$$\frac{m\omega^2 R}{\sin\alpha} = \frac{mg}{\cos\alpha} \qquad tg\,\alpha = \frac{R}{l}$$

$$l = \frac{R}{tg\,\alpha}$$

$$\omega^2 R \cos\alpha = g\sin\alpha$$

$$R = \frac{g\sin\alpha}{\omega^2\cos\alpha} = \frac{g}{\omega^2}\,tg\,\alpha \qquad l = \frac{g\,tg\,\alpha}{\omega^2\,tg\,\alpha} = \frac{g}{\omega^2}$$

$\boxed{l = \frac{g}{\omega^2}}$ Ответ

Д/3 N 4

$$\vartheta = 2\sqrt{x}$$

1) $\vartheta = \alpha\sqrt{x}$

$\alpha = const$

$t = 0$

$x = 0$

а) $\vartheta(t) - ?$ $a(t) - ?$

б) $\langle V \rangle - ?$

$$V = V_x = \frac{dx}{dt} = \alpha\sqrt{x} \qquad \vartheta = \frac{dx}{dt}$$

$$\int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\, dt \qquad \vartheta(t) = \frac{dx}{dt} = \frac{\alpha^2}{2}t$$

$$2\sqrt{x}\,\Big|_0^x = \alpha t\,\Big|_0^t \qquad a(t) = \frac{\alpha^2}{2}$$

$$2\sqrt{x} = \alpha t$$

$$4x = \alpha^2 t^2 \Rightarrow x = \frac{\alpha^2}{2}\cdot\frac{t^2}{2}$$

$$\langle \vartheta \rangle = \frac{S}{t}$$

$$S = x - x_0 = x = \frac{\alpha^2 t^2}{4}$$

$$t^2 = \frac{4S}{\alpha^2} \Rightarrow t = \frac{2\sqrt{S}}{\alpha}$$

$$\langle \vartheta \rangle = \frac{S\alpha}{2\sqrt{S}} = \frac{\alpha}{2}\sqrt{S}$$

② $a = k\sqrt{v}$

$k = const > 0$

v_0

$S_{ост}$ - ?

$t_{ост}$ - ?

$-a = k\sqrt{v}$

$-\frac{dv}{dt} = k\sqrt{v}$ (1)

$-\frac{dv}{dx} \cdot \frac{dx}{dt} = k\sqrt{v}$

$-\frac{v\,d}{\sqrt{v}} = k\,dx$

$\int_{v_0}^{v} \sqrt{v}\,dv = \int_0^x k\,dx$

$\frac{2}{3} v^{\frac{3}{2}} \Big|_{v_0}^{v} = kx \Big|_0^x$

$-\frac{2}{3} v^{\frac{3}{2}} + \frac{2}{3} v_0^{\frac{3}{2}} = kx$

$v = \left(-\frac{3}{2} kx + v_0^{\frac{3}{2}}\right)^{\frac{3}{2}}$

Время до остановки:

Надо найти путь до остановки значить ($v = 0$).

$0 = \left(-\frac{3}{2} k x_{ост} + v_0^{\frac{3}{2}}\right)^{\frac{3}{2}}$

$x_{ост} = \frac{\frac{2}{3} v_0^{\frac{3}{2}}}{k}$

$S_{ост} = x_{ост} = \frac{2 v_0^{\frac{3}{2}}}{3k}$

Теперь преобразуем выражение (1)

$\frac{-dv}{\sqrt{v}} = k\,dt$

$-\int_{v_0}^{v} \frac{dv}{\sqrt{v}} = k \int_0^t dt$

$-2\sqrt{v} \Big|_{v_0}^{v} = kt \Big|_0^t$

$\boxed{-2\sqrt{v} + 2\sqrt{v_0} = kt}$

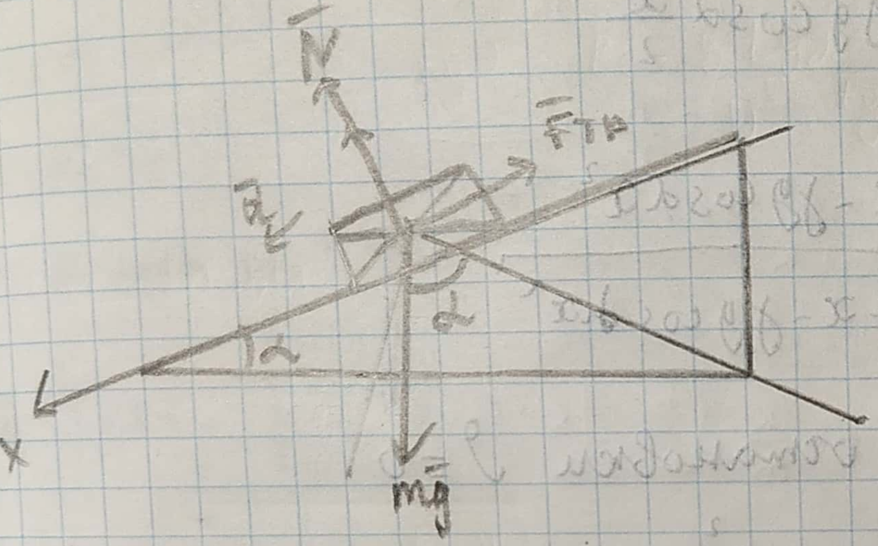
Сканировано с CamScanner

$k t_{ост} = 2\sqrt{V_0}$

$t_{ост} = \frac{2\sqrt{V_0}}{k}$

Ответ: $S_{ост} = \frac{2V_0^{\frac{3}{2}}}{3k}$; $t_{ост} = \frac{2\sqrt{V_0}}{k}$

