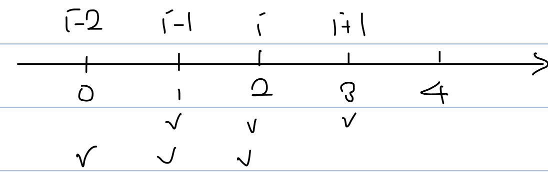
(a) $\frac{dx_i}{dt} = x_{i-1}(x_{i+1} - x_{i-2})$

$$\sum_{i=1}^{N} \frac{dx_i^2}{dt} = \sum_{i=1}^{N} 2x_i \frac{dx_i}{dt} = \sum_{i=1}^{N} 2x_i\, x_{i-1}(x_{i+1} - x_{i-2})$$

$$= \sum_{i=1}^{N} 2(x_i x_{i-1} x_{i+1} - x_i x_{i-1} x_{i-2})$$



Due to the periodicity,

$$\sum_{i=1}^{N} x_{i-1} x_i x_{i+1} = \sum_{i+1=1}^{N} x_{(i+1)-2}\, x_{(i+1)-1}\, x_{(i+1)}$$

Therefore, $\frac{dx_i^2}{dt} = 0$, the energy is conserved.

Q2: $\det(J) = \frac{dx}{da}\frac{dx'}{db}\frac{dx''}{dc} + \frac{dx}{db}\frac{dx'}{dc}\frac{dx''}{da} + \frac{dx}{dc}\frac{dx'}{da}\frac{dx''}{db}$

$$- \frac{dx}{dc}\frac{dx'}{db}\frac{dx''}{da} - \frac{dx}{db}\frac{dx'}{da}\frac{dx''}{dc} - \frac{dx}{da}\frac{dx'}{dc}\frac{dx''}{db}$$

since $x''' = P = \frac{dx''}{dt}$

$$\frac{d(\det(J))}{dt} = \frac{dx}{da}\frac{dx'}{db}\frac{dP}{dc} + \frac{dx}{db}\frac{dx'}{dc}\frac{dP}{da} + \frac{dx}{dc}\frac{dx'}{da}\frac{dP}{db}$$

$$- \frac{dx}{dc}\frac{dx'}{db}\frac{dP}{da} - \frac{dx}{db}\frac{dx'}{da}\frac{dP}{dc} - \frac{dx}{da}\frac{dx'}{dc}\frac{dP}{db}$$

$$= \frac{dx}{da}\frac{dx'}{db}\frac{dP}{dx''}\frac{dx''}{dc} + \frac{dx}{db}\frac{dx}{dc}\frac{dP}{dx''}\frac{dx''}{da} + \frac{dx}{dc}\frac{dx'}{da}\frac{dP}{dx''}\frac{dx''}{db}$$

$$- \frac{dx}{dc}\frac{dx'}{db}\frac{dP}{dx''}\frac{dx''}{da} + \frac{dx}{db}\frac{dx'}{da}\frac{dP}{dx''}\frac{dx''}{dc} - \frac{cx}{da}\frac{dx'}{dc}\frac{dP}{dx''}\frac{dx''}{db}$$

$$= \det(J)\frac{\partial P}{\partial x}$$

HW3

Q1

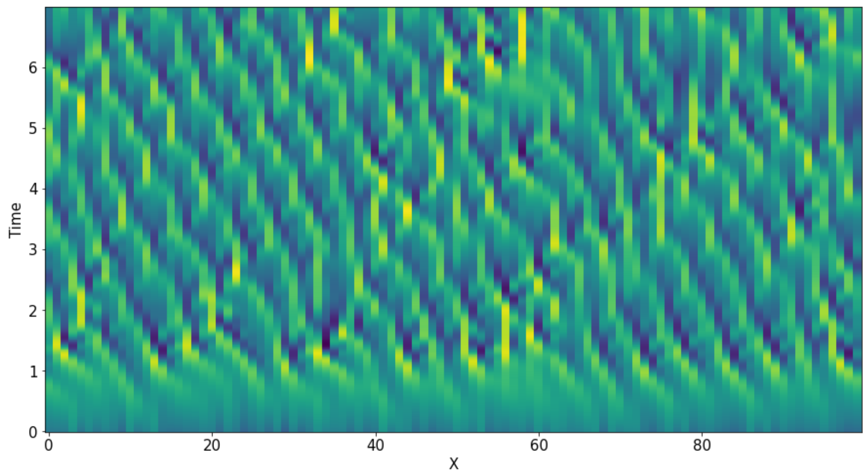
(b)

The Lorenz96 is set up with

- dt = 0.01; total time step = 700 (previous 200 is spin up)
- force = 8; number of grid points = 100
- The initial state is 0 + Gaussian noise with standard deviation = 1.0

