BOSTON UNIVERSITY

College of Engineering

Thesis Proposal

3-Dimensional Model-Based Dynamic Feedback Control for Soft Robots

by

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Contents

1 Abstract

The physical characteristics of soft robots inherently promise an ability to perform complex motions, as well as to safely and compliantly interact with sensitive environments. While trajectory tracking and environmental interaction control strategies for planar motion have been developed along with motion plans for it, it has yet to be robustly translated to three dimensions. This thesis thus aims to develop a three-dimensional, model-based, closed loop dynamic controller for continuous soft robots. To develop a robust formulation of this controller, gravitational loads that could potentially violate model assumptions must be dynamically accounted for. Kinematic singularities inherent to the dynamic model used must also be analytically or numerically managed. A suitable dynamic model to underpin the control system must then either be formulated, augmented from an existing one, or selected. Then the model must be validated either analytically or simulatively, before the control system can subsequently be built around it. The controller may then finally be validated through hardware implementation.

2 Topic Background

2.1 Model-based Controls in Soft Robotics

Much of controls theory as a field has progressed in parallel to the advances in our understanding of dynamic modeling. In fact, much of the advancements in controls theory were driven by the need to supplement the gaps in our dynamic models. Starting from the frequency domain and linear controllers, to nonlinear controllers, and most recently machine learning.

The implementation of control strategies for standard rigid robots has followed this line of progression. Algorithms built around the dynamical model of the robot itself came first, then only recently did strategies employing machine learning began to be implemented–mostly to handle unpredictable scenarios beyond the models' assumptions. Control strategies of the former kind are known as *model*-based controls, where controllers are designed based on models that mathematically represent the robot's dynamics [1]. It should be apparent that this has been the case since the dynamics of a rigid-bodied robot can more readily be modelled.

On the other hand, the development of control strategies for *soft* robots have followed the opposite direction: much of the early works in soft robotics controls employed machine learning strategies due to the high level of complexity and dimensionality required in the dynamical models. Thus, *learning*-based controls were the model for soft robot control strategies through the formative years of the field. Only in recent years did this precedent began to be re-assessed, particularly with the advent of *finite*-dimensional modelling (FDM) techniques compatible with the continuum dynamics of soft robots [2].

2.2 Model- vs. Learning-based Controls

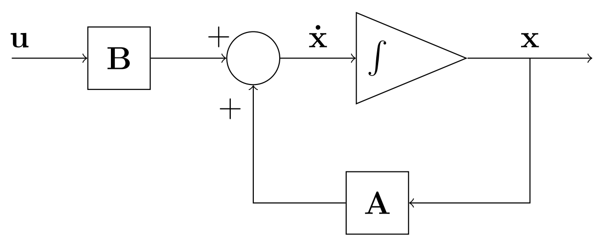

Figure 1: A generic steady-state dynamic system, with an input $\mathbf{u} \neq f(\mathbf{x})$ making it open-loop.

Model- and learning-based controls differ in the fundamental methodology that underpin *how* each approach steers their dynamic systems. Consider the steady-state open-loop dynamic system expressed in state space as $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ from Figure 1, the application of a model- and learning-based controller–respectively–to the system may be represented according to Table 1.

Model-based	Learning-based
$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$ $= \bar{\mathbf{f}}(\mathbf{x}, \mathbf{u})$ where, $\mathbf{u} = f(\mathbf{x}, \mathbf{e}, \mathbf{t}, ...)$	$\dot{\mathbf{x}} = \bar{\mathbf{f}}(\mathbf{x}, \mathbf{u}_{\text{learned}})$ or $\dot{\mathbf{x}} = \mathbf{g}(\mathbf{x}, \mathbf{u})$

Table 1: High-level generalization of model- vs. learning-based controllers.

The distinction between model- and learning-based control approaches may be described using the functional framework presented above. In both approaches, some input $\mathbf{u}$ is now introduced to the previously steady-state system. The system becomes *closed*-loop when a controller that uses the system output to determine the control input is implemented (see Figure 2). In this scenario, $f(\mathbf{x}, \mathbf{e}, \mathbf{t}, \ldots)$ can be considered the controller. It functionally relates the system output/current state–and typically error $\mathbf{e}$ and time as well–to the control input.

