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Linear Kalman Filter:

1.What is the difference between a 'control' u_t, a 'measurement' z_t and the state x_t? Give examples of each?

Ans 1.

The control is always in our hand i.e an input like fuel for a car or fuel for a rocket, the measurement is done by a device may it be an odometer or gps or any other measurement setup and the state is what is calculated using the control applied to a model and measurement of the sensors.

$dx/dt = A.x + B.u$
$Y = C.x$
Here, x is the internal state variable, Y is the output and u is the input for linear systems. As input is given by us hence we say that we can control ut where as the output is obtained through a model which in the above case is y(predicted) = z(predicted). Thus we compare the z(predicted) to z(measured) and minimize it which is equivalent to minimizing the error in prediction of internal states

2.Can the uncertainty in the belief increase during an update? Why (or not)?

Ans2.

No, the uncertainty in the belief cannot be increased during an update because the prior estimate error covariance term is always positive semidefinite.

3.During update what is it that decides the weighing between measurements and belief?

Ans 3

The uncertainty between predicted state estimate or prior estimate and the measurement uncertainty becomes the weighing factors. For instance in extreme cases if there is a perfect model(prior estimate covariance is zero) the posterior estimate becomes equal to prior estimate and if there is a perfect measument (covariance in measurement is zero) then the posterior estimate is equal to the measurement.This can be shown mathematically.

4.What would be the result of using a too large a covaraince (Q matrix) for the measurement model?

Ans 4

Having a very large Covariance (Q matrix) will lead to a larger uncertainty in the prior estimation step and as we see that the prior estimate convarince will have a higher value and after considering the measurement covariance and as we multiply both the probability distribution functions to obtain the posterior estimate thus the uncertainty in posterior estimate will depend on both the measurement covariance and the prior estimate covariance. Thus the estimates would be pessimistic (conservative) and convergence will be slower.

5.What would give the measurements an increased effect on the updated state estimate?

Ans 5
Measurement uncertainty is less or prior estimate uncertainty is very high.

6.What happens to the belief uncertainty during prediction? How can you show that?

Ans 6.

It usually Increases as the dynamic model adds noise to the state and the uncertainty is the sum of a positive definite matrix and the noise. However in theory it is possible to have less uncertainty but in real systems it almost always increases as the noise covariance is significantly large.

7.How can we say that the Kalman Filter is the optimal and minimum least square error estimator in the case of independent Gaussian noise and Gaussian priori distribution? (Just describe the reasoning not a formal proof.)

Ans7.
We know that the Kalman filter gives us the true posterior distribution for a linear gaussian system. If there exist a mean better than gaussian mean we can integrate it by multiplying with gaussian pdf and then we can put the minimizing condition which will give the result that such mean assumed if exist is the same as the gaussian mean, Hence it is true that gaussian is the optimal error estimator. Also we see in the extreme cases as follows:

Case 1 . Perfect instrument (Covariance of instrument measurement is zero)
Posterior estimate converges to output measurement
Case 2 Perfect Model(model has zero covariance)
Posterior estimate converges to prior prediction estimate
Thus as we see that in Ideal conditions the special cases are covered by Kalman filter hence it is most optimal and minimum least square error estimate.

8.In the case of Gaussian white noise and Gaussian priori distribution, is the Kalman Filter a MLE and/or MAP estimator?

Ans.8 Gaussian White Noise is MLE and Gaussian priori distribution is MAP.

Extended Kalman Filter:

9.How does the extended Kalman filter relate to the Kalman filter?

Ans.9

A Kalman filter is only used for linear systems as applying for non-linear systems do not produce a gaussian. An extended Kalman filter is used in case of non-linear systems by linearizing the system around a point. For the non-linear system In extended Kalman filter jacobians are found (numerical solution) for matrices updates and hence not necessarily an analytical solution is obtained. Furthermore ,jacobians must be invertible and high computational cost in general. In the extended Kalman filter, the state transition and observation models don't need to be linear functions of the state but may instead be differentiable functions.

$$\boldsymbol{x}_k = f(\boldsymbol{x}_{k-1}, \boldsymbol{u}_k) + \boldsymbol{w}_k$$
$$\boldsymbol{z}_k = h(\boldsymbol{x}_k) + \boldsymbol{v}_k$$

In Extended Kalman Filter the non linear functions like f and h are just replaced by linearearised functions at the point at which we need to evaluate the predictions. For instance $f(x_{k-1}, u_k)$ will correspond to $A_k x_{k-1} + B_k u_k$ and $h(x_k)$ will correspond to C_k.

