BOSTON UNIVERSITY

College of Engineering

Thesis Proposal

3-Dimensional Model-Based Dynamic Feedback Control for Soft Robots

by

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Contents

1 Abstract

The physical characteristics of soft robots inherently promise an ability to perform complex motions, as well as to safely and compliantly interact with sensitive environments. While trajectory tracking and environmental interaction control strategies for planar motion have been developed along with motion plans for it, it has yet to be robustly translated to three dimensions. This proposed thesis thus aims to develop a three-dimensional, model-based, closed loop dynamic controller for continuous soft robots. To develop a robust formulation of this controller, gravitational loads that could potentially violate model assumptions must be dynamically accounted for. Kinematic singularities inherent to the dynamic model used must also be analytically or numerically managed. A suitable dynamic model to underpin the control system must first either be formulated, augmented from an existing one, or selected. Then the model must be validated either analytically or simulatively, before the control system can subsequently be built around it. The controller may then finally be validated through hardware implementation.

2 Topic Background

2.1 Model-based Controls in Soft Robotics

Much of controls theory as a field has progressed in parallel to the advances in our understanding of dynamic modeling. In fact, much of the advancements in controls theory were driven by the need to supplement the gaps in our dynamic models. Starting from the frequency domain and linear controllers, to nonlinear controllers, and most recently machine learning.

The implementation of control strategies for standard rigid robots has followed this line of progression. Algorithms built around the dynamical model of the robot itself came first, then only recently did strategies employing machine learning began to be implemented–mostly to handle unpredictable scenarios beyond the models' assumptions. Control strategies of the former kind are known as *model*-based controls, where controllers are designed based on models that mathematically represent the robot's dynamics [1]. It should be apparent that this has been the case since the dynamics of a rigid-bodied robot can more readily be modelled.

On the other hand, the development of control strategies for *soft* robots have followed the opposite direction: much of the early works in soft robotics controls employed machine learning strategies due to the high level of complexity and dimensionality required in the dynamical models. Thus, *learning*-based controls were the model for soft robot control strategies through the formative years of the field. Only in recent years did this precedent began to be reassessed, particularly with the advent of *finite*-dimensional modelling (FDM) techniques compatible with the continuum dynamics of soft robots [2].

2.2 Model- vs. Learning-based Controls

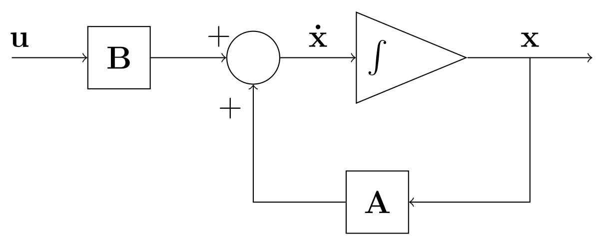


Figure 1: A generic steady-state dynamic system, with an input $\mathbf{u} \neq f(\mathbf{x})$ making it open-loop.

Model- and learning-based controls differ in the fundamental methodology that underpin *how* each approach steers their dynamic systems. Consider the steady-state open-loop dynamic system expressed in state space as $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ from Figure 1, the application of a model- and learning-based controller–respectively–to the system may be represented according to Table 1.

Model-based	Learning-based
$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$ $= \bar{\mathbf{f}}(\mathbf{x}, \mathbf{u})$ where, $\mathbf{u} = f(\mathbf{x}, \mathbf{e}, \mathbf{t}, ...)$	$\dot{\mathbf{x}} = \bar{\mathbf{f}}(\mathbf{x}, \mathbf{u}_{\text{learned}})$ or $\dot{\mathbf{x}} = \mathbf{g}(\mathbf{x}, \mathbf{u})$

Table 1: High-level generalization of model- vs. learning-based controllers.

The distinction between model- and learning-based control approaches may be described using the functional framework presented above. In both approaches, some input $\mathbf{u}$ is now introduced to the previously steady-state system. The system becomes *closed*-loop when a controller that uses the system output to determine the control input is implemented (see Figure 2). In this scenario, $f(\mathbf{x}, \mathbf{e}, \mathbf{t}, ...)$ can be considered the controller. It functionally relates the system output/current state–and typically error $\mathbf{e}$ and time as well–to the control input.

