Lab 2

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1.What are the particles of the particle filter?

Ans:

Particles in the particle filter are samples used to represent the posterior distribution of some stochastic process given noisy and/or partial observations. It is like the representation of the distribution with finite set of points. Particles are a representation of probability density function. These particles are possible paths/histories of the state over time. Thus the particles are an adaptive discrete representation of the posterior.

2. What are importance weights, target distribution, and proposal distribution and what is the relation between them?

Ans :

Wt = $P(z_t / x(hat)_t)$

They are the ration of target to proposal distribution, evaluated at the particles state. They (loosely) are the probability of the particle but cannot be 100% certain. They are the probability of resampling a particle needed to return the particle set to a sample from the posteriori.

3.What is the cause of particle deprivation and what is the danger?

Ans:

Resampling causes the density of particles to become thin in regions where there is significant probability that the true state may lie. Sample variance can cause there to be no particle in the important regions of the state space.

4. Why do we resample instead of simply maintaining a weight for each particle always ?

Ans:

The weights of some particles would drop to nearly zero while others would become very large. The diffusion(predict) state will not be able to populate the likely regions with the dense samples unless we make copies of the good particles and eliminate the bad ones.

5. Give some examples of the situations which the average of the particle set is not a good representation of the particle set.

Ans:

The average would be a bad representation for if we have a sample of extremely high values and extremely low values. Like for example we can have a case when the average is a bad estimate in a sample where the values are either very high or very low and thus very few or no values lie in the average region at all.

6. How can we make inferences about states that lie between particles.

Ans:

We can make inferences by Density extraction and by fitting a gaussian to the mean and variance of the particle set. We can create bins and count how many particles are in each bin to form a histogram. We can place so called kernel around each particle like that of a gaussian kernel.

7. How can sample variance cause problems and what are two remedies?

Ans:

The two sampling steps add randomness to the probability function. This is a sort of noise on our solution. Eventually repeated sampling will cause large errors. We can use more particles to delay the problem. We can inject random samples at each iteration to maintain particle diversity. We can use stratified sampling, that is resample in groups of clustered particles. We can also use low variance sampling. We can also delay the resampling step maintaining and updating the weights instead. Thus we resample when the variance exceeds a threshold.

8. For robot localization for a given quality of posterior approximation, how are the pose uncertainty (spread of the true posteriori) and number of particles we chose to use related.

Ans:

As the spread increases, we need more particles to maintain the quality of our estimate. As in the KLD adaptive resampling scheme.

PART II - MATLAB Exercises

2.1 Warm up problem with the Particle Filter

Question 1: What are the advantages/drawbacks of using (6) com-pared to (8)? Motivate.

Ans:

(6) has a constant angle where as (8) has a variable angle for defining the state. Thus, we see that (8) is little more complicated than (6) and is prone to errors, but it has the advantage of not only localising the robot but also can tell its orientation.

Question 2: What types of circular motions can we model using (9)? What are the limitations (what do we need to know/ x in advance)?

Ans:

$$\bar{u}_t = \begin{bmatrix} dx_t \\ dy_t \\ d\theta_0 \end{bmatrix} = dt \begin{bmatrix} v_0 \cos x_{t-1,\theta} \\ v_0 \sin x_{t-1,\theta} \\ \omega_0 \end{bmatrix}$$

With (9) we can only model the circular motion that is uniform and we need to fix the constant speed value as well as the constant angular velocity value beforehand. (this can be seen in the above image; the angular velocity and speed is kept constant)

Question 3: What is the purpose of keeping the constant part in the denominator of (10)?

Ans:

$$p(z|x, \Sigma_Q, \bar{C}) = \frac{1}{2\pi|\Sigma_Q|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}[z - \bar{C}x]^T \Sigma_Q^{-1} [z - \bar{C}x]\right)$$

The constant part in the denominator is kept for the purpose of normalisation.

Question 4: How many random numbers do you need to generate for the Multinomial re-sampling method? How many do you need for the Systematic re-sampling method?

Ans:

For the Multinomial re-sampling we need to generate M random numbers while for the Systematic resampling we need to generate only 1. As we see in the pseudo code for the multinomial re-sampling the random number generation is within the for loop where as it is not the case with the Systematic re-sampling method.

