Improved swarm formation through relationship-based cooordination and gap reduction in self-healing swarms.

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***Abstract*—Perimeter Compression is a technique where by a void reducing effect can be added to a basic swarming algorithm. The effect is dependant upon perimeter identification and is controlled by applying three weighting factors to the existing swarming formulae. One to the cohesion calculation, one the repulsion calculation, and one to the repulsion field size.**

I. Introduction

When cohesion and repulsion field effects (sometimes referred to as potential fields [2], [9], [12], [22], [23], [17]) are used to create a swarming effect, the stable structures that develop are limited to either straight edges or partial lattices [8]. The maintenance of a well-structured swarm is crucial to effective deployment for applications such as reconnaissance or artificial pollination, where 'blind spots' are best eliminated [7], and containment, where the swarm is used to surround an object or region [5]. Over time swarms form regular shapes [19] and perimeters form of partial lattices that may contain so-called *anomalies*, such as concave 'dents' or convex 'peaks' [10]. These anomalies contribute to the disruption of an otherwise well-structured swarm. The key, therefore, is to ensure that these *anomalies* are dynamically removed from a swarm.

Perimeter compression is a technique that creates a 'pull' effect between perimeter agents. It is dependant upon perimeter agent identification as discussed by Eliot et. al. in [8], [9], [10] and discussed in Section IV-A.

The aim of this new algorithm is to reduce the spacing between perimeter-based agents. Figure 1) shows an agent and it fields. S_b is the sensor field. O_b is the obstacle field. C is the cohesion field and R is the repulsion field. The implementation involves introducing three controlling matrices; k_c which can be used to increases the magnitude of the cohesion vector. k_r which can be used to modify the repulsion vector and r_b which can be used to alter the repulsion fields of agents.

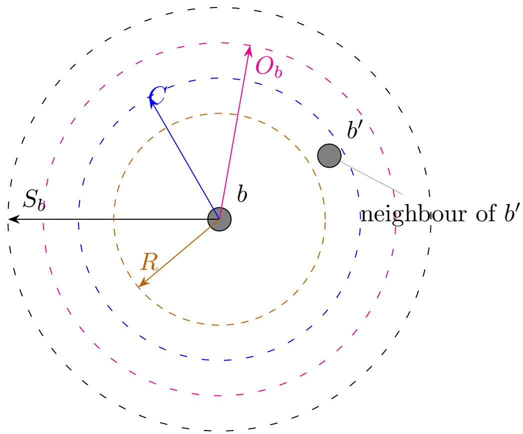


Fig. 1: Agent Fields

II. Related work

As far back as 1987 swarm theory has adopted the use of field effects/potential fields to coordinate agents [20] and this has continued since then in an attempt to improve the structure of a swarm, coordinate obstacle avoidance, and improve navigation [1], [2], [3], [4], [9], [12], [14], [22], [23]. Improvements to the basic structure of swarms has developed through the likes of a prototype framework for self-healing swarms that was developed by Dai et al. They considered how to manage agent failure in hostile environments [6]. This was similar to work by Vassev and Hinchey, who modelled swarm movement using the ASSL (Autonomic System Specification Language) [26]. This technique was employed by NASA (US National Aeronautics and Space Administration) for use in asteroid belt exploration as part of their ANTS (Autonomous Nano Technology Swarm) project. However, this work is focused towards failure of an agent's internal systems, rather than on the removal of anomalies in a swarm distribution. This need for formation control is also discussed by Speck and Bucci with respect to the diverse applications of swarms and the need to control a swarms structure [24].

In the context of swarm structure maintenance, Roach et al. focussed on the effects of sensor failure, and the impact that has on agent distribution [21]. Lee and Chong identified the issue of concave edges within swarms in an attempt to create regular lattice formations [16], and the main focus of their work is the dynamic restructuring of inter-agent formations. Ismail and Timmis demonstrated the use of *bio-inspired* healing using *granuloma formation*, a biological method for encapsulating an antigen [15]. They have also considered the effect failed agents can have on a swarm when traversing a terrain [25].

This paper proposes an alternative approach to agent coordination that can be used to induce a void reduction effect through perimeter compression. This is an extension of the work presented by Eliot et al. [10], Ismail and Timmis [15], [25], and on the work of McLurkin and Demaine on the detection of perimeter types [18]. However, perimeter type identification requires a communications infrastructure to allow the perimeter angle to be calculated. Communications within swarm formations limits swarm sizes and introduces performance problems [11]. The technique employed in this paper does not explicitly require the identification of the perimeter type as it would limit the size of the swarm[10], [16] and is therefore a reduced perimter detection algorithm to identify *any* perimeter.

III. Basic swarming model

In the Original work by Eliot et. al. the resultant vector of an agent was calculated using Equation 1. Where k_c, k_r, k_d, k_o are weighting factors for the summed vectors associated with each interaction. i.e. v_c, v_r, v_d, v_o for cohesion, repulsion, direction and object avoidance respectively.

$$v(b) = k_c v_c(b) + k_r v_r(b) + k_d v_d(b) + k_o v_o(b) \quad (1)$$

Equation 1 shows the movement vector as a linear combination of a cohesion vector v_c tending to move b towards its neighbours, a repulsion vector v_r tending to move b away from its neighbours, a direction vector v_d tending to move b towards a goal, and a vector v_o tending to steer it away from obstacles. $k_c, k_r, ...$ are the scalar coefficients of the the linear combination.

This paper does not consider goals or obstacles so we assume $k_d = k_o = 0$ and omit the third and fourth terms.

