Since the voltage and current of each resistor are related by Ohm's law as shown, we are really looking for three things: (v_1, v_2, v_3) or (i_1, i_2, i_3). At node a, KCL gives

$$i_1 - i_2 - i_3 = 0 \tag{2.8.2}$$

Applying KVL to loop 1 as in Fig. 2.27(b),

$$-30 + v_1 + v_2 = 0$$

We express this in terms of i_1 and i_2 as in Eq. (2.8.1) to obtain

$$-30 + 8i_1 + 3i_2 = 0$$

or

$$i_1 = \frac{(30 - 3i_2)}{8} \tag{2.8.3}$$

Applying KVL to loop 2,

$$-v_2 + v_3 = 0 \quad \Rightarrow \quad v_3 = v_2 \tag{2.8.4}$$

as expected since the two resistors are in parallel. We express v_1 and v_2 in terms of i_1 and i_2 as in Eq. (2.8.1). Equation (2.8.4) becomes

$$6i_3 = 3i_2 \quad \Rightarrow \quad i_3 = \frac{i_2}{2} \tag{2.8.5}$$

Substituting Eqs. (2.8.3) and (2.8.5) into (2.8.2) gives

$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

or $i_2 = 2$ A. From the value of i_2, we now use Eqs. (2.8.1) to (2.8.5) to obtain

$$i_1 = 3 \text{ A}, \quad i_3 = 1 \text{ A}, \quad v_1 = 24 \text{ V}, \quad v_2 = 6 \text{ V}, \quad v_3 = 6 \text{ V}$$

Practice Problem 2.8

Find the currents and voltages in the circuit shown in Fig. 2.28.

Answer: $v_1 = 6$ V, $v_2 = 4$ V, $v_3 = 10$ V, $i_1 = 3$ A, $i_2 = 500$ mA, $i_3 = 1.25$ A.

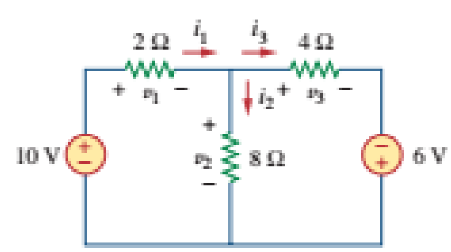

Figure 2.28
For Practice Prob. 2.8.

2.5 Series Resistors and Voltage Division

The need to combine resistors in series or in parallel occurs so frequently that it warrants special attention. The process of combining the resistors is facilitated by combining two of them at a time. With this in mind, consider the single-loop circuit of Fig. 2.29. The two resistors

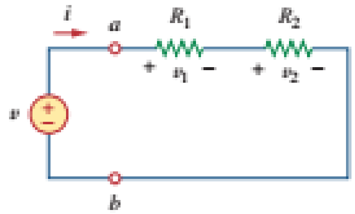

Figure 2.29
A single-loop circuit with two resistors in series.

are in series, since the same current i flows in both of them. Applying Ohm's law to each of the resistors, we obtain

$$v_1 = iR_1, \qquad v_2 = iR_2 \tag{2.24}$$

If we apply KVL to the loop (moving in the clockwise direction), we have

$$-v + v_1 + v_2 = 0 \tag{2.25}$$

Combining Eqs. (2.24) and (2.25), we get

$$v = v_1 + v_2 = i(R_1 + R_2) \tag{2.26}$$

or

$$i = \frac{v}{R_1 + R_2} \tag{2.27}$$

Notice that Eq. (2.26) can be written as

$$v = iR_{eq} \tag{2.28}$$

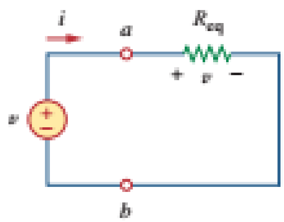

Figure 2.30
Equivalent circuit of the Fig. 2.29 circuit.

implying that the two resistors can be replaced by an equivalent resistor R_{eq}; that is,

$$R_{eq} = R_1 + R_2 \tag{2.29}$$

Thus, Fig. 2.29 can be replaced by the equivalent circuit in Fig. 2.30. The two circuits in Figs. 2.29 and 2.30 are equivalent because they exhibit the same voltage-current relationships at the terminals *a-b*. An equivalent circuit such as the one in Fig. 2.30 is useful in simplifying the analysis of a circuit. In general,

> The **equivalent resistance** of any number of resistors connected in series is the sum of the individual resistances.

