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Portfolios Under Construction

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Exploring The Risk Horizon

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The big picture

The choice of risk model horizon, or responsiveness, can dramatically affect portfolio performance. The class of portfolio, be it long-only, long-short, or optimised, is the most important factor when selecting the horizon.

Portfolio class is key

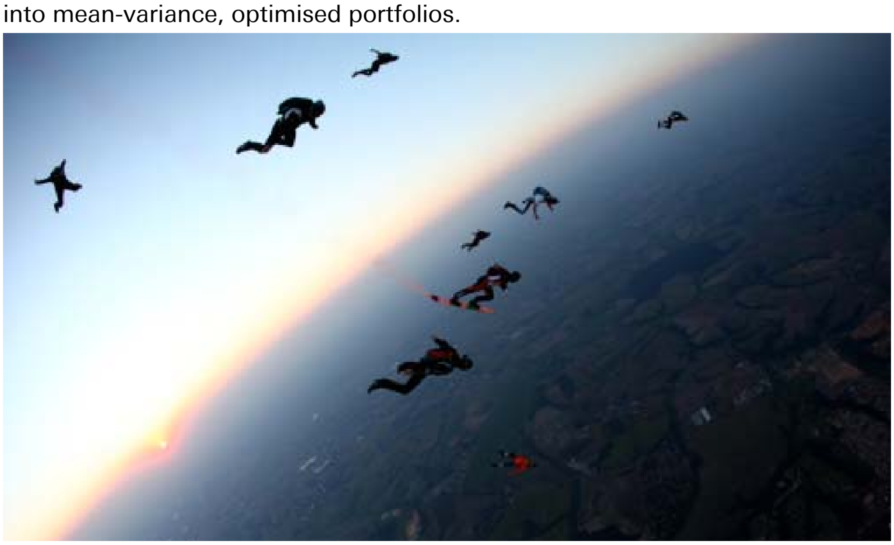
We show how the responsiveness of a risk model can significantly affect the accuracy of the volatility forecasts. However, the accuracy differs for different classes of portfolio. Our results show that more responsive risk models are typically more suited to long-only portfolios, while longer horizon models are normally more appropriate for long-short and optimised portfolios. Rebalancing frequency and transaction costs are also examined.

The effects of forecasting error

We illustrate how under- or over-forecasting of risk naturally affects portfolio leverage when risk-targeting. This in turn can affect the performance of all classes of portfolio. It can also affect the integration of alpha into mean-variance optimised portfolios. Further, our results illustrate how longer horizon risk models can over-forecast risk following a market shock, which in turn affects performance at times when volatility is often trending down and markets are trending up.

Horizon selection algorithm

We have developed a simple prototype algorithm to select the horizon depending on market conditions. The algorithm identifies periods following market shocks where less responsive risk models tend to over-forecast risk. Results indicate the algorithm can be of particular use with regard to the performance of long-only portfolios, and can increase the integration of alpha into mean-variance, optimised portfolios.



Source: Getty Images

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Introduction

Risk models have become ubiquitous in the investment community. Having initially been the exclusive domain of hard-core quants, they are now used extensively from high-frequency traders through to actuaries and pension funds, and are the cornerstone of investment risk management and regulatory reporting. Their uses include portfolio construction, stock screening, risk and return attribution, stress-testing, the determination of trading limits, and risk reporting. The models can range from simple, spreadsheet based calculations, to intricate multi-factor models that incorporate a plethora of advanced numerical techniques. Vendors typically offer risk models labelled with different 'horizons', such as 'short', 'medium' or 'long', which reflect how quickly a model responds to changing market conditions, and in turn are often assumed to imply suitability for an investor's respective horizon and turnover mandate. In this report we investigate to what extent this is true, exploring the effects model responsiveness has on risk forecasts for single-stock, long-only, long-short, and minimum-volatility and mean-variance portfolios, and the implications for portfolio construction, performance, risk management, and reporting.

There is a large volume of research on risk forecasting and the assessment of its accuracy in the academic and professional literature. Risk model vendors have developed tools to assess risk models and forecasting accuracy (Briner and Connor, 2008; Connor, 2000; Alvarez et al., 2012; Menchero et al., 2013), while in academia there is a large volume of research on volatility forecasting (Andersen et al., 2010; Diebold and Mariano, 1995; Taylor, 1994) and identifying unbiased volatility accuracy measurements (Patton, 2011). Further, modern portfolio theory and numerous portfolio construction techniques have stemmed from Markowitz' (1952) pioneering work, and are a staple of professional investment management. Much research in this field has concentrated on minimum volatility and mean-variance optimized portfolios (Ledoit & Wolf, 2003; Michaud, 1989). However, to our knowledge there is little research connecting risk forecasting accuracy and the ensuing effects on portfolio construction and performance.

In the first part of this report (page 3) we examine the forecasting accuracy of single stock portfolios when using risk models with different levels of responsiveness. We also briefly examine approaches to evaluating risk forecasting accuracy with the aid of a simulation study. We then analyse multivariate long-only, long-short (dollar neutral), and mean-variance optimised portfolios. We then extend the analysis to look at different rebalancing frequencies, moving from monthly to annual rebalancing.

The second part of this report (page 32) moves from looking at volatility forecasting accuracy to the implications for portfolio performance. The implications for portfolio turnover and transaction costs are also examined.

In the final section (page 40) we develop a simple prototype algorithm to determine appropriate model responsiveness based on market conditions. We then analyse the effects on performance and volatility forecasting accuracy.

Risk forecasting

The primary function of a risk model is to provide a forecast of the uncertainty of returns, typically in the form of the standard deviation. The calculation of the standard deviation is straightforward for a set of data that has no missing values, and is independent and identically distributed (i.i.d.), or stationary. However, financial returns time-series are known to be non-stationary, with market-wide volatility regimes including the relatively tranquil periods of 2003-2006, highly volatile periods such as the 2008 financial crisis and the recurring European sovereign debt crisis, and firm-specific events such as the recent Volkswagen emissions scandal and Glencore's debt-cutting program. Accordingly, the true volatility of a stock or portfolio is a somewhat enigmatic, latent variable.

There is an abundance of models to estimate equity volatility in the literature, including a huge range of ARCH and GARCH models (Bollerslev, 1986; Engle, 2002) and various stochastic volatility models (Taylor, 1994; Taylor, 2007) as well as other techniques. However, in this report we restrict our attention to exponential weighting as it is the staple of volatility estimation in most commercially available risk models, and is the principle determinant of the responsiveness, or 'horizon' of these models.

In this report we restrict our attention to volatility estimated with exponential weighting.

Exponential weighting is central to volatility forecasting for most commercial risk models.

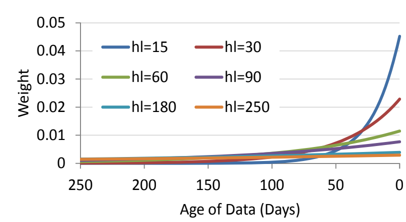
Exponential weighting

Exponential weighting is a simple, transparent method to compensate for non-stationarity in time-series data. When estimating the statistics at point T, exponential weighting applies greater weight to more recent observations. For a given data sample, the weight of the point at time t is given by:

$$w_t = 0.5^{t-T/hl} \tag{1}$$

where hl is the half-life[1], which determines the number of days counting into the past where the weight will halve. For example, with a half-life of 120 days the weights at points 120, 240 and 360 days into the past will have weights relative to that at point T of 0.5 w_t, 0.25 w_t and 0.125 w_t respectively. A shorter half-life corresponds to a greater degree of responsiveness. Various exponential weighting sets are illustrated in Figure 1.

Figure 1: Exponential weights with various half-lives



Source: Deutsche Bank

The variance at time T+1 is then given by:

$$\hat{\sigma}^2_{T+1} = a\hat{\sigma}^2_T + (1-a)r^2_T$$
$$a = 0.5^{1/hl} \tag{2}$$

where $\hat{\sigma}^2_T$ is the variance estimate for time T, r^2_T is the excess returns of an asset, and a is a weighting parameter.

[1] Unless explicitly stated otherwise, the half-life will always refer to the weighting used to estimate variance, or by extension, volatility. The half-lives used to estimate correlations are briefly discussed later.

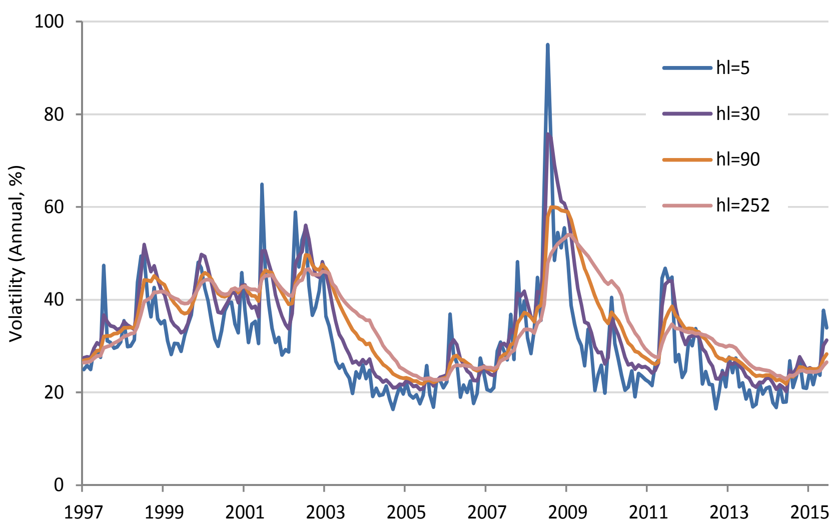
When estimating volatility with an exponential weighting scheme we can term an 'effective' estimation window of around 4 half-lives, as older data points will have less than 6.25% of the weight of the most recent observation. Accordingly, a volatility estimate with a 5-day half-life will have an effective estimation window of around 20 days, and would thus be sensitive to estimation error and outliers. Conversely, a volatility estimate with a half-life of 252 days implies an effective estimation window of approximately 1,000 days, and would tend to be more robust to outliers.

The 'effective' estimation window can be approximated as four times the half-life

Single stock portfolios

We first investigated single stock portfolios. This simplifies the analysis by restricting the estimation of asset volatilities to be a univariate process without the need to consider correlations. We based our analysis on assets in the MSCI Europe universe, and estimated the asset volatilities with exponential weights with half-lives of 5, 10, 15, 30, 60, 90, 180 and 252 (business) days. At all times care was taken to ensure we avoided survivorship bias. Figure 2 displays the average asset-level volatility when estimated with a sub-set of these half-lives. It is immediately clear that the volatility estimates can vary when using different half-lives.

Figure 2: Average asset volatility of MSCI Europe assets as measured with exponential weights with half-lives of 5, 30, 90 and 252 days



Source: Factset, Deutsche Bank

Bias statistic

Portfolio volatility forecasting accuracy is often evaluated using the bias statistic (Connor, 2000, Alvarez et al., 2012). In this report, in the context of equity risk forecasting, the bias statistics measures the standard deviation of asset returns that are standardized by the forecast volatility. The standardized returns are the z-scores:

The bias statistic measures the standard deviation of returns that are standardised by their forecast volatilities, thus the ideal value is 1

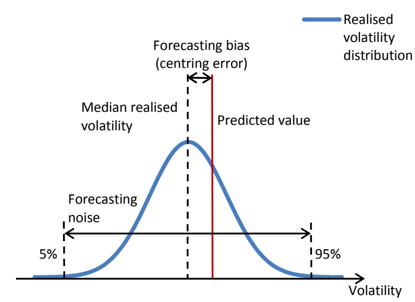
$$z_t = \frac{r_t}{\hat{\sigma}_t} \quad (3)$$

where r_t is the asset return for period t, and $\hat{\sigma}_t$ is the forecast volatility for the same period. The bias statistic at time T is then given by the standard deviation of the z-scores:

$$B_t = \sqrt{\frac{1}{N}\sum_{t=T-N}^{T}\left(z_t - \bar{z}\right)^2} \quad (4)$$

If the volatility forecasts are accurate, the bias statistic will be equal to one. Bias statistic values above (below) one indicate that volatility is over- (under-) forecast.

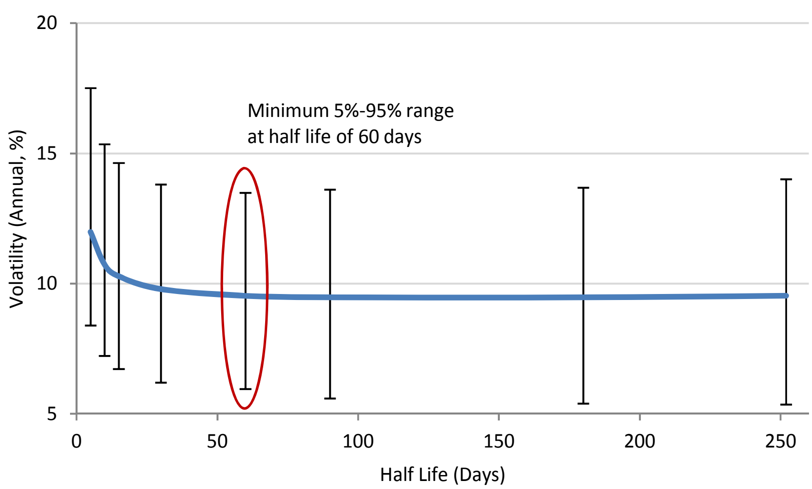
Figure 3: Schematic distribution of realised volatilities vs. forecast volatility



Source: Deutsche Bank

We first use a simplified version of the bias statistic, by rebalancing every month and adjusting the leverage of portfolios such that they have an ex-ante volatility of 10%. We can then simply measure the realised volatility of the portfolios and compare it to the targeted 10%.

Figure 4: Median realised annual volatility of European assets (within MSCI Europe; June 1995-December 2015) rebalanced monthly to target 10% ex-ante volatility vs. half-lives. Error bars: 5%-95% range



Source: Axioma, Factset, Deutsche Bank

Figure 4 displays the average realised volatility of the single stocks when leverage was adjusted to achieve an ex-ante volatility of 10%. We see that with short half-lives the realised volatility is above 10%, indicating that volatility is, on average, under-forecast, whereas for half-lives of 30 days or more the volatility is below 10%, implying a tendency to over-forecast risk.

The error bars in Figure 4 correspond to the variation in the volatility estimates, depicting the 90% range from 5% to 95%. While the average realised volatility corresponds to the overall bias in the risk forecasts, the spread of the error bars indicates the noise in the forecasts. This is similar to estimation error, and is akin to the risk associated with a risk estimate, termed 'second order risk' by Shepard (2009). The noise is slightly larger when using very short half-lives of 5 or 10 days or longer half-lives of 125 or 252 days. The minimum noise is found when using a half-life of 60 days.

While Figure 4 indicates that a half-life of 30 days can give the most accurate aggregate volatility forecasts, this does not tell the whole story. Measuring the realised volatility over a full history lacks temporal resolution; it is possible that periods of over-forecasting can cancel periods of under-forecasting.

Simulation Study

A problem with non-stationary data such as equity returns is that the true volatility can never be known, thus the evaluation of the accuracy of equity volatility forecasting is not straightforward. To address this problem we performed a study based on simulated equity returns with known volatility.

The simulation study was performed to gain insight into volatility forecasting accuracy. Exact historical replication of historical equity data was not required of the simulation; rather a reasonable proxy.

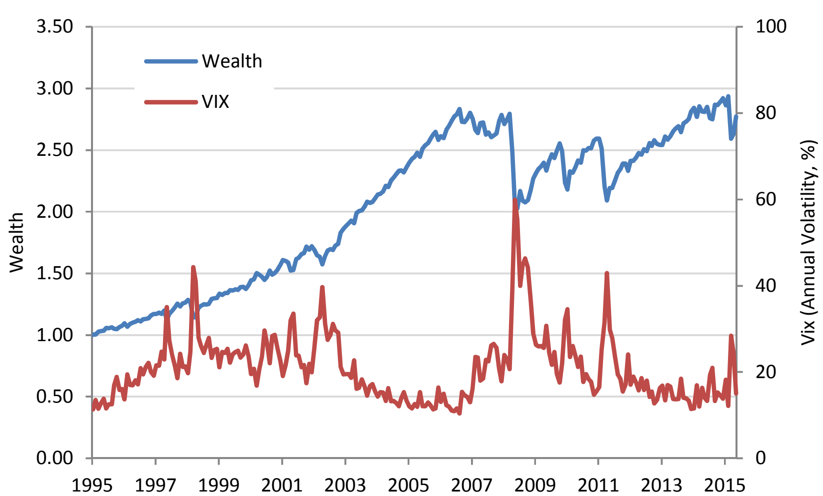
We sought to simulate asset returns that are a reasonable proxy for realised equity returns over the previous 15-20 years. To this end we used the Heston stochastic volatility model (Heston, 1993) and VIX data from 1997-December 2015. The returns were generated using Equation 5:

$$r_t = qdt + V_t\left(\sqrt{1-\rho^2}\,dW + \rho dV_t\right)\sqrt{dt} \quad (5)$$

where q is the average return, V_t is the annual volatility at time t taken from the VIX, dW is a random draw from a standard normal distribution ($N(0,1)$), dV is the standardised relative change in the VIX, and ρ is the correlation between the change in volatility and the asset returns, which is set at -0.46 in accordance with Shu and Zhang (2003). The negative correlation is intuitive, in that increases in volatility are typically associated with negative returns, and vice-versa (Zanutto, 2014).

Figure 5 illustrates the average wealth curve of 1,000 asset returns simulated according to Equation 5 from May 1997-present. The wealth curve illustrates that the simulated returns are akin to real historical returns. Exact replication of historical equity returns was not required here; rather, we sought an approximation of the returns generating process to gain insight into volatility forecasting accuracy.

