Stan

Software Ecosystem for Modern Bayesian Inference

github.com/jgabry/lander-stan-class-2021

Lander Analytics
Stan Workshop
July 2021

Instructors

Jonah Gabry
Columbia University

linkedin
website

Rob Trangucci
University of Michigan

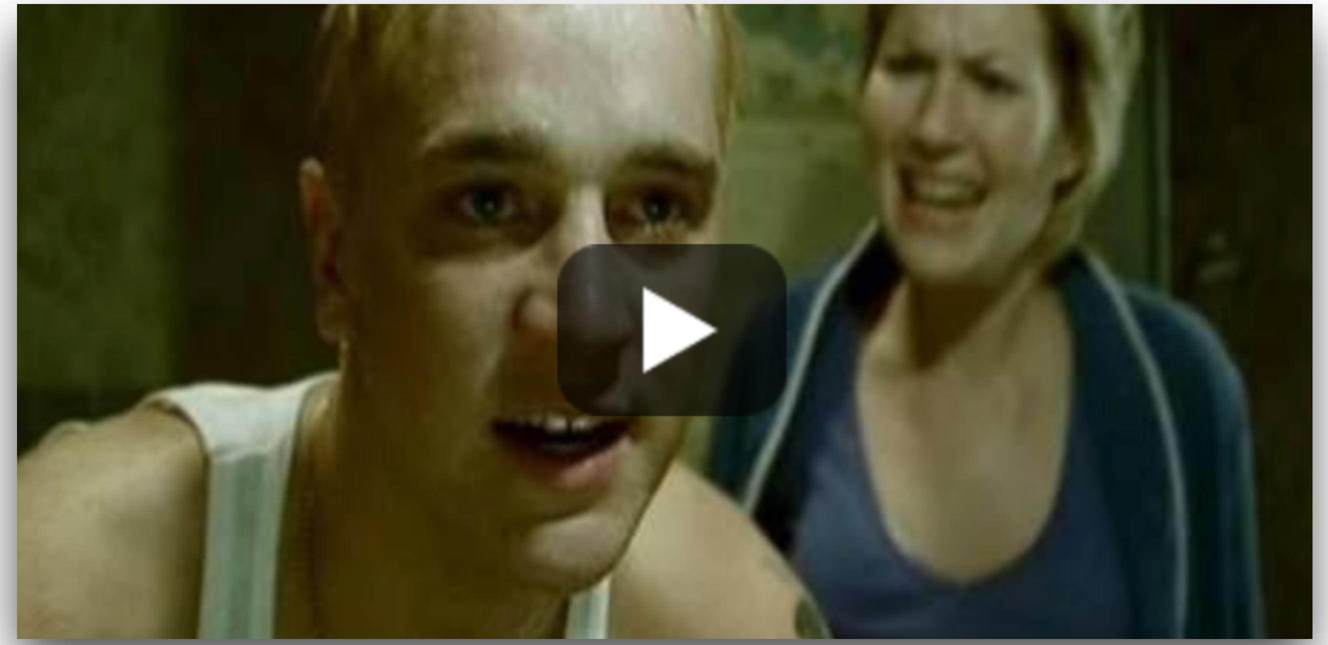
linkedin

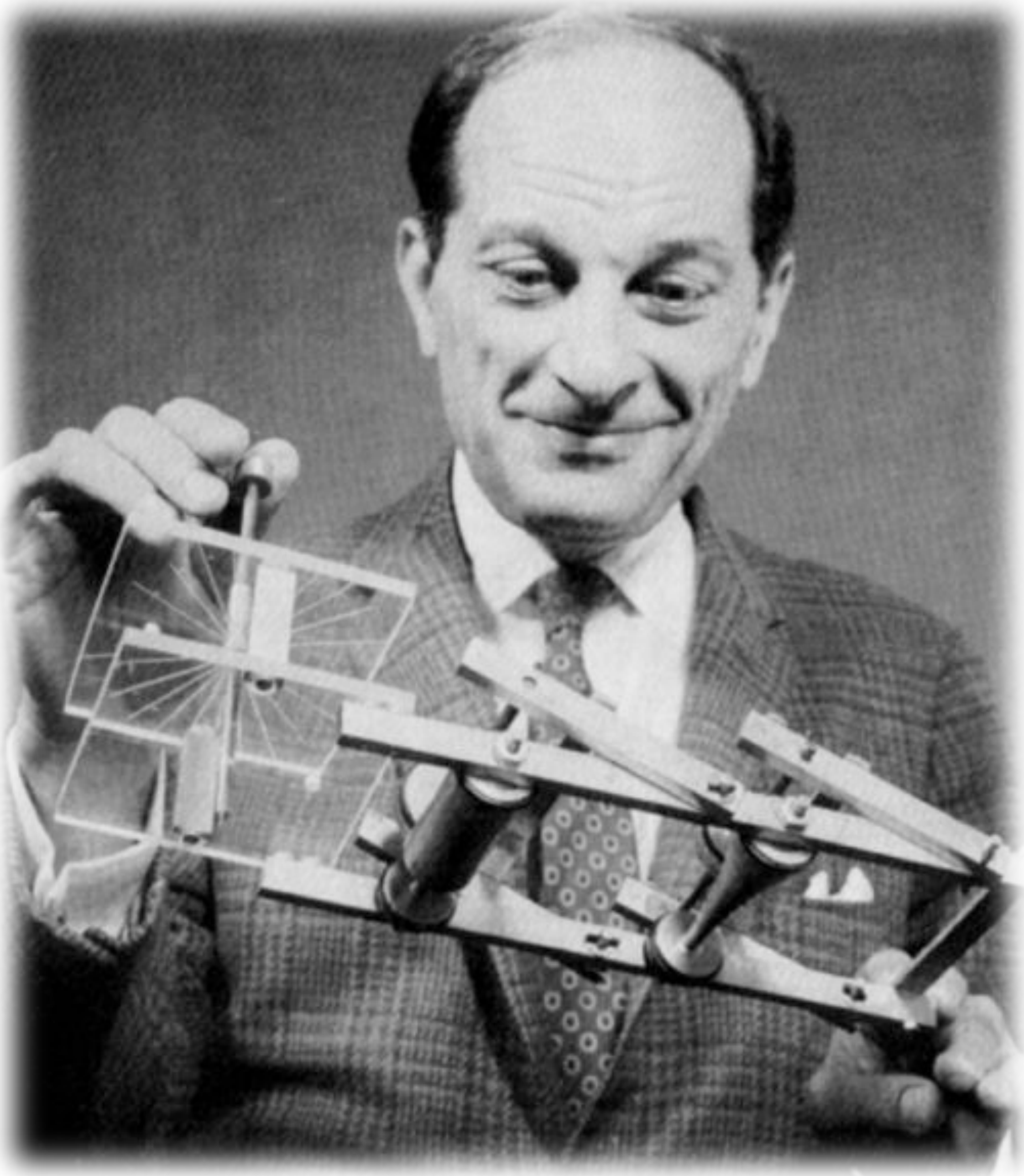
Scott Spencer
Columbia University

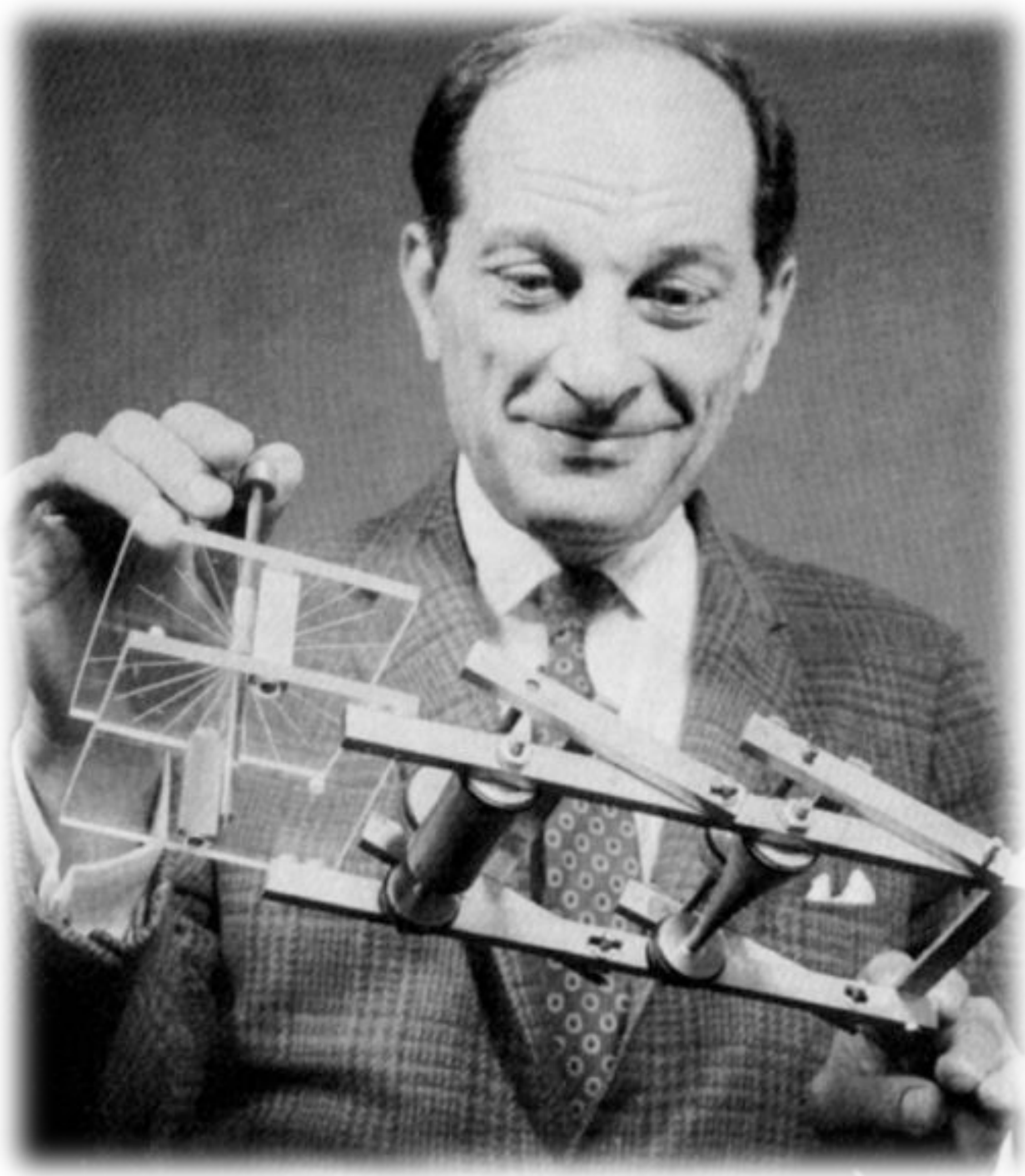
linkedin
Columbia faculty page

Why “Stan”?

