Д/З - 21

$y'' + 4y' + 4y = \sin x$

Сделаем замену:

$y = e^{\lambda x} \quad y' = \lambda e^{\lambda x}, \; y'' = \lambda^2 e^{\lambda x}$

$\lambda^2 e^{\lambda x} + 4\lambda e^{\lambda x} + 4e^{\lambda x} = 0$

$\lambda^2 + 4\lambda + 4 = 0$

$\lambda = \frac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot 4}}{2} = -2$

$\lambda_1 = -2 \quad \lambda_2 = -2$

$y = C_1(x)\, e^{-2x} + C_1(x) \cdot x \cdot e^{-2x}$

$$\begin{cases} C_1'(x)e^{-2x} + C_2'(x)\cdot x\cdot e^{-2x} = 0 & (1) \\ C_1'(x)\cdot(-2e^{-2x}) + C_2'(x)(e^{-2x} - 2xe^{-2x}) = \sin x & (2) \end{cases}$$

1) $C_1'(x) + C_2'(x)\cdot x = 0 \qquad C_1'(x) = -x\,C_2'(x)$

2) $-2e^{-2x}C_1'(x) + e^{-2x}(1-2x)C_2'(x) = \sin x$

$-2(-x\,C_2'(x)) + (1+2x)\,C_2'(x) = \sin x \quad e^{2x}$

$2x\,C_2'(x) + C_2'(x) - 2x\,C_2'(x) = \sin x\cdot e^{2x}$

$\underline{C_2'(x) = \sin x\cdot e^{2x}}$

$C_2(x) = \int \sin x\, e^{2x}\,dx = \left\{\begin{matrix} f = e^{2x} & df = 2e^{2x}dx \\ dg = \sin x\,dx & g = -\cos x \end{matrix}\right\} =$

$= -\cos e^{2x} + 2\int \cos x\, e^{2x}\,dx =$

$= \left\{\begin{matrix} f = e^{2x} & dF = 2e^{2x}dx \\ dy = \cos x\,dx & y = \sin x \end{matrix}\right\} = -\cos x\, e^{2x} +$

$+2\left(\sin x\, e^{2x} - 2\int \sin x\, e^{2x}\,dx\right) =$

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$= -\cos x\, e^{2x} + 2\sin x\, e^{2x} - 4\int \sin x\, e^{2x}\, dx$

$C_2(x) = -\frac{1}{5}\cos x\, e^{2x} + \frac{2}{5}\sin x\, e^{2x}.$

$C_1'(x) = -x\, C_2'(x) = -x\sin x\, e^{2x}$

$$C_1(x) = -\int x \sin x\, e^{2x}\, dx = \left\{\begin{array}{l} f = x \\ dg = \sin x\, e^{2x}\, dx \\ df = dx \\ g = -\frac{1}{5}\cos x\, e^{2x} + \frac{2}{5}\sin x\, e^{2x} \end{array}\right\} =$$

$$= \frac{1}{5} x\cos x\, e^{2x} - \frac{2}{5} x\sin x\, e^{2x} + \int\left(-\frac{1}{5}\cos x\, e^{2x} + \frac{2}{5}\sin x\, e^{2x}\right)dx = \frac{1}{5} x\cos x\, e^{2x} - \frac{2}{5} x\sin x\, e^{2x} -$$

$$- \frac{1}{5}\left(\frac{1}{5}\sin x\, e^{2x} + \frac{2}{5}\cos x\, e^{2x}\right) + \frac{2}{5}\left(-\frac{1}{5}\cos x\, e^{2x} + \frac{2}{5}\sin x\, e^{2x}\right) = \frac{5}{25} x\cos x\, e^{2x} - \frac{10}{25} x\sin x\, e^{2x} +$$

$$+\frac{3}{25}\sin x e^{2x}-\frac{4}{25}\cos x e^{2x}=\frac{1}{25}e^{2x}\left(\cos x(5x-4)+\right.$$

$$\left.+\sin x(3-10x)\right)$$

$$y=C_1(x)\cdot e^{-2x}+C_2(x)e^{-2x}=$$

$$=\frac{1}{25}\left(\cos x(5x-4)+\sin x(3-10x)\right)-$$

$$-\frac{1}{5}\cos x+\frac{2}{5}\sin x=\frac{1}{25}\left(\cos x(5x-9)+\sin x(13-10x)\right)$$

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