A. Cohesion

The cohesion component is calculated based on the proximity of neighbours. Where $n_c(b)$ is the set of neighbour agents for b (Eq. 2). The inclusion of an agent from a swarm (S) in by the agent's cohesion field (C).

$$n_c(b) = \{b' \in S \ : \ b' \neq b \wedge \|b' - b\| \leq C_b\} \quad (2)$$

The effect of an agent being within this set is that it will generate a vector that should 'encourage' agents to maintain their proximity. i.e. generate a cohesive swarm. The general weighted (k_c) formula for agents to maintain their proximity is to direct their motion towards the central point of all neighbouring agents as shown in Equation 3. Where $|n_c(b)|$ denotes the cardinality of $n_c(b)$. This formula includes the k_c quotient that allows the cohesion effect to be 'balanced' with respect to other vector influences as described in [8], [9], [10].

$$v_c(b) = \frac{1}{|n_c(b)|} \sum_{b' \in n_c(b)} (b' - b) \quad (3)$$

B. Repulsion

The repulsion component of an agent's movement is calculated from interaction with its neighbours $n_r(b)$ (Eq. 4) in a swarm ($\mathcal{S}$) that are within the agent's (b) repulsion field (R_b).

$$n_r(b) = \{b' \in \mathcal{S} : b \neq b' \wedge |b' - b| \leq R_b)\} \quad (4)$$

The repulsion is then calculated as the average of all the vectors created by the agent (b) to the neighbours (b') (Eg. 5) and its proximity ($\|b' - b\| - R$). Where $|n_r(b)|$ denotes the cardinality of $n_r(b)$.

$$v_r(b) = \frac{1}{|n_r(b)|} \sum_{b' \in n_r(b)} (\|b' - b\| - R)\, (\widehat{b' - b}) \quad (5)$$

Here, $\widehat{b' - b}$ denotes $b' - b$ normalized to unit length.

IV. New Inter-agent Model

In this paper, we propose that the behaviour of an agent should be modified depending on whether or not it is on a *perimeter*. These perimeter-based agents may form part of an outer (green) or inner (red) boundary (Fig. 2).

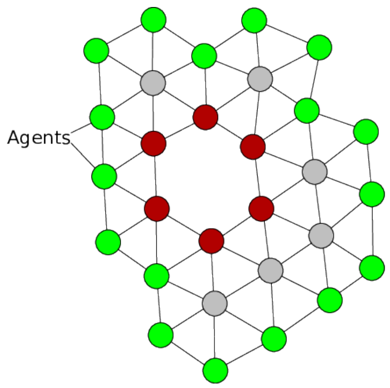


Fig. 2: Outer and inner swarm perimeters.

A. Perimeter detection

The detection process is achieved using a cyclic analysis of the agents that surround an agent (Fig. 3). Ghrist et al. discusses a similar technique using sweep angles [13] as does McLurkin et al [18].

When detecting a perimeter it is useful to define an ordering on an agent's cohesion neighbours. We choose to order the cohesion neighbours of an agent b by their *polar angle* with respect to b (Fig. 3).

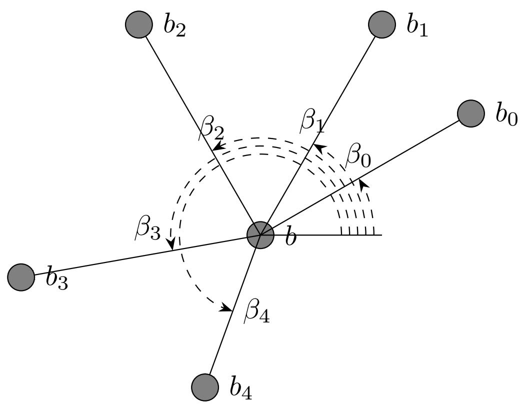


Fig. 3: Agent neighbours

The polar angle with respect to b of b', $\alpha(b, b')$, is the counterclockwise angle that vector $\vec{bb'} = b' - b$ makes with the positive x axis as shown in Figure 3 and described by Equation 6.

$$\alpha(b, b') = \mathsf{atan2}((b' - b)_y, (b' - b)_x) \tag{6}$$

A partial ordering of agents by polar angle with respect to a specific agent, b, is denoted $\leq_{\alpha_b}$, and is defined by:

$$b' \leq_{\alpha_b} b'' \iff \alpha(b, b') \leq \alpha(b, b'') \tag{7}$$

We denote by $\langle b_0, b_1, .., b_{n-1} \rangle_{\leq_{\alpha_b}}$ a bijection from $\{0, .., n-1\} \rightarrow n_c(b)$ that is ordered by polar angle as shown in Figure 4 and more formally in Equation. 8.

$$\forall i, j \ : \ 0 \leq i, j, < n \cdot i \leq j \implies b_i \leq_{\alpha_b} b_j \tag{8}$$

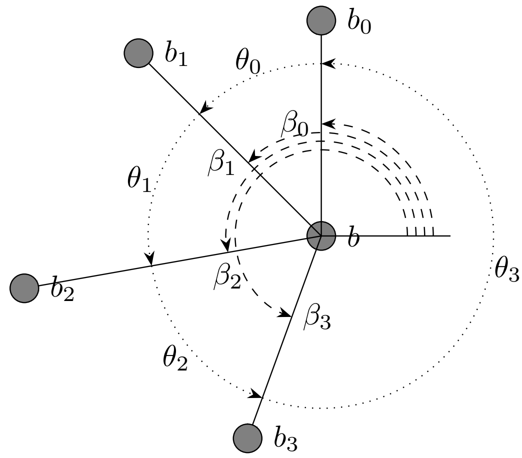


Fig. 4: Agent neighbour angles

An agent b is on a perimeter if it satisfies any one of three conditions:

1) consecutive neighbours are not within each other's cohesion field, or
2) consecutive neighbours subtend a reflex angle, or
3) the agent has too few neighbours.

A function, $\mathsf{prm}(b)$, specifies these conditions formally. Let b be the agent of interest and b', b'' any pair of consecutive neighbours of b in the angle-sorted list $\langle b_0, b_1, .., b_{n-1} \rangle_{\leq_{\alpha_b}}$, i.e. $b' = b_i, b'' = b_{(i+1)\%n}$ for some $i \in \{0, .., n-1\}$. Then $\mathsf{prm}(b)$ iff any one of the following conditions is satisfied:

1) $b' \notin n_c(b'')$,
2) $\delta > \pi$, where $\delta = \alpha(b, b'') - \alpha(b, b')$ (or $\delta = \alpha(b, b'') - \alpha(b, b') + 2\pi$ if the former is negative), or
3) $\|n_c(b)\| < 3$.