10.Is the EKF guaranteed to converge to a consistent solution?

Ans.10
No.EKF model depends highly on the linearization of the system. If the system is not linearizable then EKF may not converge or if the system is highly non linear EKF cannot give optimal solution.

11.If our filter seems to diverge often can we change any parameter to try and reduce this?

Ans 11.
Yes we might change our modelled uncertainties Q and R. Typically divergence occurs on update and increasing the relative size of the measurement covariance Q. If the divergence was due to poor data association, we can change our matching threshold.

12.If a robot is completely unsure of its location and measures the range r to a know landmark with Gaussain noise what does its posterior belief of its location p(x; y; θ :r) look like? So a formula is not needed but describe it at least.

Ans 12.

It will have a uniform distribution over θ which is from -pi to pi.The position will be like a donut /ring so a gaussian on ϱ in radial coordinate with uniform distribution on the angle ϕ. Thus the equations will be as follows:

$x = \varrho * \cos(\phi)$

$y = \varrho * \sin(\phi)$

exp{ –[($\varrho - r)^2 /(2\sigma r^2) + (b - \phi + \theta)^2/(2\sigma b^2)$]} will be the term for the gaussian exponential.

13. If the above measurement also included a bearing how would the posterior look?

Ans 13.

The same as the above answer except that the heading and angle around the ring would be Gaussian with a completely correlated covariance.

14.If the robot moves with relatively good motion estimation (prediction error is small) but a large initial uncertainty in heading _ how will the posterior look after traveling a long distance without seeing any features?

Ans 14.
It will look like an arc.The heading θ will be correlated with position along the arc.

15.If the above robot then sees a point feature and measures range and bearing to it how might the EKF update go wrong?

Ans 15.
The linearized jacobian will produce an update direction which is a straight line that cannot move the estimate along the curved arc. As then gaussian will not be able to produce a crescent shape and the update might go wrong.

PART II - Matlab Excercises
3.1 Warm up problem with Standard Kalman Filter

1.What are the dimensions of ϵk andδk ? What parameters do you need to define in order to uniquely characterize a white Gaussian?

Ans 1.
Both noises are modelled as 2*2 matrices with only diagonal elements.

2. Make a table showing the roles/usages of the variables(x,xhat, P, G, D, Q, R, wStdP, wStdV, vStd, u, PP). To do this one must go beyond simply reading the comments in the code to seeing how the variable is used. (hint some of these are our estimation model and some are for simulating the car motion).

Ans 2.

Variables	Explaination
X	state space contains position and speed
X hat	state variable estimate
P	Belief uncertainty of estimate x
G	4*4 Identity matrix
D	coefficient of white noise in measurement (1*1 Identity matrix
Q	measurement noise covariance
R	Process noise variance
wStdP	Standard deviation of Noise on simulated position
wStdV	Standard deviation of Noise on simulated velocity
vStd	Standard deviation of Simulated measurement noise on position
u	Input controls
PP	Covariance matrix for state variable estimate

3.Please answer this question with one paragraph of text that summarizes broadly what you learn/deduce from changing the parameters in the code as described below. Choose two illustrative sets of plots to include as demonstration. What do you expect if you increase/decrease the covariance matrix of the modelled (not the actual simulated) process noise/measurement noise 100 times(one change in the default parameters each time) for the same underlying system? Characterize your expectations. Confirm your expectations using the code (save the corresponding figures so you can analyse them in your report). Do the same analysis for the case of increasing/decreasing both parameters by the same factor at the same time. (Hint: It is the mean and covariance.

Ans 3.