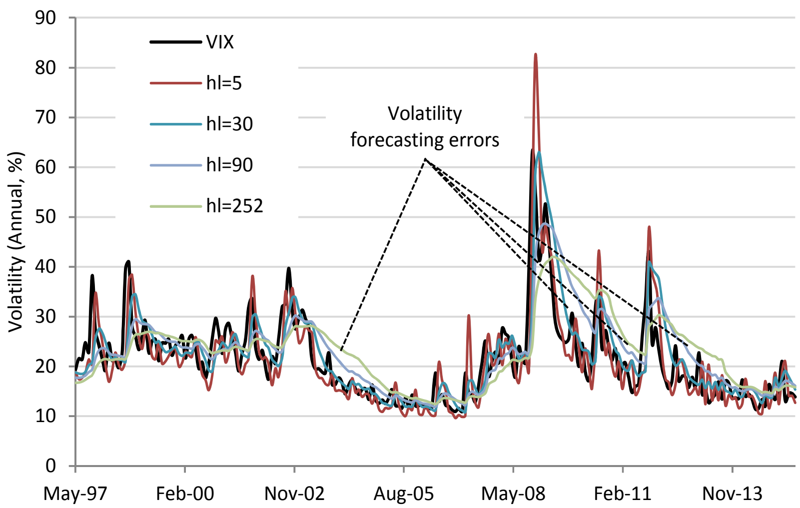
Figure 5: Average wealth curve of simulated assets and the VIX



Source: FactSet, Deutsche Bank

We investigated the volatility forecasts of 1,000 simulated asset returns generated using Equation 5, with the VIX as the underlying volatility. At the end of each month the volatility of each stock was measured using Equation 2, with half-lives of 5, 10, 15, 30, 60, 90, 125 and 252 days.

Figure 6: VIX and one month lagged volatility measurements (forecasts) of simulated assets as calculated with different half-lives

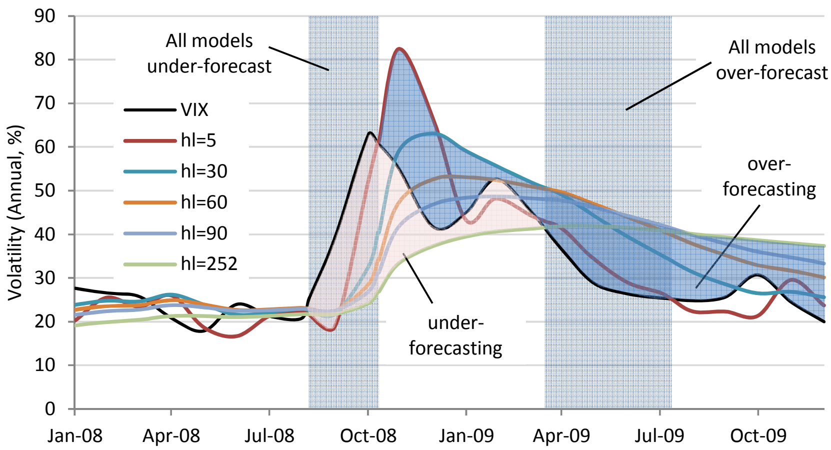


Source: Factset, Deutsche Bank

In Figure 6 the VIX corresponds to the known underlying volatility of the simulated assets.

Figure 6 illustrates the average of the estimated volatility of the 1,000 simulated assets for the full history from May 1997 to December 2015 when using different half-lives, along with the VIX. We see spikes in the VIX associated with periods such as the global financial crisis and the European sovereign debt crisis. As in this case we know the true volatility of the simulated assets, we can explicitly see how errors in the volatility estimation manifest around market shocks. It is also clear that the volatility estimates diverge at these points.

Figure 7: VIX and one-month lagged average volatility measurements (forecasts) of simulated assets as calculated with different half-lives around the 2008-2009 financial crisis



Source: FactSet, Deutsche Bank

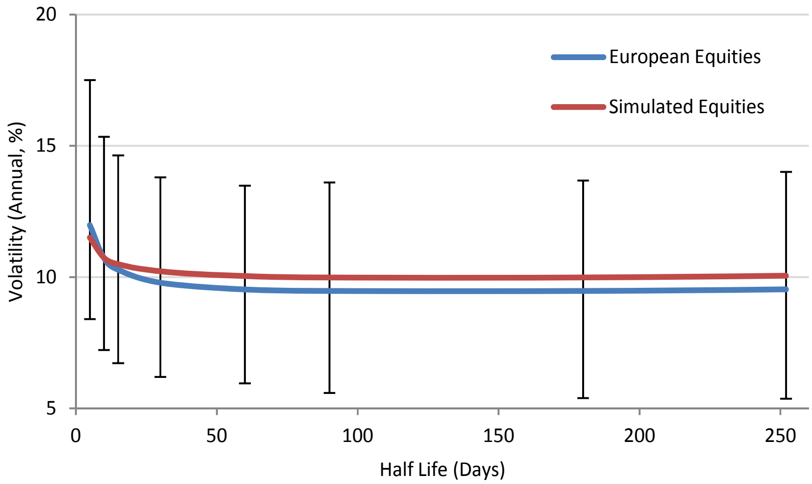
Figure 7 illustrates how volatility is typically under- then over-forecast around a market shock, indicating that the errors can cancel out over a long period, and how over-forecasting can persist following a shock, particularly with long half-lives.

Figure 7 focuses on the period surrounding the peak of the financial crisis in 2008, which more clearly illustrates typical behaviour of volatility estimates based on exponential weighting around a volatility shock. Although the data is based on simulations, it is nonetheless informative:

- All models under-forecast the onset of the shock in Sep '08-Nov '08
- The most responsive model, with a half-life of 5 days, takes approximately 2 months 'to 'catch up' with the true volatility, and then over-shoots
- The least responsive model, with a half-life of 252 days, under forecasts for approximately 7 months Sep '08-Apr '09, with a maximum under-forecast of approximately 40%
- Following April 2009, when the volatility drops below 40%, all models over-forecast for at least 4 months
- The model with a 252 day half-life over-forecasts volatility from April 2009-July 2011, when the European sovereign debt crisis occurred
- The over-forecasting of the model with a 252 day half-life is of lower magnitude than the under-forecasting, at an average of 10%

Given the under-forecasting around the market shock followed by over-forecasting as the shock dissipates, we can infer that the errors may cancel each other out over a long horizon when using realized volatility or the bias statistic. Accordingly, alternative methods to assess periods of significant under- or over-forecasting are warranted.

Figure 8: Average realised annual volatility of 1,000 simulated and European assets (within MSCI Europe; June 1995-October 2015) rebalanced monthly to target 10% ex-ante volatility vs. half-lives. Error bars for European equities: 5%-95% range



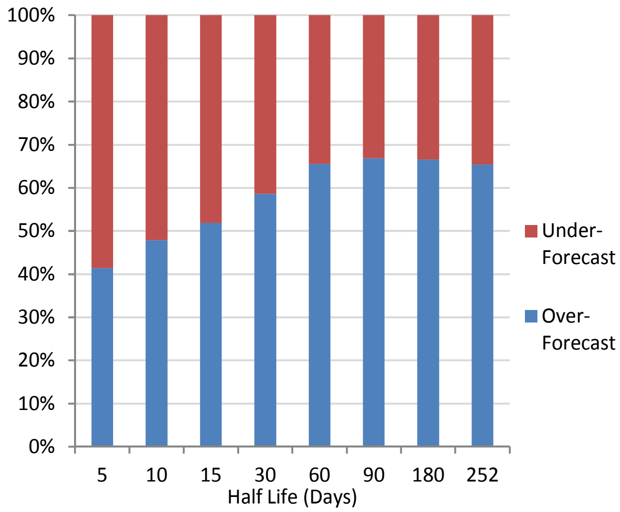
Source: Axioma, Factset, Deutsche Bank

Results for simulated equities are in good agreement with European equities, indicating the simulations can provide useful insights

Figure 8 is similar to Figure 4, but includes the realised volatility of the simulated assets when rebalanced to an ex-ante volatility of 10% based upon the different volatility estimates. There is close agreement in the measurements with the simulated and European equities, suggesting that the mechanisms responsible for driving the volatility estimation errors in the simulated data may be the same as those driving the volatility estimation errors in the European equity data.

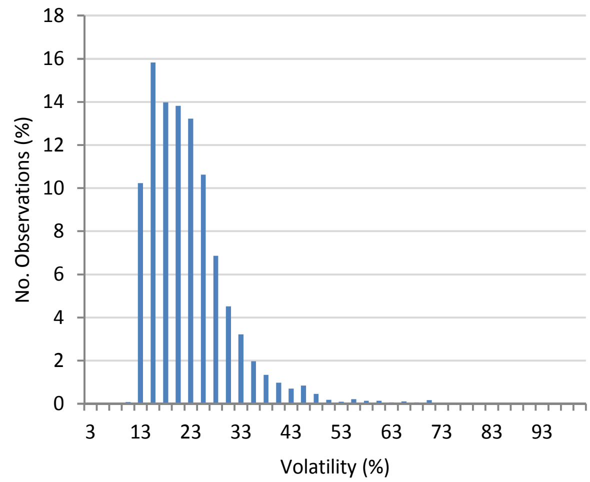
Having seen how risk is under- and then over-forecast around market shocks from the simulated data in Figure 6 and Figure 7 we have further evidence that realised volatility alone, and by extension the bias statistic, provides limited insight into volatility forecasting accuracy. Using the simulated data we can look at other methods to assess volatility forecasting accuracy.

Figure 9: Percentage of observations under- or over-forecasting risk vs. half-life for simulated equities



Source: Factset, Deutsche Bank

Figure 10: Distribution of VIX levels



Source: FactSet, Deutsche Bank

Figure 9 illustrates the percentage of months where volatility of the simulated equities is under- or over-forecast. Here we see that for very short half-lives of 5 and 10 days it is more likely that risk will be under-forecast, while as the half-lives get longer the risk is more likely to be over-forecast. From this perspective the optimal half-life would appear to be between 10 and 15 days, although this does not take into account the magnitude of the forecast error at each point.

The tendency to over-forecast is caused by volatility outliers or 'clusters'

Figure 10 depicts the histogram of volatilities observed in the VIX. We see the high degree of right skew due to short-lived high volatility bursts around market shocks. This is consistent with observations that price volatility tends to be mean-reverting, with clustering of volatility spikes. With respect to Figures 6-10, we can infer that instances of over-forecasting risk are generally of a small magnitude, and can be persistent, whereas under-forecasting tends to be more severe and short-lived around the onset of market shocks.

Using the wrong horizon can be misleading for risk reporting and control

Taking the above results together we can see that the timing and duration of measurement window can be critical from a risk reporting perspective. For example, if a reporting window is one year long and incorporates a volatility shock at the start, a portfolio manager who is too conservative, and does not take on enough active risk following the shock, may misleadingly appear to do a good job in adhering to a tracking error mandate.

Rolling bias statistics could also result in errors of opposite sign cancelling out

Considering this problem it is prudent to employ another function to evaluate the accuracy of risk forecasts. Using bias statistics with a rolling window would revert to the problem of errors cancelling each other out when the data is aggregated; e.g. a period with a bias statistic of 0.9, indicating over-forecasting, averaged with a period with a bias statistic of 1.1, indicating under-forecasting, yields a potentially misleading bias score of 1.

Using the absolute deviation of the bias statistic (from one) prevents positive and negative errors from cancelling each other out. Moreover, the absolute deviation is a loss function, and thus the interpretation is simple in that the lower the value, the better[2]:

Rolling absolute deviation of bias statistic prevents errors of opposite sign cancelling out

$$AD_t^{(Bias)} = |B_t - 1| \quad (6)$$

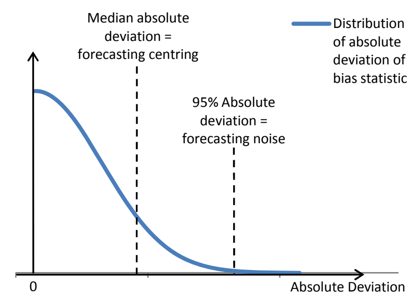
The absolute deviations can be aggregated across a cross-section, and then in turn across time-series when ensuring non-overlapping samples of z-scores are used. Here we use a sample of 12 months for the bias statistics, which allows both temporal resolution and a sample size that is in-line with typical industry practice for volatility measurement.

The simulations also allow us to evaluate the absolute deviation of the forecast volatility from the true (VIX) volatility:

$$AD_t^{(VIX)} = |\hat{\sigma}_t - VIX_t| \quad (7)$$

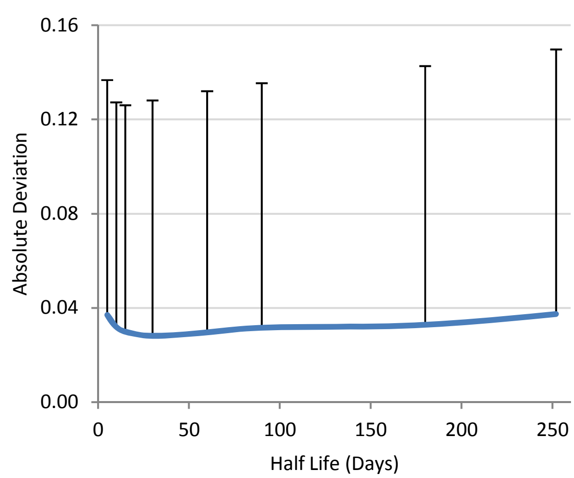
Figure 12 illustrates the median absolute deviations of the volatility forecasts from the true volatility (VIX) for different half-lives. Here we see a minimum average value at a half-life of 30 days, while the 95th percentile of the absolute deviations is at a minimum with a half-life of 15 days. However, the 95 per cent absolute deviation values are very similar at half-lives of 10 - 30 days.

Figure 11: Schematic distribution of absolute deviation of bias statistic



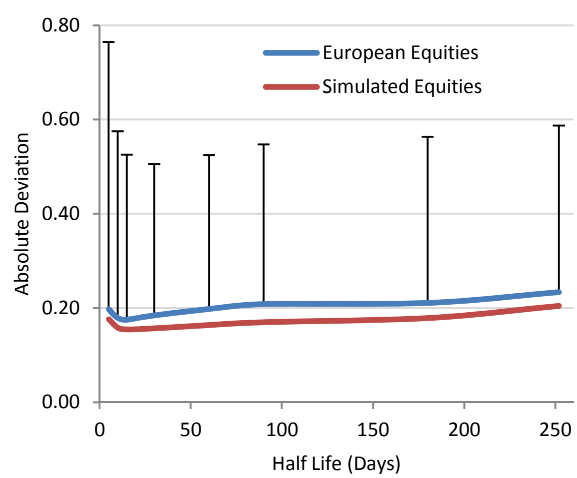
Source: Deutsche Bank

Figure 12: Median absolute deviation from true volatility (VIX) of simulated equities vs. half-life. Error bars: 95% values



Source: Factset, Deutsche Bank

Figure 13: Median absolute deviation of bias scores vs. half-life. Error bars: 95% values



Source: Axioma, Factset, Deutsche Bank

[2] The evaluation of volatility forecasts is contentious, with numerous approaches described in the literature. Patton (2011) has addressed the issue from a rigorous theoretical perspective and defined a set of 'loss functions' to evaluate volatility forecasting accuracy, one instance of which has been adopted by Menchero et al (2013). However, we find the cross-sectional and time-series data generated by these functions to be noisy, with very high kurtosis and skew, to be difficult to interpret, and to vary according to subjective user-defined parameters. Accordingly, we base the forecasting evaluation on the bias statistic and the derived absolute deviation as these metrics provide intuitive and interpretable results, and rely on measurements that are in-line with industry practice.

Figure 13 displays the median absolute deviations of the Bias scores for different half-lives. The error bars correspond to the European, rather than simulated data. Here the lowest median absolute deviation is observed with a half-life of 15 days for both the European and simulated data[3]. When considering the 95% value, as depicted by the error bars, the minimum occurs when using a half-life of 30 days; both for European and simulated equities.

Comparing Figure 12 and Figure 13 again illustrates that the simulated and European equities share similar characteristics. The error measurements relative to the 'true' volatility of the VIX of the simulation studies in Figure 12, and the absolute deviation of the bias statistics of the simulated and European equity data in Figure 13 produce comparable results with respect to optimal half-life. The median and 95th percentile errors increase gradually with half-lives from 30 days onwards for both simulated and European equities, and with respect to $AD^{(VIX)}$ and $AD^{(Bias)}$. Further, for $AD^{(Bias)}$ the minima occur at half-lives of 15 days for the median and 30 days for the 95% levels for simulated and European data alike. The comparability supports the validity of using the absolute deviation of the bias scores to evaluate volatility forecasting accuracy.

'Forecasting centring' versus 'forecasting noise'

Results so far have been presented in terms of average (median) levels of the bias statistic and absolute deviations, and a spread measurement, either with a 5th-95th percentile spread with regard to realised volatility and bias scores, or a 95th percentile level with regard to the absolute deviation. In this report we colloquially refer to the average accuracy of the forecasts as 'forecasting centring' and the range measurements of the forecasts as forecasting noise.

An interpretation of 'forecasting centring' is simply the average, or median value of the forecasting accuracy. The 'forecasting noise' refers to the spread. We have seen that for a given measurement (e.g. the absolute deviation in Figure 13), the optimal half-lives for forecasting centring and forecasting noise are different. This is similar to the so-called 'bias-variance tradeoff' in the statistical literature. However, we use forecasting centring and forecasting noise here to avoid possible confusion with the bias statistic.

