Scalar Product

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1 An introduction

We want to modelise the effect of a force when there a displacement of a mobile. Suppose we ride a bicycle. Firstly, we go upstairs. We see that the gravity is againt us (which is negative from the point of view of the rider). If we modelise the action of gravity on us by the weight $\vec{P}$(which is indeed a force and so a vector pointing down) and our trajectory by the vector $\overrightarrow{AB}$ going from our starting point A to ending point B we see that our usual scalar product is negative. With the same king of modelling that our usual scalar product is positive if we ride downstairs. Indeed, the gravity plays in favour of us. If we go by a flat road, the scalar is null due to orthogonality of $\vec{P}$ and $\overrightarrow{AB}$. In this special case, gravity has no effect.

2 A definition

We begin by an undergraduate definition of the scalar product. We want to compute the scalar product of two vectors u and v denoted by $u.v$. A scalar product is a product so it has the following properties as a product of two numbers: $u.(v+w) = u.v+u.w$ (the effect of the sum of two forces is the sum of the effect like say the superposition principle) and also $(u+v).w$ for three vectors u, v and w . Don't forget (λu).. For the moment, we keep in mind the usual collegian notion of vectors. A (planar) vector is a couple of two (real) numbers, whose set is designed by $\mathbb{R}^2$. We usually denote a couple of two numbers by (u_1, u_2).