

This leads to a new equation for loop 1. Simplifying leads to

$$(4 + 2 - 8)i_1 + (-2 + 8)i_2 = 0$$

or

$$-2i_1 + 6i_2 = 0 \quad \text{or} \quad i_1 = 3i_2$$

$$-2i_1 + 11i_2 = -10$$

Substituting the first equation into the second gives

$$-6i_2 + 11i_2 = -10 \quad \text{or} \quad i_2 = -10/5 = -2 \text{ A}$$

Using the Thevenin equivalent is quite easy since we have only one loop, as shown in Fig. 4.35(d).

$$-4i + 9i + 10 = 0 \quad \text{or} \quad i = -10/5 = -2 \text{ A}$$

6. **Satisfactory?** Clearly we have found the value of the equivalent circuit as required by the problem statement. Checking does validate that solution (we compared the answer we obtained by using the equivalent circuit with one obtained by using the load with the original circuit). We can present all this as a solution to the problem.

Obtain the Thevenin equivalent of the circuit in Fig. 4.36.

Answer: $V_{\text{Th}} = 0 \text{ V}$, $R_{\text{Th}} = -7.5 \Omega$.

Practice Problem 4.10

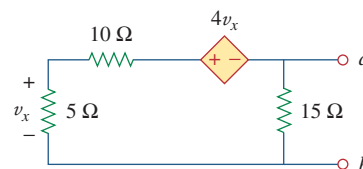


Figure 4.36
For Practice Prob. 4.10.

4.6 Norton's Theorem

In 1926, about 43 years after Thevenin published his theorem, E. L. Norton, an American engineer at Bell Telephone Laboratories, proposed a similar theorem.

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N , where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

Thus, the circuit in Fig. 4.37(a) can be replaced by the one in Fig. 4.37(b).

The proof of Norton's theorem will be given in the next section. For now, we are mainly concerned with how to get R_N and I_N . We find R_N in the same way we find R_{Th} . In fact, from what we know about source transformation, the Thevenin and Norton resistances are equal; that is,

$$R_N = R_{\text{Th}} \quad (4.9)$$

To find the Norton current I_N , we determine the short-circuit current flowing from terminal a to b in both circuits in Fig. 4.37. It is evident

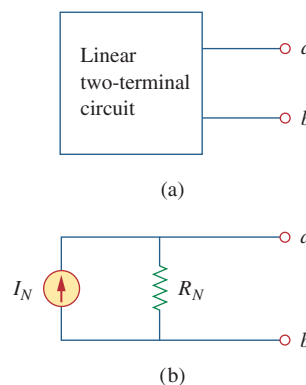
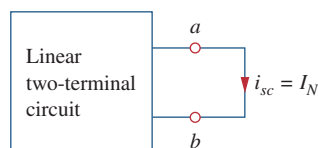


Figure 4.37
(a) Original circuit, (b) Norton equivalent circuit.

**Figure 4.38**Finding Norton current I_N .

The Thevenin and Norton equivalent circuits are related by a source transformation.

that the short-circuit current in Fig. 4.37(b) is I_N . This must be the same short-circuit current from terminal a to b in Fig. 4.37(a), since the two circuits are equivalent. Thus,

$$I_N = i_{sc} \quad (4.10)$$

shown in Fig. 4.38. Dependent and independent sources are treated the same way as in Thevenin's theorem.

Observe the close relationship between Norton's and Thevenin's theorems: $R_N = R_{Th}$ as in Eq. (4.9), and

$$I_N = \frac{V_{Th}}{R_{Th}} \quad (4.11)$$

This is essentially source transformation. For this reason, source transformation is often called Thevenin-Norton transformation.

Since V_{Th} , I_N , and R_{Th} are related according to Eq. (4.11), to determine the Thevenin or Norton equivalent circuit requires that we find:

- The open-circuit voltage v_{oc} across terminals a and b .
- The short-circuit current i_{sc} at terminals a and b .
- The equivalent or input resistance R_{in} at terminals a and b when all independent sources are turned off.