Question 5: With what probability does a particle with weight w=$\frac{1}{M}$+∈ survive the resampling step in each type of re-sampling (vanilla and systematic)? What is this probability for a particle with $0 \leq w < \frac{1}{M}$? What does this tell you? (Hint: it is easier to reason about the probability of not surviving, that is M failed binary selections for vanilla, and then subtract that amount from 1.0 to find the probability of surviving.

Ans:

As there will be M random numbers selection, we can see this problem as given M chances of drawing a particle, what is the probability of it being drawn which is equal to the particle surviving. Hence assuming that the particle is not drawn in M changes gives the value $(1-w)^M$Hence the survival probability would be $1 - (1-w)^M$ and it will be same for both cases. In the case of systematic resampling, the survival probability would be 1 as the value added for every iteration is $\frac{1}{M}$ hence the at least case is always satisfied as the there will be 1 particle always whose weight will be greater than separation. Where as in the case of vanilla resampling the probability of surviving will be proportional both to the weight of the particle and to the number of particles being drawn M, and thus the survival probability has to be Mw.

2.1.4. Experiments

Question 6: Which variables model the measurement noise/process noise models?

Ans:

Sigma_Q and Sigma_R are the variables used in the model to depict the measurement noise and the process noise respectively

Question 7: What happens when you do not perform the diffusion step? (You can set the process noise to 0)

Ans:

If we set the process noise to zero, all the particles that were sampled from previous steps would make the exact same movement and this process will continue with iterations until we have only one particle whose weight would be the highest and all other particles will converge to it.

Question 8: What happens when you do not re-sample? (set RESAM-PLE MODE=0)

Ans :

In the beginning we have a uniform distribution and then in futher steps the particles move according to motion model and noise and thus we get a different distribution, however the particles never converge to the real location.

Question 9: What happens when you increase/decrease the standard deviations (diagonal elements of the covariance matrix) of the observation noise model? (try values between 0.0001 and 10000)

Ans:

If the measurement covariance is very low the particles do not converge however if the value is more than 1 the convergence takes place and also we observe that for values very large, the noise also becomes large thus the variance is too high and particles are very spread.

Question 10: What happens when you increase/decrease the standard deviations (diagonal elements of the covariance matrix) of the process noise model? (try values between 0.0001 and 10000)

Ans:

If the value of process covariance is low it needs several time steps to converge, while if the process covariance is high the convergence is fast but the variance is high and the particles are very much spread.

Question 11: How does the choice of the motion model affect a reasonable choice of process noise model?

Ans:

Logically if we are very confident about my motion model, we will have to add less process noise where as if our motion model is approximate, we can add more process noise to keep the possibility of all the true positions around our estimation. Thus, the noise model depends on our confidence of the motion model.

Question 12: How does the choice of the motion model affect the precision/accuracy of the results? How does it change the number of particles you need?

Ans:

If our motion model is not very accurate, we need to take into account more particles spread so that all possible true positions are covered like we said in the Q.11, thus more the particles better is the prediction and coverage of all possible states however the computational power also increases if the number of particles are high and more particles also lead to higher accuracy for the real position.

Question 13: What do you think you can do to detect the outliers in third type of measurements? Hint: what happens to the likelihoods of the observation when it is far away from what the Filter has predicted?

Ans:

We can set a threshold for the likelihood of the observation if the threshold is too low we can reject it by considering it as an outlier.

Question 14: Using 1000 particles, what is the best precision you get for the second type of measurements of the object moving on the circle when modelling a fixed, a linear or a circular motion (using the best parameter setting)? How sensitive is the filter to the correct choice of the parameters for each type of motion?

Ans:

After playing with various parameters in the model, I am getting bad results for the fixed motion, where as in the case of circular and the linear give better results and more or less similar output errors, but circular giving much better result. Also I have concluded that process noise and number of particles should be very high for getting a better result from all three models but especially significantly high for the fixed motion model.

Question 15: What parameters affect the mentioned outlier detection approach? What will be the result of the mentioned method if you model a very weak measurement noise $|Q| \rightarrow 0$?

Ans:

The measurement noise and the threshold selected both affect the outlier detection problem. If we consider the instrument as very precise then a weak measurement noise would correspond to incorrect detection of outliers as the noise model would have large peak and very narrow deviations.

Question 16: What happens to the weight of the particles if you do not detect outliers?

Ans:

When the outlier is undetected the filter will give more weights to the outlier, which was incorrect and the algorithm would take more time to converge.