③



α

$\mu = \gamma x$ $\qquad \sum \bar{F} = m\bar{a}$

$\gamma = const$ $\qquad m\bar{g} + \bar{N} + f_{тр} = m\bar{a}$

$S_{ост} = ?$ $\qquad x: \; mg\sin\alpha - f_{тр} = ma$

$V_{max} = ?$ $\qquad y: \; -mg\cos\alpha + N = 0$

$V(x) = ?$ $\qquad N = mg\cos\alpha$

$f_{тр} = \mu N = \mu mg\cos\alpha = \gamma x mg\cos\alpha$

$\cancel{m}a = \cancel{m}g\sin\alpha - \gamma x \cancel{m}g\cos\alpha$

$a = g(\sin\alpha - \gamma x\cos\alpha)$

$\frac{dv\,dx}{dt\,dx} = g\sin\alpha - g\gamma x\cos\alpha \Big| \frac{dx}{dx}$

$\frac{v\,dv}{dx} = g\sin\alpha - g\gamma x\cos\alpha$

$$\int_0^V V\,dV = \int_0^x g\sin\alpha\,dx - \int_0^x \gamma\, x\, g\cos\alpha\,dx$$

$$\frac{V^2}{2} = g\sin\alpha\, x - \gamma g\cos\alpha\,\frac{x^2}{2}$$

$$V^2 = 2g\sin\alpha\, x - \gamma g\cos\alpha\, x^2$$

$$V = \sqrt{2g\sin\alpha\, x - \gamma g\cos\alpha\, x^2}$$

В момент остановки $\vartheta = 0$

$$2g\sin\alpha\, x = \gamma x^2 g\cos\alpha$$

$$S_{ост} = x = \frac{2\,tg\,\alpha}{\gamma}$$

ϑ принимает max-зн. когда $2g\sin\alpha x - \gamma g\cos\alpha x^2$ max

$$\max(2g\sin\alpha\, x - \gamma g\cos\alpha\, x^2) = (2g\sin\alpha\, x - \gamma g\cos\alpha\, x^2)\Big|_{x=\frac{tg\alpha}{\gamma}}$$

$$= 2g\frac{\sin\alpha\, tg\,\alpha}{\gamma} - g\frac{\sin\alpha\, tg\,\alpha}{\gamma}$$

(4) ϑ_0, m $\quad F = ma$

$t = 0$ $\quad -2V = ma$

$f = -2V$ $\quad -2V = m\frac{dV}{dt}$

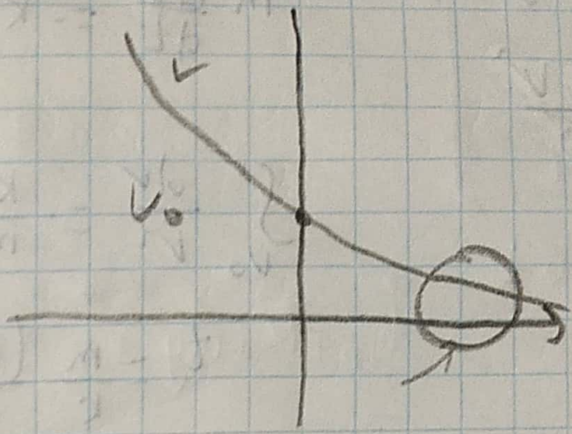
a) t - ?

б) $V(S)$ - ?

$S_{ост}$ - ?

Сканировано с CamScanner

$$-\frac{2}{m}\int_0^t dt = \int_{v_0}^{v} \frac{dv}{v}$$



$$\ln v = -\frac{2}{m}t$$

$$\ln\frac{v}{v_0} = \ln e^{\frac{2}{m}t}$$

$$v = v_0 \cdot e^{-\frac{2}{m}t}$$

В момент ост $v = 0 \Rightarrow 0 \Rightarrow v_0 e^{-\frac{2}{m}t}$

$t \to \infty$

$$S = \int_0^t v\,dt = -\frac{m}{2} v_0 e^{-\frac{2}{m}t}\Big|_0^t = -\frac{m}{2} v_0 \left(e^{-\frac{2}{m}t} - 1\right) =$$

$$= \frac{m}{2}(v_0 - v)$$

$$v = v_0 - \frac{s2}{m}$$

В момент полной остановки $v = 0$

$$0 = v_0 - \frac{2s}{m} \qquad S_{ост} = \frac{v_0 m}{2}$$

5) $F = F_0 \cos wt$ $\quad F = ma$ $\quad \frac{F_0}{m}\frac{\sin wt}{w}\Big|_0^t = v\Big|_0^v$