If the measurement noise variance increases the Kalman gain shall intuitively decrease but however there will be convergence but it would be a slower process. In this particular case due to the problem of data association, the covariance doesnot tend to zero. On the other hand if the measurement uncertainty decreases the Kalman gain will be determined more by the measurement as it is assumed that measurement noise will be very minimal. And thus the covariance in the final state estimator will converge to minimum value almost zero if measurement certainty is very very high.In the experiement below we have increased the process noise by 1000 times and in Figure 2 we have decreased it by 1000 times. We observe that the position estimate is more closer to true estimate when the process noise is high as the Kalman gain is dependent on the measurement as it weighs the measurements high and also In the inverse case when the position estimate is low, the error in position increases and the case is vice versa for velocity estimates as seen in the figures(1 and 2).

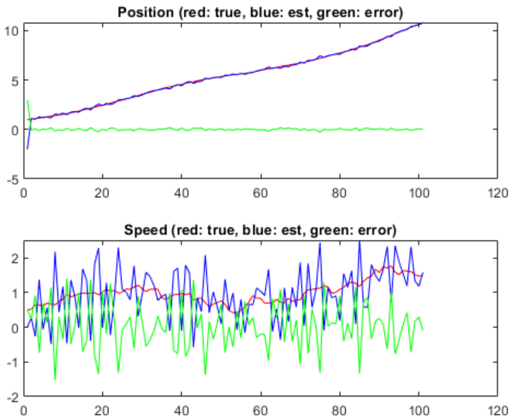

Figure 1: estimated state variable and error when process noise is made 1000 times the initial

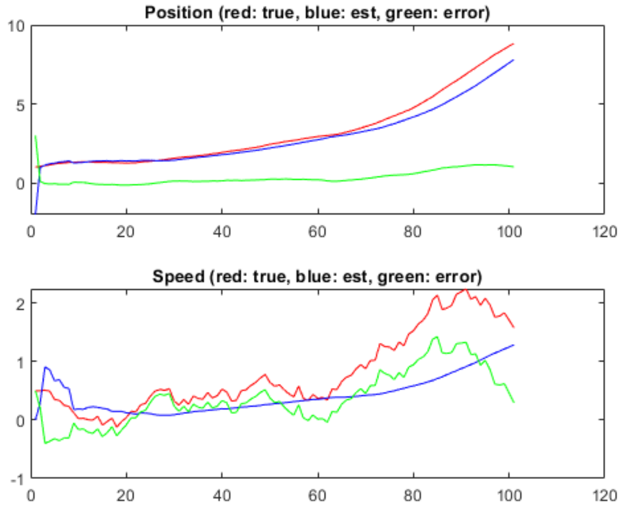

Figure 2: estimated state variable and error when process noise is made 0.001 times the initial

4.How do the initial values for P and xhat affect the rate of convergence and the error of the estimates (try both much bigger and much smaller)?

Ans:

We observe that if P is higher the error minimization happens quickly as the Kalman gain becomes higher and estimates are positive.Similarly making xhat more certain by lowering the process noise in calculating estimate has a similar effect of converging faster.Making both smaller will be like a pessimistic estimates, the mean will converge at a faster rate but the error covariance takes time to converge.

2.2 Main problem: EKF Localization

5.Which parts of (2) and (3) are responsible for prediction and update steps?

Ans:

For the prediction step the second equation in (3) is the responsibe where as for the update step the first equation in (3) is responsible.

6. In the maximum likelihood data association, we assumed that the measurements are independent of each other. Is this a valid assumption? Explain why.

Ans

No. It is not a valid assumption. The values may be conditionally Independent but not necessarily independent to each other. For localisation problem we conclude that landmark association becomes less prone to error if we have dynamic landmarks than static ones, therefore conditional independence is a good assumption rather than total independence for maximum likelihood data association.

7.What are the bounds for δm in (8)? How does the choice of δm affect the outlier rejection process? What value do you suggest for λm when we have reliable measurements all arising from features in our map, that is all our measurements come from features on our map? What about a scenario with unreliable measurements with many arising from so called clutter or spurious measurements?