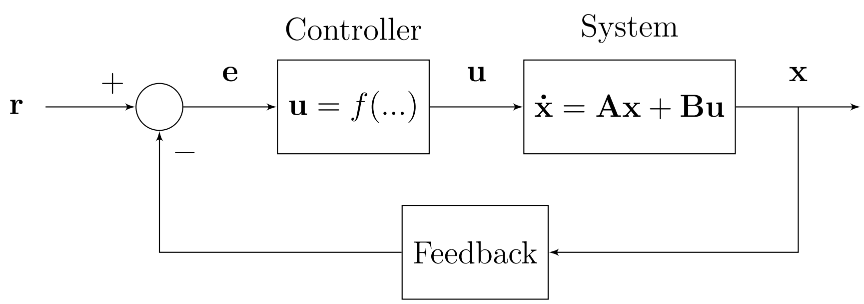


Figure 2: A generic closed-loop system, the "System" block was illustrated in Figure 1.

For model-based controls, designing the controller based on the dynamics of the systems means tuning $\mathbf{u}$ to the $\mathbf{A}$ and $\mathbf{B}$ matrices. A conveniently acceptable way of interpreting $\bar{\mathbf{f}}$ is to regard $\mathbf{A}$ as the system's steady-state behavior, and $\mathbf{B}$ as the system's response to control inputs. The two matrices *must* sufficiently describe the system dynamics before $\mathbf{u}$ can be designed to steer the system towards the desired configuration. Formulating $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{u}$ while respecting their interdependency thus becomes inherent to the model-based approach.

Formulating models that allow the input to be solved in a mathematically manageable way, while simultaneously remaining faithfully representative of the *actual* system behavior becomes one of model-based controls' biggest challenges. Learning-based controls circumvent this by using data-driven techniques and learning algorithms to arrive at $\mathbf{u}$. In some cases, $\bar{\mathbf{f}}$ may even be left unknown as the full system is wholly formulated by the learning medium ($\mathbf{g}(\mathbf{x}, \mathbf{u})$ in Table 1).

A general assessment of the benefits and drawbacks between the two approaches is presented in Table 2. While both approaches bring different things to the table–and extensive research continue to be conducted on both, the author has decided to explore the model-based approach for this proposed thesis.

Analysis	Model-Based	Learning-Based
Advantages	Meaningful information regarding the dynamics and inputs of the system are preserved	Known to achieve higher maneuvering performance and better robustness [1]
Disadvantages	Less robust and adaptable at handling system configurations in fringe scenarios [1], [3]	More complex and typically lacks formal guarantees of safety [3]

Table 2: An assessment of Model- and Learning-Based controls.

3 Prior Work

3.1 The Piece-wise Constant-Curvature Model

It was mentioned earlier that in the formulation of the dynamical model to develop a controller around, towing the balance between mathematical simplicity and physical accuracy is one of the major challenges in model-based controls. While this continues to be true, continuum dynamics-compatible FDM techniques has also indeed broken considerable ground regarding this issue. In their exact formulation, the dynamics of a soft robot is effectively an infinite-dimensional system. Such a system cannot be described without the use of partial differential equations (PDEs). However, by applying the appropriate assumptions, approximations, and/or discretization to the dynamical model of the soft robot, the system's description may be reliably "minimized" into a *finite*-dimensional system that can be described with ordinary differential equations (ODEs) instead.

One of the most widely-implemented family of FDM techniques is the Piece-wise Constant Strain (PCS) approximations (see Figure 3). It is a family of discretization methods applied to a type of approximation for soft robot dynamics known as "rod models". It is extremely common for soft robots to be "thin" and/or "elongated": to have one physical dimension dominate the other two. In that regard, such soft robots may be approximated as a rod with the continuum mechanics of one too–hence rod models. At the heart of this methodology is the assumption that volumetric deformations may be neglected, and modeling the dynamics around the dominant central axis is sufficient [2].

