№ - 3

$$(y'+1)\ln\frac{y+x}{x+3}=\frac{y+x}{x+3}$$

$$t=\frac{y+x}{x+3}\Rightarrow\frac{dt}{dx}=\frac{(x+3)\,d(y+x)-(y+x)\,d(x+3)}{(x+3)^2\,dx}=$$

$$=\frac{dy+dx}{(x+3)dx}-\frac{(y+x)\,dx}{(x+3)^2dx}=\frac{1}{x+3}\cdot\frac{dy}{dx}+\frac{1}{x+3}-\frac{y+x}{(x+3)^2}=$$

$$=\frac{1}{x+3}\cdot\frac{dy}{dx}+\frac{3-y}{(x+3)^3}\Rightarrow\frac{dy}{dx}=(x+3)\frac{dt}{dx}-\frac{3-y}{x+3}$$

$$\left((x+3)t' - \frac{3-y}{x+3} + 1\right)\cdot \ln t = t \Rightarrow$$

$$\left((x+3)t' + \frac{x+y}{x+3}\right)\ln t = t \Rightarrow$$

$$\left((x+3)t' + t\right)\ln t = t$$

$$(x+3)\ln t \frac{dt}{dx} + t\ln t = t$$

$$(x+3)\ln t\, dt = (t - t\ln t)\, dx$$

$$\frac{\ln t\, dt}{t - t\ln t} = \frac{dx}{x+3}$$

$$\int \frac{\ln t\, dt}{t - t\ln t} = \begin{cases} u = t - t\ln t \\ du = (1 - \ln t - t\cdot\frac{1}{t})\, dt = -\ln t\, dt \end{cases} =$$

$$= \int -\frac{du}{u} = -\ln|u| = -\ln|t - t\ln t| = -\ln|t(1-\ln t)| =$$

$$= -\ln|t| - \ln|1-\ln t|$$

$$\int \frac{dx}{x+3} = \ln|x+3|$$

$$-\ln|t| - \ln|1-\ln t| = \ln(x+3)$$

$$-\ln\left|\frac{y+x}{x+3}\right| - \ln\left|1 - \ln\left(\frac{y+x}{x+3}\right)\right| = \ln|x+3|$$

$$-\ln\left|\frac{y+x}{x+3} - \frac{y+x}{x+3}\ln\left(\frac{y+x}{x+3}\right)\right| = \ln|x+3|$$

$$\left|\ln\left|\frac{y+x}{x+3} - \frac{y+x}{x+3}\ln\left(\frac{y+x}{x+3}\right)\right| = \ln\left|\frac{1}{x+3}\right|\right.$$

$$\frac{y+x}{x+3} - \frac{y+x}{x+3}\ln\left(\frac{y+x}{x+3}\right) = \frac{1}{x+3}$$

$$y+x-(y+x)\ln\left(\frac{y+x}{x+3}\right) = 1$$

Отв: $1 - \ln\left(\frac{y+x}{x+3}\right) = \frac{1}{y+x}$