B. r_b, k_r and k_c

In this section we will discuss the application of the new r_b, k_r and k_c matrices which are structured as shown in Equation 9 which are indexed via a `true` (1) / `false` (0) reference.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{9}$$

The new model requires each agent to modify their inter-agent repulsion and cohesion vectors based upon their perimeter status and each neighbour's perimeter status. The basic perimeter control technique is shown in Equation 10 where the cohesion and repulsion matrices (k_c, k_r, r_b) are integrated into $v_c(b)$ and $v_r(b)$.

$$v(b) = v_c(b) + v_r(b) \tag{10}$$

1) Cohesion vector:

$$v_c(b) = \frac{1}{|n_c(b)|} \sum_{b' \in n_c(b)} k_c[p_b, p_{b'}](b' - b) \tag{11}$$

where $|n_c(b)|$ denotes the cardinality of $n_c(b)$, $p_b = \mathsf{prm}(b)$, $p_{b'} = \mathsf{prm}(b')$, and k_c is a 2x2 boolean-indexed array of constants that determine the weight of a component of the cohesion vector according to whether the interaction between b, b' is between non-perimeter agents, non-perimeter–perimeter, perimeter–non-perimeter, or perimeter–perimeter agents.

2) Repulsion vector:

The set of repellers of b are defined as Equation 12.

$$n_r(b) = \{b' \in \mathcal{S} : b \neq b' \wedge b' - b \leq R[p_b, p_{b'}]\} \tag{12}$$

where $p_b = \mathsf{prm}(b)$, $p_{b'} = \mathsf{prm}(b')$, and R is a 2x2 boolean-indexed array of constants that determine the radius of the *repulsion field* for agents in the swarm, according to whether the interaction between b, b' is between non-perimeter agents, non-perimeter–perimeter, perimeter–non-perimeter, or perimeter–perimeter agents.

Now $v_r(b)$ is defined by Equation 13

$$v_r(b) = \frac{1}{\|n_r(b)\|} \sum_{b' \in n_r(b)} k_r[p_b, p_{b'}] \left(1 - \frac{R[p_b, p_{b'}]}{b' - b}\right)(b' - b) \tag{13}$$

where $p_b = \mathsf{prm}(b)$, $p_{b'} = \mathsf{prm}(b')$, and k_r is a 2x2 boolean-indexed array of constants that determine the weight of a component of the repulsion vector according to whether the interaction between b, b' is between non-perimeter agents, non-perimeter–perimeter, perimeter–non-perimeter, or perimeter–perimeter agents.

C. Gap-filling

In addition to cohesion and repulsion vectors, a *gap-filling* vector can also be used to contribute to agent behaviour. Gap-filling vectors have proven useful in quickly reducing internal voids and in controlling the shape of the external perimeter.

A gap-filling vector for b contributes a motion of b towards the midpoint of a gap identified in the perimeter test for b.

Let $\langle b_0, b_1, .., b_{n-1}\rangle_{\leq \alpha_b}$ be the cohesion neighbours of b in polar angle order, and let $b' = b_i$ and $b'' = b_{(i+1)\%n}$ be the first pair of consecutive neighbours that satisfy either condition (1) or condition (2) of the perimeter function $\mathsf{prm}()$, then the gap-filling vector, $v_g(b)$, for agent b is defined in Equation 14.

$$v_g(b) = k_g \left(\frac{b' + b''}{2} - b\right) = k_g \frac{b' - b + b'' - b}{2} \tag{14}$$

If there is no such pair of consecutive neighbours then $v_g(b) = 0$.

k_g is a weighting for the gap-filling vector allowing the combination of it with the other motion vectors (cohesion, repulsion, ...) to be "tuned".

A stricter alternative to this is to choose the first consecutive neighbour pair b', b'' that satisfy condition (1), ignoring condition (2). Again, $v_g(b)$ is defined by eq (14) if such a pair exists, or 0 otherwise.

V. Resultant vector

The resultant vector is simply the sum of the cohesion, repulsion and gap-filling vectors:

$$v(b) = v_c(b) + v_r(b) + v_g(b) \tag{15}$$

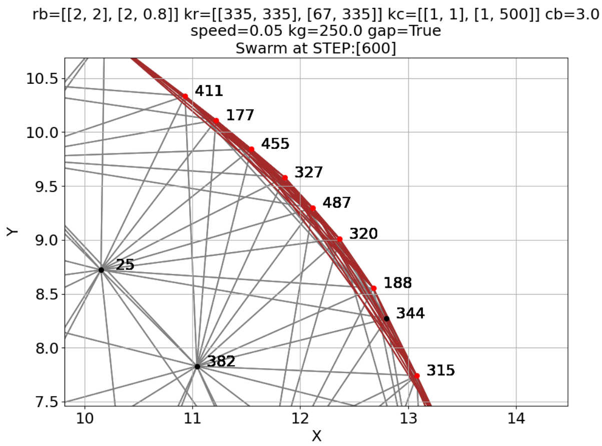


Fig. 5: Perimeter compression.

==== STILL TO BE WORKED ON =====

The repulsion component of the swarm model is now applied as shown in Equations **??**, 16 and 17.

$$v_r(b) = \frac{1}{|n_r(b)|} \sum_{b' \in n_r(b)} \mathsf{ekr}(b, b') \left(\|b' - b\| - \mathsf{erf}\,(b, b')\right) (\widehat{b' - b}) \tag{16}$$

$$n_r(b) = \{b' \in \mathcal{S} : b \neq b' \wedge \|b' - b\| \leq \mathsf{erf}(b, b')\} \quad (17)$$

