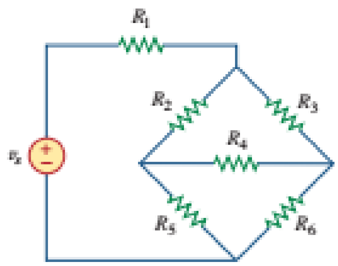


**Figure 2.46**
The bridge network.

## 2.7 †Wye-Delta Transformations

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series. For example, consider the bridge circuit in Fig. 2.46. How do we combine resistors $R_1$ through $R_6$ when the resistors are neither in series nor in parallel? Many circuits of the type shown in Fig. 2.46 can be simplified by using three-terminal equivalent networks. These are the wye (Y) or tee (T) network shown in Fig. 2.47 and the delta (Δ) or pi (Π) network shown in Fig. 2.48. These networks occur by themselves or as part of a larger network. They are used in three-phase networks, electrical filters, and matching networks. Our main interest here is in how to identify them when they occur as part of a network and how to apply wye-delta transformation in the analysis of that network.

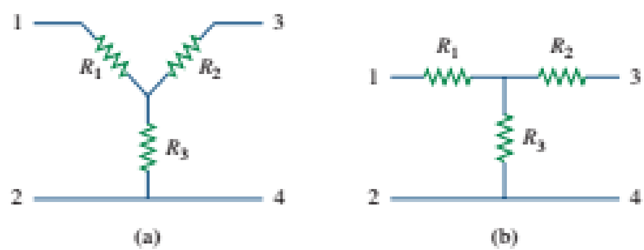


**Figure 2.47**
Two forms of the same network: (a) Y, (b) T.

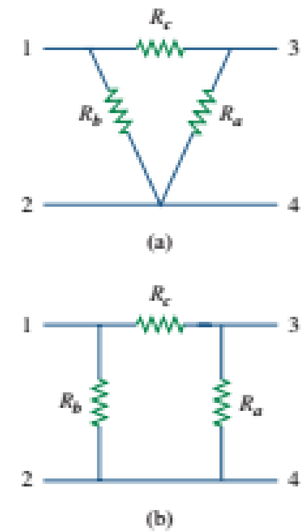


**Figure 2.48**
Two forms of the same network: (a) Δ, (b) Π.

## Delta to Wye Conversion

Suppose it is more convenient to work with a wye network in a place where the circuit contains a delta configuration. We superimpose a wye network on the existing delta network and find the equivalent resistances in the wye network. To obtain the equivalent resistances in the wye network, we compare the two networks and make sure that the resistance between each pair of nodes in the Δ (or Π) network is the same as the resistance between the same pair of nodes in the Y (or T) network. For terminals 1 and 2 in Figs. 2.47 and 2.48, for example,

$$R_{12}(\mathrm{Y}) = R_1 + R_3 \tag{2.46}$$
$$R_{12}(\Delta) = R_b \parallel (R_a + R_c)$$

Setting $R_{12}(\mathrm{Y}) = R_{12}(\Delta)$ gives

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \tag{2.47a}$$

Similarly,

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \tag{2.47b}$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \tag{2.47c}$$

Subtracting Eq. (2.47c) from Eq. (2.47a), we get

$$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c} \tag{2.48}$$

Adding Eqs. (2.47b) and (2.48) gives

$$\boxed{R_1 = \frac{R_b R_c}{R_a + R_b + R_c}} \tag{2.49}$$

and subtracting Eq. (2.48) from Eq. (2.47b) yields

$$\boxed{R_2 = \frac{R_c R_a}{R_a + R_b + R_c}} \tag{2.50}$$

Subtracting Eq. (2.49) from Eq. (2.47a), we obtain

$$\boxed{R_3 = \frac{R_a R_b}{R_a + R_b + R_c}} \tag{2.51}$$

We do not need to memorize Eqs. (2.49) to (2.51). To transform a Δ network to Y, we create an extra node $n$ as shown in Fig. 2.49 and follow this conversion rule:

Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

One can follow this rule and obtain Eqs. (2.49) to (2.51) from Fig. 2.49.

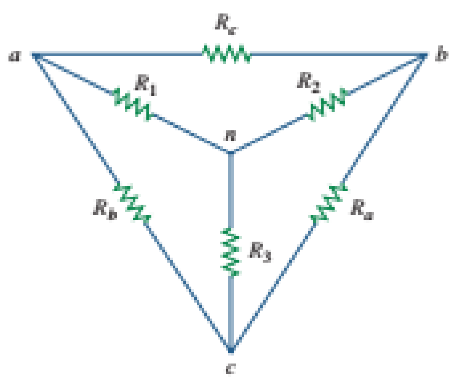


**Figure 2.49**
Superposition of Y and Δ networks as an aid in transforming one to the other.

## Wye to Delta Conversion

To obtain the conversion formulas for transforming a wye network to an equivalent delta network, we note from Eqs. (2.49) to (2.51) that

$$\begin{aligned} R_1R_2 + R_2R_3 + R_3R_1 &= \frac{R_aR_bR_c(R_a + R_b + R_c)}{(R_a + R_b + R_c)^2} \\ &= \frac{R_aR_bR_c}{R_a + R_b + R_c} \end{aligned} \tag{2.52}$$

