ELECTRIC CIRCUIT

- An electric circuit is an interconnection of electrical elements.
- It can be as simple as a flash light having just a battery, lamp, and connecting wires.
- Most of the real life circuits are much more complex and have many components.
- The objective of this course is –
    - The analysis of circuits by studying the behaviour of the circuit
    - Understand how does it respond to a given input
    - How the various elements in the circuit interact, etc.

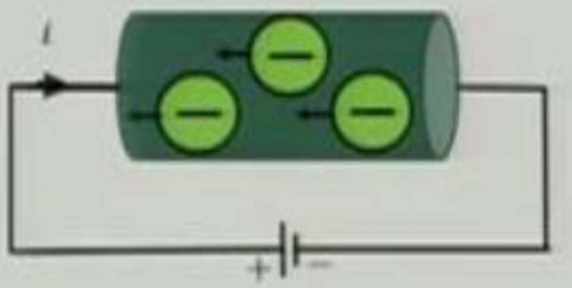
CHARGE AND CURRENT

ELECTRIC CHARGE:

- Charge is an electrical property of the atomic particles of which matter consists.
- Each atom consists of electrons, protons, and neutrons.
- Electric charge is the most basic quantity in a circuit required to explain all electrical phenomena.
- Electric charge is measured in coulombs (C).
- The charge on an electron is negative and has a magnitude equal to 1.602×10^{-19}.
- Proton on the other hand has a positive charge and has the same magnitude as that of an electron.
- A coulomb of charge has $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons.
- Electric charge follows the law of conservation which states that charge can neither be created nor destroyed.

ELECTRIC CURRENT:

- The battery acts as a source of electromotive force (emf).
- Connecting a conducting wire to a battery causes the electrons to move in one direction.
- This motion of charges create an electric current.
- Although, the current in a metallic conductor is due to the flow of electrons it is conventional to take the current as the net flow of positive charge in a direction opposite to the flow of electrons.



ELECTRIC CURRENT (CONT...)

- Hence, electric current can be defined as the time rate of change of charge.
- Mathematically, the relationship between charge, q and current, i can be expressed as,

$$i \triangleq \frac{dq}{dt}$$

- The unit of current is ampere (A), where 1 ampere = 1 coulomb / second.
- The charge transferred between time t_0 and t is evaluated by integrating the above equation

$$Q \triangleq \int_{t_0}^{t} i \, dt$$

EXAMPLE:

- How much charge is represented by 4,600 electrons?

  SOLUTION: Each electron has -1.602×10^{-19} C.

  Hence 4,600 electrons will have -1.602×10^{-19} C/electron $\times$ 4,600 electrons = -7.369×10^{-16} C

- How much charge is represented by 6 million protons?

  SOLUTION: Each electron has 1.602×10^{-19} C.

  Hence 6 million protons will have 1.602×10^{-19} C/proton $\times$ 6×10^{6} protons = 9.612×10^{-13} C

EXAMPLE:

- The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate the current at $t = 0.5$ s?

SOLUTION: $i = \frac{dq}{dt} = \frac{d}{dt}(5t \sin 4\pi t) = 5 \sin 4\pi t + 20\pi t \cos 4\pi t$

At $t = 0.5$

$$i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42\text{mA}.$$

EXAMPLE:

- Determine the total charge entering a terminal between $t = 1\text{s}$ and $t = 2\text{s}$ if the current passing the terminal is $i = (3t^2 - t)\text{A}$?

SOLUTION:

$$Q = \int_{t=1}^{2} i\,dt = \int_{t=1}^{2} (3t^2 - t)\,dt$$

$$= \left(t^3 - \frac{t^2}{2}\right)\Big|_1^2$$

$$= 5.5\text{C}$$

VOLTAGE

- Energy transfer or work is needed to move electrons in a particular direction.
- This work is performed by an external emf generally provided by the battery.
- The emf is also known as voltage or potential difference.
- Voltage is defined as the energy required to move a unit charge through an element.
- It is expressed mathematically as,

$$v_{ab} \triangleq \frac{dw}{dq}$$

- Here, v_{ab} is the voltage between two points a and b, w is the energy in joules (J), and q is the charge in coulombs (C).

VOLTAGE (CONT…):

- Voltage is measured in volts (V).
- Here, 1 volt = 1 joule / coulomb = 1 newton-meter / coulomb.
- The + and – signs are used to represent the voltage polarity or the reference direction.
- It follows logically that $v_{ab} = -v_{ba}$.
- Current and voltage are considered as the two basic variables in the electric circuit.

POWER AND ENERGY

- Although current and voltage are the two basic variables in an electric circuit, they are not sufficient by themselves.
- For practical purposes, we need to know how much power an electric device can handle.
- When we pay our bills to the electric utility companies, we are paying for the electric energy consumed over a certain period of time.
- Thus, power and energy calculations are also important in circuit analysis.