The introduction of the matrices allows for specific relationships to effect the movement of agents. Using a J matrix results no perimeter repulsion change and all agents use the same repulsion field and the repulsion effect is monolithic within the swarm. However if $P_r[1,1] < 0$ as shown in Equation 18) and both agents are perimeter-based i.e. $\mathsf{prm}(b)$ and $\mathsf{prm}(b')$ $(true, true)$ result in $\mathsf{erf}(b, b')$ returning 0.4 from the matrix. This effectively reduces the R_b field allowing neighbouring agents to move more closely together which effects a compression effect. This is then combined with a weighting effect from the p_{kr} matrix which is used by $\mathsf{ekr}(b, b')$. This function returns a weighting value for the overall inter-agent repulsion relationship. This can be used to reduce the effect of specific vectors in the reopulsion calculations. When this effect is combined with a perimeter-based cohesion (§ V-A) increase a sustainable compression is possible as shown in figure 5.

$$p_r = \begin{pmatrix} 1 & 1 \\ 1 & 0.4 \end{pmatrix} \quad (18)$$

A. *Inter-agent cohesion (p_{kc}, $\mathsf{ekc}(b, b')$)*

The cohesion component of the swarming effect is applied when an agent (b) and its neighbour (b') are both within range (C_b). $\mathsf{ekc}(b, b')$ works in a similar manor to $\mathsf{ekr}(b, b')$. $\mathsf{ekc}(b, b')$ references the p_{kc} matrix to obtain a weighting that is based upon the two agents statuses. e.g. If both the agents are perimeter-based then the agents cohesion vector is scaled by the scalar value that is referenced in the matrix (Eq. **??**) i.e. using equation 20 $p_{kc}[1, 1]$ would return the value 500.0. In the case described above the effect of the additional cohesion-compression weighting will be increase the generated cohesion-vector for that individual relationship (Eq. 19) which causes the agent to have a tendency to move more strongly towards that neighbour.

$$v_c(b) = \frac{1}{|n_c(b)|} \sum_{b' \in n_c(b)} \mathsf{ekc}(b, b') \ (b' - b) \quad (19)$$

$$p_{kc} = \begin{pmatrix} 1 & 1 \\ 1 & 500.0 \end{pmatrix} \quad (20)$$

VI. Experimental results

A. *Baseline*

For all the experiments the parameters used to create the basic swarming effect are shown in Table I. Where C_b is the cohesion field, k_c is the cohesion weighting, R_b is the repulsion field, k_r is the repulsion weighting and k_g is the weighting factor applied in the comparison of the gap reduction algorithm discussed in [10]. The swarm consists of 200 agents which are distributed with a void at the centre. These initial parameters create a hexagonal-based distribution of agents that stabilise as shown in Figure 6. This basic swarm is used as the initial state for all the experiments.

Swarming Variable	Value
C_b	3.00
k_c	0.15
R_b	2.00
k_r	50.00
k_g	25.00

TABLE I: Swarming effect parameters

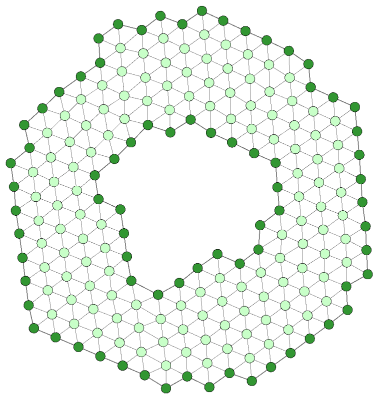


Fig. 6: Baseline swarm in stabilised configuration.

When the simulation is ran with no compression the changes are identified using a magnitude-based metric [9]. The resultant magnitudes generated are shown in figure 7. These states are used as the baseline for the experiments to measure the effects of the compression algorithm and compare the new algorithm to the existing void reduction algorithm.

Figure 8 shows how the compression effect can remove a void from a swarm by surround an obstacle in a similar manner to the method described in [10].

B. *Inner compression model*

$$p_{kr} = \begin{pmatrix} 1 & x \\ 1 & 1 \end{pmatrix} \quad (21)$$

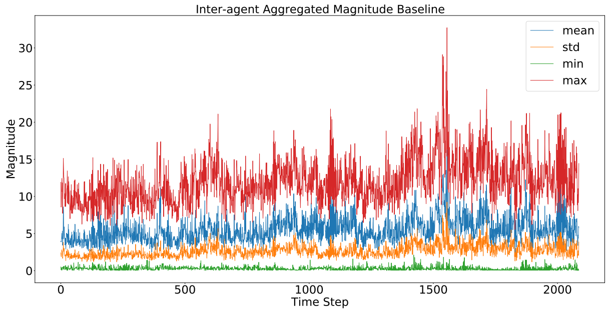
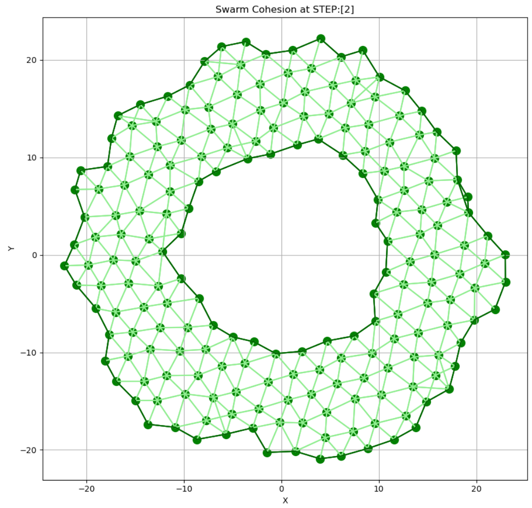
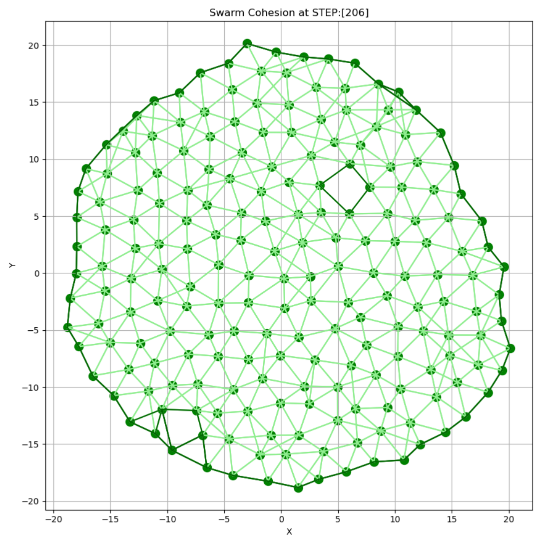


Fig. 7: Baseline swarm in stabilised configuration.