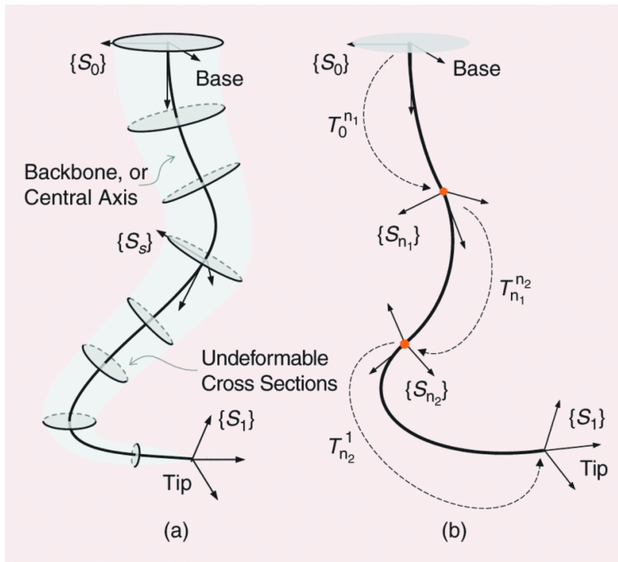


Figure 3: a) "Elongated" soft robots as described in rod models. b) PCS discretization applied to the rod model. Image taken from [2].

Among the many implementations of PCS, the planar Piece-wise Constant Curvature (PCC) model has been extensively used in soft robotics throughout the last decade. Its approximation of the robot as pieces of constant-curvature arcs linked together in series with mutually tangent connection points makes the kinematics and Jacobian formulation for the model *closed*-form [4]. A dynamic feedback controller using this model was developed in [5], with an implementation of a trajectory generator for the controller formulated in [6].

There are many viable models out there that can be used as the basis for the controller that this proposed thesis seeks to develop, a selection of them that seems promising with respect to the scope of this proposed thesis will be outlined below.

3.2 PCC in an Augmented Rigid Body Model

The controller from [5] mentioned earlier uses the kinematics of PCC matched to the dynamics of an augmented rigid body, allowing control strategies typically implemented for rigid-body robots to be used in this scenario. This augmented formulation was initially proposed in [7] by Della Santina et al. as well.

Recall that the PCC Model essentially approximates a rod-like soft robot as a series of constant-curvature (CC) arc segments with mutually-tangent connections. Della Santina et al. proposed that for a given CC segment, the dynamics of the segment can be matched to a rigid-robotic structure comprised of Revolute-Prismatic-Prismatic-Revolute (RPPR) joints in series (see Figure 4). A point mass is located between the two prismatic joints and also matched to the CC segment's center of mass, ensuring the inertial properties of the rigid robot matches the CC segment [7].

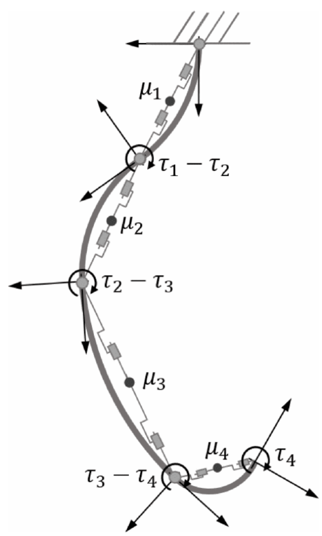


Figure 4: A series of CC Segments matched to a series of RPPR Robots. Image taken from [7].

Using this model, Della Santina et al. were able to develop two feedback controllers aimed at different implementations. The first controller is intended for trajectory tracking, while the second for Cartesian impedance control. While the model *and* controller does not cover three-dimensional motion, preliminary work into translating the model to three-dimensions is explored in [8]. Implementing this "augmented rigid body" model for this proposed thesis will involve expanding the preliminary work into the three-dimensional translation of the model, and then implementing the control design methodology already laid out in [5]. The body of literature available to build on regarding this model is one of the biggest appeals in using it for this proposed thesis.

However, the kinematic singularity found in the three-dimensional straight configuration remains a challenge without solutions previously attempted.

3.3 PCC Using an Alternative State Parametrization

Another one of Cosimo Della Santina's works using the PCC model can be found in [9]. In this work, Della Santina et al. proposes an *alternative* state parametrization for PCC kinematics that describes the CC segments based on the arc lengths of four arcs that bound the surface of the segment.