Resistors in series behave as a single resistor whose resistance is equal to the sum of the resistances of the individual resistors.

For N resistors in series then,

$$\boxed{R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n} \tag{2.30}$$

To determine the voltage across each resistor in Fig. 2.29, we substitute Eq. (2.26) into Eq. (2.24) and obtain

$$\boxed{v_1 = \frac{R_1}{R_1 + R_2}v, \qquad v_2 = \frac{R_2}{R_1 + R_2}v} \tag{2.31}$$

Notice that the source voltage v is divided among the resistors in direct proportion to their resistances; the larger the resistance, the larger the voltage drop. This is called the *principle of voltage division*, and the circuit in Fig. 2.29 is called a *voltage divider*. In general, if a voltage divider has N resistors ($R_1, R_2, \ldots, R_N$) in series with the source voltage v, the nth resistor (R_n) will have a voltage drop of

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N}v \tag{2.32}$$

2.6 Parallel Resistors and Current Division

Consider the circuit in Fig. 2.31, where two resistors are connected in parallel and therefore have the same voltage across them. From Ohm's law,

$$v = i_1 R_1 = i_2 R_2$$

or

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2} \tag{2.33}$$

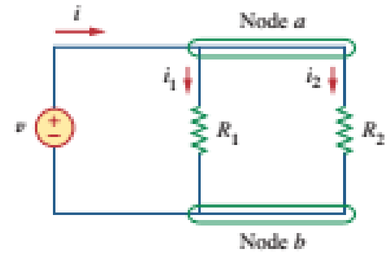

Figure 2.31
Two resistors in parallel.

Applying KCL at node *a* gives the total current *i* as

$$i = i_1 + i_2 \tag{2.34}$$

Substituting Eq. (2.33) into Eq. (2.34), we get

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}} \tag{2.35}$$

where R_{eq} is the equivalent resistance of the resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{2.36}$$

or

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

or

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}} \tag{2.37}$$

Thus,

> The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

It must be emphasized that this applies only to two resistors in parallel. From Eq. (2.37), if $R_1 = R_2$, then $R_{eq} = R_1/2$.

We can extend the result in Eq. (2.36) to the general case of a circuit with N resistors in parallel. The equivalent resistance is

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}} \tag{2.38}$$

Note that R_{eq} is always smaller than the resistance of the smallest resistor in the parallel combination. If $R_1 = R_2 = \cdots = R_N = R$, then

$$R_{eq} = \frac{R}{N} \tag{2.39}$$

For example, if four 100-Ω resistors are connected in parallel, their equivalent resistance is 25 Ω.

It is often more convenient to use conductance rather than resistance when dealing with resistors in parallel. From Eq. (2.38), the equivalent conductance for N resistors in parallel is

Conductances in parallel behave as a single conductance whose value is equal to the sum of the individual conductances.

$$\boxed{G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N} \quad \textbf{(2.40)}$$

where $G_{eq} = 1/R_{eq}$, $G_1 = 1/R_1$, $G_2 = 1/R_2$, $G_3 = 1/R_3$, $\ldots$, $G_N = 1/R_N$. Equation (2.40) states:

> The equivalent conductance of resistors connected in parallel is the sum of their individual conductances.

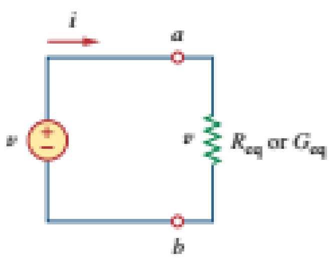

Figure 2.32
Equivalent circuit to Fig. 2.31.

This means that we may replace the circuit in Fig. 2.31 with that in Fig. 2.32. Notice the similarity between Eqs. (2.30) and (2.40). The equivalent conductance of parallel resistors is obtained the same way as the equivalent resistance of series resistors. In the same manner, the equivalent conductance of resistors in series is obtained just the same way as the resistance of resistors in parallel. Thus the equivalent conductance G_{eq} of N resistors in series (such as shown in Fig. 2.29) is

$$\boxed{\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} + \cdots + \frac{1}{G_N}} \quad \textbf{(2.41)}$$

Given the total current i entering node a in Fig. 2.31, how do we obtain current i_1 and i_2? We know that the equivalent resistor has the same voltage, or

$$v = iR_{eq} = \frac{iR_1R_2}{R_1 + R_2} \quad \textbf{(2.42)}$$

Combining Eqs. (2.33) and (2.42) results in

$$\boxed{i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}} \quad \textbf{(2.43)}$$

which shows that the total current i is shared by the resistors in inverse proportion to their resistances. This is known as the *principle of current division*, and the circuit in Fig. 2.31 is known as a *current divider*. Notice that the larger current flows through the smaller resistance.

