Just the bare bones of the simplified model

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1 Swarms, agents, cohesion neighbours and perimeter

A swarm S comprises a set of *agents*, b, b', b'', b_0, b_1, etc. An agent is modelled simply as a point in the 2-D Euclidean plane, specified by a position vector in some coordinate system. Notice that, by definition, two different agents cannot occupy the same position.

Assume a global constant, C, associated with a swarm, that determines the radius of the *cohesion field* of each agent in the swarm.

For each agent, $b \in S$, its *cohesion neighbours*, is the set of agents, $n_c(b)$, defined by

$$n_c(b) = \{b' \in S \ : \ b' \neq b \wedge \|b' - b\| \leq C\} \tag{1}$$

It is useful to define an ordering on an agent's cohesion neighbours. We choose to order the cohesion neighbours of an agent b by their *polar angle* with respect to b. The polar angle with respect to b of b', $\alpha(b, b')$, is the counterclockwise angle that vector $\overrightarrow{bb'} = b' - b$ makes with the positive x axis:

$$\alpha(b, b') = \mathsf{atan2}((b' - b)_y, (b' - b)_x) \tag{2}$$

A partial ordering of agents by polar angle with respect to a specific agent, b, is denoted $\leq_{\alpha_b}$, and is defined by:

$$b' \leq_{\alpha_b} b'' \iff \alpha(b, b') \leq \alpha(b, b'') \tag{3}$$

We denote by $\langle b_0, b_1, .., b_{n-1}\rangle_{\leq_{\alpha_b}}$ a bijection from $\{0, .., n-1\} \to n_c(b)$ that is ordered by polar angle, i.e. $\forall i, j \ : \ 0 \leq i, j, < n \cdot i \leq j \implies b_i \leq_{\alpha_b} b_j$.

In this paper, we propose that the behaviour of an agent should be modified depending on whether or not it is on a *perimeter*. An agent b is on a perimeter if it satisfies any one of three conditions:

1. consecutive neighbours are not within each other's cohesion field, or
2. consecutive neighbours subtend a reflex angle, or
3. the agent has too few neighbours.

A function, $\mathsf{prm}(b)$, specifies these conditions formally. Let b be the agent of interest and b', b'' any pair of consecutive neighbours of b in the angle-sorted list $\langle b_0, b_1, .., b_{n-1}\rangle_{\leq \alpha_b}$, i.e. $b' = b_i, b'' = b_{(i+1)\%n}$ for some $i \in \{0, .., n-1\}$. Then $\mathsf{prm}(b)$ iff any one of the following conditions is satisfied:

1. $b' \notin n_c(b'')$,
2. $\delta > \pi$, where $\delta = \alpha(b, b'') - \alpha(b, b')$ (or $\delta = \alpha(b, b'') - \alpha(b, b') + 2\pi$ if the former is negative), or
3. $n_c(b) < 3$.

2 Cohesion vector

$$v_c(b) = \frac{1}{|n_c(b)|} \sum_{b' \in n_c(b)} k_c[p_b, p_{b'}](b' - b) \tag{4}$$

where $|n_c(b)|$ denotes the cardinality of $n_c(b)$, $p_b = \mathsf{prm}(b)$, $p_{b'} = \mathsf{prm}(b')$, and k_c is a 2x2 boolean-indexed array of constants that determine the weight of a component of the cohesion vector according to whether the interaction between b, b' is between non-perimeter agents, non-perimeter–perimeter, perimeter–non-perimeter, or perimeter–perimeter agents.

3 Repulsion vector

The set of repellers of b is

$$n_r(b) = \{b' \in \mathcal{S} : b \neq b' \wedge \|b' - b\| \leq R[p_b, p_{b'}]\} \tag{5}$$

where $p_b = \mathsf{prm}(b)$, $p_{b'} = \mathsf{prm}(b')$, and R is a 2x2 boolean-indexed array of constants that determine the radius of the *repulsion field* for agents in the swarm, according to whether the interaction between b, b' is between non-perimeter agents, non-perimeter–perimeter, perimeter–non-perimeter, or perimeter–perimeter agents.

Now $v_r(b)$ is defined by

$$v_r(b) = \frac{1}{|n_r(b)|} \sum_{b' \in n_r(b)} k_r[p_b, p_{b'}] \left(1 - \frac{R[p_b, p_{b'}]}{\|b' - b\|}\right)(b' - b) \tag{6}$$

where $p_b = \mathsf{prm}(b)$, $p_{b'} = \mathsf{prm}(b')$, and k_r is a 2x2 boolean-indexed array of constants that determine the weight of a component of the repulsion vector according to whether the interaction between b, b' is between non-perimeter agents, non-perimeter–perimeter, perimeter–non-perimeter, or perimeter–perimeter agents.

4 Gap-filling vector

In addition to cohesion and repulsion vectors, a *gap-filling* vector can also be used to contribute to agent behaviour. Gap-filling vectors have proven useful in quickly reducing internal voids and in controlling the shape of the external perimeter.

A gap-filling vector for b contributes a motion of b towards the midpoint of a gap identified in the perimeter test for b.

Let $\langle b_0, b_1, .., b_{n-1}\rangle_{\leq \alpha_b}$ be the cohesion neighbours of b in polar angle order, and let $b' = b_i$ and $b'' = b_{(i+1)\%n}$ be the first pair of consecutive neighbours that satisfy either condition (1) or condition (2) of the perimeter function $\mathsf{prm}()$, then the gap-filling vector, $v_g(b)$, for agent b is defined

$$v_g(b) = k_g \left(\frac{b' + b''}{2} - b \right) = k_g \frac{\overrightarrow{bb'} + \overrightarrow{bb''}}{2} \tag{7}$$

If there is no such pair of consecutive neighbours then $v_g(b) = 0$.

k_g is a weighting for the gap-filling vector allowing the combination of it with the other motion vectors (cohesion, repulsion, ...) to be "tuned".

A stricter alternative to this is to choose the first consecutive neighbour pair b', b'' that satisfy condition (1), ignoring condition (2). Again, $v_g(b)$ is defined by eq (7) if such a pair exists, or 0 otherwise.

5 Resultant vector

The resultant vector is simply the sum of the cohesion, repulsion and gap-filling vectors:

$$v(b) = v_c(b) + v_r(b) + v_g(b) \tag{8}$$