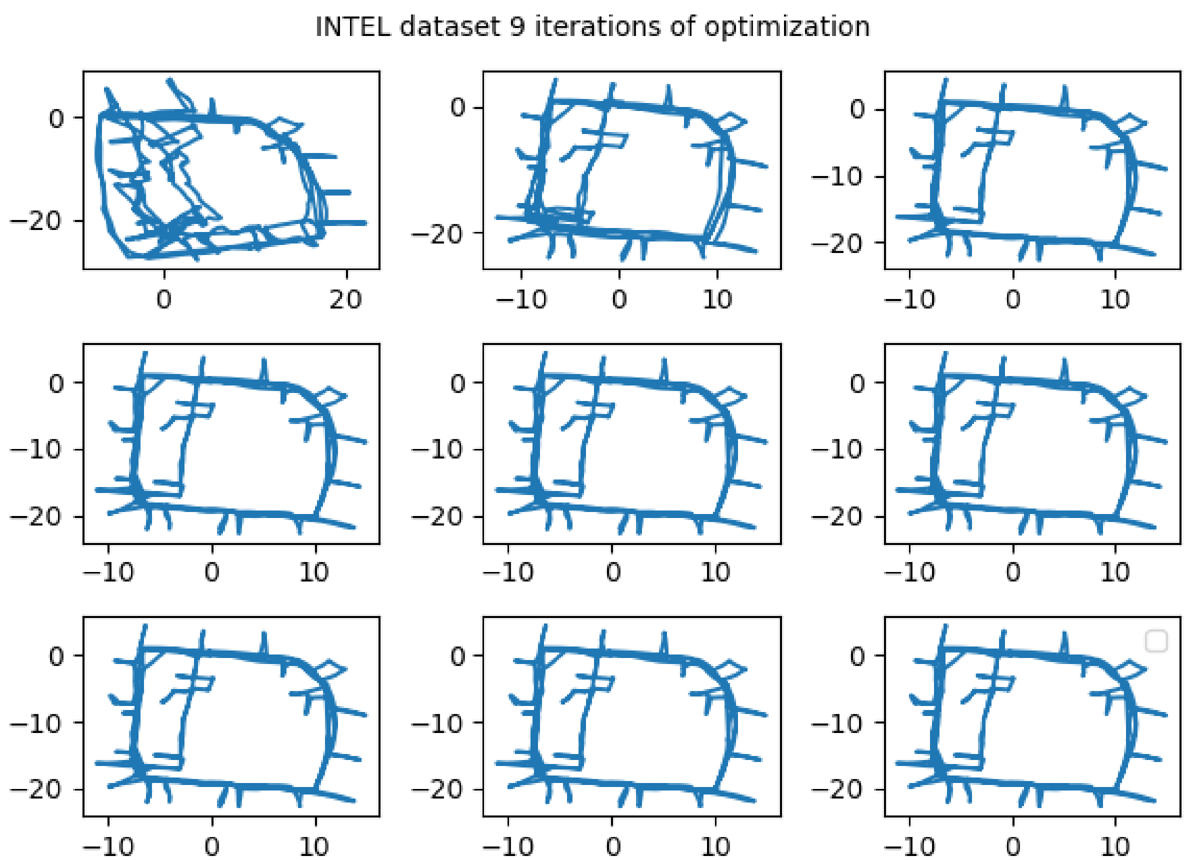

Figure 1: Convergence obtained after 9 iterations in INTEL dataset

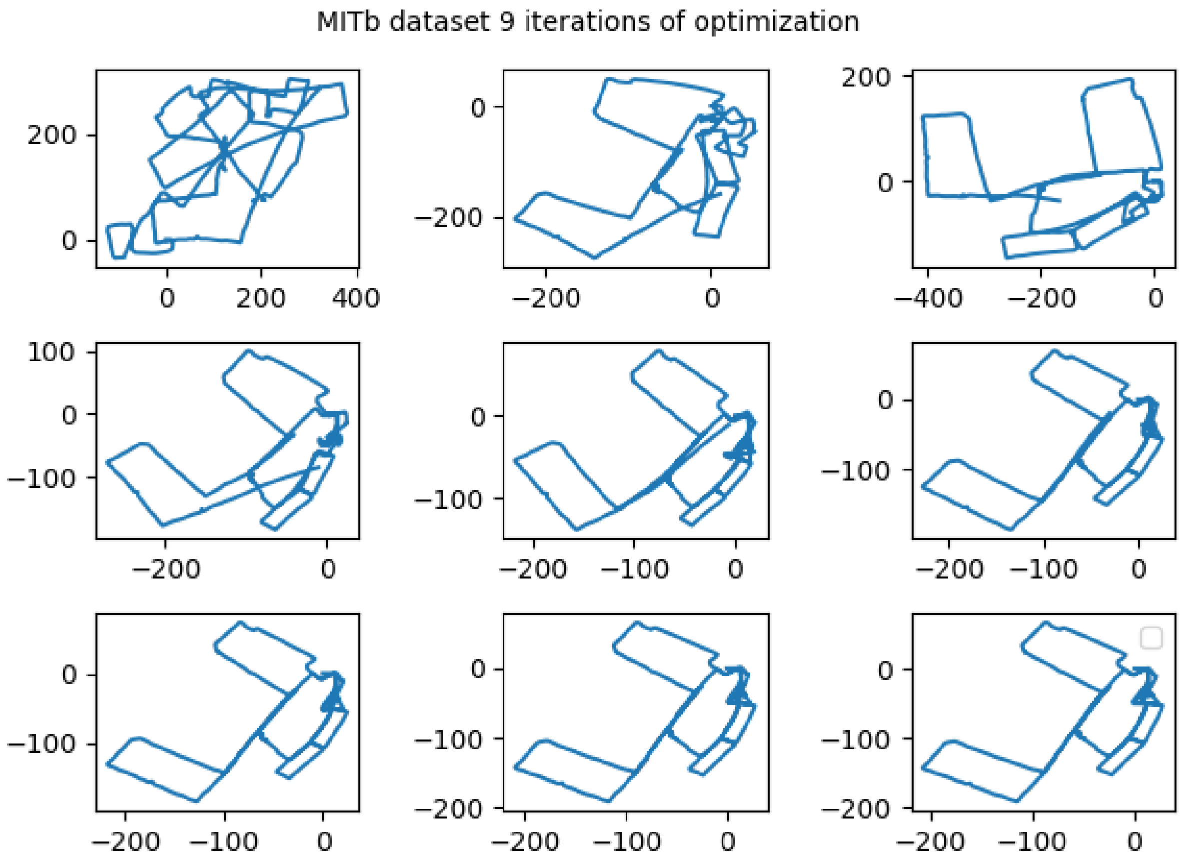

Figure 2: Convergence obtained after 9 iterations in MIT_B dataset

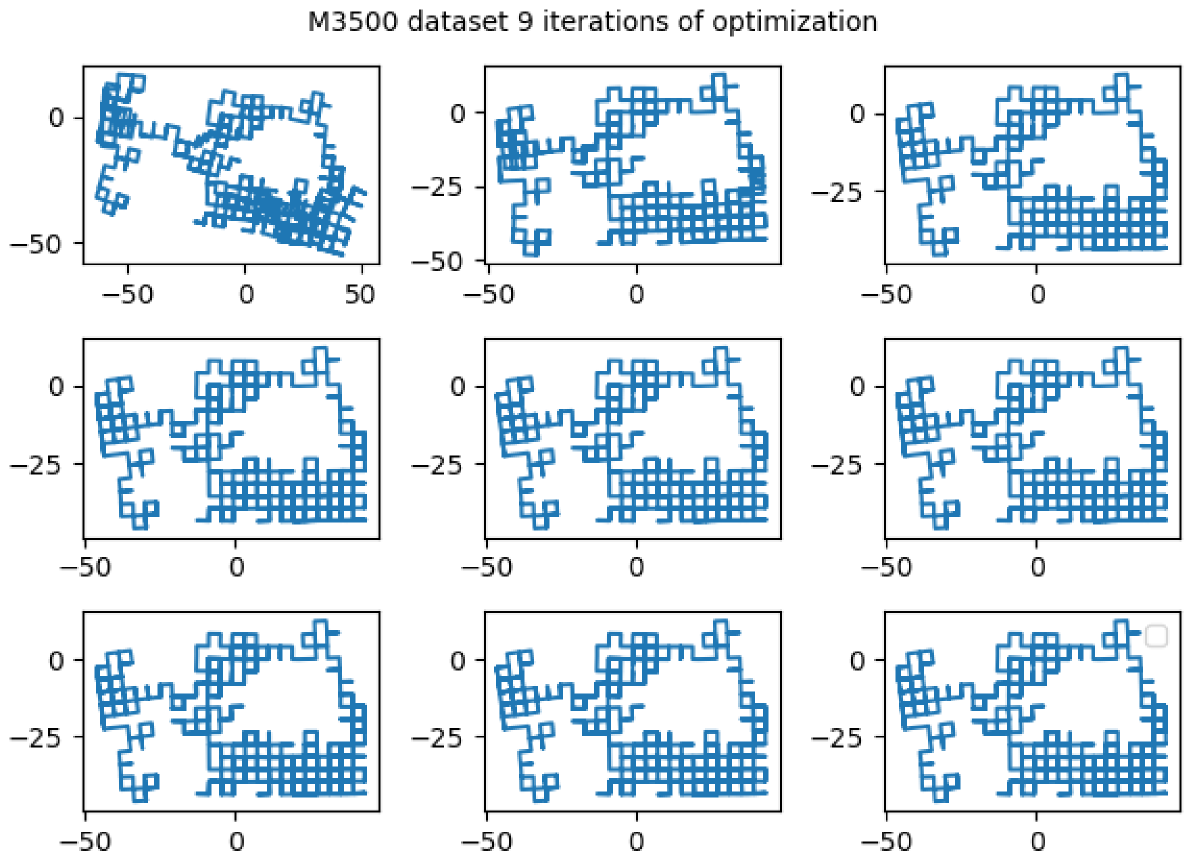

Figure 3: Convergence obtained after 9 iterations in M3500 dataset

In all the three datasets the convergence was obtained and maps were built which were clear and without any distortion. However, experimental datasets with variant of M3500 dataset were made by adding gaussian noise with standard deviation 0.1 rad, 0.2 rad and 0.3 rad with regards to the relative orientation measurements for the variants $M3500\_A$, $M3500\_B$ and $M3500\_C$ respectively and the algorithm was tested for 100 iterations, The results obtained are presented below.

