Lecture - 2 (Extra)

Conditional and BiConditional Statements

Conditional Statement

If P and Q are two statements then "if P then Q" is a compound statement, denoted by $P \rightarrow Q$ and referred to as a conditional statement, or implication. The implication $P \rightarrow Q$ is false only when P is true, and Q is false; otherwise, it is always true. In this implication, P is called the hypothesis (or antecedent) and Q is called the conclusion (or consequent).

P	**Q**	**$P \rightarrow Q$**
True	True	True
True	False	False
False	True	True
False	False	True

For Example: The following are conditional statements.

1. If a = b and b = c, then a = c.
2. If I get money, then I will purchase a computer.

Variations in Conditional Statement

Contrapositive: The proposition $\neg Q \rightarrow \neg P$ is called contrapositive of $P \rightarrow Q$.

Converse: The proposition $Q \rightarrow P$ is called the converse of $P \rightarrow Q$.

Inverse: The proposition $\neg P \rightarrow \neg Q$ is called the inverse of $P \rightarrow Q$.

Example1: Show that P →Q and its contrapositive ¬Q→ ¬P are logically equivalent.

Solution: Construct the truth table for both the propositions:

P	**Q**	**¬P**	**¬Q**	**P → Q**	**¬P → ¬Q**
True	True	False	False	True	True
True	False	False	True	False	False
False	True	True	False	True	True
False	False	True	True	True	True

As, the values in both cases are same, hence both propositions are equivalent.

Example2: Show that proposition Q→P, and ¬P→¬Q is not equivalent to P →Q.

Solution: Construct the truth table for all the above propositions:

P	**Q**	**¬P**	**¬Q**	**P → Q**	**Q → P**	**¬P → ¬Q**
True	True	False	False	True	True	True
True	False	False	True	False	True	False
False	True	True	False	True	False	True
False	False	True	True	True	True	True

As, the values of P →Q in a table are not equal to Q→P and ¬P→¬Q as in fig. So both of them are not equal to P →Q, but they are themselves logically equivalent.

BiConditional Statement

If P and Q are two statements then "P if and only if Q" is a compound statement, denoted as $P \leftrightarrow Q$ and referred as a biconditional statement or an equivalence. The equivalence $P \leftrightarrow Q$ is true only when both P and Q are true or when both P and Q are false.

P	**Q**	**$P \leftrightarrow Q$**
True	True	True
True	False	False
False	True	False
False	False	True

For Example: (i) Two lines are parallel if and only if they have the same slope.
(ii) You will pass the exam if and only if you will work hard.

Example: Prove that $P \leftrightarrow Q$ is equivalent to $(P \rightarrow Q) \wedge (Q \rightarrow P)$.

Solution: Construct the truth table for both the propositions:

P	**Q**	**$P \leftrightarrow Q$**
True	True	True
True	False	False
False	True	False
False	False	True

P	**Q**	**$P \rightarrow Q$**	**$Q \rightarrow P$**	**$(P \rightarrow Q) \wedge (Q \rightarrow P)$**
True	True	True	True	True
True	False	False	True	False
False	True	True	False	False
False	False	True	True	True

Since, the truth tables are the same, hence they are logically equivalent. Hence Proved.