Results

Firstly, particle filter gets affected by several choices which are as follows:

1. How often do we resample.: Efficiency of resampling can be measured by the variance of weights. The more varied the weights are the more we need to resample.
2. Sometimes we need to add noise to the observation and prediction model as highly peaked observations is like being too confident on our measurements. Over estimating noise is always better than underestimating it.
3. Recovery from failure : Sometimes particle filter wrongly estimates the object position because of symmetric maps or similarity in landmarks.If the particles disappear and there are no particles where measurement is observed. We can never track the object. Hence for every iteration we must randomly put out some particles everywhere.

map_sym2.txt + so_sym2 nk:

In this kind of scenario, we observe that the map has 4 symmetric landmarks. In this case the number of particles and particle bound has to be such that all the hypothesis survives. When we take 1000 particles, we observe some particle deprivation occurs and therefore filter doesn't keep all hypothesis as reliable. It is true that when we use more particles, we avoid the risk of particle deprivation and consider all possible hypothesis.

Regarding sampling, in systematic resampling more hypothesis survive as compared to the multinomial resampling as systematic resampling only considers one random number and then adds 1/N for the resampling step for every particle performed where as in multinomial resampling we consider random number for every particle which leads to high chances of missing some hypothesis as the process is completely randomised.

For measurement noises, stronger measurement noise is always beneficial than being too confident on the measurement by setting weaker measurement noise.

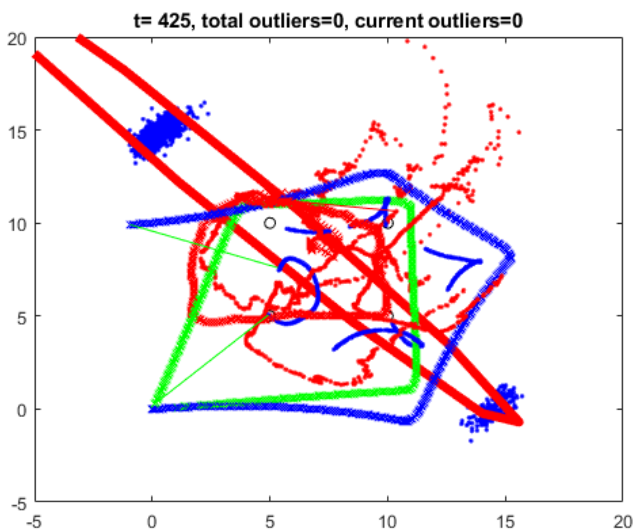

Figure: 1000 particles systematic resampling when R = 0.1 and Q =1 part bound 20 and global localisation

As we increase the particles we find that more hypothesis is preserved and we get better results as shown below.

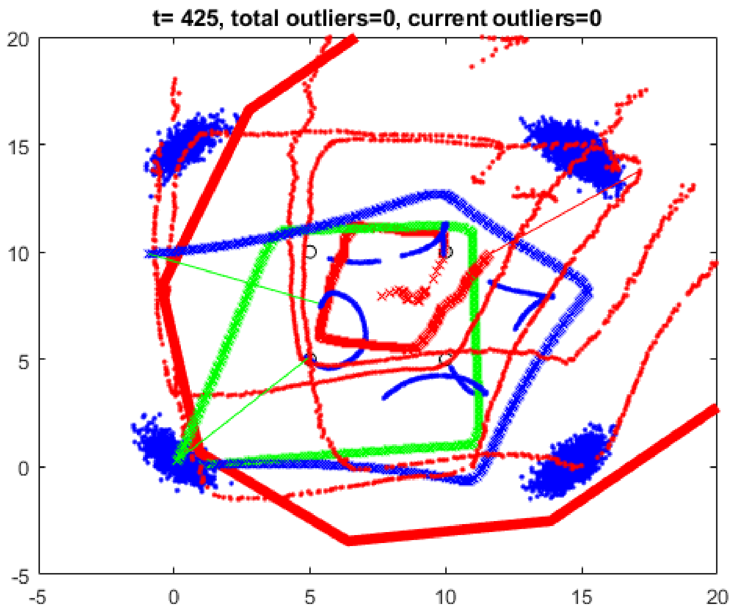

Figure: 10,000 particles Systematic resampling when R = 0.1 and Q =1 and part bound = 40 and global localisation

We observe that in systematic resampling when more particles are spread more hypothesis is preserved. And clearly, we see from the Image below that using same parameters except changing the sampling from systematic to multinomial we get better hypothesis in the case of systematic resampling as shown below.

