№3

Д3 - 2 v

$y'' = e^y \; (*)$

Заменим:

$z(y) = y' \Rightarrow y'' = \frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx} = \frac{dz}{dy} \cdot z(y)$

$(*): \; z \cdot z' = e^y$

$\frac{dz}{dy} \cdot z = e^y$

$z\,dz = e^y dy$

$\frac{z^2}{2} = e^y + C_1 \Rightarrow z = \pm\sqrt{2} \cdot \sqrt{e^y + C_1} = y' = \frac{dy}{dx}$

$\frac{dy}{\sqrt{e^y + C_1}} = \pm\sqrt{2}\,dx$

$\int \frac{dy}{\sqrt{e^y + C_1}} = \left| \begin{array}{l} u = \sqrt{e^y + C_1} \\ du = \frac{1}{2\sqrt{e^y + C_1}} \cdot e^y dy \end{array} \right| =$

$= 2\int \frac{1}{u^2 - C_1} du = 2 \cdot \frac{1}{2\sqrt{C_1}} \cdot \ln\left| \frac{u - \sqrt{C_1}}{u + \sqrt{C_1}} \right| =$

$= \frac{1}{\sqrt{C_1}} \ln \left| \frac{\sqrt{e^y + C_1} - \sqrt{C_1}}{\sqrt{e^y + C_1} + \sqrt{C_1}} \right|$

$$\frac{1}{\sqrt{C_1}} \ln \left| \frac{\sqrt{e^y + C_1} - \sqrt{C_1}}{\sqrt{e^y + C_1} + \sqrt{C_1}} \right| = \pm x\sqrt{2} + C_2$$