$F_0, w = const$ $\quad a = \frac{dv}{dt}$ $\quad v = \frac{F_0}{wm}\sin wt$

m

$t_0 = 0$ $\quad F_0 \cos wt = m\frac{dv}{dt}$ $\quad$ max. значение скорости

$v_0 = 0$ $\quad \int_0^t \frac{F_0}{m}\cos wt\,dt = \int_0^v dv$ $\quad v_{max} = \frac{F_0}{wm}$

$t_{ост}$ $\quad \sin wt = 0$

$S - ?$ $\quad wt = \pi$

$v_{max} - ?$ $\quad t = \frac{\pi}{w}$

6) $V, v_0, h,$

$t - ?$

$$m\frac{dv}{dt} = -kv^2 \qquad m\frac{dv}{v^2} = -k\,dt$$

$$\int_{v_0}^{v}\frac{dv}{v^2} = -\frac{k}{m}\int_0^t dt$$

$$t = \frac{m}{k}\frac{(v_0 - v)}{v_0 v} \quad (1)$$

$$mv\frac{dv}{ds} = -kv^2$$

$$\int_{v_0}^{v}\frac{dv}{v} = -\frac{k}{m}\int_0^h ds$$

$$k = \frac{m}{h}\ln\frac{v_0}{v} \quad (2)$$

$$t = \frac{h(v_0 - v)}{v_0 v \ln\frac{v_0}{v}}$$

7) $m; \tau$

$f = at(\tau - t)$

а) $p - ?$

б) $s = ?$

а) $$\frac{dp}{dt} = at(\tau - t)$$

$$\int_0^p dp = \int_0^\tau (a\tau t - at^2)\,dt$$

$$p = \left(a\tau\frac{t^2}{2} - a\frac{t^3}{3}\right)\Big|_0^\tau = \frac{a\tau^3}{2} - \frac{a\tau^3}{3} = \boxed{\frac{a\tau^3}{6}}$$

б) $m = const$, то $F = m\dfrac{dv}{dt}$

$$at(\tau - t) = \frac{m\,dv}{dt}$$

$$\int_0^t (a\tau t - at^2)\,dt = m\int_0^v dv$$

$$\frac{a\tau t^2}{2} - \frac{at^3}{3} = mv$$

$$v = \frac{3a\tau t^2 - 2at^3}{6m}$$

$$v = \frac{dS}{dt}$$

$$\frac{3a\tau t^2 - 2at^3}{6m} = \frac{ds}{dt}$$

$$\int_0^\tau (3a\tau t^2 - 2at^3)\,dt = 6m\int_0^s ds$$

$$3a\tau - 2a\frac{\tau^4}{4} = 6mS$$

$$a\tau^4 - \frac{a\tau^4}{2} = 6mS$$

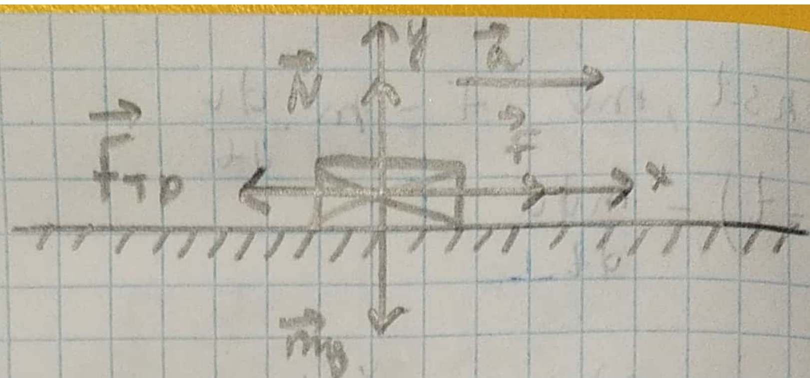
$$\boxed{S = \frac{a\tau^4}{2\cdot 6} = \frac{a\tau^4}{12}}$$

8) μ

$t=0$

$\vec{F} = \vec{a}t$

$S - ?$ за t



$$\vec{F}_{тр} + \vec{N} + m\vec{g} + \vec{F} = m\vec{a}$$

$$Ox:\ -F_{тр} + F = ma$$

$$-\mu mg + \beta t = ma$$

$$a = \frac{\beta t}{m} - g\mu$$

$$a = 0;\ \beta t_0 - g\mu = 0$$

$$t_0 = \frac{mg\mu}{\beta}$$

$$\frac{d\vartheta}{dt} = \frac{\beta t}{m} - g\mu$$

$$\int_0^{\vartheta} d\vartheta = \int_0^{t-t_0} \frac{\beta t}{m}\, dt$$

$$\vartheta_0 = \frac{\beta (t-t_0)^2}{2m}$$

$$S = \int_{t_0}^{t} \vartheta\, dt = \beta \frac{(t-t_0)^3}{6m}\Big|_{t_0}^{t} = \frac{\beta (t-t_0)^3}{6m}$$

$$t_0 = \frac{mg\mu}{\beta} \text{ если } t > t_0$$