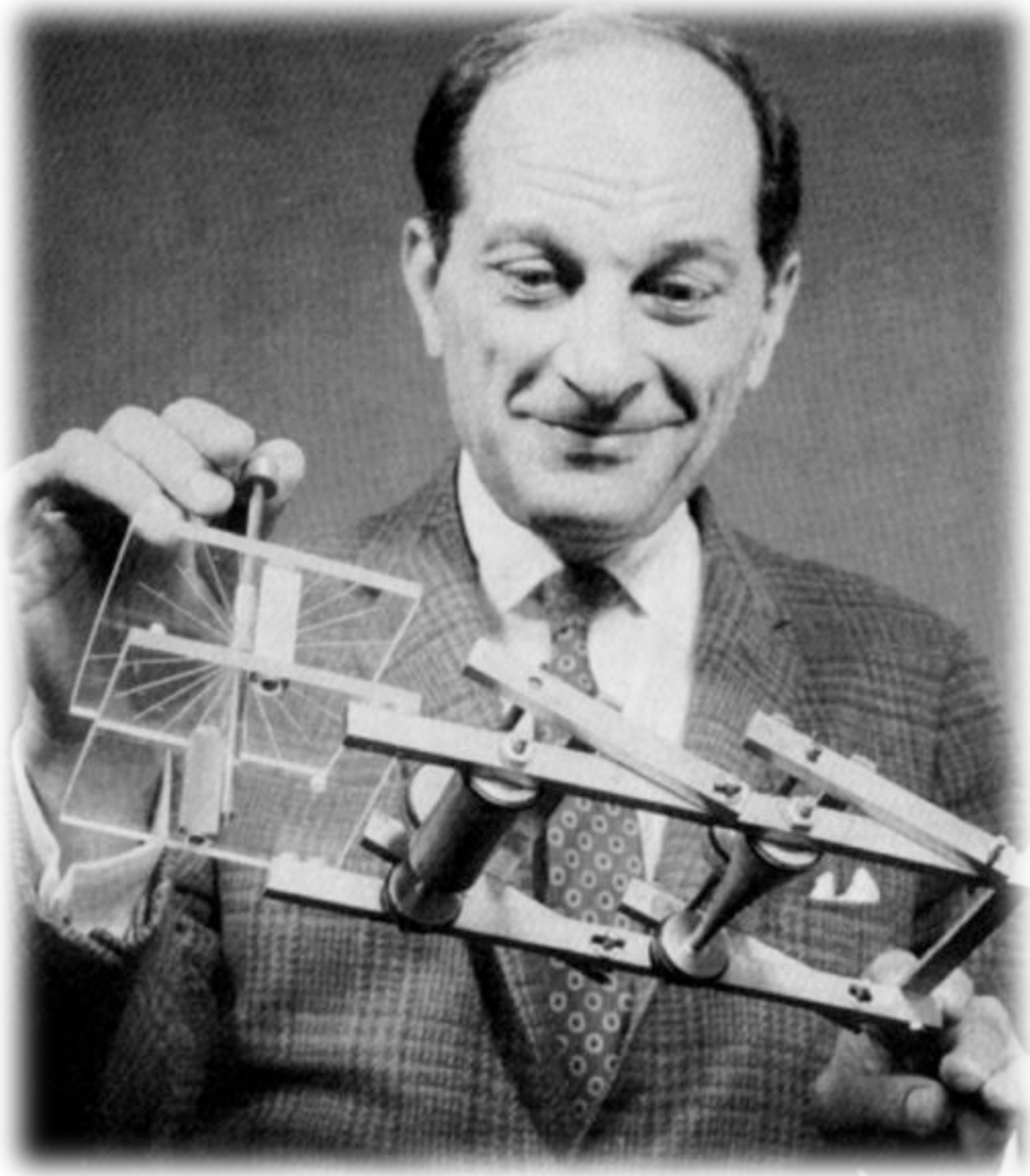
suboptimal SEO







Stanislaw Ulam
(1909–1984)



Stanislaw Ulam