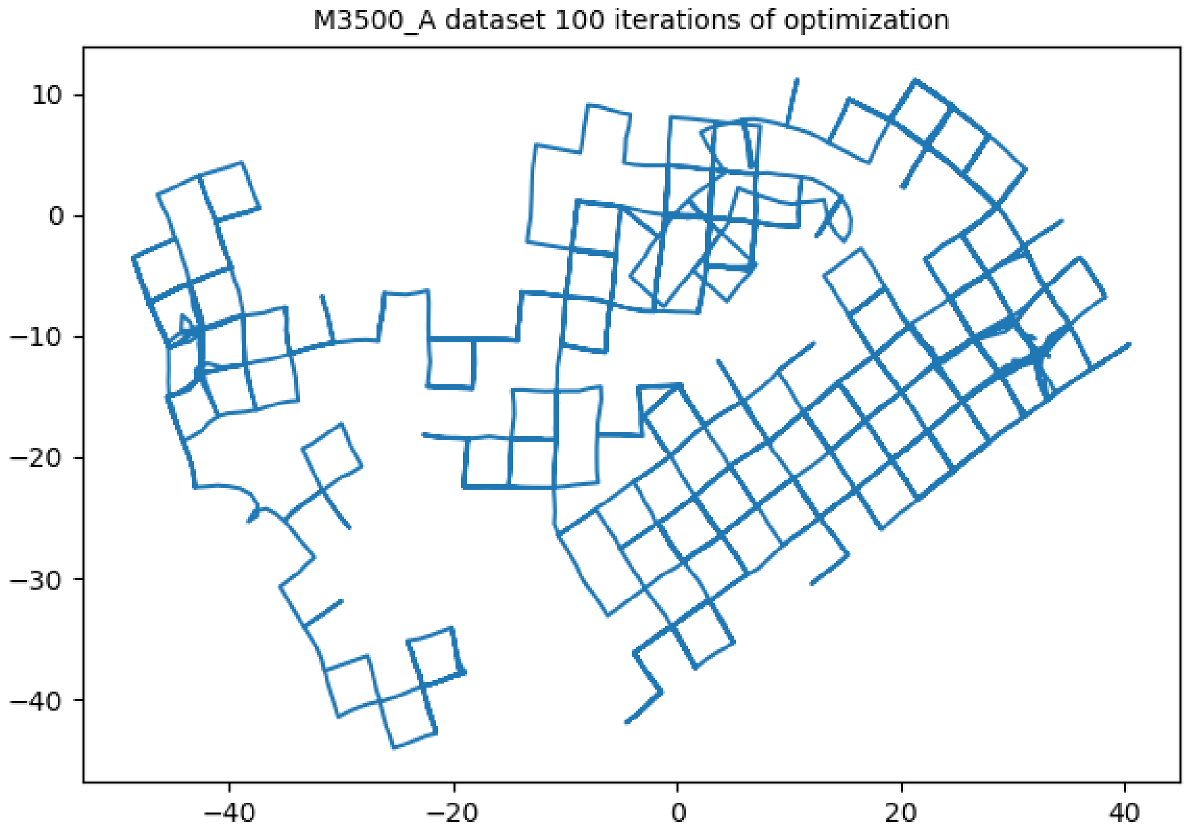

Figure 4: Convergence obtained after 100 iterations of Algorithm in $M3500\_A$ dataset

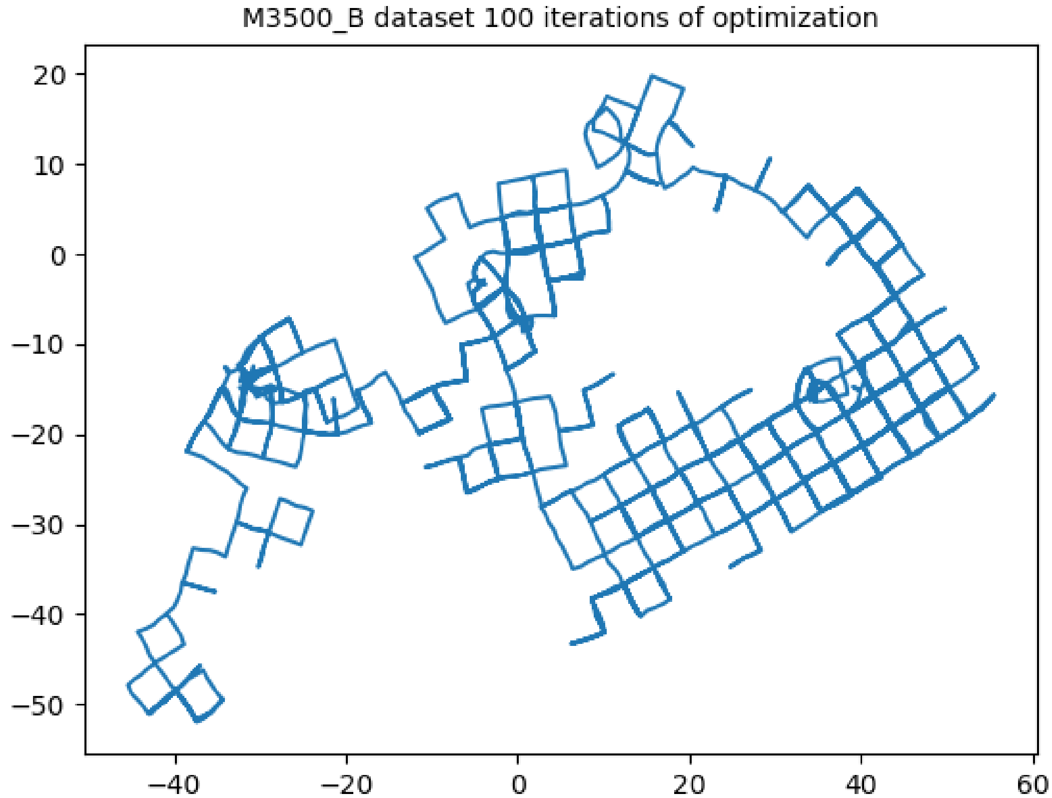

Figure 5: Convergence obtained after 100 iterations of Algorithm in $M3500\_B$ dataset