Use multiple metrics to diagnose volatility forecasting behaviour

We have seen how using realised volatility and by extension, the bias statistic, can mask periods of over- and under-forecasting. Using the 5th-95th percentile spread of the realised volatility or bias statistic will only provide insight into forecasting noise. The median and 95% levels of absolute deviation of the bias statistic also provide insights into noise in the volatility forecasting, and can overcome the problem of errors cancelling out, but do not provide information on systematic bias. Accordingly it is prudent to use all four metrics, possibly combined with attribution, to gain maximal insight into volatility forecasts.

Choosing a single parameter to optimise forecasting accuracy

We argue that optimising forecasting noise is more beneficial than optimising forecasting centring.

When considering a single metric to determine the optimal forecasting accuracy we argue that optimising the forecasting noise is of most value, and that the absolute deviation is of more value than the 5th – 95th percentile spread of realised volatilities. This is assuming there is not a gross modelling error that leads to large, systematic bias; commercial and well designed risk

[3] These are lower optimal half-lives than would be derived from the average realised volatilities seen in Figure 8

models should avoid such biases, with the possible exception of the case of minimum volatility or mean-variance optimised portfolios. When seeking to minimize forecasting noise we are looking for the forecast that is most likely to be closest to the real volatility for the majority of observations, rather than the average of many observations. With reference to the noise of the absolute deviation of the bias statistic, we are seeking the forecast where 95 per cent of the observations are closest to the true volatility.

What have we learned so far?

At this point we have three different sets of results, each providing different optimal half-lives and insights with respect to exponentially weighted volatility forecasting. The results differ because of the manner in which measurements and errors are weighted and aggregated across time. Figure 14 summarises the results so far.

Figure 14: Optimal half-lives for single stock volatility forecasting for centring or noise

Test	Optimal Half-Life (Days)		Notes
	Centring	Noise	
Realized Volatility	30	60	Realized volatility relative to target vol of 10% Errors of opposite sign can cancel each other out Good agreement with simulated data
Absolute Deviation of Bias Statistic	15	30	Errors of opposite sign do not cancel Good agreement with simulated data
Under/Over Forecast Percentage	10-15	-	Simulated equities only Percentage of months over- or under-forecast Does not consider magnitude of errors

Source: Axioma, Factset, Deutsche Bank

Long-only and long-short strategy portfolios

Having examined the implications of different levels of responsiveness on single stocks, we can expand the analysis to multi-asset portfolios. Here we separate portfolios into two classes: long-only and long-short (dollar neutral).

Portfolios

Long-only portfolios based on Axioma market, country, and industry factors

We constructed 26 long-only portfolios based on the factors in the Axioma European equity risk factor model. We restricted our universe to assets in the MSCI Europe index, and defined a set of equal-weighted country and industry portfolios. Each country portfolio was comprised of stocks with the associated country of exposure as defined by Axioma. We defined ten industry portfolios according to the classification of each stock to the ten GICS sectors. We also used a 'market' portfolio, which is an equal-weighted portfolio with the constituent assets of the MSCI Europe index.

Long-short (dollar neutral) portfolios based on Axioma style factors and 5 DB quant strategy factors

For long-short portfolios we used the eight style factors defined by Axioma, and five DB Europe Quant Strategy factors. Again we restricted the universe of assets to those within the MSCI Europe index. Each portfolio was constructed to be dollar neutral at each rebalancing point; long the equal-weighted quintile of stocks with the highest exposures to a given factor, and short the equal-weighted quintile of stocks with the lowest exposures. The list of long-only and long-short portfolios is summarized in Figure 15.

Figure 15: Long-only and long-short portfolios

Long-Only		Long-Short	
Market	Countries	Axioma Styles	DB Quant Factors
Market Factor	Austria	Growth	Growth
	Belgium	Leverage	Low Risk
	Denmark	Liquidity	Momentum
	Finland	Medium-Term Momentum	Quality
Industries	France	Short-Term Momentum	Value
Consumer Discretionary	Germany	Size	
Consumer Staples	Ireland	Value	
Energy	Italy	Volatility	
Financials	Netherlands		
Health Care	Norway		
Industrials	Portugal		
Information Technology	Spain		
Materials	Sweden		
Telecommunication Services	Switzerland		
Utilities	United Kingdom		

Source: Axioma, Deutsche Bank

Implementation

For portfolio estimation a factor model is used. Factor models estimate portfolio variance as follows:

$$\hat{\sigma}_P^2 = p^{\prime}\hat{C}p$$
$$\hat{C} = X^{\prime}\hat{F}X + \hat{S} \quad (8)$$

where $\hat{\sigma}_P^2$ is the portfolio variance, p is the N vector of portfolio asset holdings, $\hat{C}$ is the N x N asset forecasting covariance matrix, X is the K x N matrix of asset exposures to factors, $\hat{F}$ is the K x K factor forecasting covariance matrix, $\hat{S}$ is the N x N diagonal asset specific variance matrix, N is the number of assets in the given universe, and K is the number of factors.

The foundation of equity factor models is the separation of systematic drivers of risk and return, the factors, from the idiosyncratic returns and risk that are specific to each security. A core principle is that specific returns are uncorrelated with each other, and are also uncorrelated with the factor returns.

In this report we re-construct equity factor models with different levels of responsiveness based on the structure of the Axioma European factor model. We re-calculated the covariance matrices and specific risk forecasts using Axioma factor and specific returns with half-lives of 30, 60, 90, 125 and 252 days.

As is standard for factor risk models, the factor covariance matrices are calculated with the weighted factor returns:

$$\hat{F}^{(d)} = R^{\prime}WR \quad (9)$$

where R is the T x K matrix of factor returns and W is the T x T diagonal matrix of exponential weights. This is a multivariate implementation of Equation 2 with respect to factor volatility.

In order to scale the volatility estimates from a daily to an annual horizon, the serial correlation was corrected using the approach developed by Newey and West (1987):

$$\hat{\mathrm{F}}^{(\mathrm{a})} = 252\left[\hat{\mathrm{F}}^{(\mathrm{a})} + \sum_{\lambda=1}^{L}\left(1-\frac{\lambda}{L+1}\right)\left(\hat{\mathrm{F}}^{(\mathrm{d})}_{+\lambda} + \hat{\mathrm{F}}^{(\mathrm{d})}_{-\lambda}\right)\right] \tag{10}$$

where $\hat{\mathrm{F}}^{(\mathrm{a})}$ is the annual-horizon factor covariance matrix, λ is the lag in days, and $\hat{\mathrm{F}}^{(\mathrm{d})}_{+1}$ is a contemporaneous covariance matrix calculated using equation 9, with one set of daily factor returns lagged by λ days.

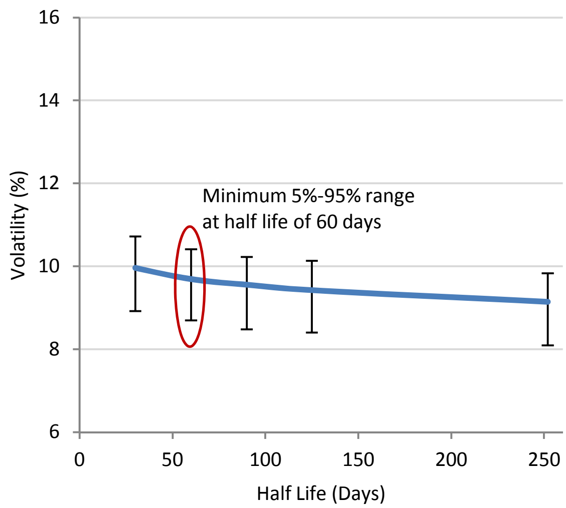
The asset-specific variance was also calculated using the specific returns weighted with the half-life used for the factor returns. Adjustment for serial correlation was performed with the univariate implementation of equation 10.

We use correlation half-lives of double the volatility half-lives. This is common industry practice.

We adopted common industry practice of using different half-lives to estimate the correlations, using correlation half-lives of double those used for the variances. It was not possible to use variance half-lives shorter than 30 days as was done for the univariate analysis, as the resulting covariance matrices were often not positive-definite, which on occasion would lead to the infeasible scenarios where some portfolios purportedly have negative risk. This problem is due to the ‘effective sample size’ described earlier; e.g. for a half-life of 5 days the effective sample size is 20 days, which is less than the number of factors. Such a short data sample relative to the number of factors leads to poorly conditioned covariance matrices that are not positive definite (or positive semi-definite).

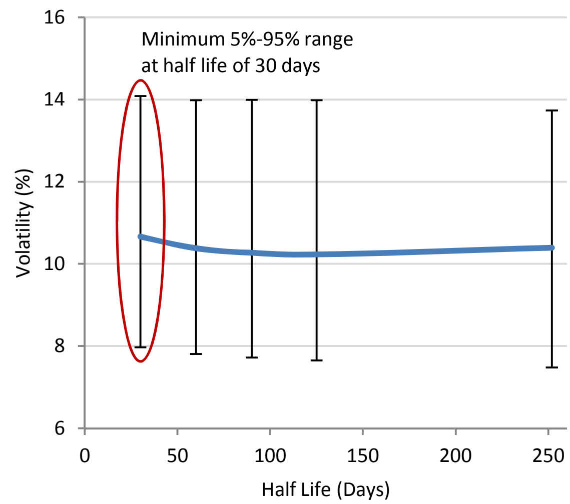
Results

Figure 16: Median realised annual volatility of long-only European equity portfolios rebalanced monthly to target 10% ex-ante volatility vs. half-lives. Error bars: 5%-95% range



Source: Axioma, Factset, Deutsche Bank

Figure 17: Median realised annual volatility of long-short European equity portfolios rebalanced monthly to target 10% ex-ante volatility vs. half-lives. Error bars: 5%-95% range



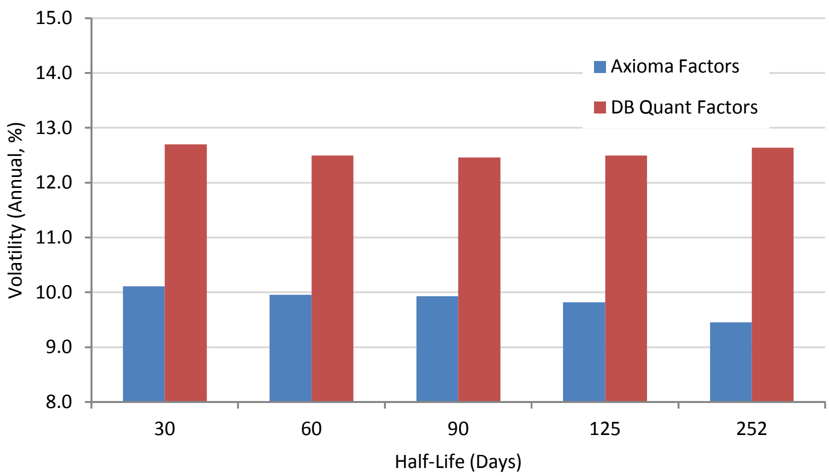
Source: Axioma, Factset, Deutsche Bank

Long-only portfolios are similar to single-stocks with regard to risk forecasting; the optimal half-life is 30 days for realised volatility centring

Figure 16 and Figure 17 illustrate the average realised volatility for the respective long-only and long-short portfolios rebalanced monthly to a target ex-ante annual volatility of 10% when using different half-lives. We see very different results for long-only and long-short portfolios. The long-only portfolios all have risk slightly over-forecast, and with a fairly narrow 5%-95% spread of around 1.8% for all half-lives, with a slight minimum at 60 days.

By contrast, the long-short portfolios have the average risk under-forecast for all half-lives. For the risk centring the relationship with the half-life is not strong, or persistent, but the realised volatility exhibits a slight minimum (indicating the least amount of under-forecasting) at a half-life of 125 days. The 5%-95% dispersion is far larger than for the long-only portfolios at around 6.2% for all half-lives. Again the relationship with the half-lives was weak with a slight minimum at a half-life of 30 days.

Figure 18: Median realised annual volatility of long-short Axioma and DB Quant portfolios rebalanced monthly to 10% ex-ante volatility vs half-life

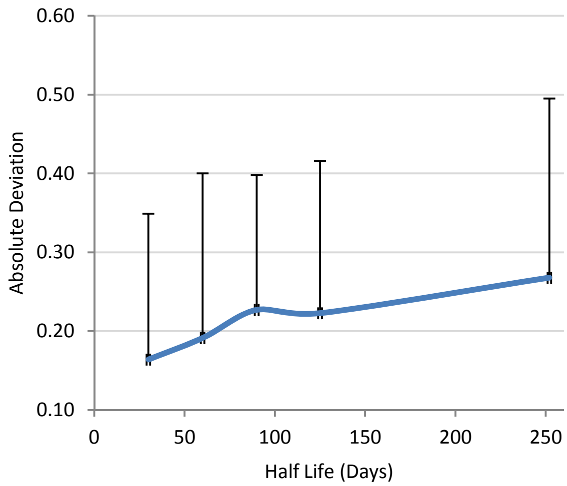


Source: Axioma, Factset, Deutsche Bank

Digging deeper into the long-short portfolios we see a different picture for the portfolios based on the Axioma factors to those based on the DB Quant factors. The average realised volatility is displayed in Figure 18. We see the risk forecasts for the portfolios based on Axioma factors are close to unbiased, with a slight tendency to over-forecast as the half-life increases. By contrast the risk of portfolios based on the DB Quant factors is under-forecast, with little dependence on the level of model responsiveness. The difference in the forecasting accuracy for the portfolios based on Axioma and DB factors is due to the respective alignment with factors in the risk model, and is discussed in the next section.

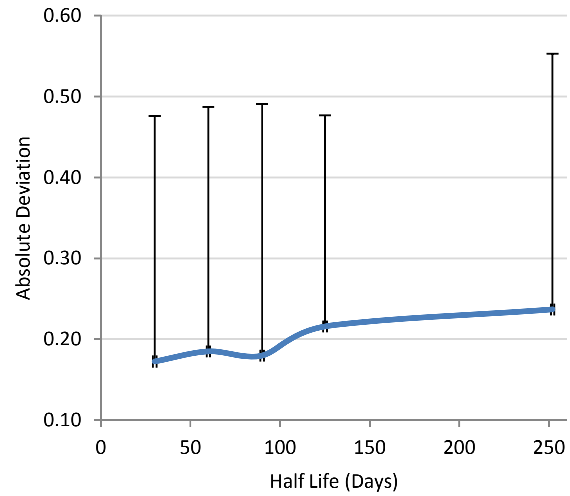
For long-short portfolios not directly aligned with Axioma factors there is no clear 'winner' for optimal half-lives, but a medium horizon model offers a good combination of forecasting accuracy, noise, and turnover.

Figure 19: Median absolute deviation of rolling bias statistics of long-only portfolios rebalanced monthly to target 10% ex-ante volatility vs. half-lives. Error bars: 95% values



Source: Axioma, Factset, Deutsche Bank

Figure 20: Median absolute deviation of rolling bias statistics of long-short portfolios rebalanced monthly to target 10% ex-ante volatility vs. half-lives. Error bars: 95% values



Source: Axioma, Factset, Deutsche Bank

Figure 19 and Figure 20 display the median and 95% levels of the absolute deviation of the bias statistic for long-only and long-short portfolios respectively. For the long-only portfolios the absolute deviation data supports the observations from the realised volatility in Figure 16; as the half-life increases, the forecasting accuracy deteriorates; this is with respect to both the centring and noise, where the optimal accuracy is seen with a half-life of 30 days.

For long-only portfolios the optimal half-life is 30 days with respect to the absolute deviation.

The long-short portfolios produce similar results to the long-only portfolios with respect to the absolute deviation of the bias statistics, although the relationship with the half-life is less strong. For centring of forecasts the optimal half-life is again 30 days, whereas for noise there is little difference between half-lives of 30, 60, 90, or 125 days, with a slight minimum with a half-life of 125 days.

The relationship between forecasting accuracy and half-life is less strong for long-short portfolios.

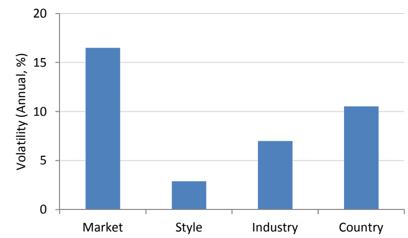
Digging deeper: risk attribution of long-only and long-short portfolios

To help gain insight into the differences in the risk forecasts for long-only and long-short portfolios, and the differences between forecasts for portfolios based on Axioma or DB Quant factors, we look to risk attribution.

In Figure 22 we see that almost all risk contribution for the long-only portfolios comes from the factors (96.8%), and that for the factor risk, the largest attribution is to the 'Market' factor. Note we use 'contribution' to separate factor and specific risk, and 'attribution' to decompose the risk from the different factors. Risk attribution is equivalent to the portfolio exposure, or beta, to a given factor or portfolio, multiplied by the volatility of the factor or portfolio. This is described in more detail by Wang et al (2014).