Dividing Eq. (2.52) by each of Eqs. (2.49) to (2.51) leads to the following equations:

$$\boxed{R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}} \tag{2.53}$$

$$\boxed{R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}} \tag{2.54}$$

$$\boxed{R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}} \tag{2.55}$$

From Eqs. (2.53) to (2.55) and Fig. 2.49, the conversion rule for Y to Δ is as follows:

Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

The Y and Δ networks are said to be *balanced* when

$$R_1 = R_2 = R_3 = R_Y, \qquad R_a = R_b = R_c = R_\Delta \tag{2.56}$$

Under these conditions, conversion formulas become

$$\boxed{R_Y = \frac{R_\Delta}{3} \qquad \text{or} \qquad R_\Delta = 3R_Y} \tag{2.57}$$

One may wonder why $R_Y$ is less than $R_\Delta$. Well, we notice that the Y-connection is like a "series" connection while the Δ-connection is like a "parallel" connection.

Note that in making the transformation, we do not take anything out of the circuit or put in anything new. We are merely substituting different but mathematically equivalent three-terminal network patterns to create a circuit in which resistors are either in series or in parallel, allowing us to calculate $R_{eq}$ if necessary.

## Example 2.14

Convert the Δ network in Fig. 2.50(a) to an equivalent Y network.

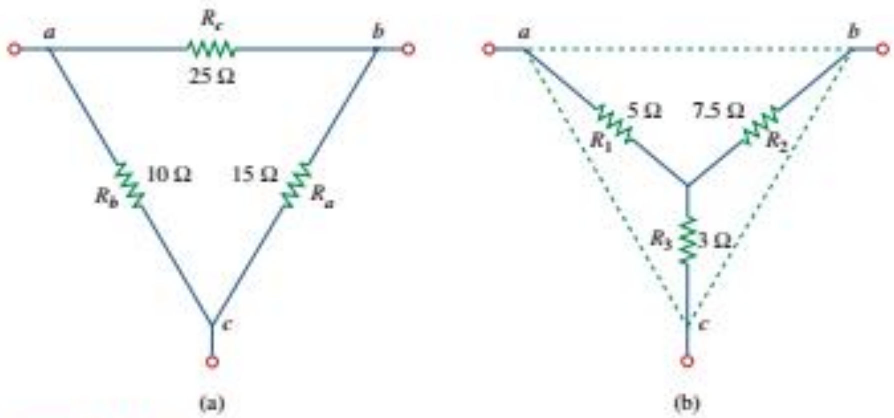


**Figure 2.50**
For Example 2.14: (a) original Δ network, (b) Y equivalent network.

**Solution:**
Using Eqs. (2.49) to (2.51), we obtain

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

The equivalent Y network is shown in Fig. 2.50(b).

## Practice Problem 2.14

Transform the wye network in Fig. 2.51 to a delta network.

**Answer:** $R_a = 140\ \Omega$, $R_b = 70\ \Omega$, $R_c = 35\ \Omega$.

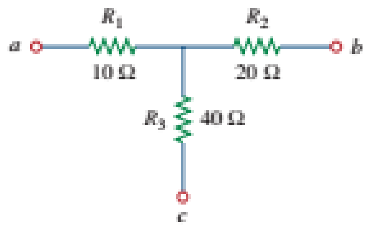


**Figure 2.51**
For Practice Prob. 2.14.

## Example 2.15

Obtain the equivalent resistance $R_{ab}$ for the circuit in Fig. 2.52 and use it to find current $i$.

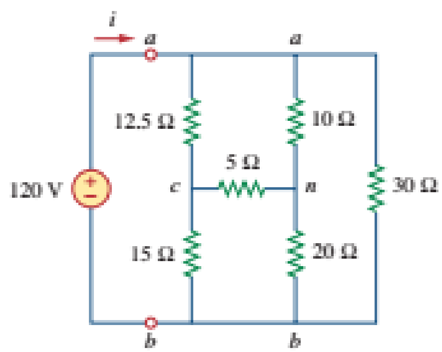


**Figure 2.52**
For Example 2.15.

**Solution:**

1. **Define.** The problem is clearly defined. Please note, this part normally will deservedly take much more time.
2. **Present.** Clearly, when we remove the voltage source, we end up with a purely resistive circuit. Since it is composed of deltas and wyes, we have a more complex process of combining the elements together. We can use wye-delta transformations as one approach to find a solution. It is useful to locate the wyes (there are two of them, one at $n$ and the other at $c$) and the deltas (there are three: *can, abn, cnb*).
3. **Alternative.** There are different approaches that can be used to solve this problem. Since the focus of Sec. 2.7 is the wye-delta transformation, this should be the technique to use. Another approach would be to solve for the equivalent resistance by injecting one amp into the circuit and finding the voltage between $a$ and $b$; we will learn about this approach in Chap. 4.