We can calculate any two of the three using the method that takes the least effort and use them to get the third using Ohm's law. Example 4.11 will illustrate this. Also, since

$$V_{Th} = v_{oc} \quad (4.12a)$$

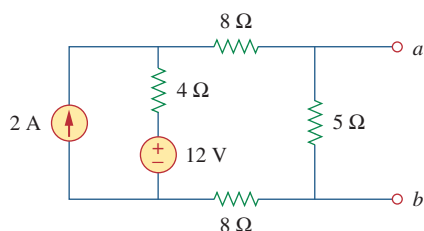
$$I_N = i_{sc} \quad (4.12b)$$

$$R_{Th} = \frac{v_{oc}}{i_{sc}} = R_N \quad (4.12c)$$

the open-circuit and short-circuit tests are sufficient to find any Thevenin or Norton equivalent, of a circuit which contains at least one independent source.

Example 4.11

Find the Norton equivalent circuit of the circuit in Fig. 4.39 at terminals a - b .

**Figure 4.39**

For Example 4.11.

Solution:

We find R_N in the same way we find R_{Th} in the Thevenin equivalent circuit. Set the independent sources equal to zero. This leads to the circuit in Fig. 4.40(a), from which we find R_N . Thus,

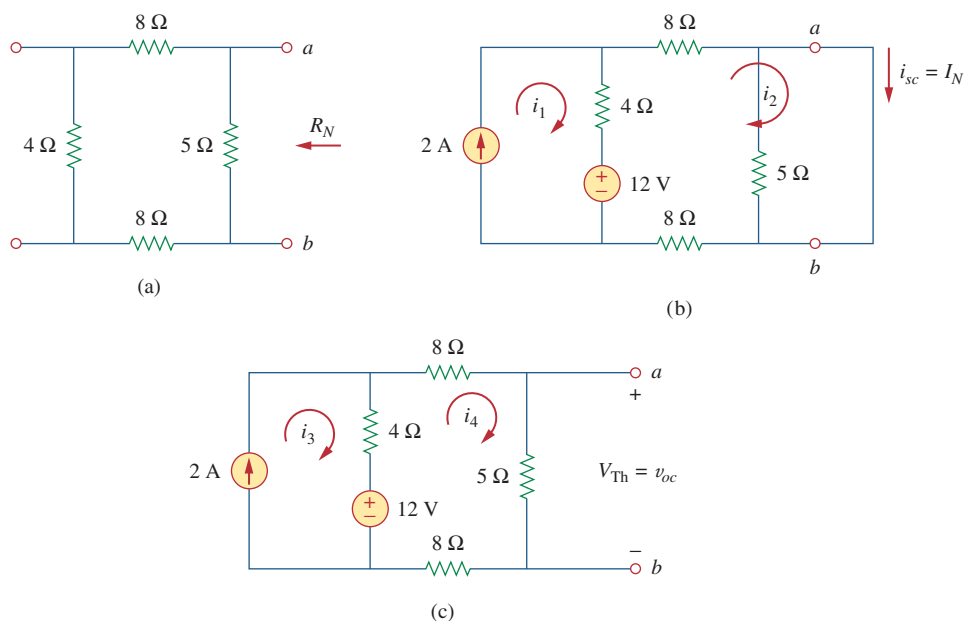
$$R_N = 5 \parallel (8 + 4 + 8) = 5 \parallel 20 = \frac{20 \times 5}{25} = 4 \Omega$$

To find I_N , we short-circuit terminals a and b , as shown in Fig. 4.40(b). We ignore the $5\text{-}\Omega$ resistor because it has been short-circuited. Applying mesh analysis, we obtain

$$i_1 = 2 \text{ A}, \quad 20i_2 - 4i_1 - 12 = 0$$

From these equations, we obtain

$$i_2 = 1 \text{ A} = i_{sc} = I_N$$

**Figure 4.40**

For Example 4.11; finding: (a) R_N , (b) $I_N = i_{sc}$, (c) $V_{Th} = v_{oc}$.

Alternatively, we may determine I_N from V_{Th}/R_{Th} . We obtain V_{Th} as the open-circuit voltage across terminals a and b in Fig. 4.40(c). Using mesh analysis, we obtain

$$i_3 = 2 \text{ A}$$

$$25i_4 - 4i_3 - 12 = 0 \quad \Rightarrow \quad i_4 = 0.8 \text{ A}$$

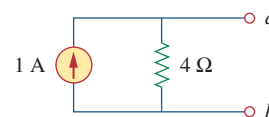
and

$$v_{oc} = V_{Th} = 5i_4 = 4 \text{ V}$$

Hence,

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1 \text{ A}$$

as obtained previously. This also serves to confirm Eq. (4.12c) that $R_{Th} = v_{oc}/i_{sc} = 4/1 = 4 \Omega$. Thus, the Norton equivalent circuit is as shown in Fig. 4.41.