Ans

Outliers are notoriously high or lower values from sensors due to malfunctioning or other problems. While we need to create a threshold on the likelihood estimates and the actual measurements. We can consider Mahalanobis distance as a threshold if crossed the measurements would be rejected. If we have unrealiable sensors or more likelihood of outliers we can keep a smaller value for δm. In that case the outliers will be rejected and if we have really good and reliable measurements we can keep the δm close to 1. We must know that making δm smaller is loss of information and should be done only when measurement failure is very highly likely and thus we need to have a trade off between how much information is important for our estimation to that of failure likelihood of our sensors.

8.Can you think of some down-sides of the sequential update approach(Alg 3)? Hint: How does the first [noisy] measurements affect the intermediate results?

Ans:

The sequential update works under the pretext that the previous estimate is a good estimate. However,if the previous estimate is by an outlier or noise the further estimates will be affected. This is a major drawback of Sequential update approach.

9. How can you modify Alg 4 to avoid redundant re-computations?

Ans:

We can modify Alg 4 by searching for symmetries in the uncertainty matrices. Thus by filling some of the matrix values without computing them as we have symmetries in there.

10. What are the dimensions of Vt and Ht in Alg 4? What were the corresponding dimensions in the sequential update algorithm? What does this tell you?

Ans:

Vt is a vector with length M*N, where N is the number of observation or (inlier indices) and M is the length of the measurement vector. In the sequential update algorithm it was a vector of the same length as the state and measurement vector. H_t has size MN*A where A= length of state variable vector and afterward its size becomes M*A.

Data Sets

1. map o3.txt + so o3 ie.txt

Ans1.

mean error (x, y, theta) = (0.000225, 0.000072, 0.001210)
mean absolute error= (0.002656, 0.003222, 0.002296)
total time =17.683149

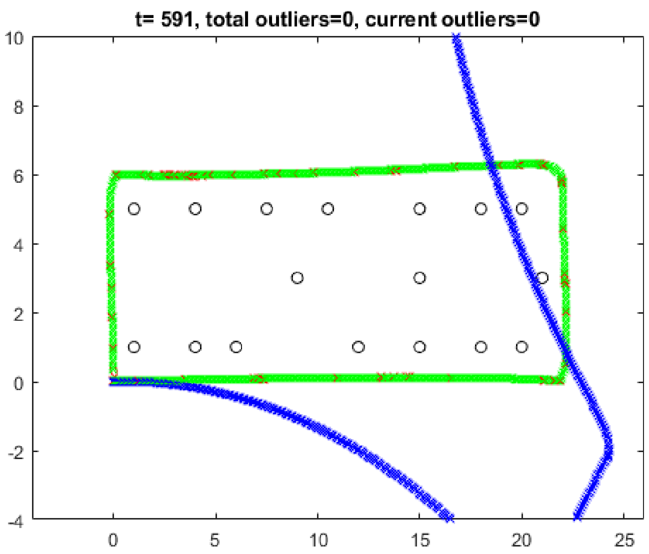

Figure 3: Estimate and actual motion trajectory

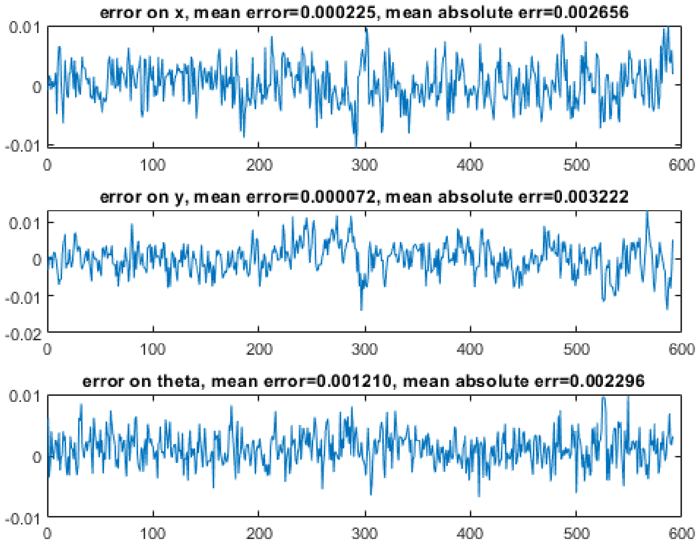

Figure 4: Error Propagation for x,y and θ.