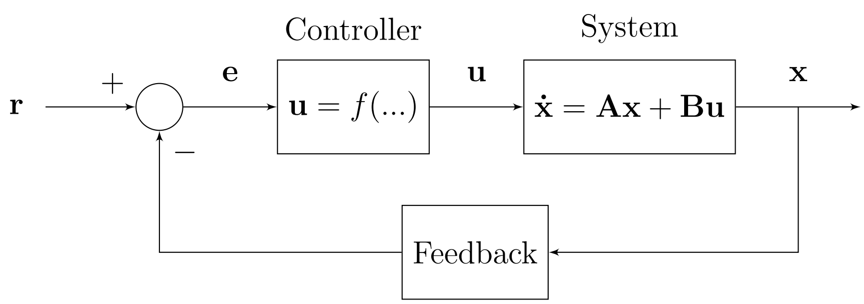

Figure 2: A generic closed-loop system, the "System" block was illustrated in Figure 1.

For model-based controls, designing the controller based on the dynamics of the systems means tuning $\mathbf{u}$ to the $\mathbf{A}$ and $\mathbf{B}$ matrices. A conveniently acceptable way of interpreting $\bar{\mathbf{f}}$ is to regard $\mathbf{A}$ as the system's steady-state behavior, and $\mathbf{B}$ as the system's response to control inputs. The two matrices *must* sufficiently describe the system dynamics before $\mathbf{u}$ can be designed to steer the system towards the desired configuration. Formulating $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{u}$ while respecting their inter-dependency thus becomes inherent to the model-based approach.

Formulating models that allow the input to be solved in a mathematically managable way, while simultaneously remaining faithfully representative of the *actual* system behavior becomes one of model-based controls' biggest challenges. Learning-based controls circumvent this by using data-driven techniques and learning algorithms to arrive at $\mathbf{u}$. In some cases, $\bar{\mathbf{f}}$ may even be left unknown and the full system is wholly formulated by the learning medium ($\mathbf{g}(\mathbf{x}, \mathbf{u})$ in Table 1).

A general assessment of the benefits and drawbacks between the two approaches is presented in Table 2. While both approaches bring different things to the table–and extensive research continue to be conducted on both, the author has decided to explore the model-based approach for this proposed thesis.

Analysis	Model-Based	Learning-Based
Advantages	Meaningful information regarding the dynamics and inputs of the system are preserved	Known to achieve higher maneuvering performance and better robustness [1]
Disadvantages	Less robust and adaptable at handling system configurations in fringe scenarios [1], [3]	More complex and typically lacks formal guarantees of safety [3]

Table 2: An assessment of Model- and Learning-Based controls.

3 Prior Work

3.1 The Piecewise Constant-Curvature Model

It was mentioned earlier that in the formulation of the dynamical model to develop a controller around, towing the balance between mathematical simplicity and physical accuracy is one of the major challenges in model-based controls. While this continues to be true, continuum dynamics-compatible FDM techniques has also indeed broken considerable ground with regards to this issue. In their exact formulation, the dynamics of a soft robot is effectively an infinite-dimensional system. Such a system cannot be described without the use of partial differential equations (PDEs). However, by applying the appropriate assumptions, approximations, and/or discretization to the dynamical model of the soft robot, the system's description may be reliably "minimized" into a *finite*-dimensional system that can be described with ordinary differential equations (ODEs) instead.

One of the most widely-implemented family of FDM techniques is the Piecewise Constant Strain (PCS) approximations (see Figure 3). It is a family of discretization methods applied to a type of approximation for soft robot dynamics known as "rod models". It is extremely common for soft robots to be "thin" and/or "elongated": to have one physical dimension dominate the other two. In that regard, such soft robots may be approximated as a rod with the continuum mechanics of one too–hence rod models. At the heart of this methodology is the assumption that volumetric deformations may be neglected, and modeling the dynamics around the dominant central axis is sufficient [2].