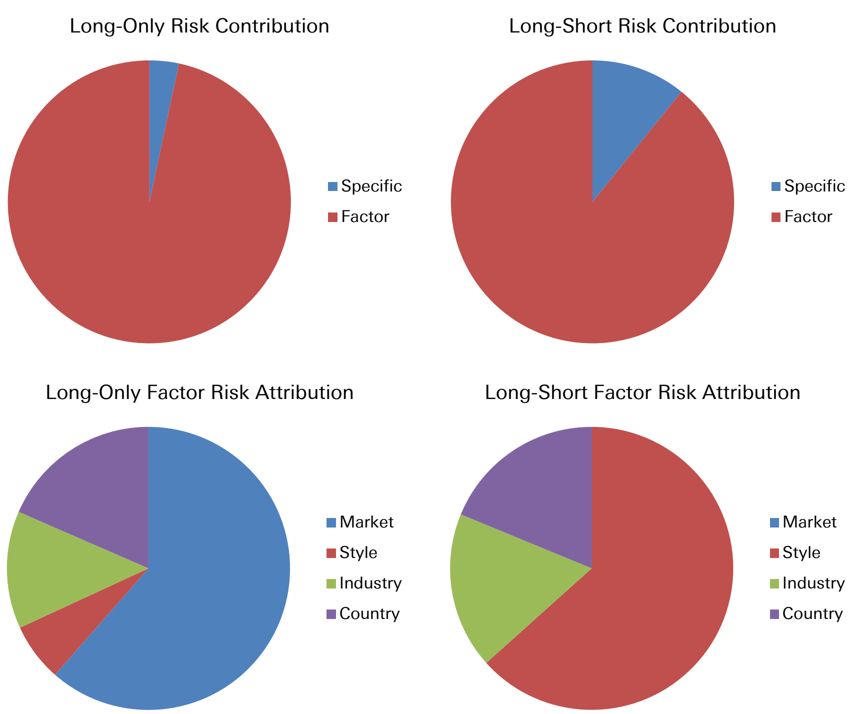
Figure 21 displays the average volatility of the style, industry, country factors, and the market factor, where we see that the market factor has the highest average risk. The high concentration of the risk in the market factor seen in Figure 22 implies that much of the risk is explained by a single factor, and is thus akin to a univariate process and observations for single stocks in the previous sections of this paper. Nevertheless, factor correlations are significant as a non-negligible portion of the risk is attributable to industry, style, and country factors.

Figure 21: Average volatility of Axioma European factor categories



Source: Axioma, Factset, Deutsche Bank

Figure 22: Analysis of average risk contribution and factor risk attribution for long-only and long-short portfolios



Source: Axioma, Factset, Deutsche Bank

Long-only portfolio volatilities are dominated by 'market' factor risk, thus the risk measurement is more akin to a univariate process, similar single stocks; correlations tend not to be significant

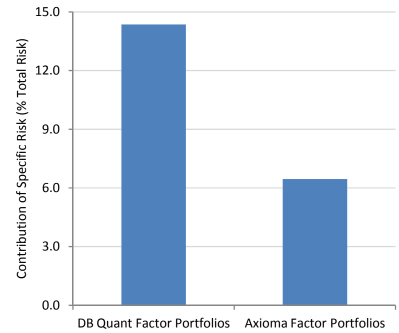
In long-short portfolios there is more specific risk, and the volatility attribution is spread more evenly across the factors; correlations are more important

The relative specific risk contribution for the long-short portfolios is large (12.5%) when compared to the long-only portfolios. In fact the portfolios based on the DB Quant factors have a greater contribution from specific risk as the portfolios are not directly aligned with the Axioma styles (see Figure 23 and Wang et al (2014) for more insights). The tendency to under-forecast the risk of the DB Quant factors suggests that some of the factor correlations may not be captured by the risk model. This would result in too much volatility being attributed to specific (idiosyncratic, diversifiable) risk, hence resulting in under-forecasting.

As the long-short portfolios are dollar-neutral and diversified, no risk is attributable to the market factor. For the portfolios based on the DB Quant factors the risk is attributed more evenly around the risk factors and specific volatility. Accordingly, the correlations between the factors and the correct measurement of specific risk are more important.

We saw earlier in Figure 18 that the realised volatility of the long-short portfolios based on the Axioma factors fell with increasing half-life. This is similar to results seen with the long-only portfolios. In this case we have portfolios that are each directly aligned with factors in the risk model. As such the risk is concentrated on a small number of factors, and is again somewhat akin to a univariate process, explaining the tendency to over-forecast when the risk model responsiveness decreases.

Figure 23: Relative contribution of specific risk for Axioma and DB Quant factor portfolios



Source: Axioma, Factset, Deutsche Bank

Optimised portfolios

Minimum- and mean-variance optimised portfolios have received a lot of attention in financial literature over recent years, particularly with the emergence of the 'low beta' anomaly (e.g. Alvarez et al (2011), Lee and Stefek (2006), Shepard (2009), Ledoit and Wolf (2003), Laloux et al (1999)).

Here we constructed 14 optimised portfolios: the minimum variance portfolio and 13 mean-variance strategies. In all cases the universe was again the stocks in the MSCI Europe index.

The minimum variance portfolio was constructed with the standard approach, which yields a portfolio that is 100% net long, or fully invested, with leverage:

$$p = \frac{\hat{\mathrm{C}}^{-1}\mathbf{1}}{\mathbf{1}'\hat{\mathrm{C}}^{-1}\mathbf{1}} \tag{11}$$

where $\mathbf{1}$ is an N-vector of 1s. Note that with regard to assessing volatility accuracy the minimum volatility portfolio leverage was adjusted such that the ex-ante annual volatility was 10%.

The mean-variance optimised portfolios look to minimize the standard utility function that balances risk with expected returns:

$$U(p) = \frac{\lambda}{2} p'\hat{\mathrm{C}}p - a'p \tag{12}$$

Each mean-variance strategy is tilted towards one of either the Axioma style factors or the DB Quant strategies listed in Figure 15.

To retain consistency we derived an analytic solution for 100% net invested, mean-variance portfolios:

$$p = \hat{\mathrm{C}}^{-1}\mathrm{B}'$$

$$\mathrm{B} = \frac{1}{\lambda}\left(a' + \gamma\mathbf{1}'\right) \qquad \gamma = \frac{\lambda - a'\hat{\mathrm{C}}^{-1}\mathbf{1}}{\mathbf{1}'\hat{\mathrm{C}}^{-1}\mathbf{1}} \tag{13}$$

where a is an N-vector of portfolio alpha scores, taken from the Axioma or DB Quant factors, and λ is the risk aversion. An iterative approach was used to scale the risk aversion such that an ex-ante annual portfolio volatility of 10% was achieved. Adopting an analytic solution allowed comparison with the results from previous sections, avoiding the need for constraints and the subsequent requirement for constraints attribution, although the leverage in the solutions could, in principle, be large.

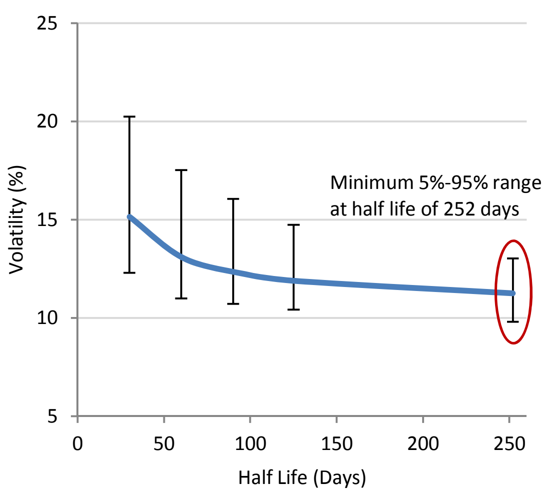
Results

Figure 24 illustrates the median realised volatility of the optimised portfolios when rebalanced every month. We see the commonly observed result that the average volatility of all optimised portfolios is significantly under-forecast, with the level of under-forecasting decreasing as the half-life increases.

Volatility is under-forecast for optimised portfolios. The level of under-forecasting decreases as the half-life increases.

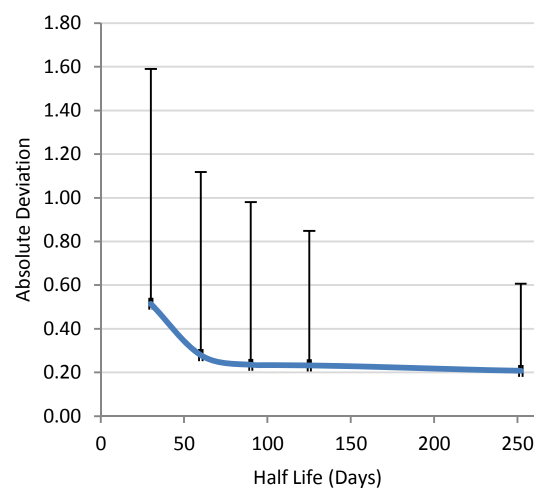
Figure 25 shows the median and 95% levels of the absolute deviation of the bias statistic of the optimised portfolios. The results are consistent with those in Figure 24, and again illustrate that longer half-lives are required to increase the forecasting accuracy.

Figure 24: Median realised annual volatility of optimised portfolios rebalanced monthly to target 10% ex-ante volatility vs. half-lives. Error bars: 5%-95% range



Source: Axioma, Factset, Deutsche Bank

Figure 25: Median absolute deviation of rolling bias statistics of optimised portfolios rebalanced monthly to target 10% ex-ante volatility vs. half-lives. Error bars: 95% values



Source: Axioma, Factset, Deutsche Bank

The results for the optimised portfolios are in direct contrast to those with the long-only portfolios, in that decreasing the model responsiveness (increasing the half-life) results in more accurate forecasts. For optimised portfolios the advantages of greater responsiveness are more than offset by the errors in the measurement.

Increasing the half-life results in more accurate forecasts for optimised portfolios.

Why is risk under-forecast for optimised portfolios?

Under-forecasting the volatility of optimised portfolios is a well known issue. Optimisation will seek out the portfolio holdings yielding the minimum variance, and will thus deliberately tilt as far as possible towards the left tail of the distribution of the covariance matrix to find the lowest volatility assets and factors, and best apparent 'hedges' between factors. This will also comprise much of the left tail of the error distribution. If stock returns are assumed to be normally distributed then the estimated covariance matrix represents a draw from a Wishart distribution (Toduka et al 2011). From this Shepard (2009) has shown that the error in the portfolio variance estimate has the following relationship:

$$\Delta\sigma_P^2 \propto (1 - N/T)^{-2} \qquad (14)$$

From Equation 14 we see that the error scalar is inversely proportional to the square of sample size, and proportional to the square of the number of assets. This is intuitive as the number of parameters in the covariance matrix to be estimated increases proportionally to the square of the number of assets: $0.5(N^2+N)$. It thus follows that the estimates with the longest half-lives, or longest effective sampling windows, will have the least under-forecasting error.

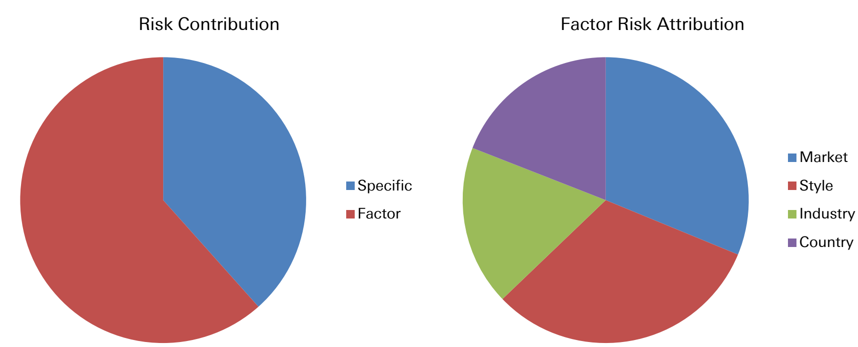
Optimising for the maximum variance portfolio would yield the equivalent problem, but with the volatility being over-forecast.

The under-forecasting of optimised portfolios has been addressed in a financial context by Menchero et al (2012), with 'Eigen-Adjusted' covariance matrices. Their approach uses Monte-Carlo simulations to determine biases in the forecasts of 'eigen portfolios', using these to correct the biases in the eigenvalues of the sample covariance matrices. However, other methods to correct bias in covariance matrices have been reported in fields such as signal and image processing, which could be utilized in this context (Muirhead (1982), Karoui (2006), Hendrikse et al (2008)).

Attribution of mean-variance optimised portfolios

Figure 26 displays the average factor and specific risk contribution for the optimised portfolios, as well as the average attribution of the factor risk. We see that specific risk accounts for a much larger fraction of the portfolio volatility than was seen with the long-only or long-short portfolios. This is intuitive as specific risk is diversifiable, thus the risk minimisation in the optimisation will reduce the percentage of risk comprised by the factors. At the same time large positions in assets with a high degree of idiosyncratic risk will be avoided.

Figure 26: Analysis of average risk contribution and factor risk attribution for mean-variance optimised portfolios



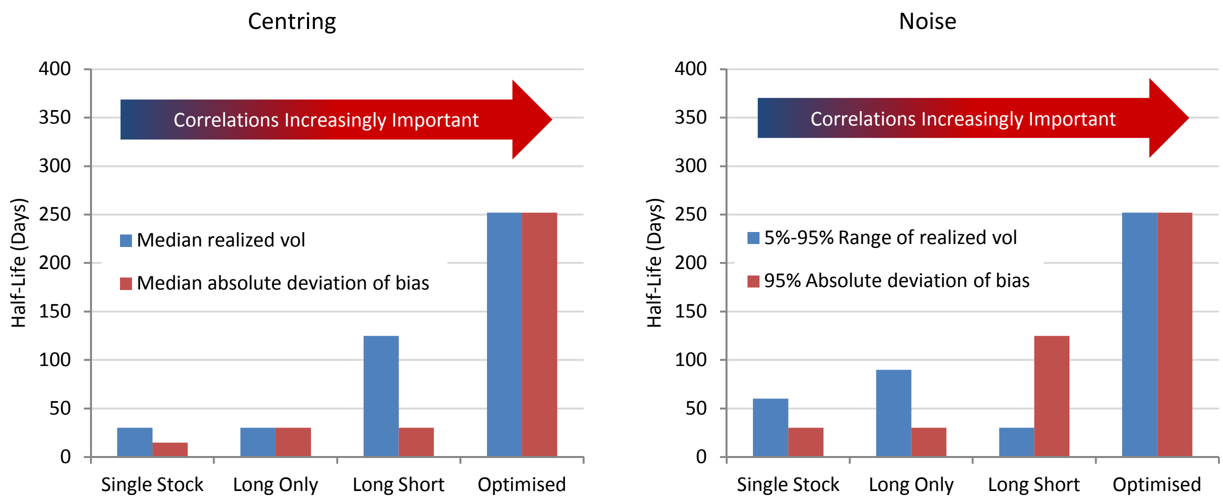
Source: Axioma, Factset, Deutsche Bank

From the factor risk attribution we see that style factors account for the largest fraction of the volatility. This follows from our use of style factor exposures as alpha scores, thus the optimisation will tilt towards assets with high exposures to the given alpha style. The attribution to the industry and country factors is low, indicating the optimisations have found combinations of stocks to diversify these ex-ante risks to a large degree. The behaviour of the risk aversion parameter and the integration of alpha into the portfolios is discussed later in this report (pages 37-39). For discussion on the makeup of minimum variance portfolios see Alvarez et al (2011).

What have we learned so far – part II?

Figure 27 summarizes the optimal half-lives for the different classes of portfolios for bias and noise measurements. We see that, on average, the optimal half-life increases as we look at single stock, long-only, long-short, and optimised portfolios in turn, with respect to both centring and noise.

Figure 27: Optimal half-lives with respect to volatility forecasting centring and noise for different portfolio classes



Source: Axioma, Factset, Deutsche Bank

From the results so far we have seen that:

The need to accurately measure correlations increases when moving from long-only to long-short to optimized portfolios.

Accurately estimating correlations requires longer 'effective' estimation windows, and so longer exponential weighting half-lives.

- The attribution to risk for long-only portfolio volatility is dominated by the 'market', and is akin to a univariate process, thus behaving much like single stocks. Correlations are of least importance. The volatility of long-only portfolios tends to be more over-forecast by longer-horizon models.
- Long-short (dollar neutral) portfolios have no exposure to the market factor, and thus the risk is attributed more evenly to the other factors, and to specific risk. Accordingly, the correlations between the factors and the correct separation of idiosyncratic and systematic risk are of more importance. Risk-forecasting accuracy of long-short portfolios has relatively little dependence on risk model responsiveness. However, correlations will generally be better estimated with a longer-horizon model, and turnover will be lower. Long-short portfolios that are explicitly aligned with a long-short factor that is incorporated in the risk model are an exception. Under these circumstances the volatility forecasting can be more akin to a univariate process.
- For optimised portfolios the estimation of correlations is of maximal importance; the optimisation process will search for the best 'hedges' between factors and assets to minimise risk. The volatility of optimised portfolios is normally significantly under-forecast. Ideally a method to correct the estimation error in the covariance matrix should be used to correct the under-forecasting.

Commercial risk models horizons

It is informative to reference the half-lives examined here to those used in commercial risk models. Two primary model vendors - Axioma and MSCI (Barra) - often market versions of their equity factor risk models with 'short', 'medium', and 'long' horizons. The exponential weighting half-lives associated with these modes are given in Figure 28. The results so far suggest that short- or medium-horizon models would be suitable for long-only or long-short portfolios, and that a long-horizon model would be more suitable for optimised portfolios.

Results so far indicate short- or medium horizon models would suit long-only or long-short portfolios, and that a long-horizon model would be most suitable for optimise portfolios.

We note that some commercial risk models employ quantitative techniques beyond simple exponential weighting to generate the final volatility forecasts. For example, Axioma uses a patented 'dynamic volatility adjustment', and MSCI (Barra) has an approach termed 'volatility regime adjustment'. Accordingly, the data presented here can only be regarded as a general guide for risk measurement solely using exponential weighting, which may not correspond to analytics provided by commercial vendors.