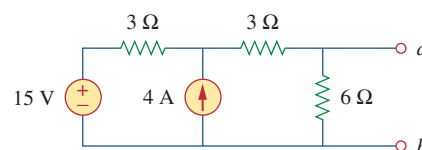
**Figure 4.41**

Norton equivalent of the circuit in Fig. 4.39.

Find the Norton equivalent circuit for the circuit in Fig. 4.42, at terminals a - b .

Answer: $R_N = 3 \Omega$, $I_N = 4.5 \text{ A}$.

Practice Problem 4.11

**Figure 4.42**

For Practice Prob. 4.11.

Example 4.12

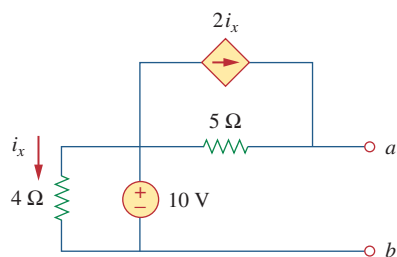


Figure 4.43

For Example 4.12.

Using Norton's theorem, find R_N and I_N of the circuit in Fig. 4.43 at terminals a - b .

Solution:

To find R_N , we set the independent voltage source equal to zero and connect a voltage source of $v_o = 1$ V (or any unspecified voltage v_o) to the terminals. We obtain the circuit in Fig. 4.44(a). We ignore the $4\text{-}\Omega$ resistor because it is short-circuited. Also due to the short circuit, the $5\text{-}\Omega$ resistor, the voltage source, and the dependent current source are all in parallel. Hence, $i_x = 0$. At node a , $i_o = \frac{1}{5\Omega} = 0.2$ A, and

$$R_N = \frac{v_o}{i_o} = \frac{1}{0.2} = 5\ \Omega$$

To find I_N , we short-circuit terminals a and b and find the current i_{sc} , as indicated in Fig. 4.44(b). Note from this figure that the $4\text{-}\Omega$ resistor, the 10-V voltage source, the $5\text{-}\Omega$ resistor, and the dependent current source are all in parallel. Hence,

$$i_x = \frac{10}{4} = 2.5\text{ A}$$

At node a , KCL gives

$$i_{sc} = \frac{10}{5} + 2i_x = 2 + 2(2.5) = 7\text{ A}$$

Thus,

$$I_N = 7\text{ A}$$

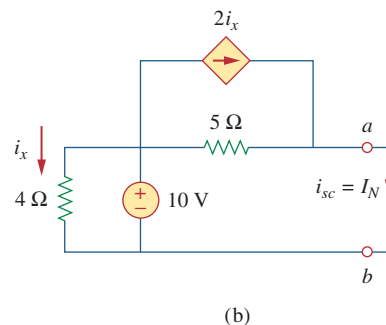
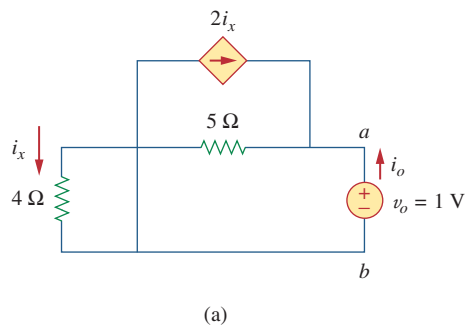


Figure 4.44

For Example 4.12: (a) finding R_N , (b) finding I_N .

Practice Problem 4.12

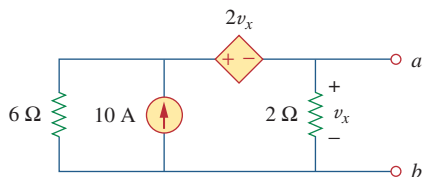


Figure 4.45

For Practice Prob. 4.12.

Find the Norton equivalent circuit of the circuit in Fig. 4.45 at terminals a - b .

Answer: $R_N = 1\ \Omega$, $I_N = 10\text{ A}$.