(a) Void removal start



(b) Void removal finish

Fig. 8: Void removal through perimeter compression

$$p_{kc} = \begin{pmatrix} 1 & y \\ 1 & 1 \end{pmatrix} \tag{22}$$

1) Inner + Gap compression model:

C. Outer compression model

$$p_{kr} = \begin{pmatrix} 1 & x \\ x & 1 \end{pmatrix} \tag{23}$$

$$p_{kc} = \begin{pmatrix} 1 & y \\ y & 1 \end{pmatrix} \tag{24}$$

1) Outer + Gap compression model:

D. Compression Effects

The compression effect parameters are shown in tables II and III

Pr/Pc	10	20	30	40	50
0.1	0.1/10	0.1/20	0.1/30	0.1/40	0.1/50
0.2	0.2/10	0.2/20	0.2/30	0.2/40	0.2/50
0.3	0.3/10	0.3/20	0.3/30	0.3/40	0.3/50
0.4	0.4/10	0.4/20	0.4/30	0.4/40	0.4/50
0.5	0.5/10	0.5/20	0.5/30	0.5/40	0.5/50
0.6	0.6/10	0.6/20	0.6/30	0.6/40	0.6/50
0.7	0.7/10	0.7/20	0.7/30	0.7/40	0.7/50
0.8	0.8/10	0.8/20	0.8/30	0.8/40	0.8/50
0.9	0.9/10	0.9/20	0.9/30	0.9/40	0.9/50

TABLE II: Experiment parameters 1

Pr/Pc	60	70	80	90	100
0.1	0.1/60	0.1/70	0.1/80	0.1/90	0.1/100
0.2	0.2/60	0.2/70	0.2/80	0.2/90	0.2/100
0.3	0.3/60	0.3/70	0.3/80	0.3/90	0.3/100
0.4	0.4/60	0.4/70	0.4/80	0.4/90	0.4/100
0.5	0.5/60	0.5/70	0.5/80	0.5/90	0.5/100
0.6	0.6/60	0.6/70	0.6/80	0.6/90	0.6/100
0.7	0.7/60	0.7/70	0.7/80	0.7/90	0.7/100
0.8	0.8/60	0.8/70	0.8/80	0.8/90	0.8/100
0.9	0.9/60	0.9/70	0.9/80	0.9/90	0.9/100

TABLE III: Experiment parameters 2

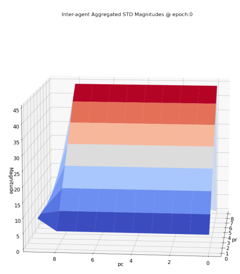


Fig. 9: Experiment Start

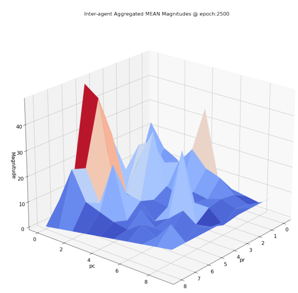


Fig. 10: Experiment Middle

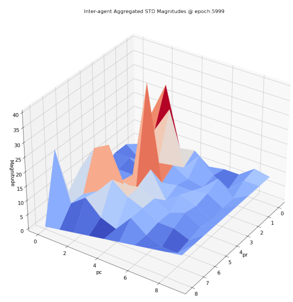


Fig. 11: Experiment End

The first area of comparison is the effect of the algorithms on the number of perimeter agents. The baseline swarm's agents oscillates but remain in a relatively stable state with a constant number of perimeter agents and the internal anomaly persists (Fig. 6). The maximum and minimum number of perimeter agents is shown in table IV.

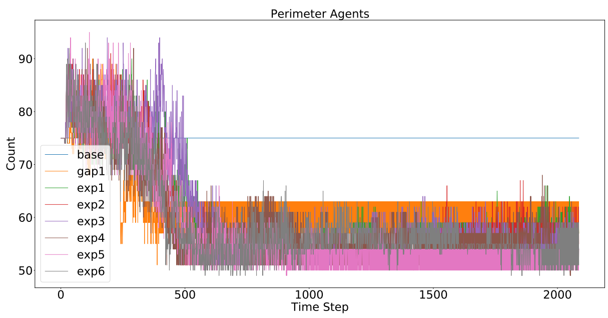


Fig. 12: Perimeter Count of baseline, gap reduction and perimeter compression.

Comp.	Base	Void	1	2	3	4	5	6
Max	75	90	90	90	94	92	95	93
Min	75	51	51	51	51	49	49	49
Mean	75	62	59	58	60	59	57	59
Std	0	6	9	10	10	8	10	8

TABLE IV: Perimeter agents

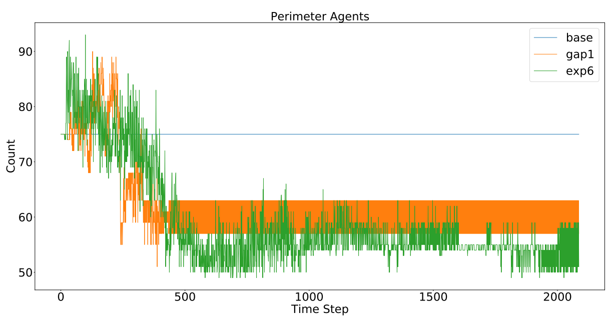


Fig. 13: Perimeter Count of baseline, gap reduction and Experiment 6.

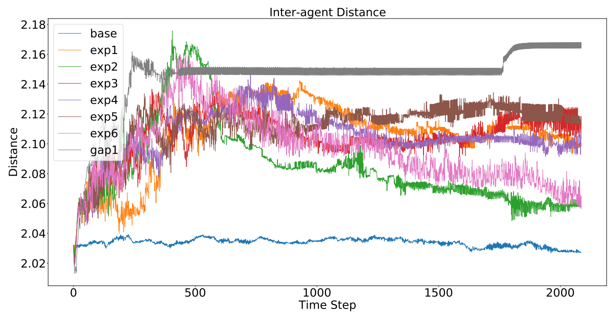


Fig. 14: Distance metric of baseline, gap reduction and perimeter compression.