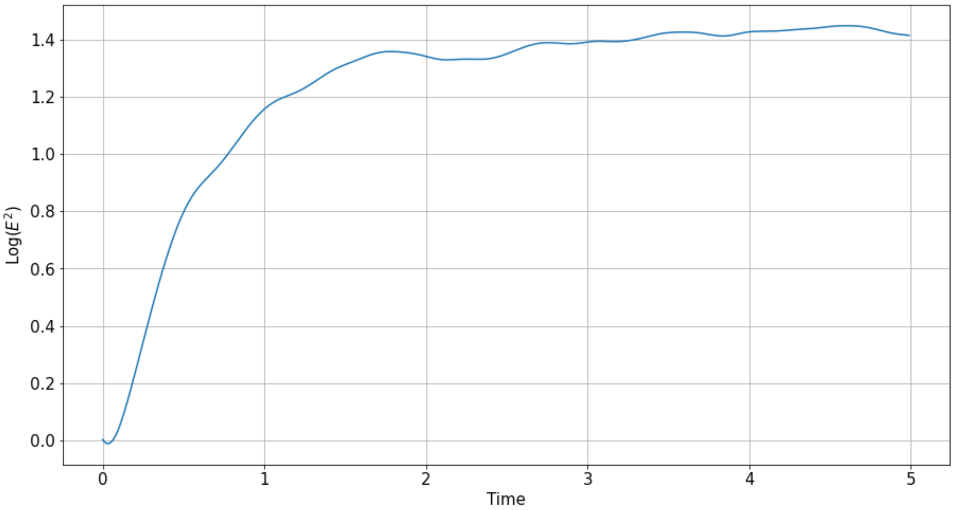
The detailed setup:
https://github.com/neko2048/ChaosAndPredictability/blob/master/hw3/code/F8.ipynb

The first attempt (as the ground truth) is shown below:



I then took this first attempt's state when time = 2 as the emsemble's initial state with noise.

The error growth of LR96 model is shown below:

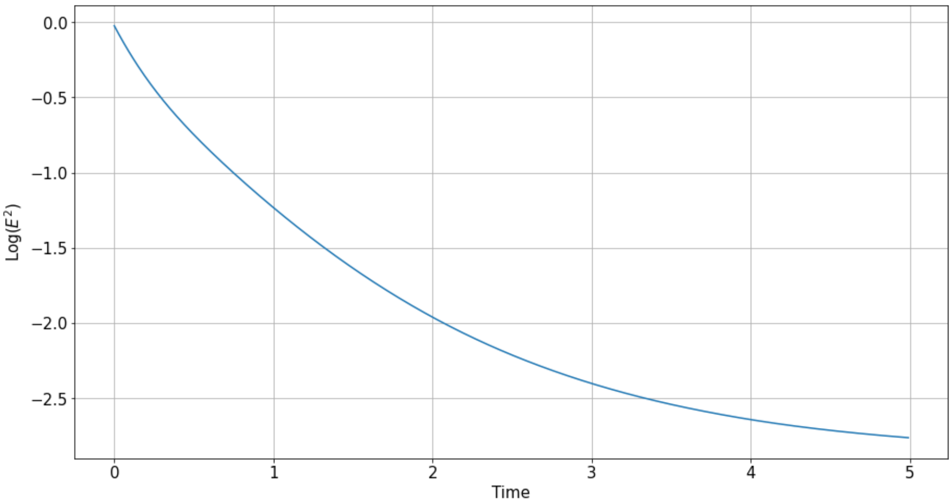


Given that $log(\frac{E(t)}{E(0)}) = log(2)$, $E(t) = 10^{log(2)+log(E(0))}$ and the corresponded time step is 38th time step ($t = 0.38$) in this case.

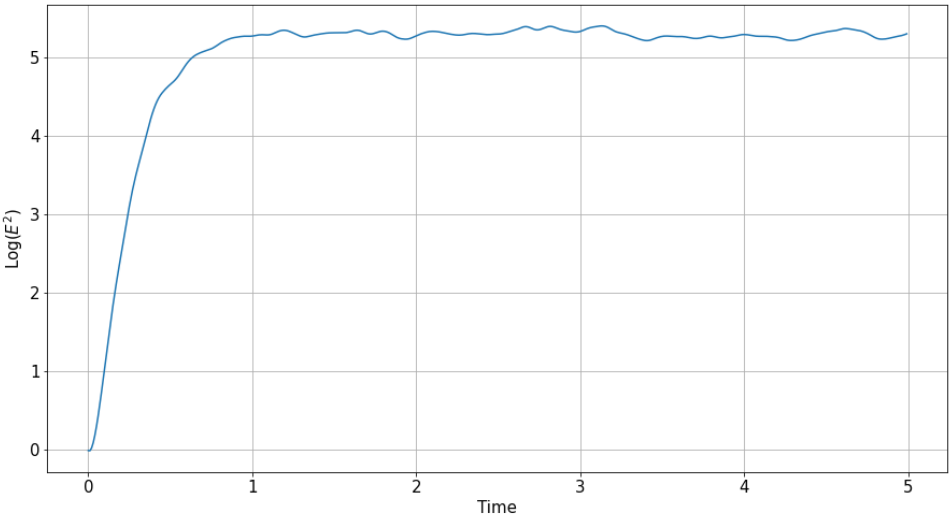
(c)

I tested other different forcing like $F = 1/8/30$,

When $F = 1$, the error growth is shown below and the error-doubling time cannot be counted as the error growth is declining:



When $F = 30$, error-doubling time is about 0.12 (12 time steps)



This indicates that with stronger forcing term, the error-doubling time increases. With smaller forcing, the system with less outer forcing tends to advect and exponentially decrease, which leads to the system become smoothed and wave damps to zero. Consequently, since all ensemble members damps to zero, there is no error growth.

However, the strong forcing term makes the system more uncertain and chaotic. Thus the error-doubling time decreases.