Lecture - 2

Propositional Logic

A **Proposition** or a statement or logical sentence is a declarative sentence which is either true or false.

Example1: The following statements are all propositions:

- Jawaharlal Nehru is the first prime minister of India.
- It rained Yesterday.
- If x is an integer, then x^2 is a +ve integer.

Example2: The following statements are not propositions:

- Please report at 11 a.m. sharp
- What is your name?
- $x^2=13$

Propositional Variables

The lower case letters starting from P onwards are used to represent propositions

Example: p: India is in Asia
q: 2 + 2 = 4

Compound Statements

Statements or propositional variables can be combined by means of logical connectives (operators) to form a single statement called compound statements.

The five logical connectives are:

Symbol	Connective	Name
~	Not	Negation
	And	Conjunction
	Or	Disjunction
	Implies or if…then	Implication or Conditional
	If and only if	Equivalence or Biconditional

Basic Logical Operations

Negation (¬)

The negation of a proposition A (written as ¬A) is false when A is true and is true when A is false.

The truth table is as follows −

A	
True	False
False	True

OR (∨)

The OR operation of two propositions A and B (written as A ∨ B) is true if at least any of the propositional variable A or B is true.

The truth table is as follows −

A	B	
True	True	True
True	False	True
False	True	True
False	False	False

AND (∧)

The AND operation of two propositions A and B (written as A∧B) is true if both the propositional variable A and B is true.

The truth table is as follows −

A	**B**	
True	True	True
True	False	False
False	True	False
False	False	False

Implication / if-then (→)

An implication A→B is the proposition “if A, then B”. It is false if A is true and B is false. The rest of the cases are true.

The truth table is as follows −

A	B	
True	True	True
True	False	False
False	True	True
False	False	True

If and only if (⇔)

A⇔B is bi-conditional logical connective which is true when p and q are same, i.e. both are false or both are true.

The truth table is as follows −

A	B	
True	True	True
True	False	False
False	True	False
False	False	True

Precedence of Logical Operators

Operator	Name	Precedence
	Negation	1
	Conjunction	2
	Disjunction	3
	Implication	4
	Biconditional	5