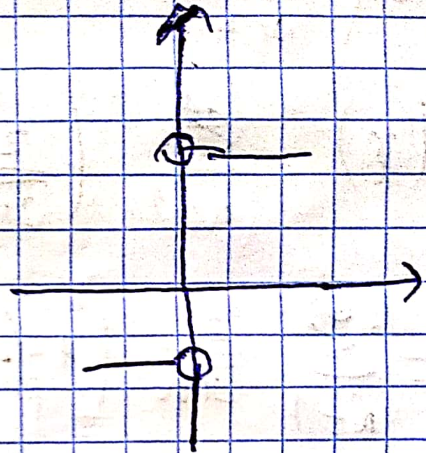
$\mathcal{D}-3$

$\operatorname{sgn}(x) : x \in [-1; 1]$



$$g(x) = \begin{cases} -1, & x \in [-1; 0] \\ 0, & x = 0 \\ 1, & x \in (0; 1) \end{cases}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos(wnx) + b_n \sin(wnx) \right)$$

$$a_n = \frac{1}{l} \cdot \int_{-l}^{l} g(x) \cos\left(\frac{\pi n x}{l}\right) dx;$$

$$b_n = \frac{1}{l} \int_{-l}^{l} g(x) \sin\left(\frac{\pi n x}{l}\right) dx$$

$$a_0 = \frac{1}{l} \int_{-l}^{l} g(x)\, dx$$

$g(x)$ - нечет $\Rightarrow a_n = 0, a_0 = 0$

$l = 1$ ; $w = \pi$

$$b_n = \int_{-1}^{1} \operatorname{sgn}(x) \sin(\pi n x)\, dx = 2 \int_{0}^{1} \operatorname{sgn}(x) \sin(\pi n x)\, dx =$$

$$= 2 \int_{0}^{1} \sin(\pi n x)\, dx = 2 \cdot \left( -\frac{1}{\pi n} \cos(\pi n x) \right) \Bigg|_0^1 =$$

$$= 2\left(-\frac{1}{\pi n}\cos(\pi n) + \frac{1}{\pi n}\right) = \frac{2}{\pi n}\left(1-\cos \pi n\right))$$

$$f(x) = \sum_{n=1}^{\infty} \left(\frac{2}{\pi n}\left(1-\cos(\pi n)\right) \sin(\pi n x)\right) =$$

$$= \sum_{n=1}^{\infty} \frac{2}{\pi n}\left(1-(-1)^{n}\right) \sin(\pi n x) = \sum_{n=0}^{\infty} \frac{2}{\pi(2n+1)}(1+1)$$

$$\sin(\pi x(2n+1)) = \sum_{n=0}^{\infty} \frac{4}{\pi(2n+1)} \sin(\pi x(2n+1))$$