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Chaos and Predictability / HW1

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In this homework, you will learn the concept of "phase dependent" error growth. i.e., given the same initial error, the spread of error grows differently at different physical spaces. You will also learn that the error growth is not only a function of physical spaces but also a function of dynamical system itself.

Question 1

We know the Lorenz 63 model is written as

$$
\begin{aligned}
\frac{dx}{dt} &= \sigma(y - x) \\
\frac{dy}{dt} &= x(\rho - z) - y \\
\frac{dz}{dt} &= xy - \beta z
\end{aligned}
\tag{1}
$$

(a) Linearizing the system about the equilibrium solution x=0,y=0,z=0, analyze the stability for the time continuous case. Assume that σ=10, $\beta = \frac{8}{3}$, and ρ=24.74.

(b) Repeat the same stability analysis at different equilibrium point, what do you see?

(c) Following (a) and dropping the z term. Using a first-order forward scheme to discretize equation (1) and assume $x^{n+1} = \lambda x^n$, $y^{n+1} = \lambda y^n$. $\|\lambda\|$ is so-called amplification factor. Compare the results between the discrete version and the continuous version in (a). Suggest a good choice for the value of the time step. (i.e., when do both cases give you similar results?)

Question 2

Program the model twice, using 1st-order forward time-differencing in one case and the fourth- order Runge-Kutta method in the other. As in part (a), use the parameter settings σ=10, $\beta = \frac{8}{3}$ and ρ=24.74. Start the model from the initial condition (x,y,z)=(0.1,0,0), which is close to the equilibrium mentioned above. Experiment to determine the longest permissible time step with each scheme. Run the model long enough to see the butterfly. Plot the results for both schemes, and compare them.