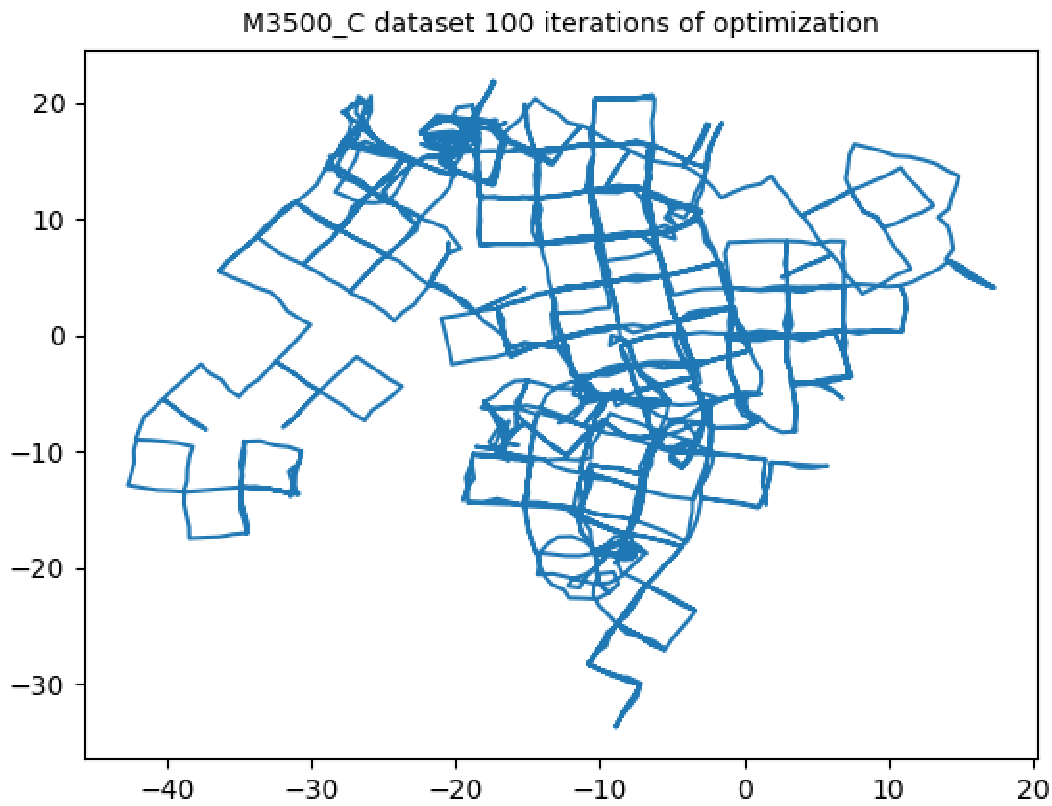

Figure 6: Convergence obtained after 100 iterations of Algorithm in $M3500\_C$ dataset

We see that the map is distorted at a few points, after adding Gaussian noise to relative orientation measurements.

6 Conclusion

In this project, the algorithm from the research work [11] was applied to the datasets for 2d pose graph SLAM that were provided on the website [4] by Luca Carlone. The results were reported on 6 datasets namely INTEL,MIT_B , M3500, and three variants of M3500 with addition of gaussian noise in relative orientation with a standard deviation of 0.1, 0.2, and 0.3 rads respectively for $M3500\_A$, $M3500\_B$ and $M3500\_C$. While the optimization resulted to clear maps for INTEL, MIT_B and M3500 datasets, however, M3500 variants with standard deviation resulted to distortion in the map. A speculated reason for this distortion could be the problem of false constraints (data association errors and false-positive loop closure detections). Future work should consider the potential effects of false constraints on optimization in pose graph SLAM. Approaches such as max-mixture, vertigo, and dynamic covariance scaling [1] [20] based SLAM techniques are suggested in the literature, can be applied against false constraints, but these algorithms also have certain limitations. This problem can also be solved by additional sensor information as described in this research work [16], where an RGB-D camera is used for identifying and removing false constraints, which could also be considered in future work.

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