   The approach we can apply here as a check would be to use a wye-delta transformation as the first solution to the problem. Later we can check the solution by starting with a delta-wye transformation.
4. **Attempt.** In this circuit, there are two Y networks and three $\Delta$ networks. Transforming just one of these will simplify the circuit. If we convert the Y network comprising the 5-Ω, 10-Ω, and 20-Ω resistors, we may select

   $$R_1 = 10\ \Omega, \qquad R_2 = 20\ \Omega, \qquad R_3 = 5\ \Omega$$

   Thus from Eqs. (2.53) to (2.55) we have

   $$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10}$$
   $$= \frac{350}{10} = 35\ \Omega$$
   $$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{350}{20} = 17.5\ \Omega$$
   $$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{350}{5} = 70\ \Omega$$

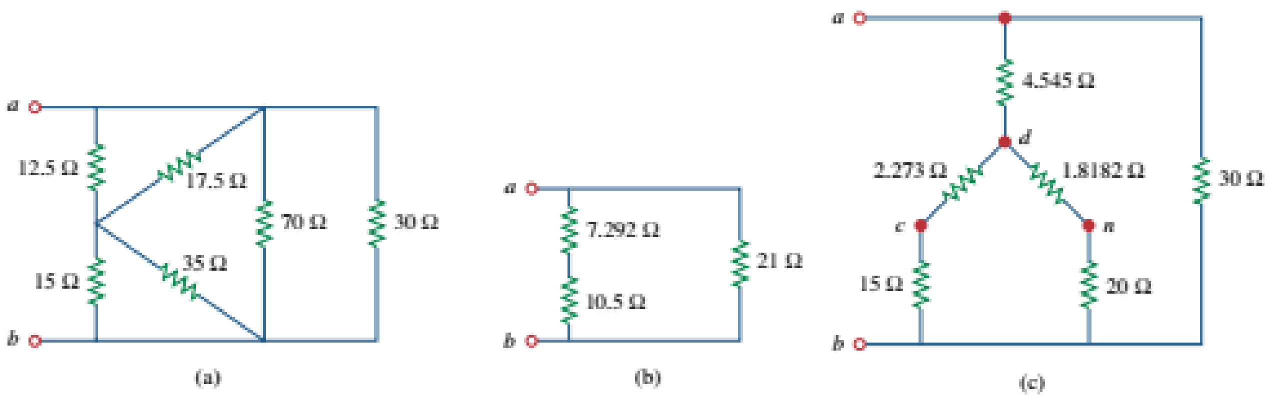


**Figure 2.53**
Equivalent circuits to Fig. 2.52, with the voltage source removed.

With the Y converted to Δ, the equivalent circuit (with the voltage source removed for now) is shown in Fig. 2.53(a). Combining the three pairs of resistors in parallel, we obtain

$$70 \| 30 = \frac{70 \times 30}{70 + 30} = 21\ \Omega$$

$$12.5 \| 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.292\ \Omega$$

$$15 \| 35 = \frac{15 \times 35}{15 + 35} = 10.5\ \Omega$$

so that the equivalent circuit is shown in Fig. 2.53(b). Hence, we find

$$R_{ab} = (7.292 + 10.5) \| 21 = \frac{17.792 \times 21}{17.792 + 21} = \mathbf{9.632\ \Omega}$$

Then

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = \mathbf{12.458\ A}$$

We observe that we have successfully solved the problem. Now we must evaluate the solution.

5. **Evaluate.** Now we must determine if the answer is correct and then evaluate the final solution.

   It is relatively easy to check the answer; we do this by solving the problem starting with a delta-wye transformation. Let us transform the delta, *can*, into a wye.

   Let $R_c = 10\ \Omega$, $R_a = 5\ \Omega$, and $R_n = 12.5\ \Omega$. This will lead to (let $d$ represent the middle of the wye):

$$R_{ad} = \frac{R_c R_n}{R_a + R_c + R_n} = \frac{10 \times 12.5}{5 + 10 + 12.5} = 4.545\ \Omega$$

$$R_{cd} = \frac{R_a R_n}{27.5} = \frac{5 \times 12.5}{27.5} = 2.273\ \Omega$$

$$R_{nd} = \frac{R_a R_c}{27.5} = \frac{5 \times 10}{27.5} = 1.8182\ \Omega$$

This now leads to the circuit shown in Figure 2.53(c). Looking at the resistance between $d$ and $b$, we have two series combination in parallel, giving us

$$R_{db} = \frac{(2.273 + 15)(1.8182 + 20)}{2.273 + 15 + 1.8182 + 20} = \frac{376.9}{39.09} = 9.642\ \Omega$$

This is in series with the 4.545-Ω resistor, both of which are in parallel with the 30-Ω resistor. This then gives us the equivalent resistance of the circuit.

$$R_{ab} = \frac{(9.642 + 4.545)30}{9.642 + 4.545 + 30} = \frac{425.6}{44.19} = \mathbf{9.631\ \Omega}$$

This now leads to

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.631} = \mathbf{12.46\ A}$$

We note that using two variations on the wye-delta transformation leads to the same results. This represents a very good check.

6. **Satisfactory?** Since we have found the desired answer by determining the equivalent resistance of the circuit first and the answer checks, then we clearly have a satisfactory solution. This represents what can be presented to the individual assigning the problem.