Appendix B. Computational Approach

Since we do not have explicit formula for B, it is natural to find some computational approach. Then it is useful to propose an algorithm searching for the optimal B with given m, k, s. For example, we can try the following (approximation) algorithm to find worst-case performance $\hat{B} \geq B^{opt}$:

Algorithm with a grid search for φ

1. Binary Search over B. Really, monotonous properties hold. As an upper bound we can take the Cho-Sahni result for the general case.
2. Assume from now that B is fixed. Check each φ over a grid on the interval $[0, 1]$.
3. Assume from now that φ is fixed. Do binary search over R for the third inequality from (1).
4. Here we assume that R is fixed. Perform conditions check. Note that z is obtained explicitly from s.

Another option is to consider all possible R, then we do not need to do the grid search for φ.

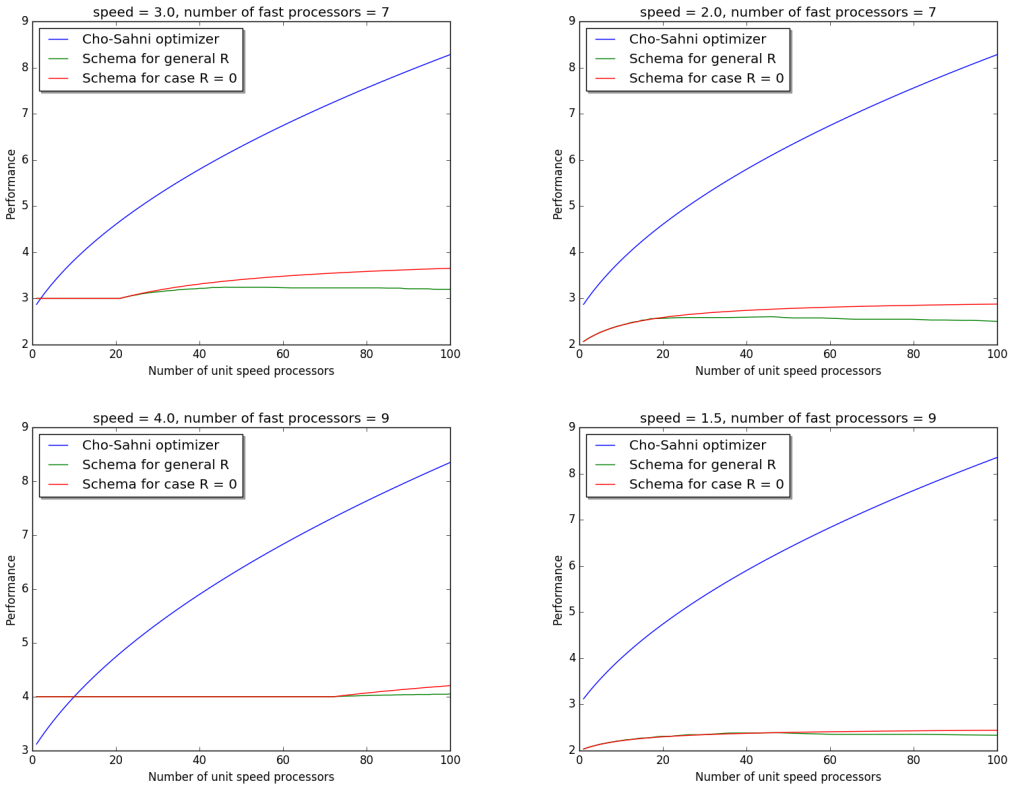
Optimization algorithm with iterating through the number of groups R

1. Binary search over B as in previous algorithm.
2. Assume from now that B is fixed. Do the following step for each R.
3. The first inequality in (1) gives linear constraint for φ. The third inequality in (1) is solved by binary search over φ.

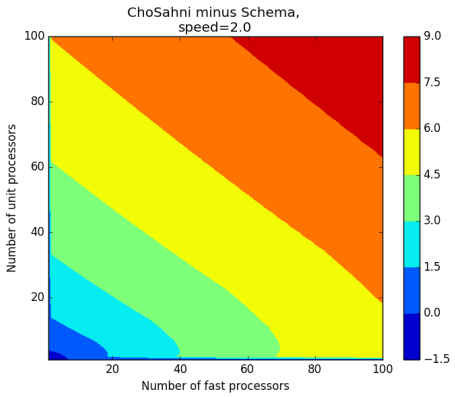
From Property 4 it follows that for fixed s, k and running m from 1 to infinity we may use the optimal result for $m - 1$ as an upper bound for B, if the properties 4 hold.

