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$$x y' = y - x e^{y/x}$$

Заменим $y = z x \rightarrow \frac{dy}{dx} = z + x \frac{dz}{dx}$

$$x \left(z + x \frac{dz}{dx} \right) = z x - x e^z$$

$$x z + x^2 \frac{dz}{dx} = z x - x e^z$$

$$x \frac{dz}{dx} = -e^z$$

$$\frac{dz}{e^z} = -\frac{dx}{x}$$

$$\int \frac{dz}{e^z} = \int e^{-z} dz = -e^{-z} + C_1$$

$$-\int \frac{dx}{x} = -\ln|x| + C_2$$

$$-e^{-z} = -\ln|x| + C$$

$$e^{-z} = \ln|x| + C$$

$$-z = \ln(\ln|x| + C)$$

$$-\frac{y}{x} = \ln(\ln|x| + C)$$

$$\underline{y = -x \ln(\ln|x| + C)}$$

Проверка:

$$y' = -\ln(\ln|x| + C) - \frac{1}{\ln|x| + C}$$

$$xy' = y - x e^{y/x} \rightarrow -x \ln(\ln|x| + C) - \frac{x}{\ln|x| + C} =$$

$$= -x \ln(\ln|x| + C) - x e^{-\ln(\ln|x| + C)} =$$

$$= -x \ln(\ln|x| + C) - \frac{x}{\ln|x| + C} - \text{const}$$