Department of Atmospheric Sciences
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Chaos and Predictability / HW2

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Question 1

We already know the PV conservation and the QG thermodynamics equation can be written as.

$$\begin{aligned} \frac{d_g q}{dt} &= 0 \\ \frac{d_g b}{dt} + wN^2 &= 0 \end{aligned} \quad (1)$$

where q is defined as $\nabla^2\psi + f + \frac{f^2}{N^2}\frac{\partial^2\psi}{\partial z^2}$, b is defined as $f\frac{\partial\psi}{\partial z}$, w is the vertical velocity, and N^2 is the square of Brunt–Väisälä frequency.

(a) Discuss what causes the instability of q from timescales of (1) shorter than the intrinsic timescales of Rossby wave (i.e., < 14 days) and (2) longer than the wave intrinsic timescales (i.e., ≥ 14 days). Which term can (cannot) be omitted in the first (second) case? Why?

(b) When an atmospheric blocking happens, the upper and lower troposphere tends to be more barotropic (i.e., high and low pressure centers over different vertical layers are spatially collocated). From the perspective of baroclinic instability, discuss why the atmospheric blocking has a much longer life cycle than a typical extratropical front?

Question 2

We know that the Rayleigh-Benard convection (instability) arises from unstable background temperature profile which generates free convection to conquer the friction. Rayleigh number, defined as the ratio of buoyancy force to friction, is center to the measurement of instability.

(a) Using Dedalus (on google colab) to simulate Rayleigh-Benard convection. Testing low Rayleigh number ($\sim 10^1$) and high Rayleigh number case ($\sim 10^8$). Run the model long enough to have a dynamical equilibrium state. Plot the results and discuss the difference.

(b) Following question (a) and equation (2), plot the X(t) against Y(t). Discuss how the choice of Rayleigh number influences the periodicity of X(t) and Y(t). An exmaple of Fourier Transform can be found here DFT

$$\begin{aligned} \zeta &= X(t)\sin(\frac{\pi a x}{H})\sin(\frac{\pi z}{H}) \\ T &= Y(t)\cos(\frac{\pi a x}{H})\sin(\frac{\pi z}{H}) - Z(t)\sin(2\frac{\pi z}{H}) \end{aligned} \quad (2)$$

(a is defined as $\frac{H}{L}$, where L is the domain size along x-axis and H is the domain size along z-axis)