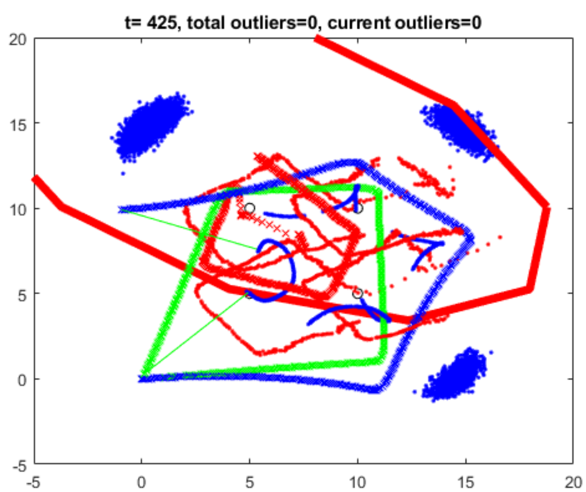

Figure: 10,000 particles multinomial resampling when R = 0.1 and Q =1 and part bound = 40 and global localisation

Now we will see how the choice of Q affects our results. We observe that for higher values of noise covariance , the particle clouds are bigger and thus particle deprivation effect is smaller. We also observe that for systematic method all of the hypothesis survived as seen in the above figure. However, if the noise covariance is lower only one hypothesis is survived in both the resampling methods.

Therefore it is necessary to have a higher value of Q for better results and here we choose Q= 10 and we see the results below

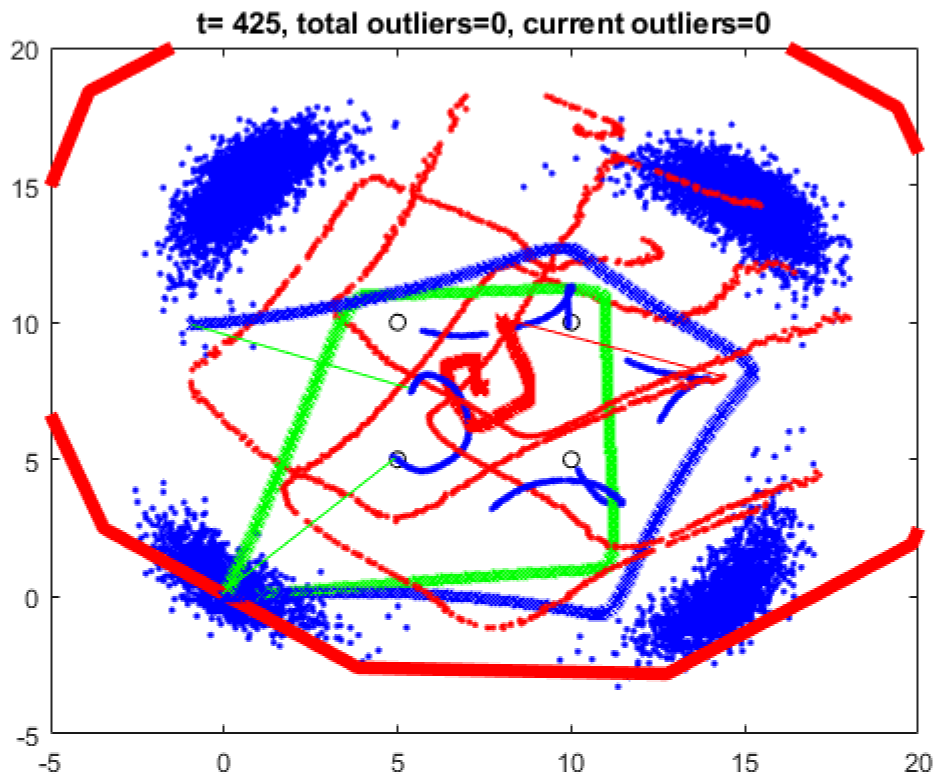

Figure : 10,000 particles multinomial resampling when R = 0.1 and Q =10 and part bound = 40 and global localisation

As we see above increasing Q gives dense particle clouds and all hypothesis is preserved even with multinomial resampling. Therefore we must keep a Q noise covariance a little more than our confidence in the model as we see that overestimation is always better than under estimation or particle deprivation where we lose hypothesis.

map sym3.txt + so sym3 nk:

Now we have a case where the map is unsymmetrical and therefore, we expect only one hypothesis to survive after the robot detects the 5th landmark. Thus, in the below figures we see as expected when the symmetry breaks only one hypothesis survives.