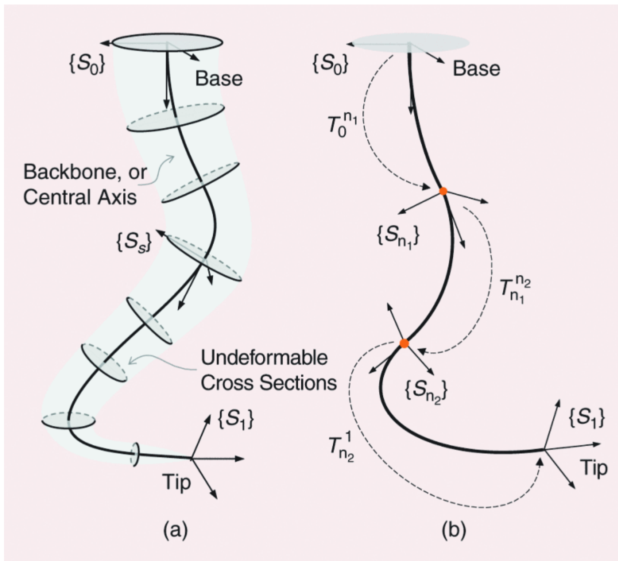

Figure 3: a) "Elongated" soft robots as described in rod models. b) PCS discretization applied to the rod model. Image taken from [2].

Among the many implementations of PCS, the planar Piecewise Constant Curvature (PCC) model has been extensively used in soft robotics throughout the last decade. Its approximation of the robot as pieces of constant-curvature arcs linked together in series with mutually tangent connection points makes the kinematics and Jacobian formulation for the model *closed*-form [4]. A dynamic feedback controller using this model was developed in [5], with an implementation of a trajectory generator for the controller formulated in [6].

There are many viable models out there that can be used as the basis for the controller that this proposed thesis seeks to develop, a selection of them that seems promising with respect to the scope of this proposed thesis will be outlined below.

3.2 PCC in an Augmented Rigid Body Model

The controller from [5] mentioned earlier uses the kinematics of PCC matched to the dynamics of an augmented rigid body, allowing control strategies typically implemented for rigid-body robots to be used in this scenario. This augmented formulation was initially proposed in [7] by Della Santina et al. as well.

Recall that the PCC Model essentially approximates a rod-like soft robot as a series of constant-curvature (CC) arc segments with mutually-tangent connections. Della Santina et al. proposed that for a given CC segment, the dynamics of the segment can be matched to a rigid-robotic structure which is comprised of Revolute-Prismatic-Prismatic-Revolute (RPPR) joints in series (see Figure 4). A point mass is located between the two prismatic joints and also matched to the CC segment's center of mass, ensuring the inertial properties of the rigid robot matches the CC segment [7].

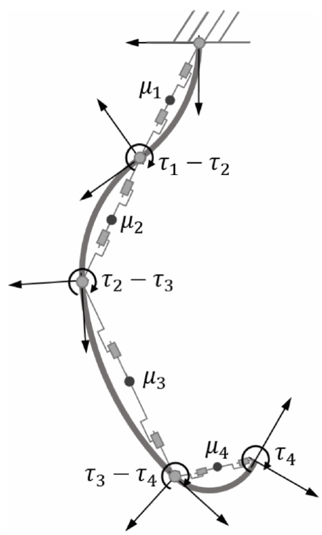

Figure 4: A series of CC Segments matched to a series of RPPR Robots. Image taken from [7].

Using this model, Della Santina et al. were able to develop two feedback controllers aimed at different implementations. The first controller is intended for trajectory tracking, while the second for Cartesian impedance control. While the model *and* controller does not cover 3-dimensional motion, preliminary work into translating the model to 3-dimensions is explored in [8]. Implementing this "augmented rigid body" model for this proposed thesis will involve expanding the preliminary work into the three-dimensional translation of the model, and then implementing the control design methodology already laid out in [5]. The body of literature available to build on regarding this model is one of the biggest appeals in using it for this proposed thesis. However, the kinematic singularity found in the 3-dimensional straight configuration remains a challenge without solutions previously attempted.

3.3 PCC Using an Alternative State Parametrization

Another one of Cosimo Della Santina's works using the PCC model can be found in [9]. In this work, Della Santina et al. proposes an *alternative* state parametrization for PCC kinematics that describes the CC segments based on the arc lengths of four arcs that bound the surface of the segment.

In the preliminary work of [8], the PCC kinematics of a soft robot were parametrized according to Figure 5. In [9], Della Santina et al. refers to this state parametrization as the "α-parametrization".

