Д - 3

1) $\sum_{n=1}^{\infty} \frac{n!}{x^n}$

Даламбер: $\lim_{n \to \infty} \left| \frac{\frac{(n+1)!}{x^{n+1}}}{\frac{n!}{x^n}} \right| = \lim_{n \to \infty} \left| \frac{n+1}{x} \right|$ - расх

2) $\sum_{n=1}^{\infty} \frac{n}{n+1} \cdot \frac{1}{(3x^2+4x+2)^n}$

Даламбер: $\lim_{n \to \infty} \left| \frac{\frac{n+1}{n+2} \cdot \frac{1}{(3x^2+4x+2)^{n+1}}}{\frac{n}{n+1} \cdot \frac{1}{(3x^2+4x+2)^n}} \right| =$

$$= \lim_{n\to\infty}\left|\frac{(n+1)^2}{n(n+2)}\cdot\frac{1}{(3x^2+4x+2)}\right| = \left|\frac{1}{3x^2+4x+2}\right| < 1 \Rightarrow$$

$$|3x^2+4x+2| > 1 \Rightarrow \left[\begin{array}{l} 3x^2+4x+2>1 \\ 3x^2+4x+2<-1 \end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} 3x^2+4x+1>0 \\ 3x^2+4x+3<0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} (x+1)(3x+1)>0 \\ \left(x+\frac{2}{3}\right)^2+\frac{5}{9}<0 \end{array}\right. \Leftrightarrow$$

$$\left[\begin{array}{l} x\in(-\infty;-1)\cup\left(-\frac{1}{3};+\infty\right) \\ \varnothing \end{array}\right.$$

На границах

$x=-1$: $\sum_{n=1}^{\infty}\frac{n}{n+1}\cdot\frac{1}{(-1)^n} = \sum_{n=1}^{\infty}(-1)^n\frac{n}{n+1}$ — не сход

$x=-\frac{1}{3}$: $\sum_{n=1}^{\infty}\frac{n}{n+1}\cdot\frac{1}{1^n} = \sum_{n=1}^{\infty}\frac{n}{n+1}$ — расх

$\sum_{n=1}^{\infty}\frac{n}{n+1}\cdot\frac{1}{(3x^2+4x+2)^n}$ — сход $\forall x\in(-\infty;-1)\cup$
$\left(-\frac{1}{3};+\infty\right)$