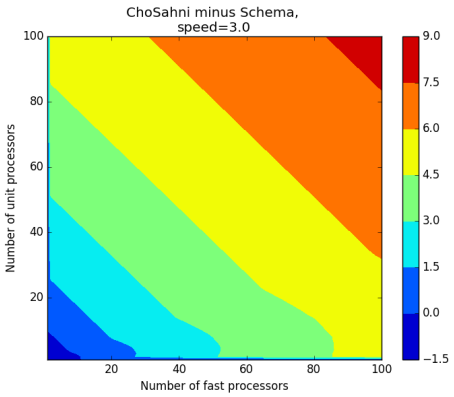
Computation Results

Here we present a plot of scheme quality for some parameters, and compare our results to the Cho-Sahni results.



Levels plot for the difference in worst-case performance between our scheme and Cho-Sahni result.





The table below illustrates performance of the scheme presented in the paper depending on the number of fast and normal machines when $s = 2$.

Table 1: Performance of Scheme

head	fast = 1	fast = 2	fast = 3	fast = 4	fast = 5	fast = 6	fast = 7	fast = 8	fast = 9	fast = 10	fast = 11	fast = 12
unit = 1	2.3364	2.2005	2.1452	2.1118	2.0916	2.0781	2.067	2.0599	2.0533	2.048	2.0436	2.0408
unit = 2	2.5	2.3364	2.2508	2.2005	2.1696	2.1452	2.1277	2.1118	2.1018	2.0916	2.0833	2.0781
unit = 3	2.5735	2.439	2.3364	2.2727	2.233	2.2005	2.177	2.1592	2.1452	2.1308	2.1219	2.1118
unit = 4	2.5873	2.5	2.4096	2.3364	2.2901	2.2508	2.2222	2.2005	2.1834	2.1696	2.1544	2.1452
unit = 5	2.5873	2.5625	2.4638	2.3864	2.3364	2.2989	2.268	2.2424	2.2222	2.2005	2.1875	2.1739
unit = 6	2.5974	2.5735	2.5	2.439	2.381	2.3364	2.3004	2.2727	2.2508	2.233	2.2183	2.2005
unit = 7	2.5882	2.5862	2.5435	2.4691	2.4129	2.3755	2.3364	2.3063	2.2824	2.2629	2.2467	2.2267
unit = 8	2.5772	2.5873	2.5641	2.5	2.4476	2.4096	2.3684	2.3364	2.3109	2.2901	2.2727	2.2508
unit = 9	2.5591	2.5873	2.5735	2.5316	2.4765	2.439	2.3964	2.3631	2.3364	2.3146	2.2964	2.2727
unit = 10	2.5502	2.5873	2.5822	2.5625	2.5	2.4638	2.4204	2.3864	2.359	2.3364	2.3176	2.2989
unit = 11	2.5502	2.5921	2.5873	2.5665	2.5316	2.484	2.4406	2.4096	2.381	2.3557	2.3364	2.3201
unit = 12	2.5323	2.5974	2.5873	2.5735	2.5559	2.5	2.4691	2.439	2.4096	2.381	2.3529	2.3364
unit = 13	2.526	2.6022	2.5873	2.5801	2.5625	2.5316	2.4894	2.4537	2.4245	2.4004	2.38	2.3529
unit = 14	2.5105	2.5882	2.5873	2.5862	2.568	2.5435	2.5	2.4691	2.439	2.4129	2.3927	2.3755
unit = 15	2.5036	2.5772	2.5873	2.5873	2.5735	2.5625	2.5294	2.4934	2.4638	2.439	2.4096	2.3864
unit = 16	2.5036	2.5772	2.5902	2.5873	2.5788	2.5641	2.5346	2.5	2.4715	2.4476	2.4272	2.4096
unit = 17	2.4882	2.572	2.5939	2.5873	2.5838	2.5689	2.5612	2.5258	2.4965	2.4691	2.439	2.4173
unit = 18	2.4764	2.5591	2.5974	2.5873	2.5873	2.5735	2.5625	2.5316	2.5	2.4765	2.4564	2.439
unit = 19	2.4764	2.5502	2.6007	2.5873	2.5873	2.5779	2.5655	2.5513	2.5229	2.4989	2.4691	2.4435
unit = 20	2.4764	2.5502	2.6016	2.5873	2.5873	2.5822	2.5696	2.5625	2.5316	2.5	2.4806	2.4638
unit = 21	2.4606	2.5502	2.5882	2.5892	2.5873	2.5862	2.5735	2.5629	2.5435	2.5207	2.5	2.4691
unit = 22	2.4477	2.5502	2.5772	2.5921	2.5873	2.5873	2.5773	2.5665	2.5625	2.5316	2.5	2.484