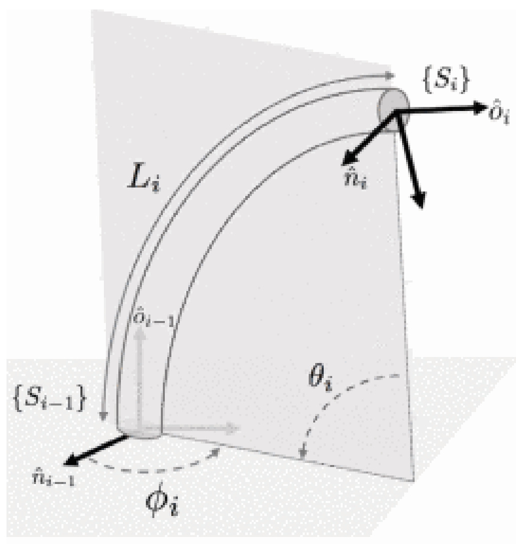

Figure 5: The standard or "old" state parametrization. Image taken from [8].

Kinematically speaking, the α-parametrization is defined as $\alpha_i = [\phi_i, \theta_i, \delta L_i]^T$ for the i-th CC segment, where $\alpha_i \in \mathbb{R}^3$. In this parametrization, ϕ_i and θ_i are the angles as indicated in Figure 5, while δL_i is the change in length of the soft robot's central axis. The issue with this parametrization–which was mentioned in our discussion of the previous model–is that for a given physical configuration, it does not *uniquely* map to a vector α_i. In the case of the straight configuration for example, this means there are infinite choices of ϕ_i for $\theta_i = 0$. Giving rise to the kinematic singularity mentioned previously.

Now in [9], Della Santina et al. refers to the alternative state parametrization they propose as the "q-parametrization". Figure 6 illustrates how the i-th segment of a PCC soft robot is represented under the alternative state parametrization.

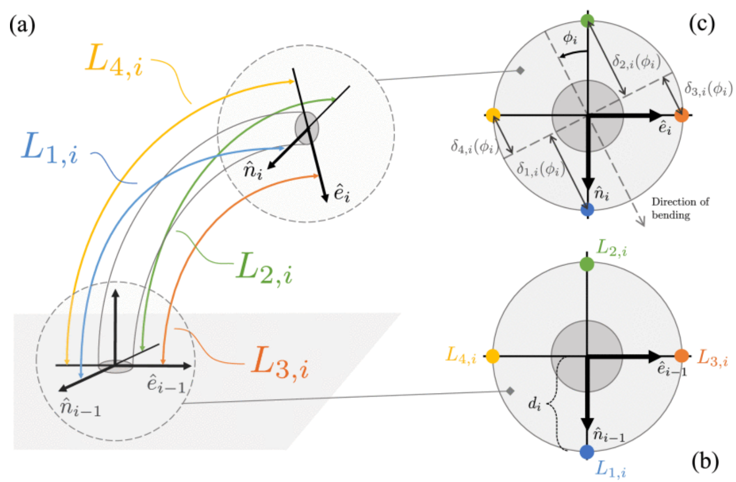

Figure 6: The alternative "Arc-Length Parametrization". Image taken from [9].

This new "Arc-Length Parametrization" defines a configuration q_i as $q_i = [\Delta_{x,i}, \Delta_{y,i}, \delta L_i]^T \in \mathbb{R}^3$. While explicit definitions of $\Delta_{x,i}$ and $\Delta_{y,i}$ will not be reported in this proposal, these two parameters essentially describe the difference in length between the two arcs whose ends are connected to the $\hat{n}$ (blue and green arcs) and $\hat{e}$ (yellow and orange arcs) axes, respectively.

While [9] proves that the q-parametrization is able to solve the limitations of the α-parametrization, a controller design methodology or framework has yet to be laid out. However, its kinematic robustness does make this model a great option to move forward with for this proposed thesis.

4 Research Approach

Formulation unlikely
Rough outline of model assessment;tabulated or other

Justify model selection based on table assessments

Analytically (?) assess model behavior?
Motor babble pneumatically-actuated soft robot at hand
Simulate output using model, and look at the error

Development of the controller
What does hardware implementation look like?
What does *validation of* the hardware implementation look like?

5 Proposed Timeline

Research Approach 1: Dec 2025
Research Approach 2: Jan 2025
Research Approach 3: Mar 2025
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6 References

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