D-3

$$\begin{pmatrix} 3 & 0 & 5 \\ 2 & 2 & 10 \\ 4 & 0 & 22 \end{pmatrix} = A$$

$$|A - \lambda E| = \begin{vmatrix} 3-\lambda & 0 & 5 \\ 2 & 2-\lambda & 10 \\ 4 & 0 & 22-\lambda \end{vmatrix} =$$

$$= -\lambda^3 + 27\lambda^2 - 96\lambda + 92 = -(\lambda - 23)(\lambda - 2)^2$$

$\lambda_1 = 2$

$\lambda_2 = 23$

1) $\left(\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 2 & 0 & 10 & 0 \\ 4 & 0 & 20 & 0 \end{array}\right) \begin{matrix} \\ (-2) \\ (-4) \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 \\ & & & 0 \end{array}\right)$

$\exists\ x_2, x_3$ - своб

$x^1 = (-5x_3;\ x_2;\ x_3) \Rightarrow \exists\ x^1 = (-5;\ 1;\ 1)$

Проверка:

$$A \cdot x^1 = \begin{pmatrix} 3 & 0 & 5 \\ 2 & 2 & 10 \\ 4 & 0 & 22 \end{pmatrix} \begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -10 \\ 2 \\ 2 \end{pmatrix} = \lambda_1 \cdot x^1$$

2)
$$\left(\begin{array}{ccc|c} -20 & 0 & 5 & 0 \\ 2 & -21 & 10 & 0 \\ 4 & 0 & -1 & 0 \end{array}\right) \begin{matrix} \uparrow \cdot 10 \\ \downarrow -2 \end{matrix} \sim \left(\begin{array}{ccc|c} 0 & -210 & 105 & 0 \\ 2 & -21 & 10 & 0 \\ 0 & 42 & -21 & 0 \end{array}\right) \begin{matrix} \uparrow \\ :5 \\ :21 \end{matrix}$$

$$\sim \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 2 & -21 & 10 & 0 \\ 0 & 2 & -1 & 0 \end{array}\right) \Rightarrow \exists\ x_3 - \text{своб} \Rightarrow \begin{cases} x_1 = \frac{1}{4} x_3 \\ x_2 = \frac{1}{2} x_3 \end{cases}$$

$$x^2 = \left(\frac{1}{4} x_3;\ \frac{1}{2} x_3;\ x_3\right) \Rightarrow \exists\ x^2 = (1;\ 2;\ 4)$$

Проверка:

$$A \cdot x^2 = \begin{pmatrix} 3 & 0 & 5 \\ 2 & 2 & 10 \\ 4 & 0 & 22 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 23 \\ 46 \\ 92 \end{pmatrix} = \lambda_2 \cdot x^2$$