Figure 28: Variance half-lives of some commercially available risk models

Vendor/Model	Variance Half Life (Days) for Horizon:		
	Short	Medium	Long
MSCI Global Equity Model	84	-	252
MSCI European Model	84	-	252
Axioma European Model	60	125	-
Axioma Global Model	60	125	-

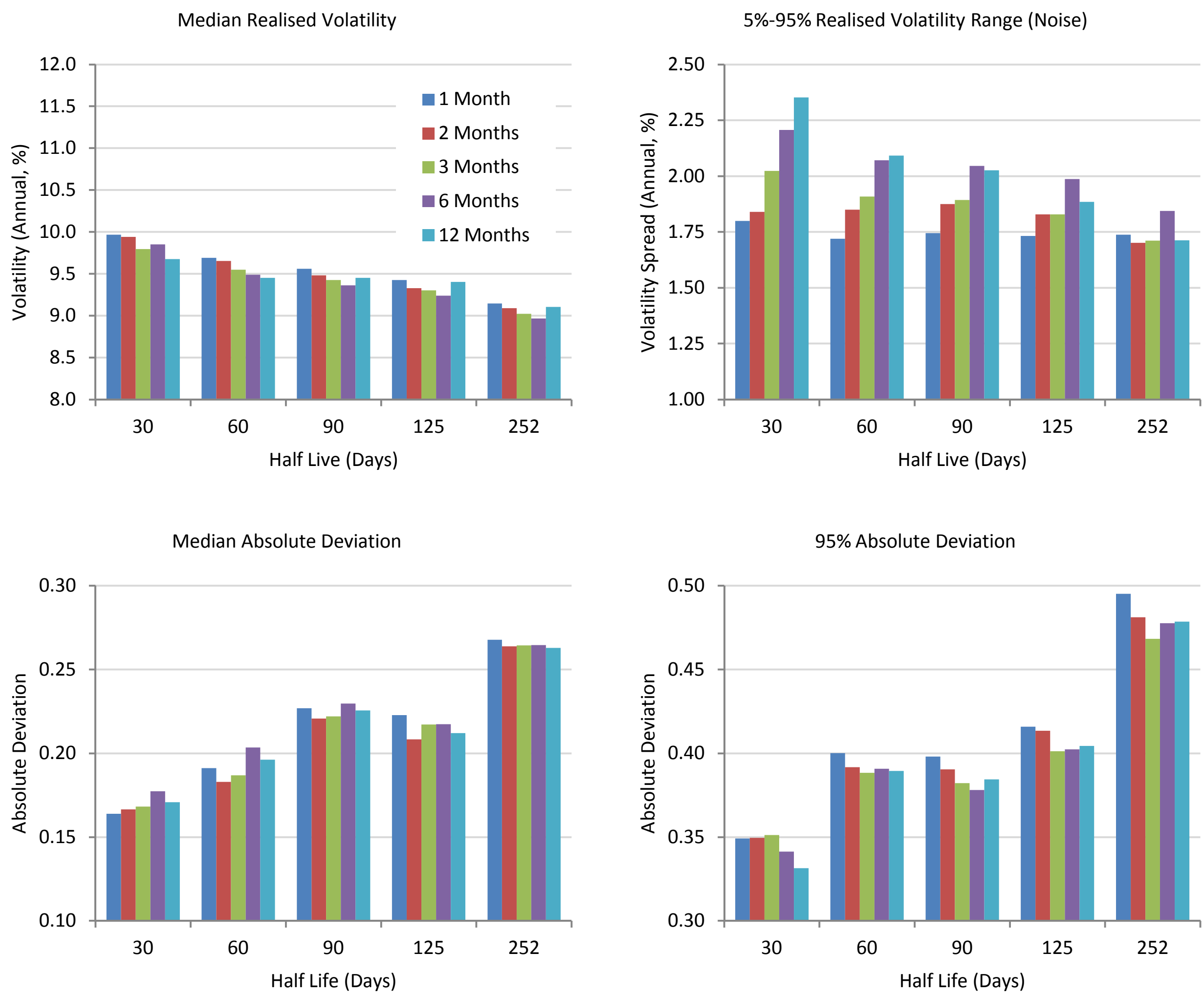
Source: Axioma, Factset, MSCI, Deutsche Bank

Going out-of-sample: the effect of rebalancing less frequently

So far we have looked at volatility forecasting in respect to monthly rebalancing. A common perspective with regard to forecasting horizon is that longer horizon models (that use longer half-lives) provide more accurate forecasts for 'longer-horizon' investors, or those that rebalance less frequently. Here we look again at the investigations performed in the previous sections, but examine the forecasting accuracy when rebalancing at intervals of 2, 3, 6, and 12 months[4]. While rebalancing at intervals such as six and twelve months may be relatively unusual, it is informative to examine the relationship between forecasting accuracy and rebalancing over a broader range of data points. In the previous section we saw that single-stock and long-only portfolios were similar from a risk-forecasting perspective. We found this to be true when extending the intervals between portfolio rebalancing points. Accordingly, in this section we omit the results on single-stock portfolios.

[4] When rebalancing at frequencies below monthly we ensured we averaged the results from overlapping back-tests with different sets of rebalancing dates to avoid market timing and seasonal artefacts. For example, when rebalancing quarterly, we ran three back-tests with rebalancing dates of Jan, Apr, ...; Feb, May, ...; Mar, Jun, ...

Figure 29: Volatility forecast evaluation for long-only portfolios when rebalanced every 1, 2, 3, 6 & 12 months



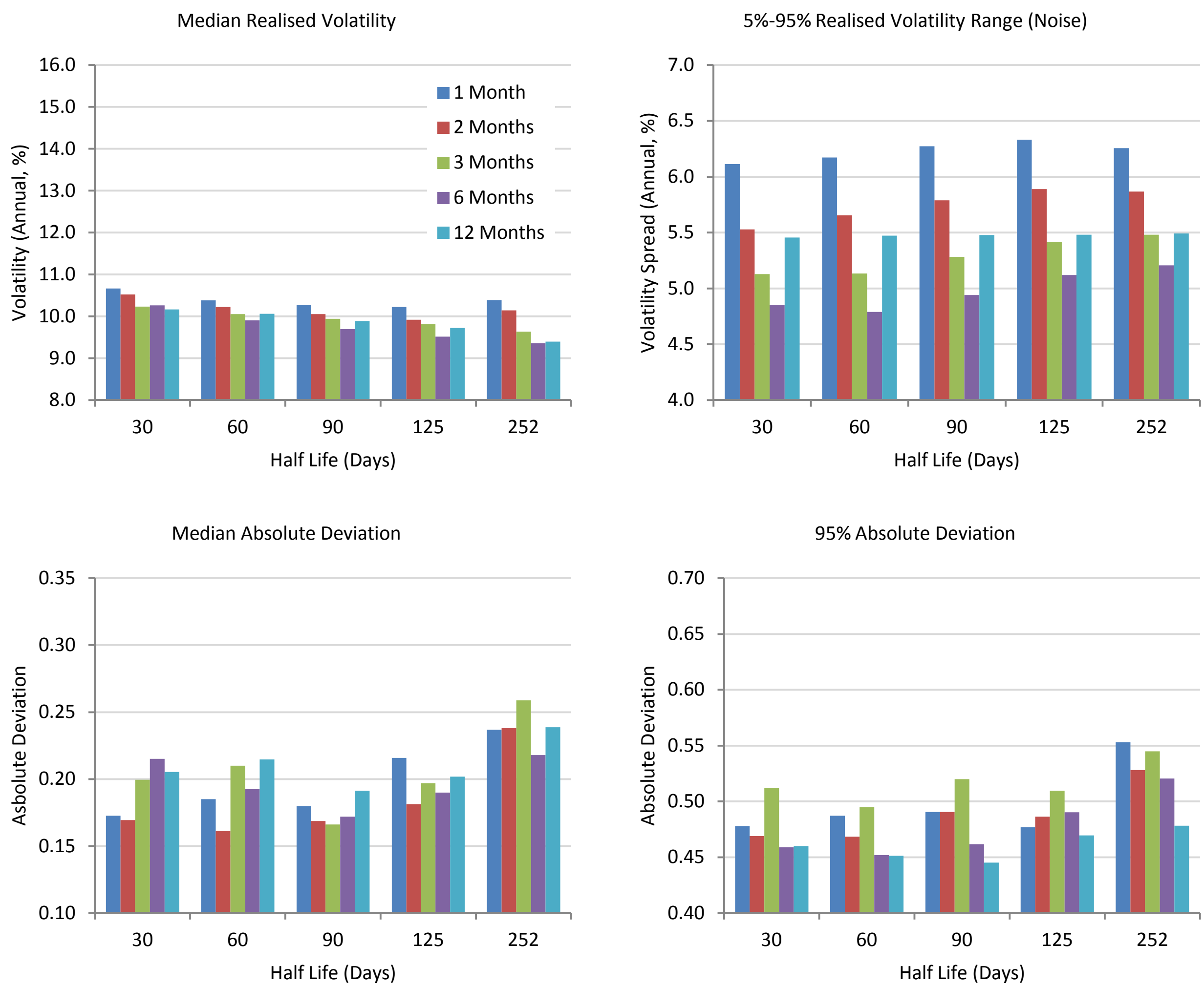
Source: Axioma, Factset, Deutsche Bank

Rebalancing infrequently results in more randomness in the realised volatility, but the systematic bias is very similar.

Figure 29 displays the results to assess forecasting accuracy for long-only portfolios when rebalancing at 1, 2, 3, 6 and 12 month intervals. When looking at the median realised volatility (top left chart), the median deviation of the bias statistic (bottom left chart), and the 95% level of the absolute deviation (bottom right chart) we see little change in the values when extending the rebalancing interval. Based on the realised volatility there is again the tendency to increasingly over-forecast risk as the model half-life is increased for all rebalancing frequencies.

We see that there is increased spread in the realised volatility (top right chart, Figure 29) when rebalancing infrequently with a model with a short half-life. When rebalancing less frequently, the risk forecast upon which the rebalancing was based will become 'out of date'. Accordingly, as the time from the last rebalancing date increases, the contemporary volatility will become more random in relation to the forecast. This results in the volatility forecast randomly becoming more or less accurate over the course of the back-testing period, and thus there is more variation in the final realised volatility. However, as the average bias remains static, the absolute deviations (bottom charts) are very similar. Moreover, as the volatility forecasts with the more responsive models follow short-term market movements, they are noisier, and more likely to capture tail volatility events. It follows that more noise is encapsulated in the realised volatility when the portfolios are rebalanced infrequently, but based on the responsive forecasts.

Figure 30: Volatility forecast evaluation for long-short portfolios when rebalanced every 1, 2, 3, 6 & 12 months

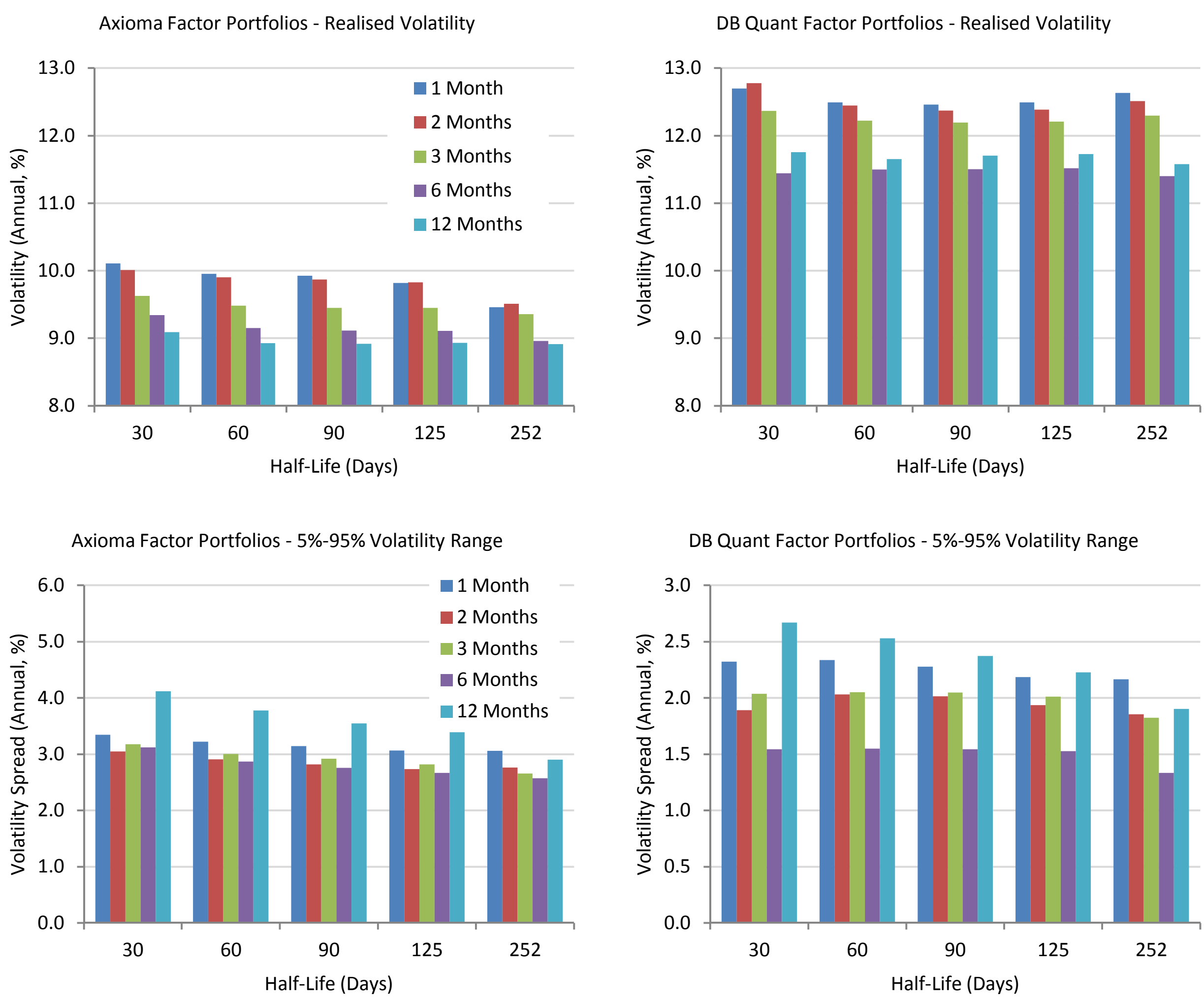


Source: Axioma, Factset, Deutsche Bank

In Figure 30 we see the forecasting accuracy measurements for long-short portfolios with various rebalancing frequencies. When looking at the absolute deviation of the bias statistic (bottom charts) we see no consistent dependency on the rebalancing frequency for the median or 95% levels.

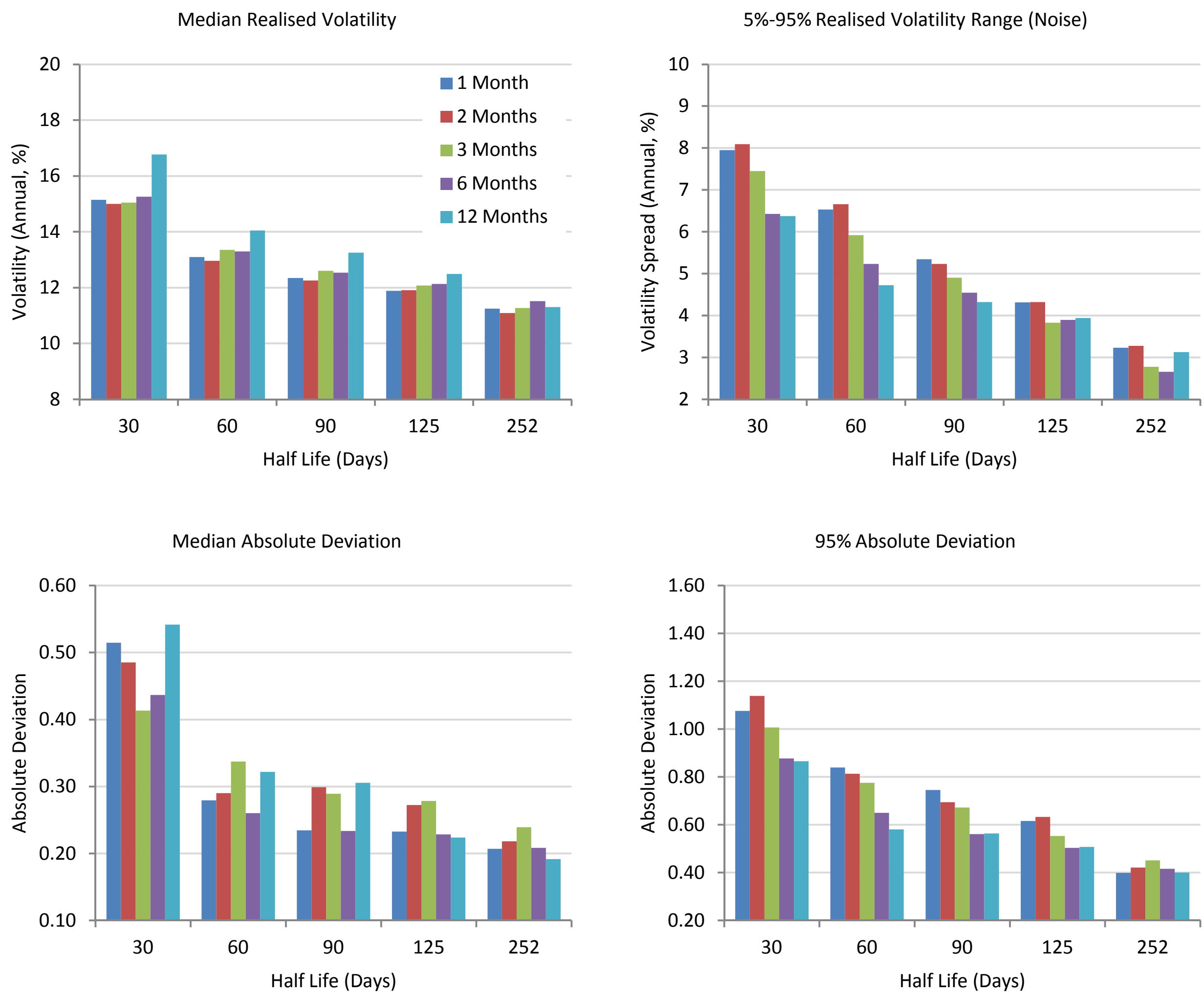
From the realised volatility (top left chart) we see the tendency to under-forecast risk reduces as we rebalance less frequently. Looking at the 5%-95% range (top right chart) we see the highest values when rebalancing monthly, which decreases to a minimum when rebalancing every six months. The reasons underlying this behaviour are subtle, and are connected to the use of portfolios based on individual Axioma and DB Quant factors discussed earlier. Separating the two sets of portfolios (Figure 31), we see that the risk forecasts of the portfolios based on Axioma factors are close to unbiased, with a slight tendency to over-forecast. The risk for portfolios based on the DB Quant factors tends to be under-forecast, indicating some factor correlations may be missing from the risk model. When rebalancing the DB Quant factor portfolios less frequently, the under-forecasting is reduced, indicating the factors may 'decay'. The relative spread between the realised volatility of the DB Quant portfolios is also reduced when rebalancing at intervals of two, three or six months (the spread is typically reduced by around 0.75% when rebalancing every 6 months rather than every month - bottom right chart), and more so than the equivalent spreads of the realised volatility of portfolios based on Axioma styles (the spread is typically reduced by around 0.25% when rebalancing every 6 months rather than every month - bottom left chart). As such, the overall spread between the aggregated set of portfolios based on Axioma and DB portfolios is reduced (top right chart of Figure 30).