POWER AND ENERGY (CONT...):

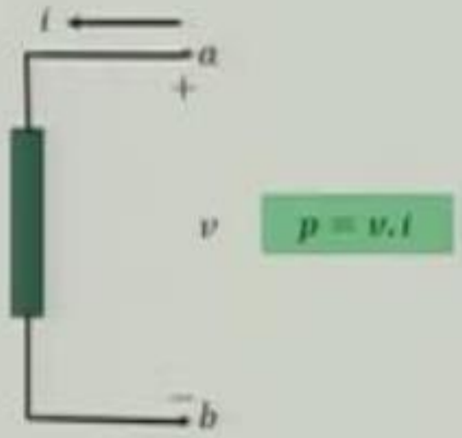
- To relate power and energy to voltage and current, we recall from physics the following:
  - Power is the time rate of expending or absorbing energy and can be expressed as

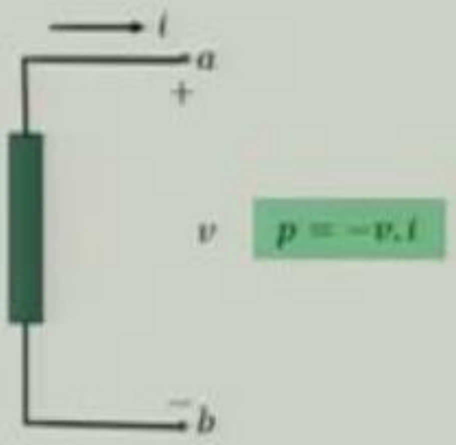
$$p \triangleq \frac{dw}{dt}$$

- Power, p, is measured in watts (W), w is the energy in joules (J), and t is the time in seconds (s).
- Using the previous equations - $$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = v.i$$
- Thus, the power absorbed or supplied by an element is the product of the voltage across the element and the current through it.
- The power p in the above equation is a time-varying quantity and is called the instantaneous power.

POWER AND ENERGY (CONT...):

- If the power has a + sign, power is being delivered to or absorbed by the element.
- If, on the other hand, the power has a − sign, power is being supplied by the element.
- Current direction and voltage polarity play a major role in determining the sign of power.





POWER AND ENERGY (CONT...):

- It is therefore important that we pay attention to the relationship between current and voltage
- The voltage polarity and current direction must conform with those shown in in the previous figure in order for the power to have a positive sign.
- This is known as the **passive sign** convention.
- Passive sign convention is satisfied when the current enters through the positive terminal of an element and $p = vi$.
- If the current enters through the negative terminal, $p = -vi$.

POWER AND ENERGY (CONT....):

- Law of conservation of energy must be obeyed in any electric circuit.
- For this reason, the algebraic sum of power in a circuit, at any instant of time, must be zero:

$$\sum p = 0$$

- This again confirms the fact that the total power supplied to the circuit must balance the total power absorbed.
- Using energy power equation, the energy absorbed or supplied by an element from time t_0 to time t is

$$w = \int_{t_0}^{t} p\,dt = \int_{t_0}^{t} vi\,dt$$

- Energy is, thus, defined as the capacity to do work and is measured in joules (J).

EXAMPLE:

- An energy source forces a constant current of 2 A for 10 s to flow through a light bulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

SOLUTION: The total charge is

$$\Delta q = i\Delta t = 2 \cdot 10 = 20C$$

The voltage drop is

$$v = \frac{\Delta w}{\Delta q} = \frac{2.3 \cdot 10^3}{20} = 115V$$

EXAMPLE:

- Find the power delivered to an element at $t = 3\text{ms}$ if the current entering its positive terminal is $i = 5\cos 60\pi t$ A and the voltage is $v = 3i$?

SOLUTION: The voltage is $v = 3i = 15\cos 60\pi t$

Hence, power $p = vi = 75\cos^2 60\pi t$ W

At $t = 3\text{ms}$,

Power, $p = 75\cos^2(60\pi * 3 * 10^{-3}) = 53.48$ W

EXAMPLE:

- For the previous problem find the power at $t = 3\text{ms}$ if the voltage is $v = 3\frac{di}{dt}$?

SOLUTION: The voltage is $v = 3\frac{di}{dt} = 3(-60\pi)5\sin 60\pi t = -900\pi \sin 60\pi t$

Hence, power $p = vi = -4500 \sin 60\pi t \cos 60\pi t$ W

At $t = 3\text{ms}$,

Power, $p = -4500\sin(60\pi * 3 * 10^{-3})\cos(60\pi * 3 * 10^{-3}) = -6.396$ kW

CIRCUIT ELEMENTS

- There are two types of elements found in electric circuits: *passive* elements and *active* elements.
- An active element is capable of generating energy while a passive element is not.
- Examples of passive elements are resistors, capacitors, and inductors.
- Typical active elements include generators, batteries, and operational amplifiers.
- The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them.

CIRCUIT ELEMENTS (CONT...):

- There are two kinds of sources: independent and dependent sources.
- An ideal independent source provides a specified voltage or current that is completely independent of other circuit elements
- An ideal independent voltage source delivers to the circuit whatever current is necessary to maintain its terminal voltage
- Physical sources such as batteries may be regarded as approximations to ideal voltage sources
- In an ideal dependent (or controlled) source, source quantity is controlled by another voltage or current

independent sources

dependent sources