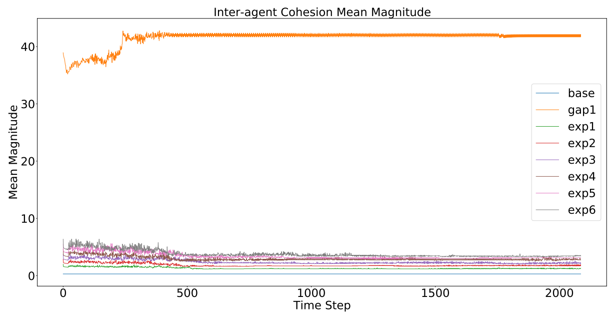


Fig. 15: Inter-agent Cohesion Mean.

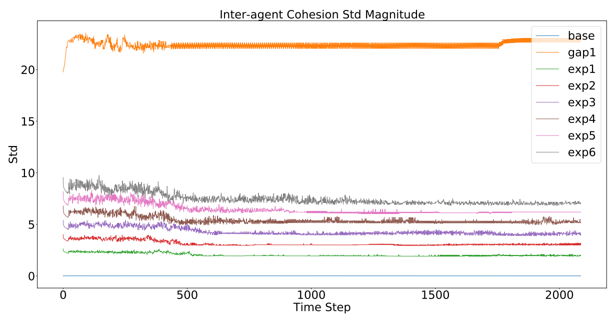


Fig. 16: Inter-agent Cohesion Std.

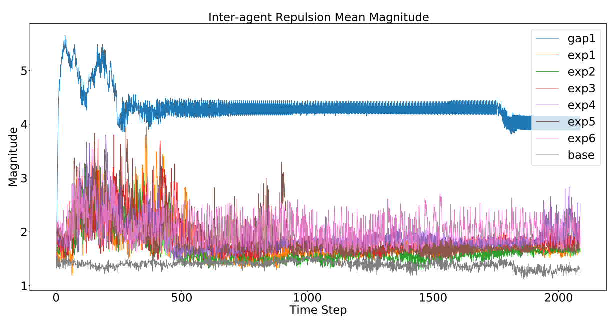


Fig. 17: Inter-agent Repulsion Mean.

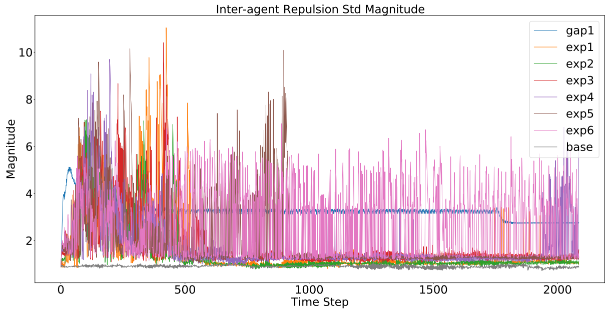


Fig. 18: Inter-agent Repulsion Std.

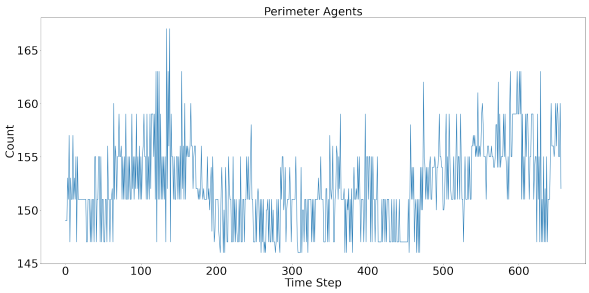


Fig. 19: Baseline swarm in stabilised configuration.

E. Gap compression

F. Perimeter compression

Compression 1

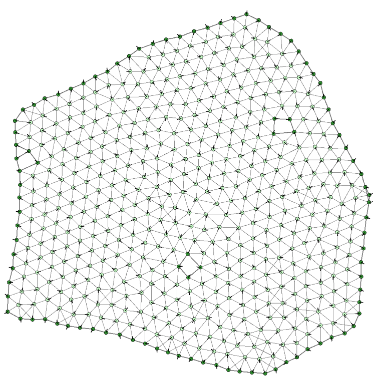


Fig. 20: Baseline swarm in with compression set 1 resultant configuration.

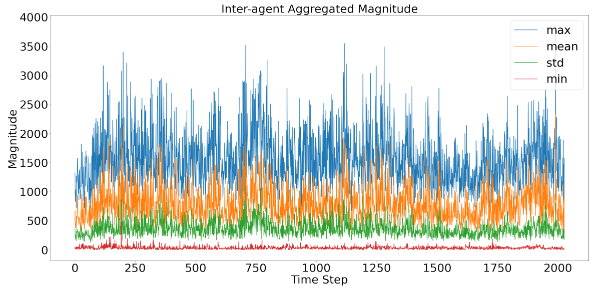


Fig. 21: Baseline swarm in with compression set 1.

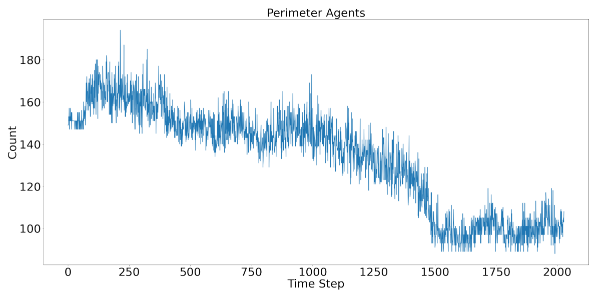


Fig. 22: Baseline swarm in with compression set 1.