Figure 31: Volatility forecast evaluation for long-short portfolios based on Axioma and DB Quant factors when rebalanced every 1, 2, 3, 6 & 12 months



Source: Axioma, Factset, Deutsche Bank

Figure 32: Volatility forecast evaluation for optimised portfolios when rebalanced every 1, 2, 3, 6 & 12 months



Source: Axioma, Factset, Deutsche Bank

Figure 32 depicts the volatility forecasting accuracy assessment for the optimised portfolios. In terms of the median realised volatility (top left chart) we see little dependence on the rebalancing frequency, with the exception of annual rebalancing, where using a shorter half-life yields the highest degree of under-forecasting. This is consistent with the results discussed in the previous section. The 5%-95% range of realised volatility (top right chart) and 95% level of the absolute deviation of the bias statistic (bottom right chart) indicate that the level of noise in the forecast accuracy is generally greatest when rebalancing on a monthly or bimonthly basis. This behaviour is consistent with estimation noise being the primary issue concerning volatility forecasts of optimised portfolios. The effects of the noise are exacerbated when using a highly responsive model and rebalancing frequently.

These results all support the notion that estimation noise is a dominant issue with regard to portfolio optimisation.

Estimation noise is exacerbated when using a short horizon risk model and rebalancing frequently.

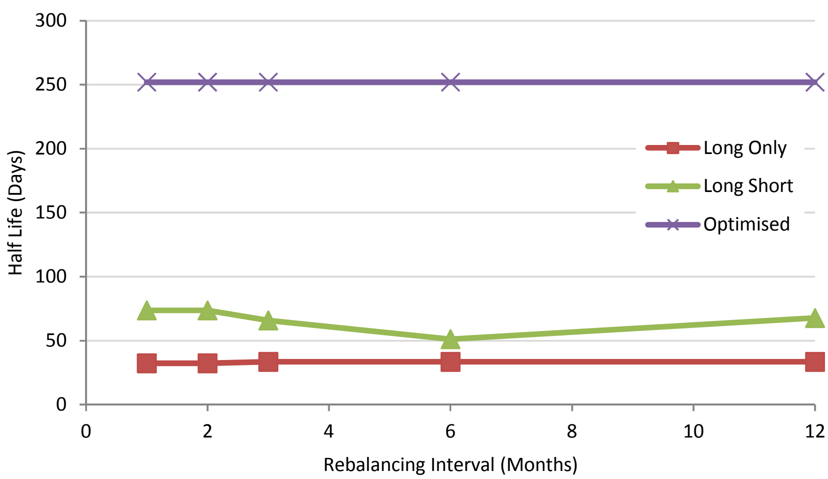
Half-life vs. rebalancing frequency

Optimal half-lives do not vary significantly for different rebalancing intervals.

Portfolio class is a significant factor at all rebalancing frequencies.

Figure 33 illustrates the average optimal half-life for each class of portfolios as determined from the 95% level of the absolute deviation of the bias statistic. As with the many of the results in this section we see almost no relationship between the rebalancing frequency and model responsiveness in terms of forecasting accuracy. However, we see that the forecasting accuracy for different classes of portfolios is dependent on model responsiveness for all rebalancing frequencies.

Figure 33: Optimal half-life for 95% absolute deviation of bias statistic for different classes of portfolio versus interval between portfolio rebalancing points



Source: Axioma, Factset, Deutsche Bank

What have we learned so far – part III?

Correlations are better estimated when using a longer half-life.

The results in this section are all consistent with the observation that the correct estimation of correlations becomes more important when moving from long-only to long-short to optimised portfolios. Correlations are better estimated when using longer half-lives, and hence effective sampling windows. This effect is more significant than the effect of rebalancing frequency with regard to the accuracy of volatility forecasting.

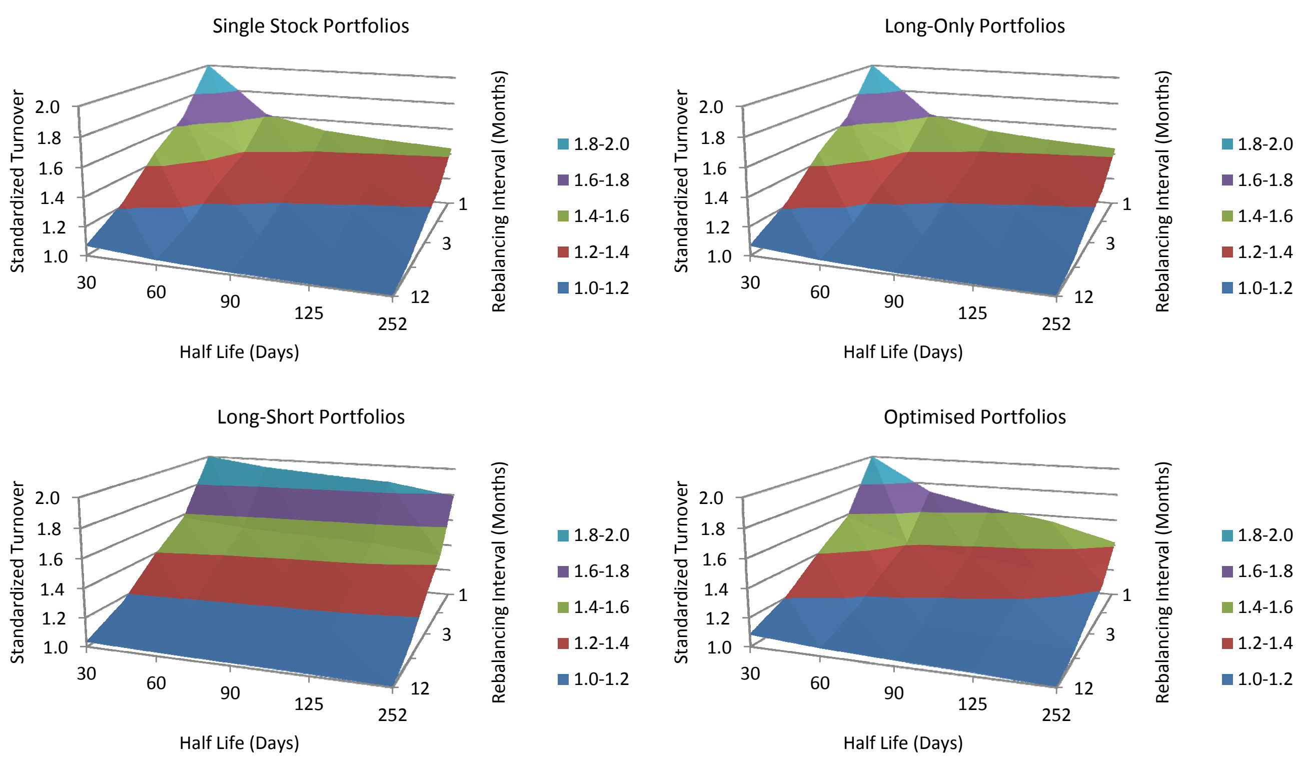
From risk forecasting to portfolio performance

In this section we look at the practical implications of risk forecasting accuracy on portfolio construction and the ensuing returns. We investigate the effect of under- or over-forecasting for investors who leverage to a risk budget, and we look at turnover and transaction costs.

Turnover

The average standardized[5] annual turnover for each class of portfolio versus half-life and rebalancing frequency is displayed in Figure 34. Here our primary focus is on the relative change in turnover due to changing the responsiveness of the risk model rather than the absolute turnover.

Figure 34: Standardized average turnover by portfolio class versus half-life and rebalancing interval



Source: Axioma, Factset, Deutsche Bank

We see that optimised portfolios are most dependent on the model half-life; the relative turnover decreases by 50-55 per cent, depending on the rebalancing frequency, as the model half-life is increased from 30 to 252 days.

[5] Turnover was standardized to vary from one to two

By contrast the long-short portfolios are least dependent on the model half-life, with a respective relative decrease of 14-17 per cent.

Turnover of optimised portfolios is most dependent on risk model responsiveness, whereas long-short portfolios are least affected.

For the optimised portfolios the construction process is particularly dependent on the risk model, with the positions based on the low-volatility tails of the covariance matrix. With shorter half-lives the positions of the low volatility tails of the covariance matrix will change rapidly, leading to high turnover for optimised portfolios. With mean-variance optimisation the turnover in the alpha signal is also significant; an alpha signal based on mean-reversions would naturally have higher turnover than one based on a more slowly evolving fundamental attribute such as size.

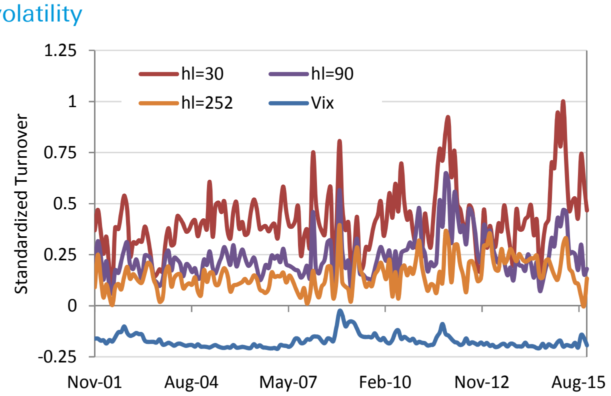
For long-short portfolios the portfolio constituents and positions are more dependent on the underlying style or strategy factors, and the risk model is used to determine the overall level of leverage for a given ex-ante volatility. As the strategies are long-short, the volatilities of the strategies and factors are less dependent on the market and its associate volatility, and thus the leverage required to hold a target volatility tends to be more stable.

Single stock and long-only factors lie between optimised and long-short portfolios with respect to the turnover's dependence on model responsiveness. The positions do not 'rotate' with movements of low-volatility pockets in the covariance matrix, but the leverage required to hold an ex-ante level of risk will depend on the overall market volatility more than long-short portfolios.

Temporal dependence of turnover

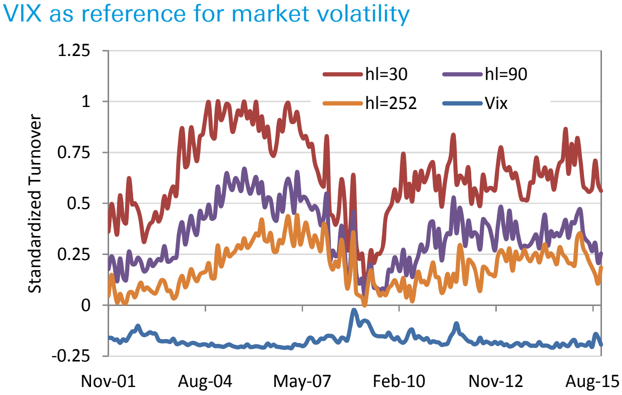
Figure 35 displays the standardized turnover against time for the minimum volatility portfolio (constituted of assets in MSCI Europe) when constructed with risk models with different levels of responsiveness and not leveraged to a volatility target. Figure 36 displays the standardized turnover for the risk-targeted mean-variance optimised portfolios. In both figures the VIX is displayed as an indication of the market volatility.

Figure 35: Average standardized turnover for minimum volatility portfolio vs. time. VIX as reference for market volatility



Source: Axioma, Factset, Deutsche Bank

Figure 36: Average standardized turnover for volatility targeted mean-variance optimised portfolios vs. time. VIX as reference for market volatility



Source: Axioma, Factset, Deutsche Bank

In Figure 35 we see intuitive behaviour where the turnover increases around periods of increased volatility, particularly around the market shocks of 2008 and 2011. We also see the higher turnover associated with the more responsive models, as previously seen in Figure 34. This behaviour would typically be observed when portfolios are constructed with a view to minimizing volatility, but with no set risk target.

However, Figure 36 illustrates behaviour that is less intuitive. Here the portfolios were constructed with a volatility target (10%). While the portfolio construction generated portfolios that were net 100 per cent invested, the long-short leverage was allowed to vary to achieve the volatility target. Accordingly, in periods when volatility was lower, such as 2003-2007, the leverage was higher to achieve the volatility target, resulting in high turnover. Conversely, in periods of high volatility such as 2008, the leverage was lower, resulting in reduced turnover.

When risk-targeting, turnover can be higher in low volatility periods due to the need for higher leverage

Risk forecasting and returns[6]

When portfolios are constructed with consideration to a risk budget, if only considering leverage as a means of risk-control, the leverage is linearly related to volatility:

$$p = p^* \frac{\sigma_\tau}{\sigma_{p^*}} \tag{15}$$

where p^* represents the portfolio holdings before risk targeting, p represents the portfolio holdings once adjusted to the volatility target, σ_{p^*} is the ex-ante volatility of portfolio p^*, and σ_τ is the risk target.

When only considering leverage, from Equation 15 the implications of over- or under-forecasting are clear: under-forecasting risk will lead to positions that are too aggressive, possibly with too much leverage; over-forecasting will result in the opposite scenario. It follows that forecasting errors will lead to positions with undesired exposures to alpha and risk.

For transaction costs we adopt a similar approach to that implemented by Garleanu and Pedersen (2013), where the costs are quadratic, and proportional to portfolio variance:

We model transaction costs as quadratic, and proportional to risk

$$TC = 2\Delta p^{\prime} \lambda \Lambda \Delta p \tag{16}$$

where Λ is an asset covariance estimate, and λ is a scalar. To ensure consistent transaction costs across all model versions we simply used the square of the VIX as the covariance estimate[7], and set λ at 0.15. The costing proportional to market volatility is intuitive, as in more volatile markets bid-ask spreads are typically higher and thus the transaction cost and market impact are typically higher, too. The multiple of two was used to reflect that transaction costs incurred are two-way.

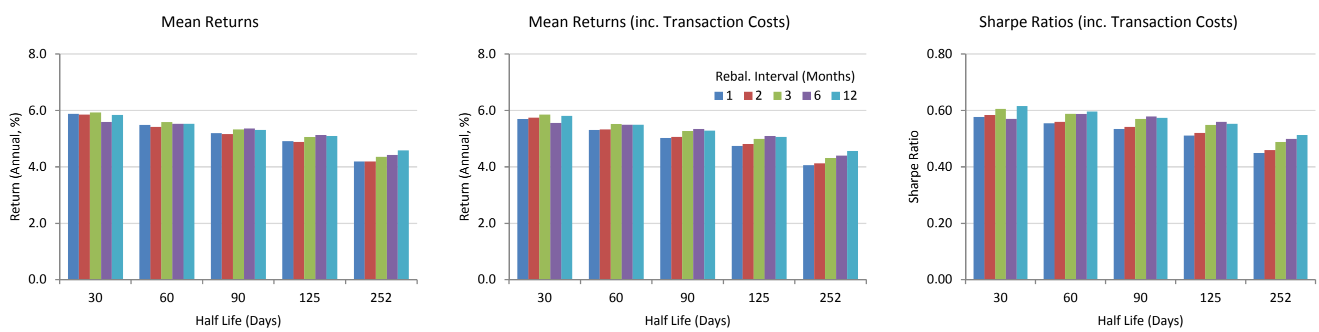
[6] In this section we omit the results of portfolios based on the Axioma short-term momentum factor. The factor is akin to a reversal factor, and has extremely high turnover which overly distorts the data when adjusting for transaction costs.

[7] i.e. Λ is a diagonal asset-by-asset covariance matrix for N assets with all diagonal elements equal to the square of the VIX index value: $\Lambda = VIX^2 I_N$

Figures 37-39 display the raw and transaction cost adjusted returns, and transaction cost adjusted Sharpe ratios for long-only, long-short, and minimum volatility portfolios when constructed with risk models with variance half-lives of 30, 60, 90, 125, and 252 days, and with rebalancing intervals of 1, 2, 3, 6, and 12 months. Note that the results for mean-variance optimised portfolios are discussed in the next section.