unit = 23	2.4477	2.5472	2.5772	2.5948	2.5873	2.5873	2.581	2.5701	2.5625	2.5373	2.5188	2.5
unit = 24	2.4466	2.5323	2.5772	2.5974	2.5873	2.5873	2.5845	2.5735	2.5641	2.5559	2.5316	2.5
unit = 25	2.4431	2.5323	2.5766	2.5999	2.5873	2.5873	2.5873	2.5769	2.5673	2.5625	2.5321	2.5172
unit = 26	2.4311	2.526	2.5676	2.6022	2.5886	2.5873	2.5873	2.5801	2.5705	2.5625	2.5492	2.5316
unit = 27	2.4202	2.5234	2.5591	2.5982	2.591	2.5873	2.5873	2.5832	2.5735	2.5651	2.5625	2.5316
unit = 28	2.4165	2.5105	2.551	2.5882	2.5932	2.5873	2.5873	2.5862	2.5765	2.568	2.5625	2.5435
unit = 29	2.4165	2.5036	2.5502	2.5788	2.5953	2.5873	2.5873	2.5873	2.5794	2.5708	2.5632	2.559
unit = 30	2.4165	2.5036	2.5502	2.5772	2.5974	2.5873	2.5873	2.5873	2.5822	2.5735	2.5659	2.5625
unit = 31	2.4138	2.5036	2.5502	2.5772	2.5994	2.5882	2.5873	2.5873	2.5849	2.5762	2.5685	2.5625
unit = 32	2.401	2.5036	2.5502	2.5772	2.6013	2.5902	2.5873	2.5873	2.5873	2.5788	2.571	2.5641
unit = 33	2.3964	2.497	2.5502	2.5772	2.6031	2.5921	2.5873	2.5873	2.5873	2.5813	2.5735	2.5665
unit = 34	2.3881	2.4882	2.5499	2.572	2.5962	2.5939	2.5873	2.5873	2.5873	2.5838	2.576	2.5689
unit = 35	2.3881	2.4798	2.5433	2.5654	2.5882	2.5957	2.5873	2.5873	2.5873	2.5862	2.5783	2.5713
unit = 36	2.3881	2.4764	2.5323	2.5591	2.5806	2.5974	2.5879	2.5873	2.5873	2.5873	2.5806	2.5735
unit = 37	2.3851	2.4764	2.5323	2.553	2.5772	2.5991	2.5896	2.5873	2.5873	2.5873	2.5829	2.5758
unit = 38	2.3756	2.4764	2.5304	2.5502	2.5772	2.6007	2.5913	2.5873	2.5873	2.5873	2.5851	2.5779
unit = 39	2.3714	2.4764	2.526	2.5502	2.5772	2.6022	2.5929	2.5873	2.5873	2.5873	2.5873	2.5801
unit = 40	2.3714	2.4764	2.5243	2.5502	2.5772	2.6016	2.5944	2.5873	2.5873	2.5873	2.5873	2.5822
unit = 41	2.3684	2.4698	2.519	2.5502	2.5772	2.5948	2.5959	2.5877	2.5873	2.5873	2.5873	2.5842
unit = 42	2.3654	2.4606	2.5105	2.5502	2.5748	2.5882	2.5974	2.5892	2.5873	2.5873	2.5873	2.5862
unit = 43	2.3654	2.4519	2.5036	2.5502	2.5693	2.5819	2.5988	2.5907	2.5873	2.5873	2.5873	2.5873
unit = 44	2.3614	2.4477	2.5036	2.5502	2.5641	2.5772	2.6002	2.5921	2.5873	2.5873	2.5873	2.5873
unit = 45	2.3614	2.4477	2.5036	2.5502	2.5591	2.5772	2.6016	2.5935	2.5873	2.5873	2.5873	2.5873
unit = 46	2.3585	2.4477	2.5036	2.5472	2.5542	2.5772	2.6029	2.5948	2.5876	2.5873	2.5873	2.5873
unit = 47	2.3524	2.4466	2.5036	2.5403	2.5502	2.5772	2.5997	2.5961	2.5889	2.5873	2.5873	2.5873
unit = 48	2.3466	2.4466	2.5036	2.5323	2.5502	2.5772	2.5939	2.5974	2.5902	2.5873	2.5873	2.5873
unit = 49	2.3435	2.4466	2.5	2.5323	2.5502	2.5772	2.5882	2.5987	2.5915	2.5873	2.5873	2.5873
unit = 50	2.3407	2.4431	2.494	2.5323	2.5502	2.5766	2.5828	2.5999	2.5927	2.5873	2.5873	2.5873