In the preliminary work of [8], the PCC kinematics of a soft robot were parametrized according to Figure 5. In [9], Della Santina et al. refers to this state parametrization as the "α-parametrization".

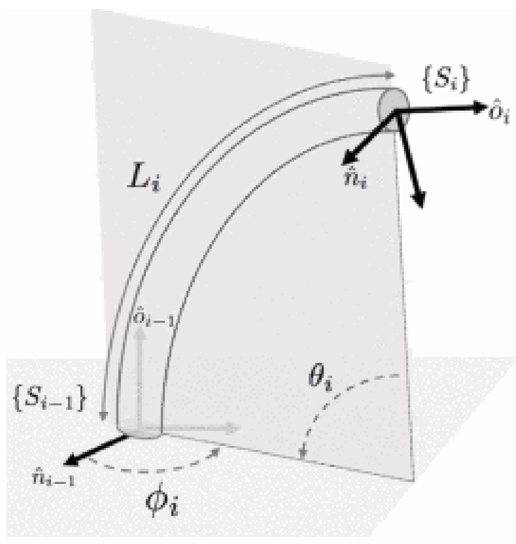


Figure 5: The standard or "old" state parametrization. Image taken from [8].

Kinematically speaking, the α-parametrization is defined as $\alpha_i = [\phi_i, \theta_i, \delta L_i]^T$ for the i-th CC segment, where $\alpha_i \in \mathbb{R}^3$. In this parametrization, ϕ_i and θ_i are the angles as indicated in Figure 5, while δL_i is the change in length of the soft robot's central axis. The issue with this parametrization–which was mentioned in our discussion of the previous model–is that for a given physical configuration, it does not *uniquely* map to a vector α_i. In the case of the straight configuration for example, this means there are infinite choices of ϕ_i for $\theta_i = 0$. Giving rise to the kinematic singularity mentioned previously.

Now in [9], Della Santina et al. refers to the alternative state parametrization they propose as the "q-parametrization". Figure 6 illustrates how the i-th segment of a PCC soft robot is represented under the alternative state parametrization.

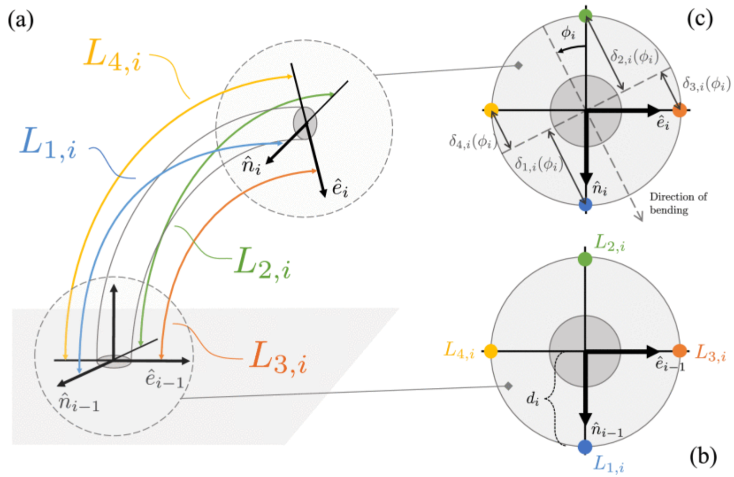


Figure 6: The alternative "Arc-Length Parametrization". Image taken from [9].

This new "Arc-Length Parametrization" defines a configuration q_i as $q_i = [\Delta_{x,i}, \Delta_{y,i}, \delta L_i]^T \in \mathbb{R}^3$. While explicit definitions of $\Delta_{x,i}$ and $\Delta_{y,i}$ will not be reported in this proposal, these two parameters essentially describe the difference in length between the two arcs whose ends are connected to the $\hat{n}$ (blue and green arcs) and $\hat{e}$ (yellow and orange arcs) axes, respectively.

While [9] proves that the q-parametrization is able to solve the limitations of the α-parametrization, a controller design methodology or framework has yet to be laid out. However, its kinematic robustness does make this model a great option to move forward with for this proposed thesis.

