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$$f(x) = \ln(1-x-20x^2) = \ln((1+4x)(1+5x)) = \\ = \ln(1+4x) + \ln(1+5x), x_0 = 0$$

$$\ln(1+x) x_0 = 0: \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$$

$$\text{Так } f(x): \sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{4x}{n}\right)^n + \sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{5x}{n}\right)^n$$

$$\cdot \frac{(4x^n) + (5x^n)}{n} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n (4^n + 5^n)}{n}$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{(5x)^n \left(\left(\frac{4}{5}\right)^n + (-1)^n \right)}{n}$$

Рассмотрим:

$$\lim_{n \rightarrow \infty} \left| \frac{(5x)^{n+1} \left(\left(\frac{4}{5}\right)^{n+1} + (-1)^{n+1} \right)}{n+1} \cdot \frac{n}{(5x)^n \left(\left(\frac{4}{5}\right)^n + (-1)^n \right)} \right| =$$

$$= \lim_{n \rightarrow \infty} \left| 5x \frac{n}{n+1} \cdot \frac{\left(\left(\frac{4}{5}\right)^{n+1} + (-1)^{n+1} \right)}{\left(\left(\frac{4}{5}\right)^n + (-1)^n \right)} \right| =$$

$\geq |5x| < 1 \Rightarrow$ область сходимости $x \in \left(-\frac{1}{5}, \frac{1}{5}\right)$