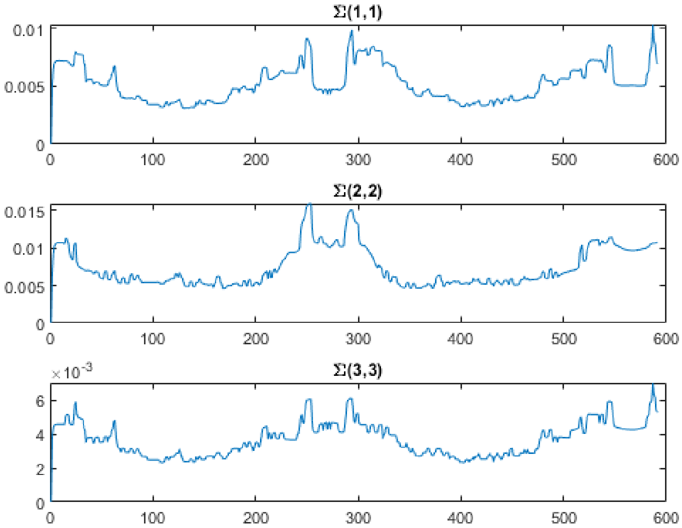

Figure 5: Covariance matrix

2. map pent big 10.txt + so pb 10 outlier.txt:

Ans 2.

We will do the following changes in init.m file : R=diag([0.1 0.1 0.1]).^2; (process noise covariance) and Q =diag([0:2; 0:2]).^2(measurement noise covariance)

mean error(x, y, theta)=(0.014452, -0.002937, -0.019592)
mean absolute error=(0.047913, 0.045565, 0.052118)
total_time =40.310067