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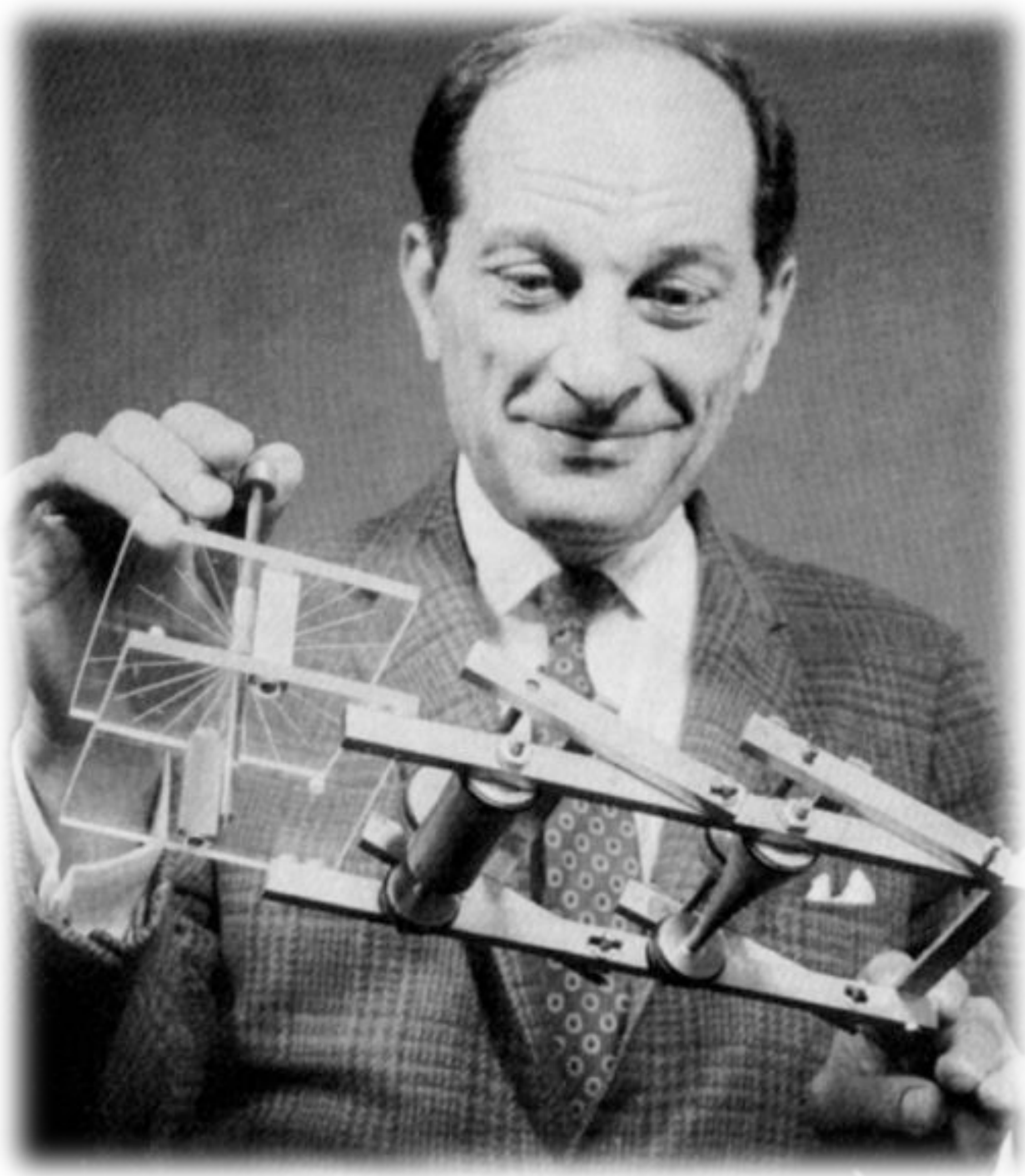
Monte Carlo
Method