For long-only portfolios (Figure 37) we see a slight decrease in returns as the risk model half-life increases. This is consistent with Figure 29, where we saw that models with longer half-lives generally provided higher volatility forecasts. Accordingly, given the holdings are scaled to 10% ex-ante volatility, it follows from Equation 15 that the portfolios constructed using risk models with shorter half-lives will have more leverage, and given long holdings, on average, yield positive returns, these portfolios will out-perform those constructed using risk models with longer half-lives.

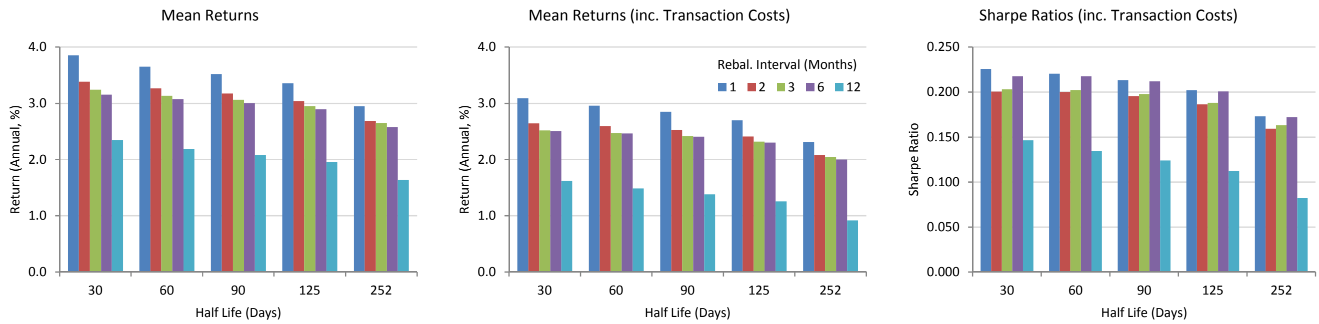
Figure 37: Performance of long-only portfolios versus half-life and rebalancing interval



Source: Axioma, Factset, Deutsche Bank

The impact of the transaction costs is negligible with all model versions for long-only portfolios. The Sharpe ratios decrease slightly with increasing model half-lives in line with the returns. Increasing the rebalancing interval does not have a consistent effect on the portfolio performance.

Figure 38: Performance of long-short portfolios versus half-life and rebalancing interval



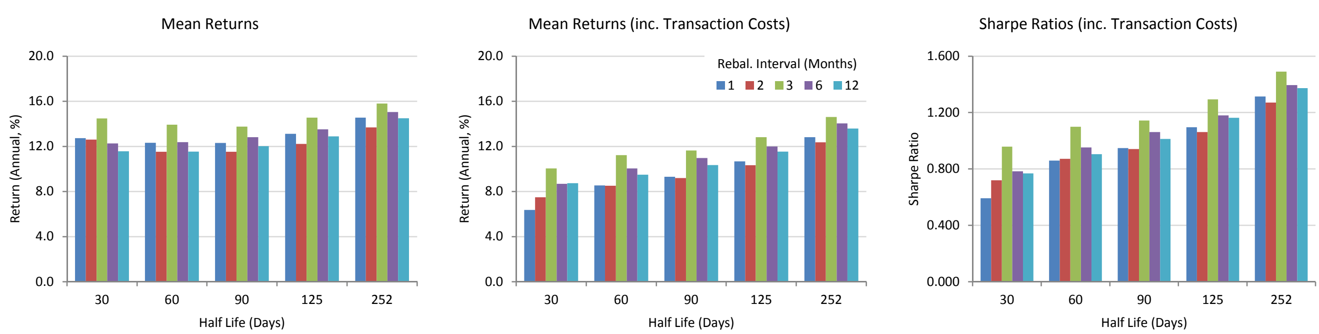
Source: Axioma, Factset, Deutsche Bank

For long-short portfolios (Figure 38) the transaction costs have a more pronounced effect in all scenarios due to the increased leverage, and on average are around one per cent of the annual returns. As seen in Figure 34, there is relatively small dependence on the model half-life.

Returns and Sharpe ratios are highest when rebalancing on a monthly basis.

There is clear benefit in more frequent rebalancing to capture contemporary updates in the factor signals; lower rebalancing frequency results in lower returns with and without accounting for transaction costs. The Sharpe ratio is inversely related to the realised volatilities observed in Figure 30, where the lowest realised volatility was observed when rebalancing every six months. Accordingly, although the highest Sharpe ratio accompanies the highest return, the Sharpe ratio when rebalancing every six months is only fractionally lower.

Figure 39: Performance of minimum volatility portfolios versus half-life and rebalancing interval



Source: Axioma, Factset, Deutsche Bank

Shorter half-lives result in substantially higher turnover and transaction costs for optimised portfolios.

When looking at the minimum volatility portfolios (Figure 39), the impact of transaction costs is largest. Further, the transaction costs are highly dependent on the risk model half-life, as previously indicated in Figure 34. When rebalancing monthly with a half-life of 30 days the transaction costs reduce the return by approximately 6%, whereas the returns are reduced by 1.7% when using a model with a half-life of 252 days. Accordingly, when optimising for portfolio construction, the responsiveness of the risk model is a critical factor with regard to performance. The Sharpe ratios when adjusted for transaction costs reflect the associated returns.

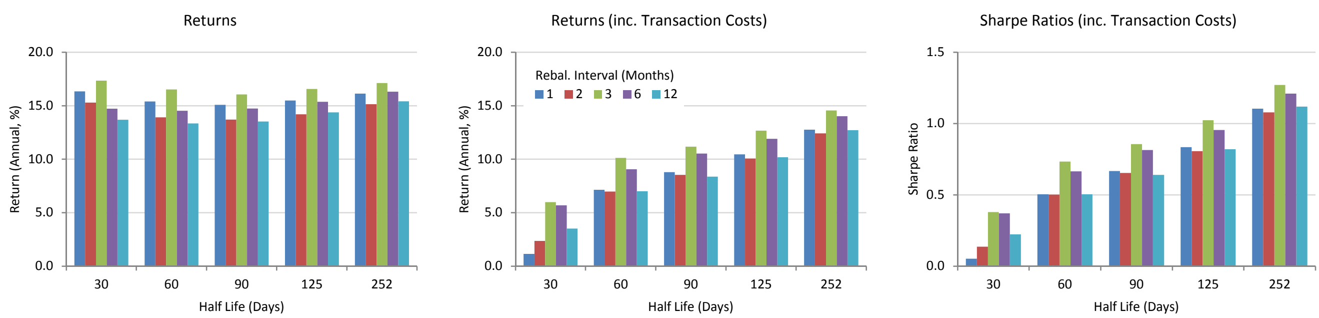
Mean-variance portfolios: alpha can be 'eaten'

The performances of mean-variance optimised portfolios (Figure 40) are, on average, more affected by transaction costs than the minimum volatility portfolios. This is due to the varying degrees of leverage to achieve the 10% ex-ante volatility target combined with the changes in the alpha signal. Turnover when using models with a shorter half-life is far higher, in-turn leading to higher transaction costs, as seen in Figure 34 and Figure 36.

The increased turnover when using risk models with a short rather than long half-life can completely negate returns from an alpha signal

Prior to transaction costs we see little systematic dependence of the performance on the model half-life. However, when including transaction costs, using a model with a half-life of 30 rather than 252 days can completely negate returns otherwise inherent in the portfolio or alpha signal.

Figure 40: Performance of mean-variance optimised portfolios versus half-life and rebalancing interval

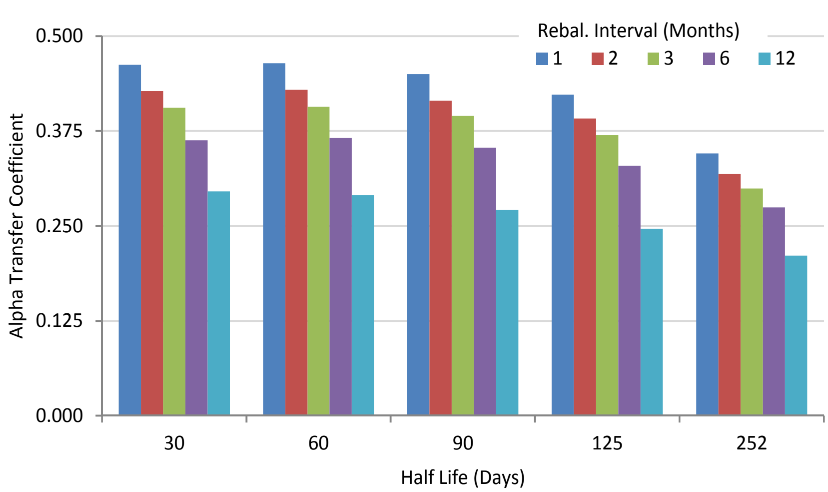


Source: Axioma, Factset, Deutsche Bank

In Figure 41 we see the average transfer coefficients, χ, of the portfolio holdings with the alpha signals. The alpha transfer coefficient is given by:

$$\chi_t = corr(a_t, p_t) \qquad (17)$$

Figure 41: Average alpha transfer coefficients for mean-variance optimised portfolios versus half-lives and rebalancing intervals



Source: Axioma, Factset, Deutsche Bank

There is a clear decay in the transfer coefficient as the portfolio rebalancing intervals increase. This would be more pronounced for rapidly decaying alpha signals such as reversal, and would be smaller for signals that evolve more slowly, such as size.

The alpha transfer coefficient decreases as the model responsiveness decreases.

The alpha transfer coefficient also decays with increasing half-lives. This is due to the tendency for risk models with longer half-lives to over-forecast discussed previously. The utility function for mean-variance optimised portfolios is described by Equation 12, and is a balance of expected returns and risk. Given a tendency to over-forecast risk it follows that the utility will tilt further away from the alpha signal, thus lowering the transfer function.

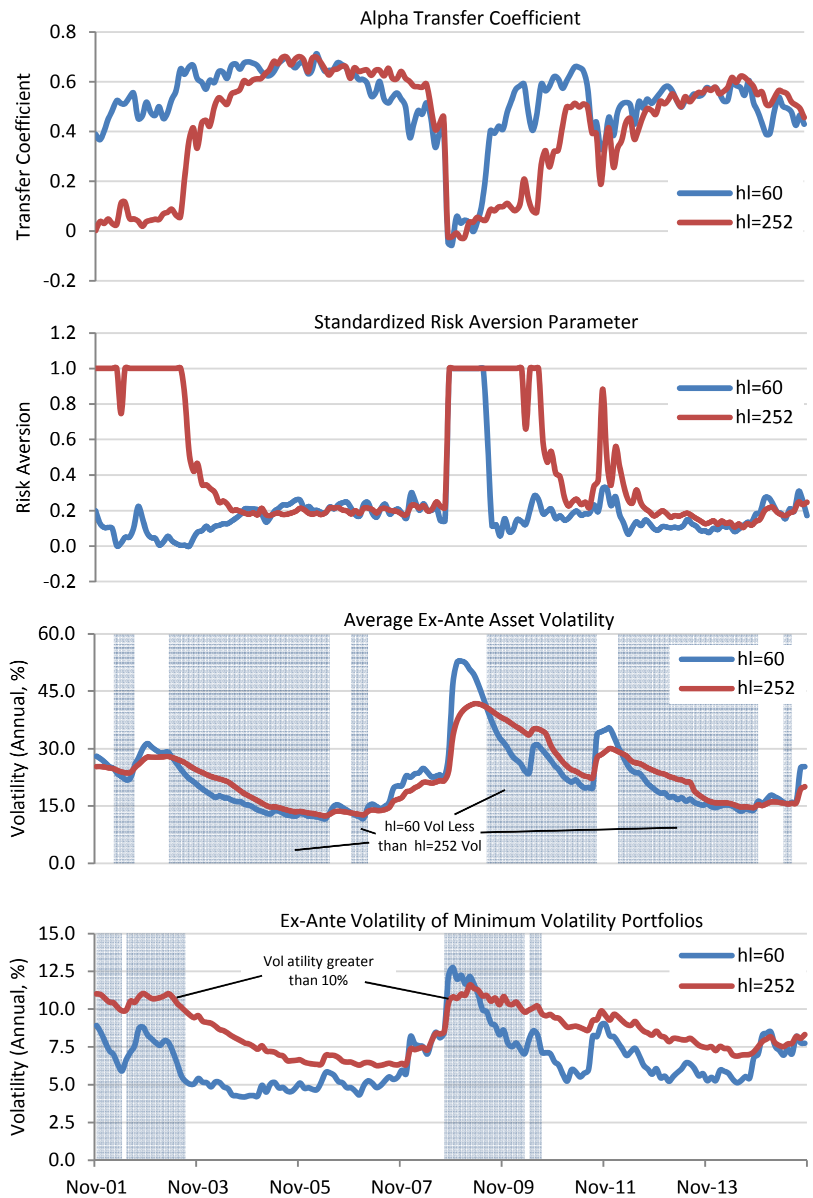
While the results in Figure 40 and Figure 41 provide useful information on the net output, it is insightful to look into the time-series properties. Figure 42 has four time-series charts related to the mean-variance portfolios generated with the risk models with half-lives of 60 and 125 days with a 10% ex ante volatility target; the first chart displays the average transfer coefficients of the mean-variance portfolios; the second chart displays the average standardized risk aversion of the mean-variance portfolios; the third chart shows the average ex-ante asset volatilities as determined with the risk models with 60 and 125 day half-lives; the fourth chart displays the volatilities of the minimum volatility portfolios calculated with Equation 11.

From Figure 42 we see in the top chart how the alpha transfer coefficients vary over time, and have markedly different values depending on the responsiveness of the risk model. The forecast (ex-ante) volatilities determine this behaviour. When looking at the average asset-level ex-ante volatility for the two models (third chart), we see the intervals shaded in grey where the less responsive model generates higher volatility forecasts. As discussed earlier, these periods tend to follow intervals of high volatility. Accordingly, the alpha transfer coefficients for the model with the 125 day half-life tend to be lower.

Over-forecasting risk results in reduced transfer of alpha into mean-variance optimised portfolios

A second factor driving the transfer coefficient is the ex-ante volatility of the minimum volatility portfolio, seen in the bottom chart of Figure 42. The intervals shaded in grey correspond to points where the minimum volatility portfolio of one the models has an ex-ante volatility greater than 10%. At these points mean-variance optimisation will not be able to converge on a portfolio with 10% ex-ante volatility. It follows that the risk aversion parameter will be driven to the highest possible value (second chart), and the mean-variance portfolio will simply be the minimum volatility portfolio; the alpha signal will be 'ignored'. Accordingly, any positive transfer coefficient at such times will be due to coincidental correlation between the alpha signal and the holding of the minimum volatility portfolio. Again, given the tendency for the model with the longer half-life to yield higher volatility forecasts, there are more points where the ex-ante volatility of the minimum volatility portfolio calculated with the risk model with a half-life of 125 days is above 10%. At these points the corresponding alpha transfer coefficient is markedly lower.

Figure 42: Average alpha transfer coefficient and standardised risk aversion for mean-variance optimised portfolios in top two charts. Bottom two charts show average asset-level ex-ante volatility and ex-ante volatility of minimum volatility optimised portfolios. In all cases illustrated when using risk models with half-lives of 60 and 125 days.



Source: Axioma, Factset, Deutsche Bank

Can we do better?

In this section we begin to explore the possibility of using a risk model with differing degrees of responsiveness. We develop a simple prototype algorithm to determine intervals to switch between using risk models with differing levels of responsiveness.

Model switching

Thus far we have seen how different levels of responsiveness in risk models can yield different degrees of forecasting accuracy depending on the nature of the portfolio in question. We have seen how under- or over-forecasting risk can lead to incorrect levels of leverage in portfolio holdings, and with respect to mean-variance optimisation can lead to the incorrect integration of an alpha signal into the final portfolio. Turnover and transaction costs are also affected.

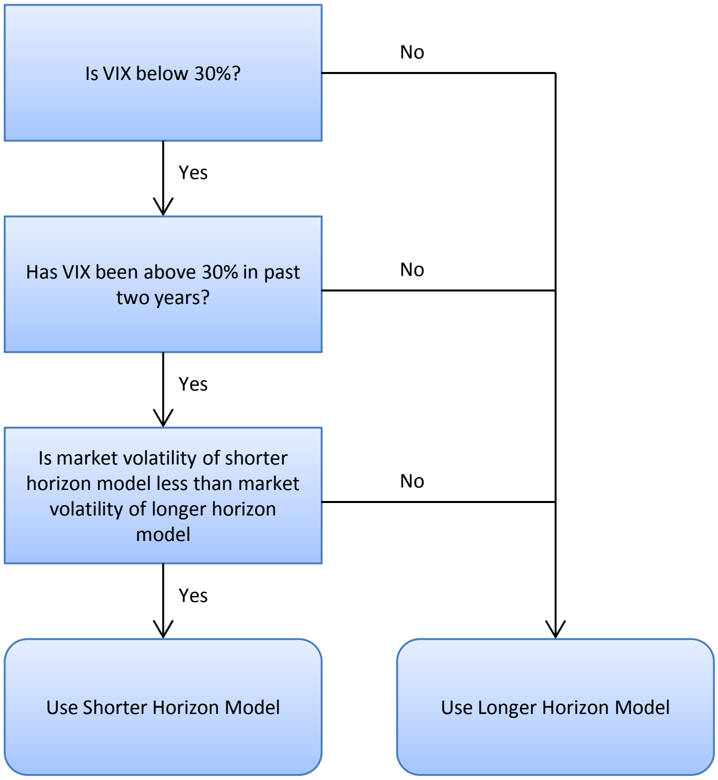
We have seen in Figure 6 and Figure 7 how volatility tends to be over-forecast to a greater extent by less responsive models following a market shock, while in more stable and lower volatility environments there is less consistent differentiation between the forecasts of risk models with different half-lives. A natural conclusion is that more responsive risk models would be more suitable in periods following market shocks.