4 Research Approach

The goal of this proposed thesis is to develop a three-dimensional, model-based, closed-loop dynamic controller for continuous soft robots. This proposal would now like to put forward a breakdown of how research in pursuit of the goal will be approached. The work shall be conducted in three phases: the Model Selection phase, Model Calibration phase, and finally the Controller Implementation phase, these phases are described below.

4.1 Model Selection

This proposal has discussed the importance and impacts of the constitutive dynamical model used when designing model-based controllers, and consequently how limitations born out of the model's assumptions can translate to implementation challenges in the controller. The Model Selection phase has been included in the research approach in light of this, specifically in the interest of making the development of the controller as practical as possible.

The majority of this phase will be dedicated to reviewing the body of literature available. We are particularly interested in identifying models that can be applied to rod-like continuous soft robots, but also models that possess at least one of the two following characteristics: 1) a model with some precedent of implementing a controller for it, or 2) a model whose underlying dynamic assumptions, kinematics, and/or parametrization has been shown to be robust. The models from [5] and [9] previously discussed in this proposal belong to each category, respectively.

Through this literature review, the author hopes to gain a better view of what the prevailing dynamic models being used in the field are. The main goal of this phase is to identify *a* model whose feasibility regarding the design of a controller for it falls within the time scope of the proposed thesis. This proposal would like to stress that the main focus of the proposed thesis is *not* dynamical modelling of soft robots, but rather *controller design* for them. As such, we are more interested in designing around a "workable" model than identifying *the* prefect model.

4.2 Model Calibration

Since one of the goals of this proposed thesis is to validate the controller designed via hardware implementation, the chosen dynamic model will need to be tuned to the test bench platform we have at hand (see Figure 7). Practically speaking, this means evaluating what the various dynamical terms and matrices are for our soft robot.

Figure 7: The 3-D pneumatically-actuated "limb" serving as our test bench platform

From the construction and physical characteristics of the robot alone, some semblance of what the dynamical terms and matrices of the robot are can be extracted. This initial application of the model to our system can be considered the "un-tuned" model. It can then be tuned by motor-babbling (passing in a series of randomized inputs–"babbles") our soft robot, simulate motor-babbling the soft robot *using* the model, and then comparing the outputs between the two. Generally speaking, a "perfect" model would simulate an exact replica of the output that the robot itself physically produced. While the tuned model cannot do the same, it should simulate sufficiently accurate outputs when operating in configurations found within the boundaries established by the model's assumptions.

The main goal of this phase is to calibrate the dynamic model to such an extent that it can sufficiently represent how the system *responds* to an input signal. Since our soft robot is *pneumatically*-actuated, that means the input signal will be in terms of pressure change. Roughly speaking, we want to ensure that for a given pressure change, the model simulates a change in configuration reflective of the change in configuration physically exhibited by the robot.

4.3 Controller Implementation

The third and final phase is meant to be the bulk of the *experimental* work to be done. The phase will be initiated by first formulating the controller using known theories and current literature. This will consume the majority of the phase, as some amount of research into the body of literature available regarding controller design for the chosen model specifically will need to be done. Any reviews of theories or concepts deemed necessary at this point will also need to be conducted. The main functionality to be pursued for this controller is task-space regulation, and then trajectory tracking. A controller that can actuate the end effector of the robot around a point or bring it to trace a trajectory is necessary before physical compliance and environmental interaction can be explored.