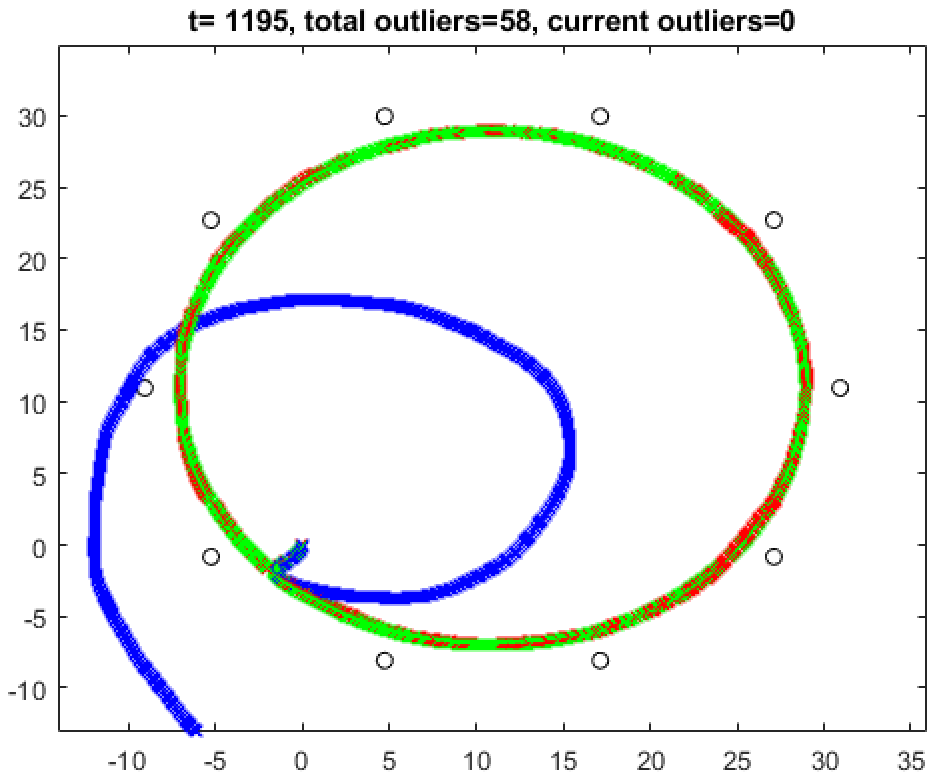

Figure 6: Estimate and actual motion trajectory

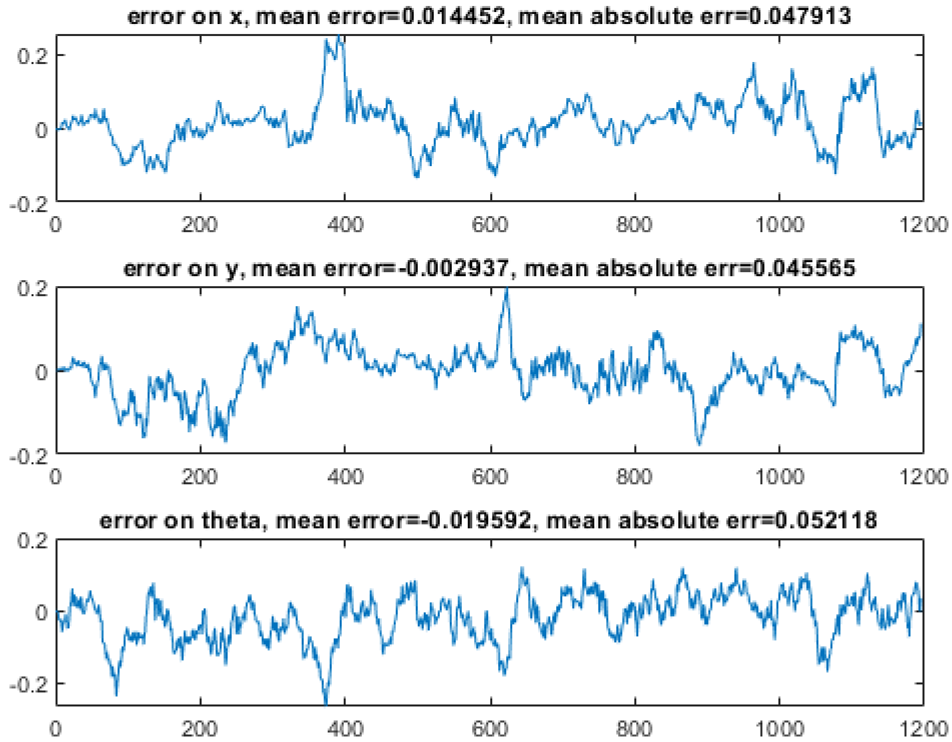

Figure 7: Error Propagation for x,y and θ.

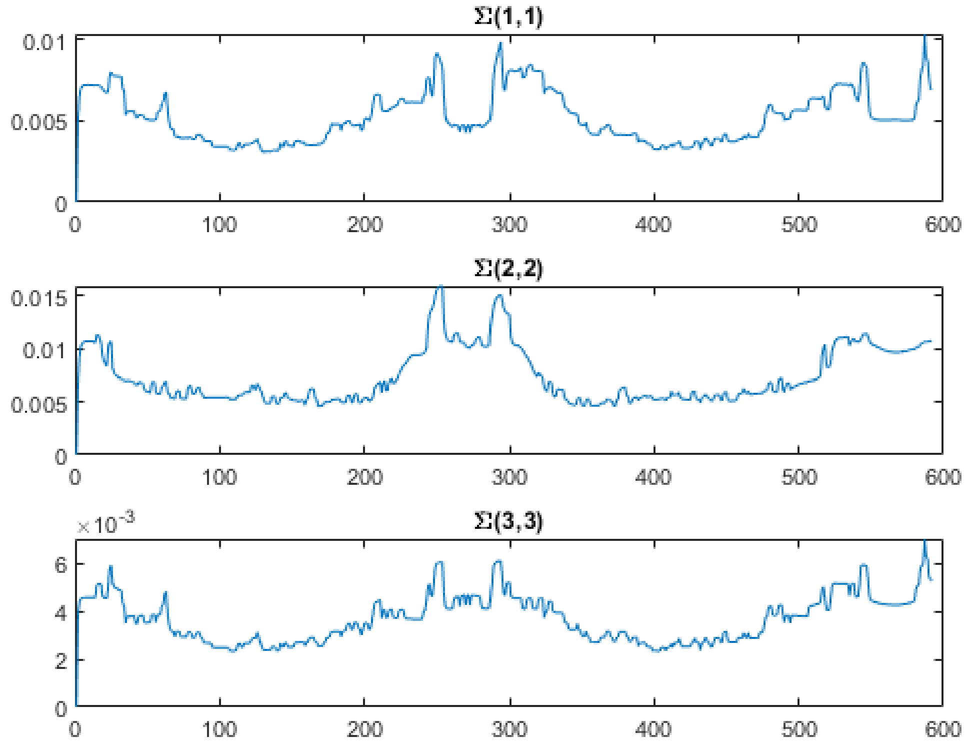

Figure 8: Covariance

3. map pent big 40.txt + so pb 40 no.txt:

We will do the following changes in init.m file :
R=diag([1 1 1]).^2;% process noise covariance
Q=diag([0.1 0.1]).^2;% measurement noise covariance

For sequential update and without detecting outliers we have the following results.

mean error(x, y, theta)=(0.131768, -0.109652, -0.036107)
mean absolute error=(0.379373, 0.413503, 0.093050)
total_time =5.096438

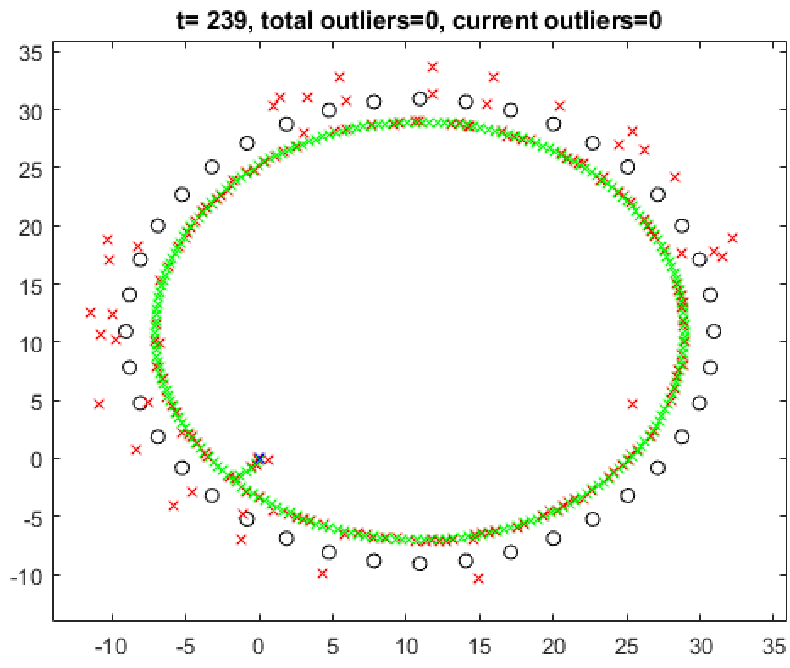

Figure 9: Estimate and actual motion trajectory

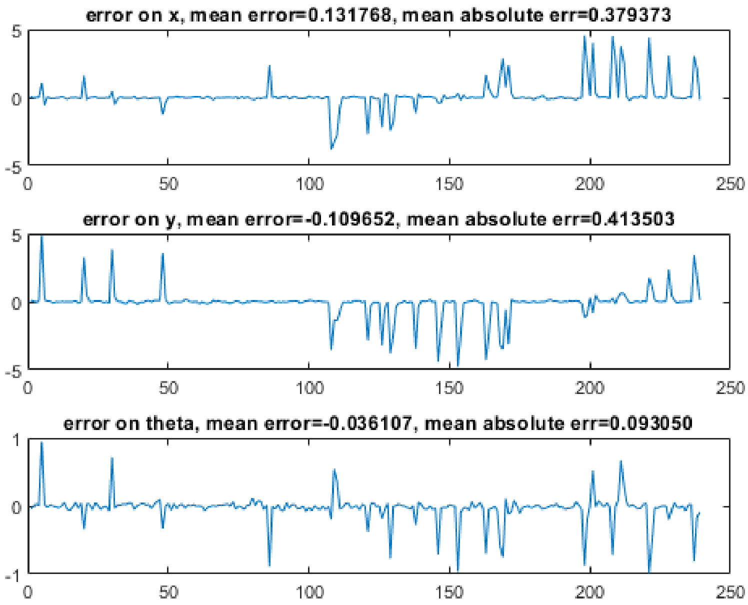

Figure 10: Error Propagation for x,y and θ.