As an extreme case, suppose one of the resistors in Fig. 2.31 is zero, say $R_2 = 0$; that is, R_2 is a short circuit, as shown in Fig. 2.33(a). From Eq. (2.43), $R_2 = 0$ implies that $i_1 = 0$, $i_2 = i$. This means that the entire current i bypasses R_1 and flows through the short circuit $R_2 = 0$, the path of least resistance. Thus when a circuit

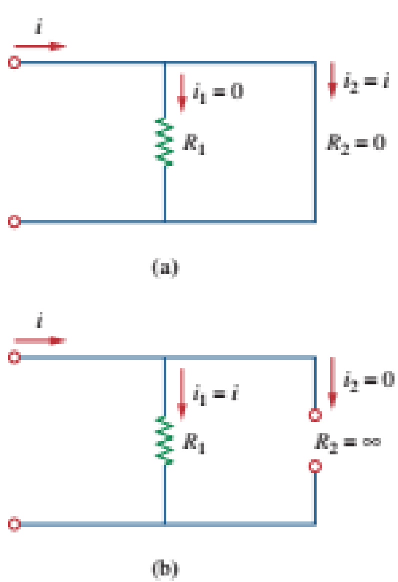

Figure 2.33
(a) A shorted circuit, (b) an open circuit.

is short circuited, as shown in Fig. 2.33(a), two things should be kept in mind:

1. The equivalent resistance $R_{eq} = 0$. [See what happens when $R_2 = 0$ in Eq. (2.37).]
2. The entire current flows through the short circuit.

As another extreme case, suppose $R_2 = \infty$, that is, R_2 is an open circuit, as shown in Fig. 2.33(b). The current still flows through the path of least resistance, R_1. By taking the limit of Eq. (2.37) as $R_2 \rightarrow \infty$, we obtain $R_{eq} = R_1$ in this case.

If we divide both the numerator and denominator by R_1R_2, Eq. (2.43) becomes

$$i_1 = \frac{G_1}{G_1 + G_2} i \tag{2.44a}$$

$$i_2 = \frac{G_2}{G_1 + G_2} i \tag{2.44b}$$

Thus, in general, if a current divider has N conductors ($G_1, G_2, \ldots, G_N$) in parallel with the source current i, the nth conductor (G_n) will have current

$$i_n = \frac{G_n}{G_1 + G_2 + \cdots + G_N} i \tag{2.45}$$

In general, it is often convenient and possible to combine resistors in series and parallel and reduce a resistive network to a single *equivalent resistance* R_{eq}. Such an equivalent resistance is the resistance between the designated terminals of the network and must exhibit the same *i-v* characteristics as the original network at the terminals.

Example 2.9

Find R_{eq} for the circuit shown in Fig. 2.34.

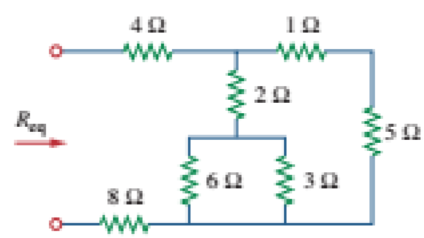

Figure 2.34
For Example 2.9.

Solution:

To get R_{eq}, we combine resistors in series and in parallel. The 6-Ω and 3-Ω resistors are in parallel, so their equivalent resistance is

$$6\ \Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

(The symbol $\parallel$ is used to indicate a parallel combination.) Also, the 1-Ω and 5-Ω resistors are in series; hence their equivalent resistance is

$$1\ \Omega + 5\ \Omega = 6\ \Omega$$

Thus the circuit in Fig. 2.34 is reduced to that in Fig. 2.35(a). In Fig. 2.35(a), we notice that the two 2-Ω resistors are in series, so the equivalent resistance is

$$2\ \Omega + 2\ \Omega = 4\ \Omega$$

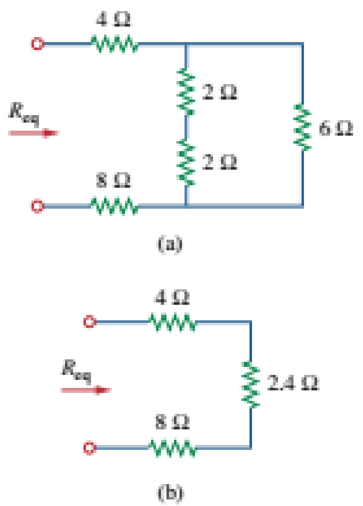

Figure 2.35
Equivalent circuits for Example 2.9.