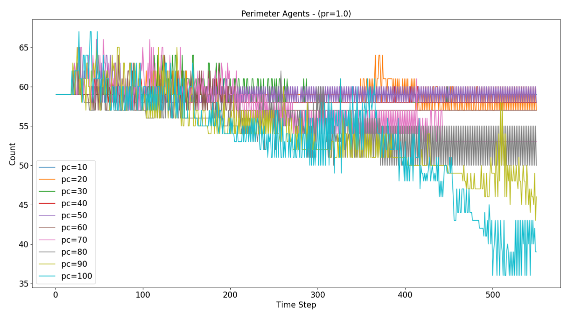


Fig. 23: Sample

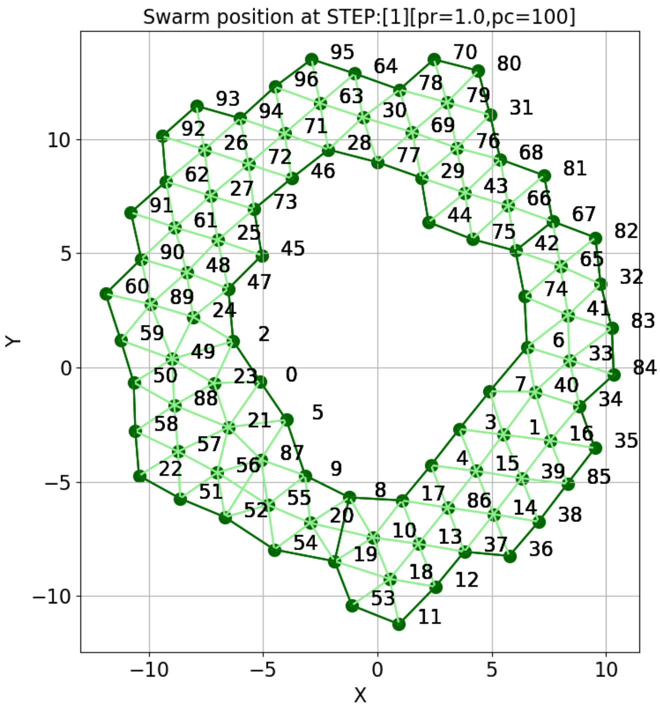


Fig. 24: Sample

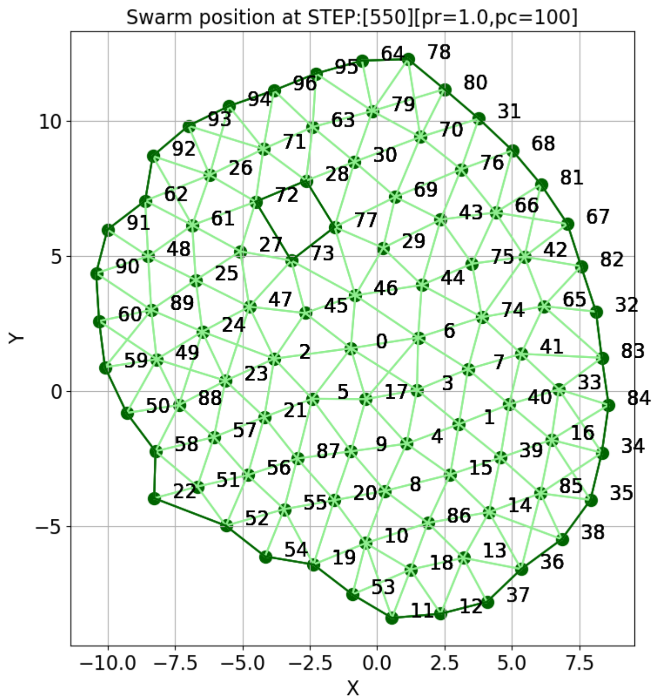


Fig. 25: Sample

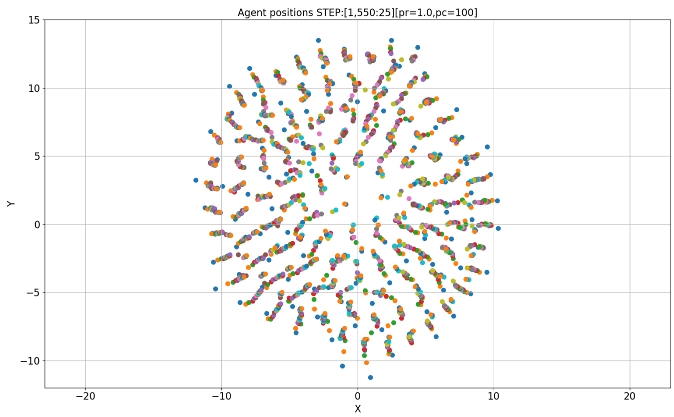


Fig. 26: Sample

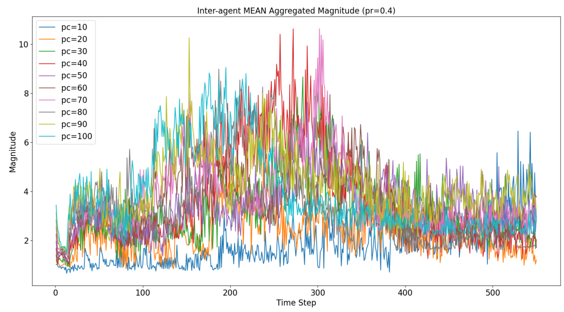


Fig. 27: Sample

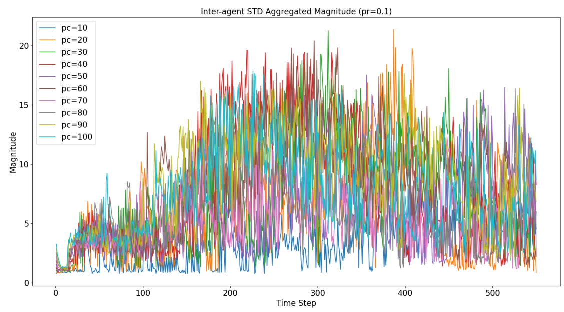


Fig. 28: Sample

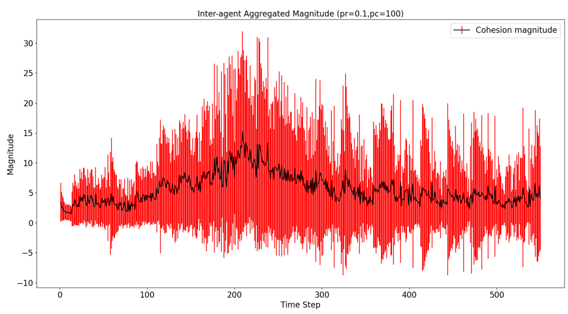


Fig. 29: Sample

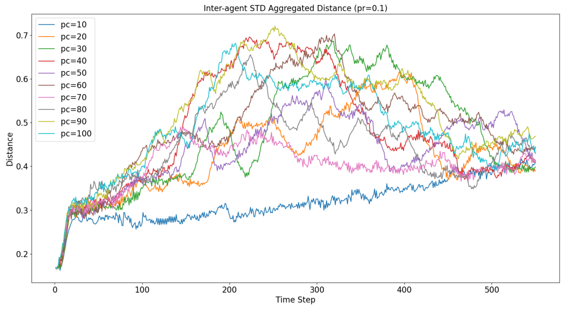


Fig. 30: Sample

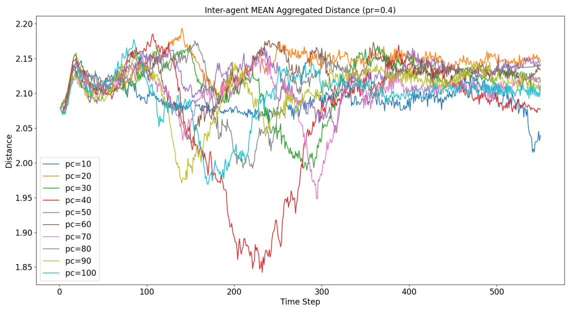


Fig. 31: Sample

Fig. 32: Sample

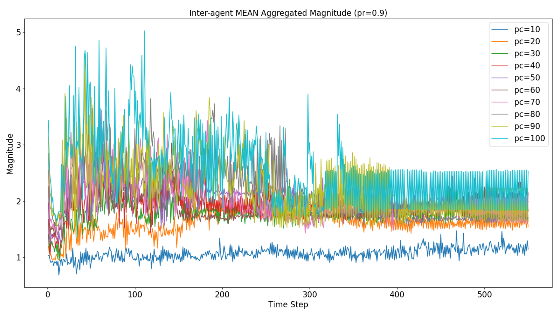


Fig. 33: MAG pr=0.9 mean