Model switching algorithm identifies periods following market shocks where a longer-horizon model is likely to over-forecast risk

Here we have developed a simple prototype algorithm to determine periods to switch between two risk models with different levels of responsiveness. We use two models with 60 and 125 day half-lives, which correspond to the volatility half-lives in the Axioma[8] short- and medium-horizon European equity factor risk models.

[8] Note we are using volatilities determined with exponentially weighted Axioma factor returns as opposed to the factor volatilities as determined by the Axioma factor risk models. As mentioned earlier, the Axioma risk factor models use methods such as their patented 'Dynamic Volatility Adjustment' to augment their factor volatility measurements, which can yield different results. Such approaches will be discussed in subsequent research.

Figure 43: Model switching algorithm



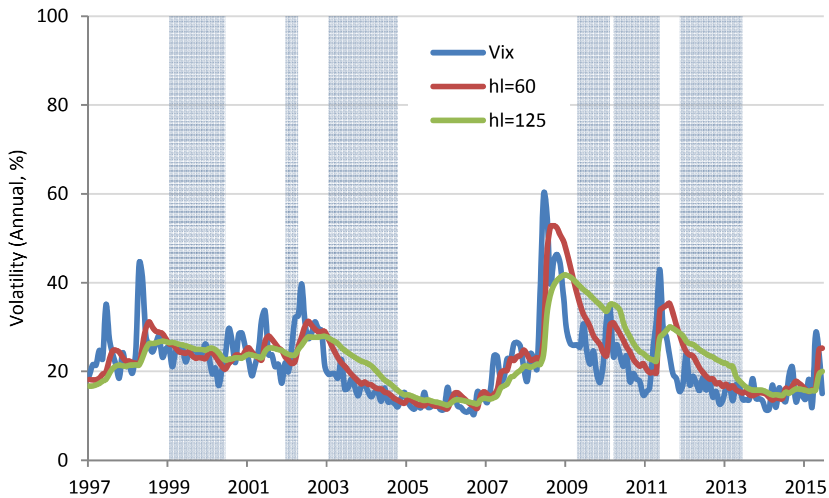
Source: Deutsche Bank

The inputs to our algorithm are the volatilities of the market factors of the 60 and 125-day models and the VIX. It is of note that while we use a short (60 day half-life) and medium (125 day half-life) horizon here, we could equally use a medium and long (252 day) horizon to elicit a similar effect. The approach is illustrated schematically in Figure 43. Briefly, the level of the VIX is used as reference; when the level is below 30%[9], but has been above 30% in the two preceding years, the algorithm checks the relative levels of the volatility of the market factors. The default model is the longer horizon model, but if the volatility of the market factor of the shorter horizon model is lower, then that model is used. A two year look-back window is used as we saw in Figure 7 how over-forecasting after a market shock can be sustained for considerably more than a year.

We use a short (60 day half-life) and medium (125 day half-life) horizon here, but could equally use a medium and long (252 day) horizon to elicit a similar effect.

[9] The choice of 30% was empirically determined as a threshold to define a market shock.

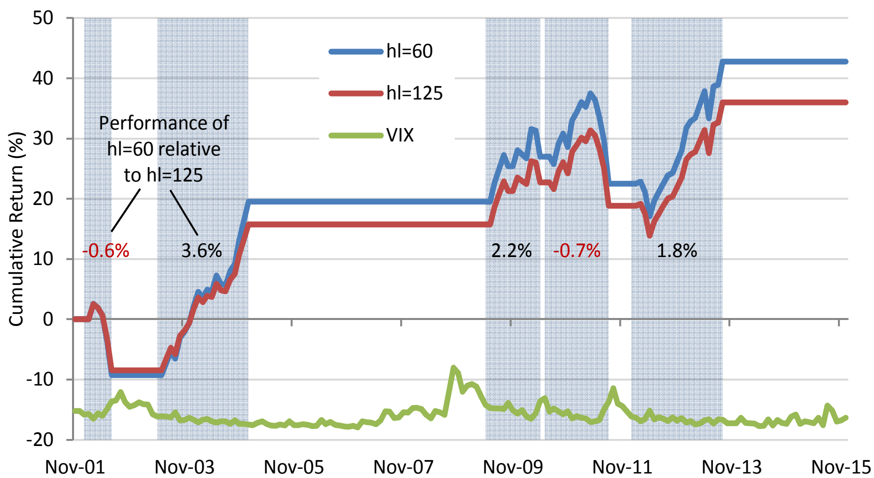
Figure 44: Volatility of market factors measured with half-lives of 60 and 252 days. VIX shown as reference. Grey shaded areas indicate periods to use more responsive risk model



Source: Axioma, Factset, Deutsche Bank

The three inputs of the model switching algorithm are illustrated in Figure 44. The grey shaded areas correspond to the intervals where the shorter (60 day half-life) model is used. We see that there are six periods where the more responsive model is used, including those following the Russian financial crisis in 1998, the post dot-com bubble downturn of 2002, the 2008 global financial crisis, and the 2011 European sovereign debt crisis. In these periods we see the volatility of the more responsive model falls significantly below that of the less responsive model, which we have seen in previous sections would tend to over-forecast risk. The market volatility as determined by the VIX can also be seen to generally be in decline. The intervals vary in length from 3 to 20 months.

Figure 45: Average wealth curves of long-only portfolios in periods where shorter horizon model is selected. VIX shown as reference

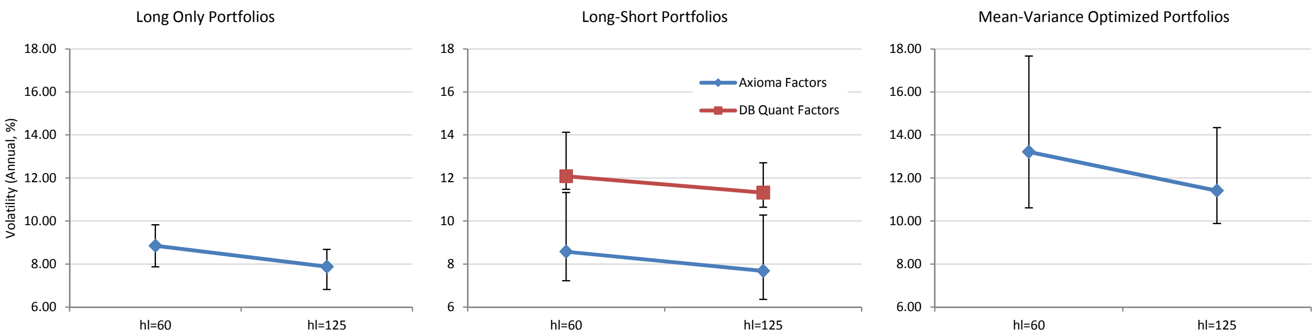


Source: Axioma, Factset, Deutsche Bank

Figure 45 illustrates the average wealth curves of long-only portfolios positioned according to risk models with 60 and 125 day half-lives when invested during the periods identified by the algorithm where the shorter horizon model should be used. In this context we see an aggregate improvement in portfolio performance when holding leverage levels determined by the shorter-horizon risk model. We see that in three of the five periods the shorter horizon model out-performed with relative gains of 3.6%, 2.2%, and 1.8%, while in two periods the holdings based on the more responsive model under-performed by smaller margins of 0.6% and 0.7%. The losses are due to the lag in responsiveness when a new market shock occurs; the associated downturns in market result in greater losses in the more leveraged portfolios. The ability to switch at intra-month points may help overcome this problem.

In these risk-targeted portfolios (10% ex-ante volatility target) more leverage is induced in the 'switched' periods. This follows from Equation 15 as the shorter horizon risk model forecasts lower volatility levels at these times. Further, the algorithm from Figure 43 will generally detect periods where risk is decreasing, as seen with the VIX in Figure 45 and previously in Figure 44. Given that changes in volatility tend to be negatively correlated with returns (Shu & Zhang, 2003; Zanutto, 2014), it follows that market direction would normally be positive, thus the long positions with more leverage generally out-perform. These results indicate the switching algorithm could be of use in market timing.

Figure 46: Median realised volatility of 10% risk-targeted long-only, long-short, and mean-variance optimised portfolios with 60 and 125 day half-life risk models in periods where shorter horizon model is selected by switching algorithm. Error bars: 5%-95% range.



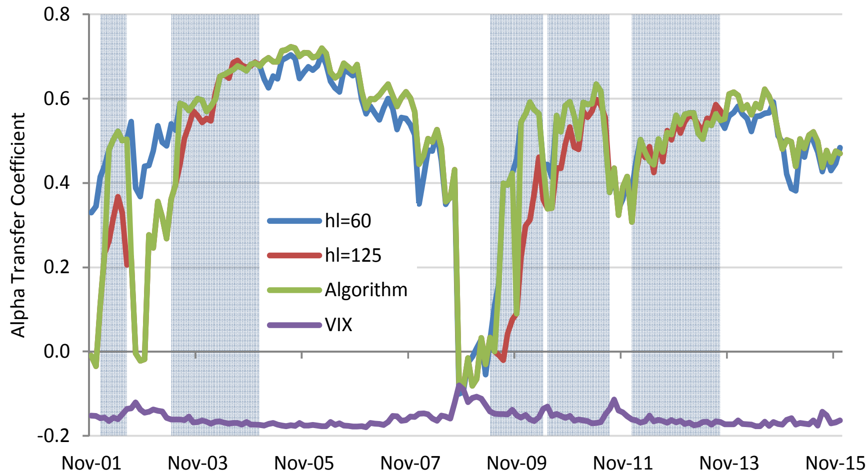
Source: Axioma, Factset, Deutsche Bank

Figure 46 illustrates the average realised (annual) volatilities of the long-only, long-short, and mean-variance optimised portfolios when constructed using the models with 60 and 125 day half-lives in the 'switched' periods. For the long-only portfolios the results are consistent with the results in Figure 16; the models tend to over-estimate the volatility, but less so with the shorter half-life. Accordingly, the realised volatility is closer to the targeted 10% when using the shorter half-life.

For the long-short portfolios we again see a difference between the realised volatility of the portfolios based on Axioma factors and those based on the DB Quant factors. The difference is consistent with Figure 18. The risk of portfolios based on the Axioma factors is, on average, over-forecast due to the predominantly univariate nature of the risk measurement. Switching to the more responsive risk model results in more accurate risk forecasts. For the portfolios based on the DB Quant factors the tendency is to under-forecast risk, likely due to correlations missed in the factor model. Switching to the shorter horizon model results in inferior volatility forecasts; the longer effective sampling window associated with the less responsive model results in better correlation measurements.

When looking at mean-variance optimised portfolios we again see that the risk models systematically under-forecast volatility, and more so for models with shorter half-lives.

Figure 47: Average alpha transfer of mean-variance optimised portfolios with risk models with half-lives of 60 and 252 days. Transfer of switching algorithm also shown. The grey shaded areas correspond to periods where the shorter horizon model is selected. VIX displayed for reference.



Source: Axioma, Factset, Deutsche Bank

For mean-variance optimised portfolios the average alpha transfer coefficient using models with 60 and 125 day half-lives and the switching algorithm is displayed in Figure 47. The results are intuitive and consistent with observations based on Figure 42 and Figure 44: the lower volatility forecasts of the shorter horizon model in the 'switched' periods result in higher alpha transfer coefficients.

We recall that the mean-variance optimised portfolios here are 100% invested, but use leverage. Accordingly, when seeking the most efficient incorporation of alpha signal with an accurate risk target, it is of interest to determine if the optimal portfolio would be similar to that yielded with the shorter horizon model, but with less leverage, or more similar to that yielded with the longer horizon model, which tilts more towards the minimum volatility portfolio. To investigate this question we need to use a method to correct the risk model for the under-forecasting of volatilities of optimised portfolios. The existence of such methods was discussed earlier. This will be investigated in subsequent research.

Conclusions

In this report we have investigated how risk model responsiveness, as determined by the half-life of the weights applied to returns time series, can affect volatility forecasting accuracy, and the implications for portfolio performance with respect to portfolio construction. We have looked at four classes of portfolio: single-stock, long-only, dollar-neutral, and optimised portfolios. We briefly looked into different approaches to evaluate the accuracy of risk forecasts. We also investigated the effect of risk model responsiveness on portfolio turnover and transaction costs. Finally, we have developed a simple prototype algorithm to determine the optimal responsiveness of a risk model for different market conditions.

Our results have illustrated how risk model responsiveness can significantly affect the accuracy of the risk forecasts. In turn, the forecasting accuracy can significantly affect portfolio performance when the risk model has been used to determine leverage, or in portfolio construction when using optimisation. Further, we have seen how forecasting accuracy varies according to the type of portfolio, and with rebalancing frequency. When considering forecasting accuracy we have seen how the optimal level of risk model responsiveness generally decreases respectively with consideration to single stock, long-only, dollar-neutral, and optimised portfolios, with optimal volatility half-lives ranging from 30-60 days for single stock and long-only portfolios, to 250+ days for optimised portfolios. This is largely due to the increasing importance of correlations in the risk attribution when considering these portfolio classes in turn. In regard to optimised portfolios there is inherent under-forecasting bias that is exacerbated when using shorter half-lives. A method to correct the estimation error in the covariance matrix should ideally be used to remedy this.

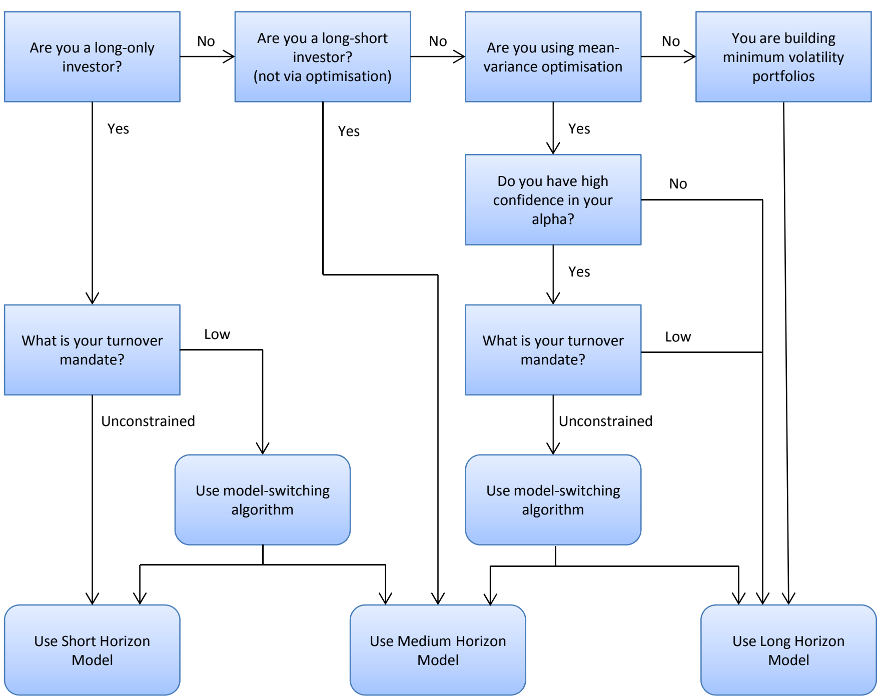
Considering portfolio turnover we have seen that risk model responsiveness has the greatest relative impact on optimised portfolios, with more responsive models instigating far higher turnover than equivalent, less responsive models. Indeed, rebalancing monthly with a very responsive risk model can almost completely negate gains from an alpha signal that would be achieved without transaction costs. Counter-intuitively we have also seen that if using leverage to achieve volatility targets, the highest turnover occurs during periods of low market volatility, as greater leverage is typically required to achieve a volatility target. Again, this is exacerbated when using more responsive risk models.

When evaluating volatility forecasts the results have shown how the metric to evaluate forecasting accuracy is of great importance. Considering the realised volatility or bias statistic (Connor, 2000) of a portfolio can mask extended periods of under- or over-forecasting. Such matters warrant consideration with regards to risk control and reporting. Using the absolute deviation of the bias statistic can help to overcome this problem. With the aid of simulated data and the absolute deviation of the bias statistic we have seen how, generally, no model is able to forecast a market volatility shock, and as such risk is under-forecast. We have also seen that lower-frequency risk models tend to over-forecast risk in the period following a market shock, with ex-ante volatilities above those of shorter horizon models. The over-forecasting of risk from long horizon models results in under-leveraged portfolios, and when using mean-variance optimisation, in lower utilization of the alpha signal.

With regard to over-forecasting volatility following a market shock we have developed a simple algorithm using the VIX and the ex-ante market volatilities of risk models with 60 and 125 day half-lives to determine periods to switch between short- and medium-horizon risk models. The algorithm identifies periods after market shocks where switching to more responsive risk models can be beneficial. Results indicate that this can aid the construction of long-only portfolios as volatility forecasts are improved, and leverage is typically increased in periods when volatility is normally decreasing and returns are trending up. The algorithm is also potentially beneficial for the construction of mean-variance optimised portfolios in that the integration of the alpha signal is increased.

Based on these conclusions we have developed the model horizon selection algorithm depicted in Figure 48. For reference we refer to a half-life of 60 days as 'short' horizon, a half-life of 125 days as 'medium' horizon, and a half-life of 252 days as a 'long' horizon.

Figure 48: Risk model horizon selection tool



Source: Deutsche Bank

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