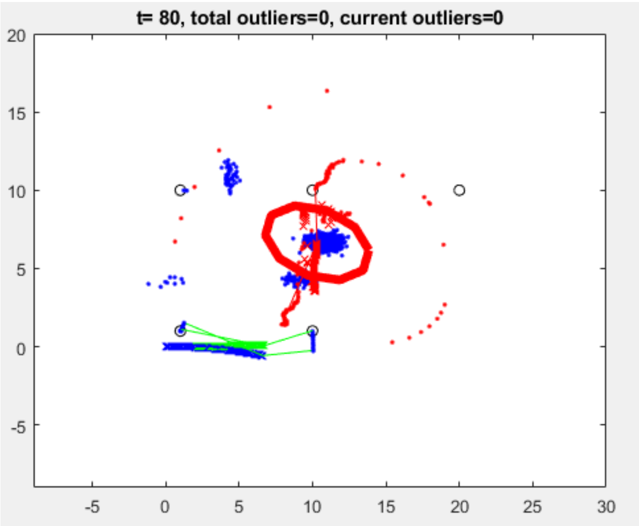

Figure: Approximately 3 hypotheses at t=80

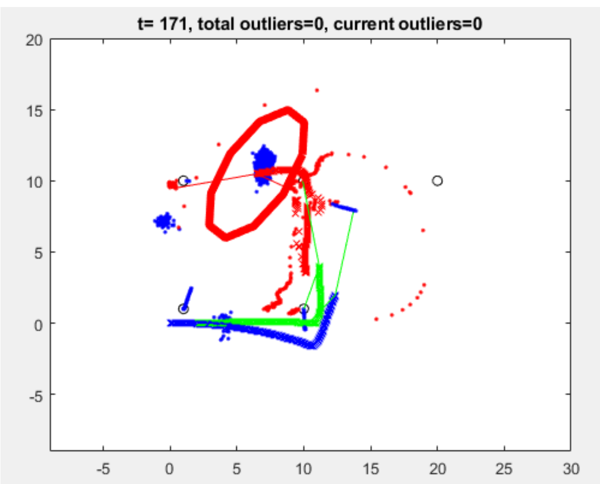

Figure: Approximately 2 hypotheses at t=171

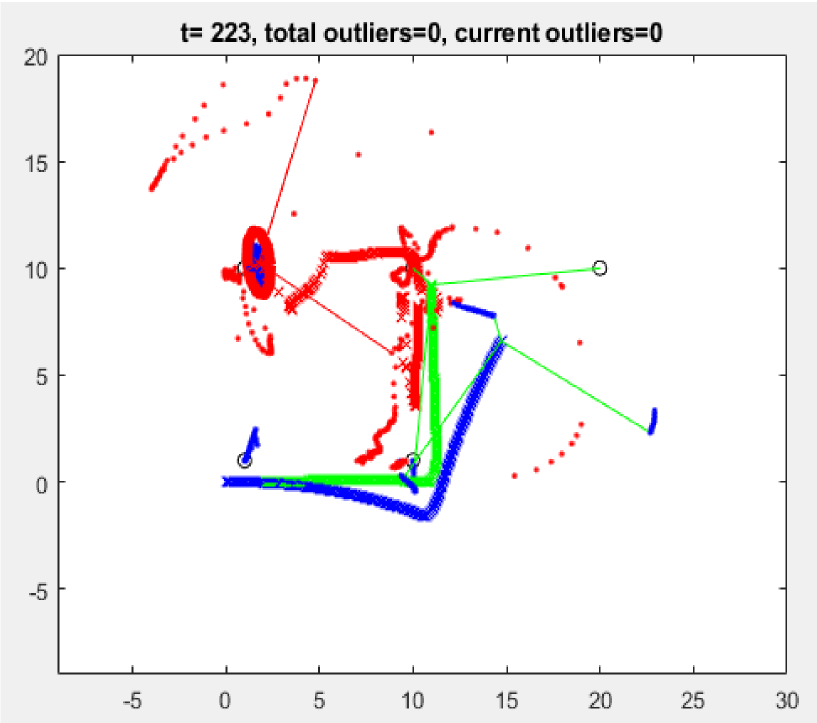

Figure: Only 1 hypotheis at t=223

Thus we see that only one hypothesis survives when the symmetry breaks and 5th landmark is observed.