

Uniform Ratios for Sectioned Line Segments (WIP)

Avery Rosenblum-O'Connor

ABSTRACT

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Introduction

Definitions

Sectioned Line Segment — A sectioned line segment is a line segment that has been divided by a number of points into two or more smaller, constituent line segments. These “sections” do not overlap one another and together cover the entirety of the sectioned line segment to which they belong.

Section — A section is one of the constituent line segments of which a sectioned line segment is comprised.

Form (of a ratio) — Any given ratio between two real numbers $A : B$ may be expressed as either of two real numbers — $\frac{A}{B}$ and $\frac{B}{A}$. These numbers are the forms of that ratio.

Canonical Form (of a ratio) — As all ratios have two forms, it is desirable to select a single form for each ratio by which that ratio will be referred to. If both forms of a ratio are equal to 1, the canonical form of that ratio is 1. In all other cases, the canonical form of a ratio is whichever of its forms is greater than 1. In this paper, ratios are always referred to as their canonical forms, and comparisons between ratios are performed on the basis of their canonical forms. Thus, if the canonical forms of two ratios are equal, those ratios are said to be equal, and if the canonical form of one ratio is greater than that of another, the former is said to be greater than the latter.

Uniform Ratio — A uniform ratio is a one that describes the relationships present between certain sectioned line segments and their constituent sections. In particular, if a uniform ratio exists for a particular sectioned line segment, then that ratio is equal to the ratio between the length of the sec-

tioned line segment and the length of its longest section, the length of the longest section to the length of the second longest section, the length of the second longest section to the length of the third longest section, and so on. If there is no ratio that satisfies this property, then there does not exist a uniform ratio for that sectioned line segment.

Uniform Sectioned Line Segment — If a uniform ratio exists for a particular sectioned line segment, that sectioned line segment is said to be uniform.

Motivation

This paper was born out of an exploration of generalizations of the so-called “golden cut,” which is a uniform sectioned line segment composed of two sections. The uniform ratio for such a uniform sectioned line segment is equal to Φ , or the golden ratio — hence the term “golden cut.” Can a similar sectioned line segment be constructed with three sections? Four? What ratio has the same properties for these sectioned line segments as Φ does for the golden cut? Do there exist generalized answers to these questions when they are asked in the case of n sections?

These questions gave rise first to the concepts, and then to the terminology, of uniform sectioned line segments and their corresponding uniform ratios. Uniform sectioned line segments represent the generalization of the golden cut to sectioned line segments with more than two sections, and in this context uniform ratios represent the generalization of Φ .

Discovery Process

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