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H-Bomb

What is Stan?

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  - MCMC for full Bayes
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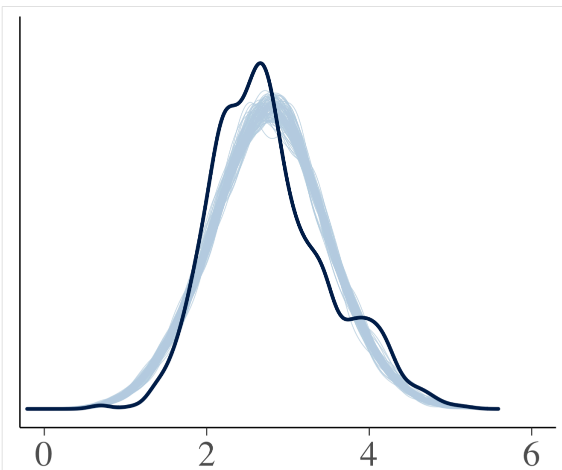
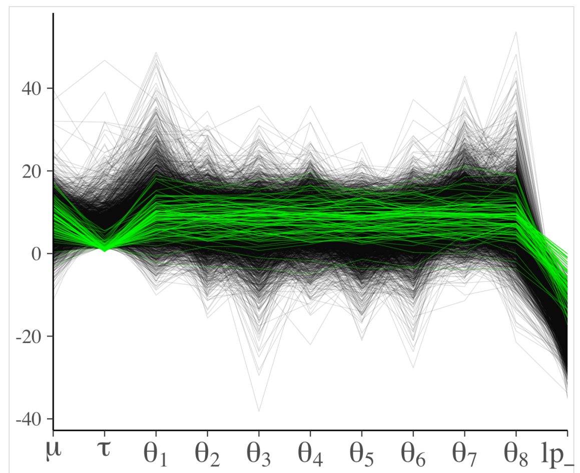
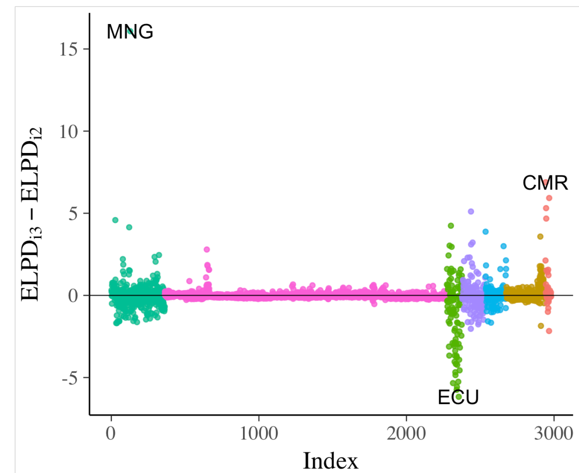
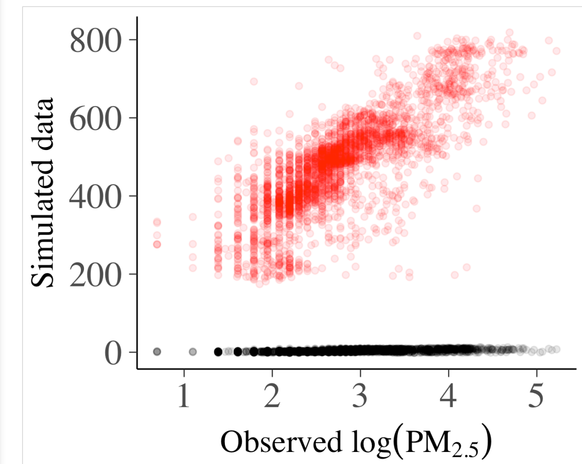
- Stan **ecosystem**
  - language, math library (C++)
  - interfaces and tools (R, Python, Julia, many more)
  - documentation (reference manual and user guide, case studies, R package vignettes, and more)
  - online community (Stan Forums on Discourse)