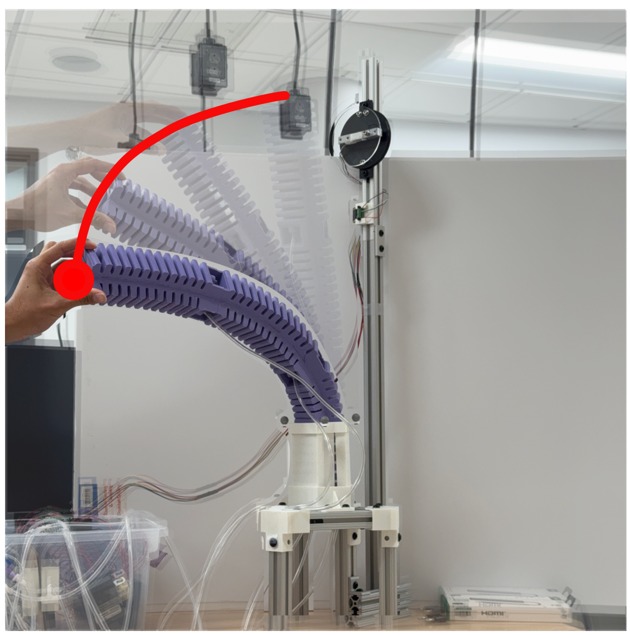
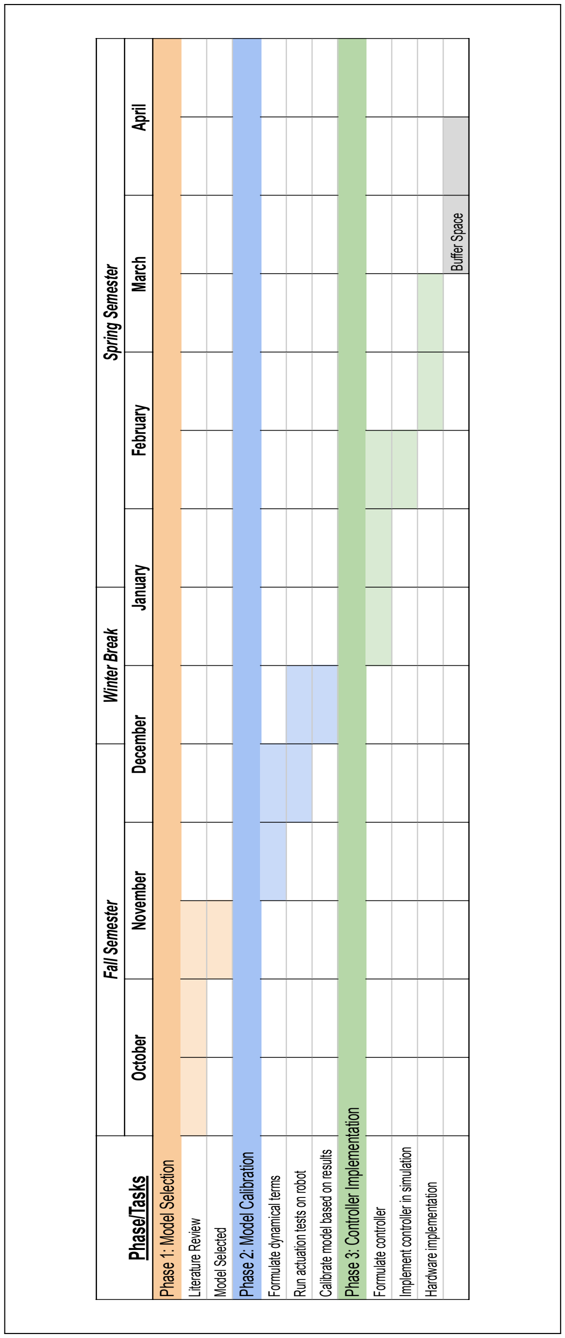


Figure 8: The soft robot stabilizing around a trajectory

As the controller takes shape, simulations will then be run by implementing the controller onto the dynamic model. At this point, we are interested in seeing how closely does the output of the system come to the desired/reference configuration $\mathbf{r}$ when using the input signal $\mathbf{u} = f(\mathbf{x}, \mathbf{r}, \mathbf{e}, \mathbf{t}, ...)$ produced by the controller. The big focus of this proposed thesis remains hardware implementation, but this phase includes a simulation step essentially as a validation method. The simulations should point out any glaring limitations or discrepancies in the controller formulated.

Finally, the controller can then be implemented on our soft robot. In this final step, any major or fundamental changes to the controller would ideally be avoided. The main goal of the experimentation to be conducted on the hardware is really to serve as an assessment of the controller formulation: how well does the methodology used to formulate the controller stand up to physical implementation? While results that inform us to change certain tuning or calibration parameters can be readily implemented, the deeper and more meaningful insights around controller design methodology for this scenario shall be left for future work.

5 Proposed Timeline



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