This 4-Ω resistor is now in parallel with the 6-Ω resistor in Fig. 2.35(a); their equivalent resistance is

$$4\ \Omega \parallel 6\ \Omega = \frac{4 \times 6}{4 + 6} = 2.4\ \Omega$$

The circuit in Fig. 2.35(a) is now replaced with that in Fig. 2.35(b). In Fig. 2.35(b), the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

Practice Problem 2.9

By combining the resistors in Fig. 2.36, find R_{eq}.

Answer: 10 Ω.

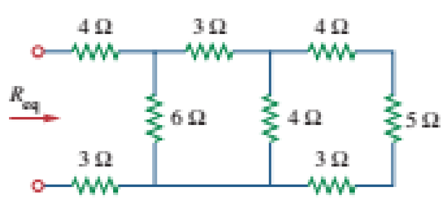

Figure 2.36
For Practice Prob. 2.9.

Example 2.10

Calculate the equivalent resistance R_{ab} in the circuit in Fig. 2.37.

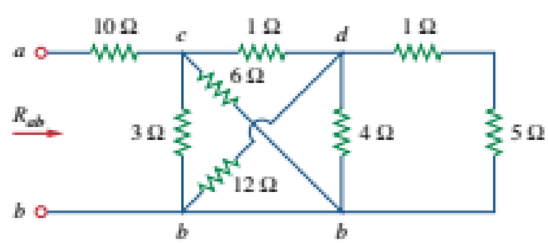

Figure 2.37
For Example 2.10.

Solution:
The 3-Ω and 6-Ω resistors are in parallel because they are connected to the same two nodes c and b. Their combined resistance is

$$3\ \Omega \parallel 6\ \Omega = \frac{3 \times 6}{3 + 6} = 2\ \Omega \qquad \textbf{(2.10.1)}$$

Similarly, the 12-Ω and 4-Ω resistors are in parallel since they are connected to the same two nodes *d* and *b*. Hence

$$12\ \Omega \parallel 4\ \Omega = \frac{12 \times 4}{12 + 4} = 3\ \Omega \qquad \textbf{(2.10.2)}$$

Also the 1-Ω and 5-Ω resistors are in series; hence, their equivalent resistance is

$$1\ \Omega + 5\ \Omega = 6\ \Omega \qquad \textbf{(2.10.3)}$$

With these three combinations, we can replace the circuit in Fig. 2.37 with that in Fig. 2.38(a). In Fig. 2.38(a), 3-Ω in parallel with 6-Ω gives 2-Ω, as calculated in Eq. (2.10.1). This 2-Ω equivalent resistance is now in series with the 1-Ω resistance to give a combined resistance of $1\ \Omega + 2\ \Omega = 3\ \Omega$. Thus, we replace the circuit in Fig. 2.38(a) with that in Fig. 2.38(b). In Fig. 2.38(b), we combine the 2-Ω and 3-Ω resistors in parallel to get

$$2\ \Omega \parallel 3\ \Omega = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$$

This 1.2-Ω resistor is in series with the 10-Ω resistor, so that

$$R_{ab} = 10 + 1.2 = 11.2\ \Omega$$

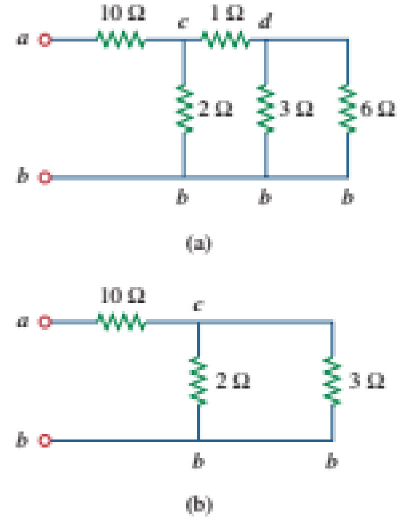

Figure 2.38
Equivalent circuits for Example 2.10.