Visualization in Bayesian workflow









Jonah Gabry

Columbia University
Stan Development Team

A Bayesian modeler commits to an a priori *joint distribution*

Likelihood x Prior

$$p(\mathbf{y}, \boldsymbol{\theta}) = p(\mathbf{y} \mid \boldsymbol{\theta})p(\boldsymbol{\theta}) = p(\boldsymbol{\theta} \mid \mathbf{y})p(\mathbf{y})$$

Posterior x Marginal Likelihood

Data
(observed)

Parameters
(unobserved)

Workflow

Bayesian data analysis

Workflow
Bayesian data analysis

- Exploratory data analysis

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- *Prior* predictive checking

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Gabry, J., Simpson, D., Vehtari, A., Betancourt, M., and Gelman, A. (2019).
Visualization in Bayesian workflow.
Journal of the Royal Statistical Society Series A

Journal version: rss.onlinelibrary.wiley.com/doi/full/10.1111/rssa.12378
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Example

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Goal Estimate global PM2.5 concentration

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Problem Most data from noisy satellite measurements (ground monitor network provides sparse, heterogeneous coverage)

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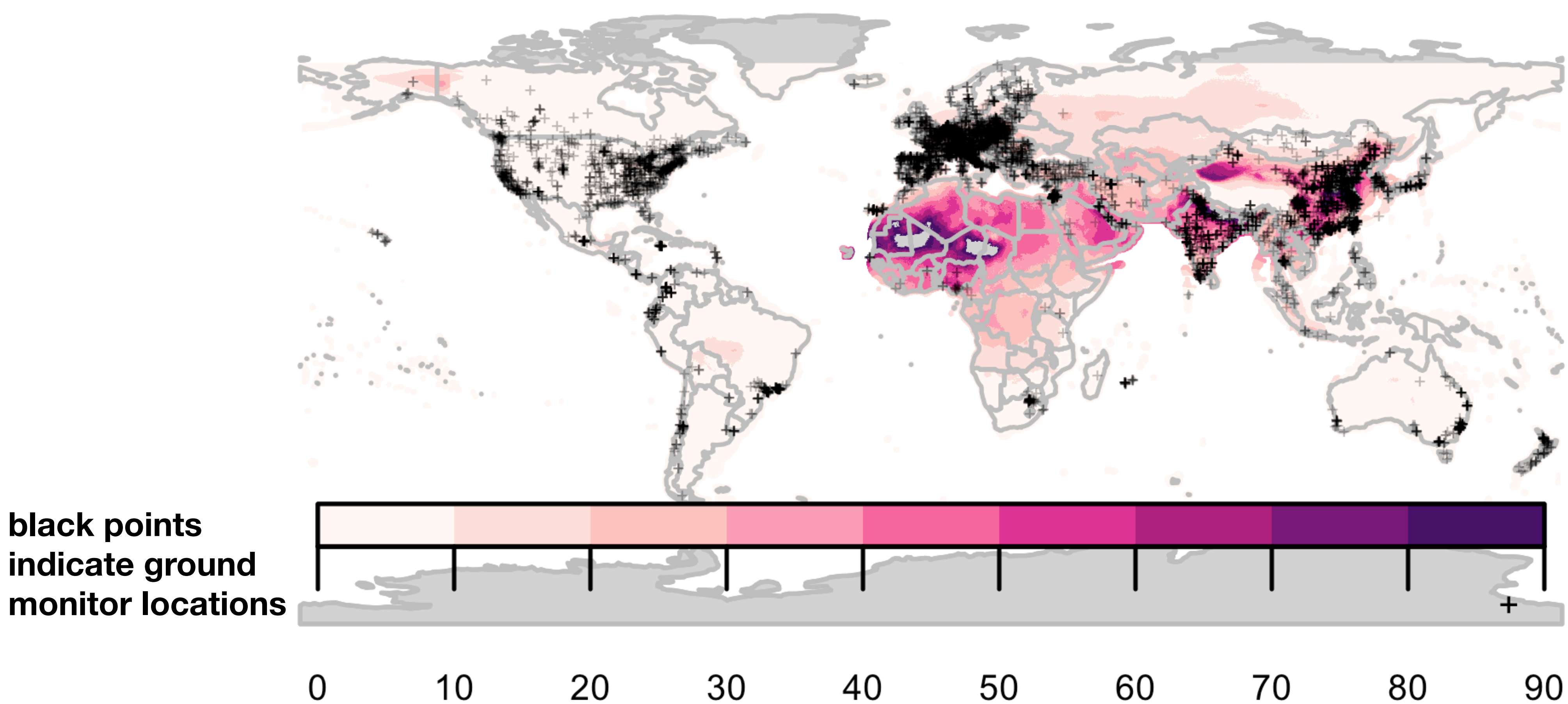


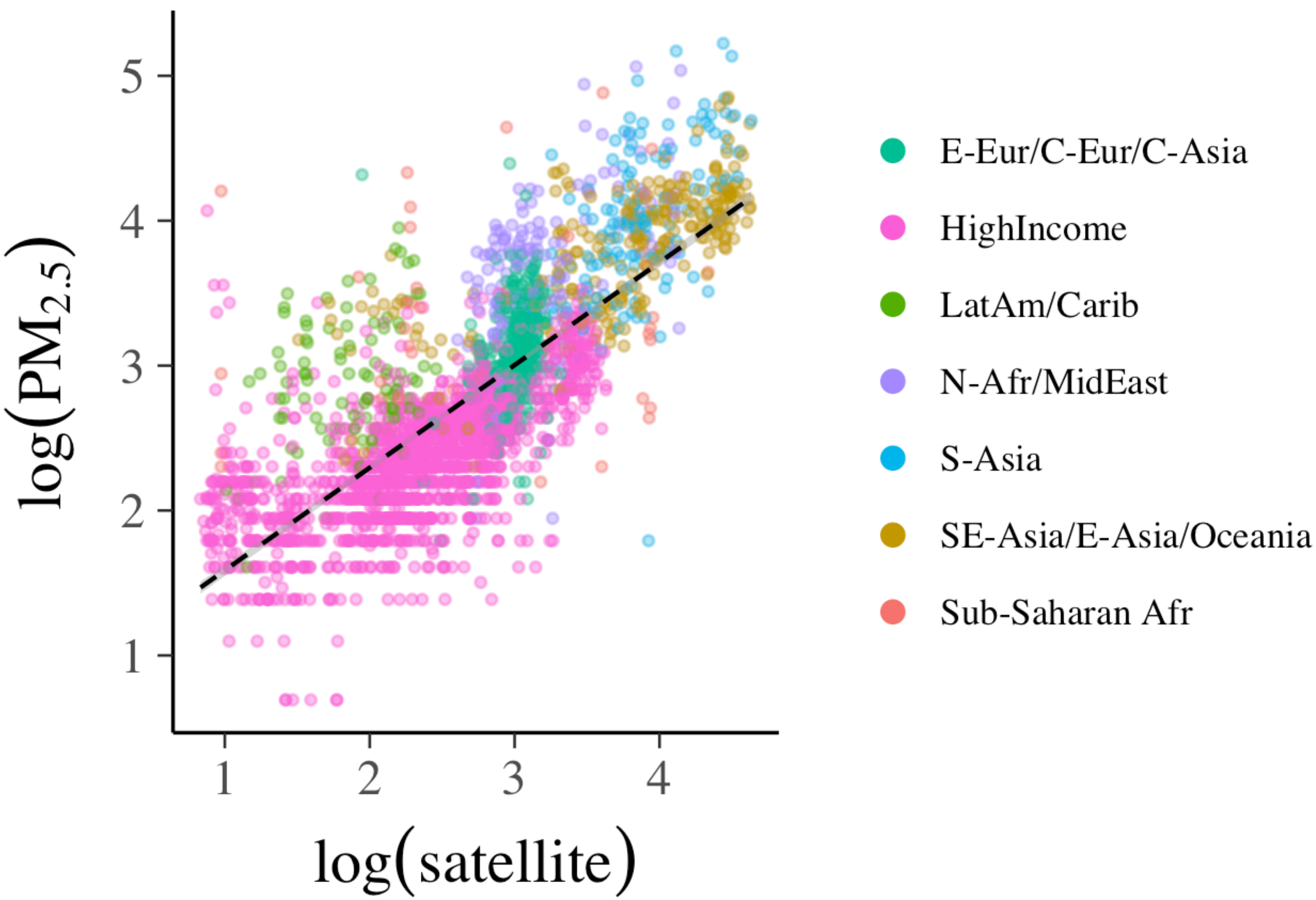
Satellite estimates of PM2.5 and ground monitor locations