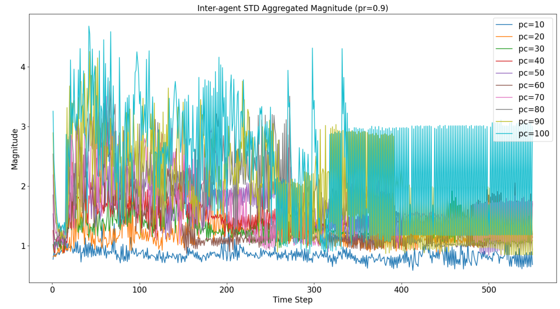


Fig. 34: MAG pr=0.9 std

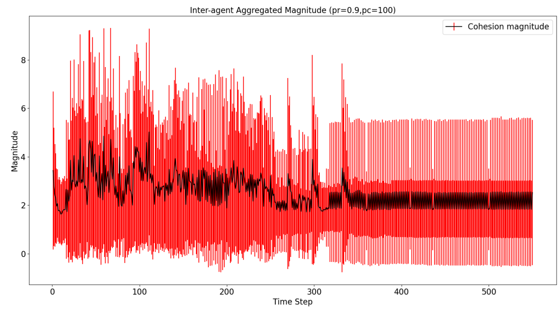


Fig. 35: MAG pr=0.9 pc=100 error

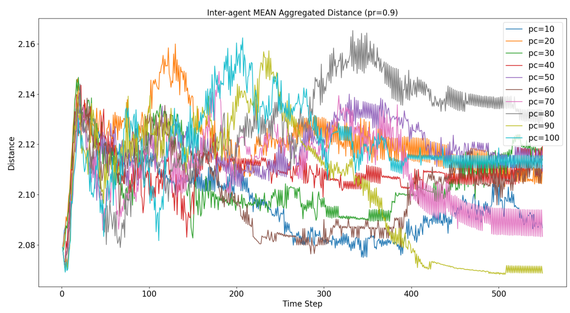


Fig. 36: DIST pr=0.9 mean

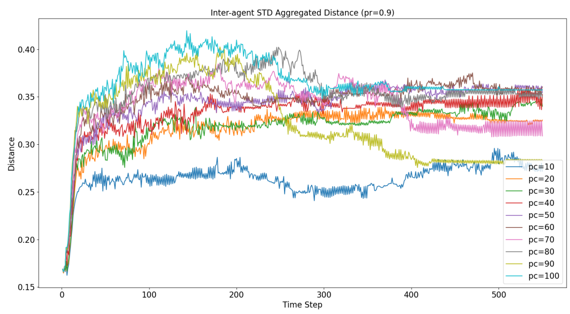


Fig. 37: DIST pr=0.9 std

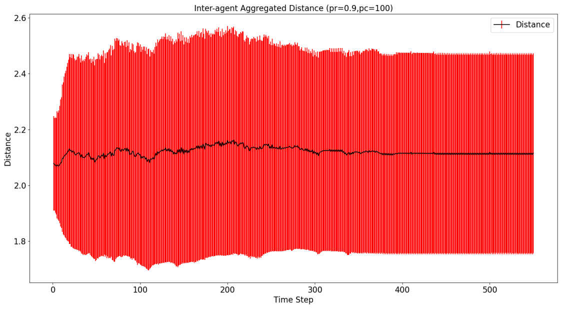


Fig. 38: DIST pr=0.9 pc=100 error

G. *Comparison*

VII. Conclusions

From the initial simulations it is possible to show that the technique is able to successfully remove voids and surround an obstacle as shown in the video https://youtu.be/3eY1vvq0JWo.

VIII. Future Work

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