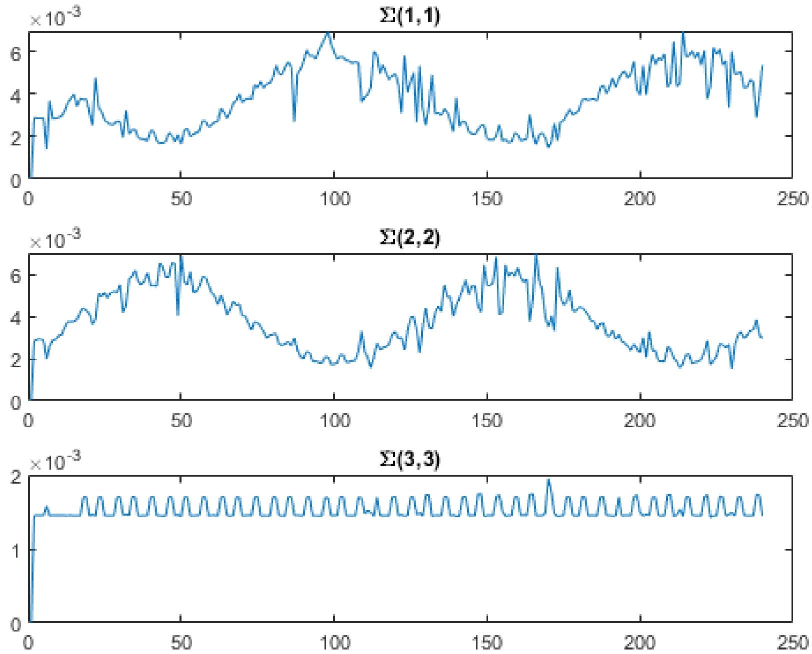

Figure 11: Covariance

For Batch update
mean error(x, y, theta)=(-0.026019, -0.022660, 0.018390)
mean absolute error=(0.080351, 0.088318, 0.047883)
total_time =6.026730

As expected, there is a significant difference (better for batch update) in the mean absolute error and also as expected the computation cost is high for the batch update.

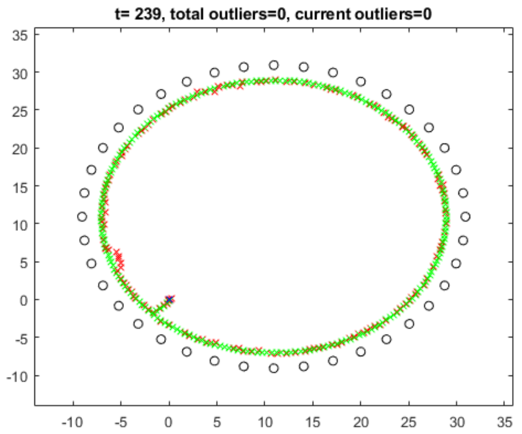

Figure 12: Estimate and actual motion trajectory

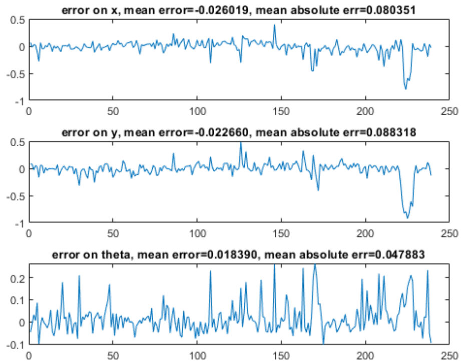

Figure 13: Error Propagation for x,y and θ.

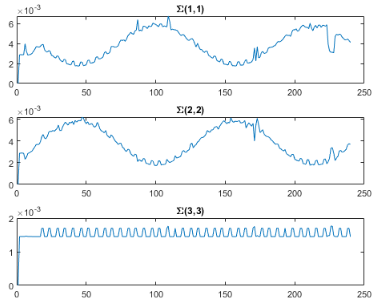

Figure 14: Covariance