MATRIX

Matrix: A set of data that is arranged in rows and columns is known as Matrix.

Order of Matrix = Number of Rows x Number of Columns.

Rank of a matrix: The rank of a matrix is defined as the maximum number of linearly independent rows or columns in the matrix. On other hand, number of non-zero rows in echelon form.

Example: $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 3 & 6 \\ 0 & 6 & 12 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 0 \end{pmatrix}$ $so\ the\ rank\ of\ the\ matrix\ is\ 2.$

Types of matrix:

There are several types of matrices, including:

- Square Matrix: A square matrix is a matrix with an equal number of rows and columns. For example, a 3x3 matrix or a 4x4 matrix.

Example:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \rightarrow\ 2 \times 2\ square\ matrix$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \rightarrow 3 \times 3\ square\ matrix$$

- Row Matrix: A row matrix is a matrix that has only one row. It can be represented as a horizontal array of elements.

Example: (1 2 3) $\rightarrow 1 \times 3\ matrix$

- Column Matrix: A column matrix is a matrix that has only one column. It can be represented as a vertical array of elements.

Example:

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \rightarrow 3 \times 1\ matrix$$

- Zero Matrix: A zero matrix, denoted by the symbol O or 0, is a matrix in which all elements are zero.
- Identity Matrix: An identity matrix, denoted by the symbol I or sometimes by the symbol E, is a square matrix with ones on the main diagonal (from the top left to the bottom right) and zeros elsewhere. The main diagonal elements are equal to 1. Defined as I.

Example: $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

- Diagonal Matrix: A diagonal matrix is a square matrix in which all elements outside the main diagonal are zero. The main diagonal elements can be zero or non-zero.

Example: $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 8 \end{pmatrix}$

- Scalar Matrix: A scalar matrix is a diagonal matrix in which all diagonal elements are equal.

Example: $\begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$

- Upper Triangular Matrix: An upper triangular matrix is a square matrix in which all elements below the main diagonal are zero.

Example: $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$

- Lower Triangular Matrix: A lower triangular matrix is a square matrix in which all elements above the main diagonal are zero.

Example: $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{pmatrix}$

- Transpose of a Matrix: The transpose of a matrix is found by interchanging its rows into columns or columns into rows. It is defined by A^T.

Example: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad then\ transopose\ of\ A\ is\ A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

- Symmetric Matrix: A symmetric matrix is a square matrix that is equal to its transpose. The elements above and below the main diagonal are mirror images of each other. (If $A=A^T$)

Example:

$$A = \begin{pmatrix} 2 & 5 & 9 \\ 5 & 1 & -3 \\ 9 & -3 & 6 \end{pmatrix} \rightarrow A^T = \begin{pmatrix} 2 & 5 & 9 \\ 5 & 1 & -3 \\ 9 & -3 & 6 \end{pmatrix}$$

- Skew-Symmetric Matrix: A skew-symmetric matrix is a square matrix that is equal to the negation of its transpose. The elements above the main diagonal are the negatives of the corresponding elements below the main diagonal. (If $A=-A^T$)

Example:

$$A = \begin{pmatrix} 0 & 5 & 9 \\ -5 & 0 & 3 \\ -9 & -3 & 0 \end{pmatrix} \rightarrow A^T = \begin{pmatrix} 0 & -5 & -9 \\ 5 & 0 & -3 \\ 9 & 3 & 0 \end{pmatrix} \rightarrow -A^T = \begin{pmatrix} 0 & 5 & 9 \\ -5 & 0 & 3 \\ -9 & -3 & 0 \end{pmatrix}$$

- Conjugate Matrix: The concept of a conjugate matrix is commonly associated with complex numbers. In complex analysis, the conjugate of a complex number involves changing the sign of its imaginary part. Similarly, we can define a conjugate matrix by taking the conjugate of each element in the matrix. Defined as $\bar{A}$

Example: $A = \begin{pmatrix} 3+2i & 4-i \\ 1+5i & 2 \end{pmatrix} \to \bar{A} = \begin{pmatrix} 3-2i & 4+i \\ 1-5i & 2 \end{pmatrix}$

- Hermitian Matrix: A Hermitian matrix is a square matrix that is equal to its conjugate transpose. (If $A = \bar{A}^T$)

Example: $A = \begin{pmatrix} 2 & 3-i \\ 3+i & 4 \end{pmatrix} \to \bar{A} = \begin{pmatrix} 2 & 3+i \\ 3-i & 4 \end{pmatrix} \to \bar{A}^T = \begin{pmatrix} 2 & 3-i \\ 3+i & 4 \end{pmatrix}$

- Skew Hermitian Matrix: A skew-Hermitian matrix is a square matrix whose conjugate transpose is equal to the negation of the original matrix. (If $A = -\bar{A}^T$)

Example:

$$A = \begin{pmatrix} 0 & 5+i \\ -5+i & 2i \end{pmatrix} \to \bar{A} = \begin{pmatrix} 0 & 5-i \\ -5-i & -2i \end{pmatrix} \to \bar{A}^T = \begin{pmatrix} 0 & -5-i \\ 5-i & -2i \end{pmatrix} \to$$
$$-\bar{A}^T = \begin{pmatrix} 0 & 5+i \\ -5+i & 2i \end{pmatrix}$$

- Orthogonal Matrix: An orthogonal matrix is a square matrix where its transpose is equal to its inverse ($A^T = A^{-1}$). In other words, if you multiply an orthogonal matrix by its transpose, you get the identity matrix ($A \times A^T = I$).