LINEAR ALGEBRA

1. If $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ then prove that $A^3 + A^2 - 21A - 45I = 0$.
2. $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ find the value of A^{-1}.
3. $B = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ Show that $B \times B^{-1} = I$.
4. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 4 & 6 \end{bmatrix}$ is it possible to find the inverse of the matrix? Explain your answer.
5. Solve the following system of linear equations:

   a. $$\begin{aligned} 2x + 3y + 5z + t &= 3 \\ 3x + 4y + 2z + 3t &= -2 \\ x + 2y + 8z - t &= 8 \\ 7x + 9y + z + 8t &= 0 \end{aligned}$$

   b. $$\begin{aligned} 2x + y - 2z &= 10 \\ 3x + 2y + 2z &= 1 \\ 5x + 4y + 3z &= 4 \end{aligned}$$

   c. $$\begin{aligned} x - y + z + t &= 0 \\ x + 2y - z - t &= 1 \\ 2x - 2y + z - t &= 0 \\ 3y - z + t &= 1 \end{aligned}$$

   d. $$\begin{aligned} x + 2y - z + 3t &= 3 \\ 2x + 4y + 4z + 3t &= 9 \\ 3x + 6y - z + 8t &= 10 \end{aligned}$$

   e. $$\begin{aligned} x_1 - x_2 + x_3 + x_4 - 2x_5 &= 0 \\ 2x_1 + x_2 - x_3 - x_4 + x_5 &= 1 \\ 3x_1 + 3x_2 - 3x_3 - 3x_4 + 4x_5 &= 2 \\ 4x_1 + 5x_2 - 5x_3 - 5x_4 + 7x_5 &= 3 \end{aligned}$$

6. Determine the value of λ such that the following system of linear equations has
   (i) no solution (ii) more than one solutions (iii) a unique solution
   $$\begin{cases} x + y - z = 1 \\ 2x + 3y + \lambda z = 3 \\ x + \lambda y + 3z = 2 \end{cases}$$
7. Determine the value of λ such that the following system of linear equations has
   (i) no solution (ii) more than one solutions (iii) a unique solution
   $$\begin{cases} x + y + \lambda z = 1 \\ x + \lambda y + z = 1 \\ \lambda x + y + z = 1 \end{cases}$$

8. Find the conditions on λ and μ so that the following system of linear equations will have (i) no solution (ii) more than one solutions (iii) a unique solution where

$$2x + 3y + z = 5$$
$$3x - y + \lambda z = 2$$
$$x + 7y - 6z = \mu$$

9. Solve the linear system using matrix(matrix inversion or augmented matrix):

a) $\begin{aligned} x + 2y + 3z &= 1 \\ 2x + 4y + 5z &= -1 \\ 3x + 5y + 6z &= 1 \end{aligned}$

b) $\begin{aligned} 3x + 5y - 7z &= 13 \\ 4x + y - 12z &= 6 \\ 2x + 9y - 3z &= 20 \end{aligned}$

c) $\begin{aligned} 2x + y - 2z &= 10 \\ 3x + 2y + 2z &= 1 \\ 5x + 4y + 3z &= 4 \end{aligned}$

d) $\begin{aligned} x + 2y - z &= -1 \\ 3x + 8y + 2z &= 28 \\ 4x + 9y - z &= 14 \end{aligned}$

10. Write the vector u as the linear combination of vectors u_1, u_2, u_3 where
    (i) u= (1,-2,5), u_1= (1,1,1), u_2= (1,2,3), u_3= (2,-1,1)
    (ii) u= (4,2,1,0), u_1= (3,1,0,1), u_2= (1,2,3,1), u_3= (0, 3, 6,6)
    (iii) u= (3,9,-4,-2), u_1= (1,-2,0,3), u_2= (2,3,-1,0), u_3= (2,-1,2,1)
11. Write the matrix U as the linear combination of matrices U_1, U_2, U_3 where
    (i) U = $\begin{pmatrix} 6 & 3 \\ 0 & 8 \end{pmatrix}, U_1 = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}, U_2 = \begin{pmatrix} 0 & 1 \\ 2 & 4 \end{pmatrix}, U_3 = \begin{pmatrix} 4 & -2 \\ 0 & -2 \end{pmatrix}$
    (ii) U = $\begin{pmatrix} 6 & -1 \\ -8 & -8 \end{pmatrix}, U_1 = \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix}, U_2 = \begin{pmatrix} -1 & 7 \\ 0 & 2 \end{pmatrix}, U_3 = \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}$
    (iii) U = $\begin{pmatrix} 2 & 1 \\ -1 & -2 \end{pmatrix}, U_1 = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}, U_2 = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}, U_3 = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$
12. Check the linear dependency of the vectors
    (i) (1,0,1), (-3,2,6), (4,5,-2)
    (ii) (6,2,3,4), (0,5,-3,1), (0,0,7,-2)
    (iii) (1,0,0), (0,1,0), (1,1,0)
    (iv) (1,-1,2), (3,-5,1), (2,7,8), (-1,1,1)
    (v) (1,-2,3), (5,6,-1), (3,2,1)
13. If the vectors v_1, v_2, v_3 are linearly independent then show that the vectors $v_1+v_2-2v_3$, $v_1-v_2-v_3$, v_1+v_3 are also linearly independent.