Exploratory Data Analysis

Building a network of models

Exploratory data analysis

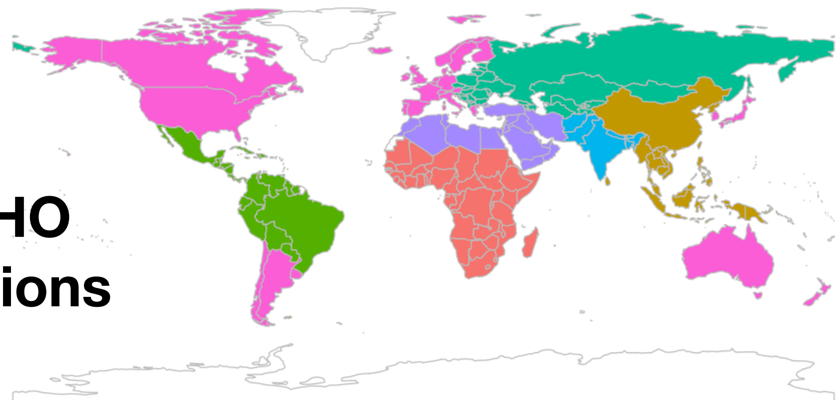
building a network of models



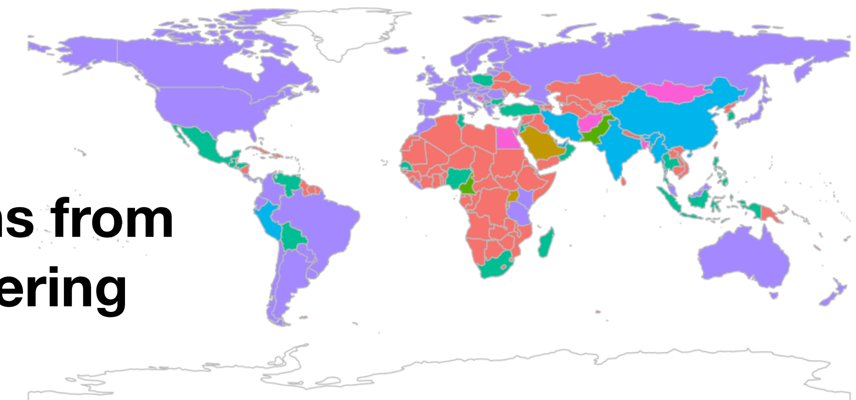
Exploratory data analysis

building a network of models

WHO Regions



Regions from clustering



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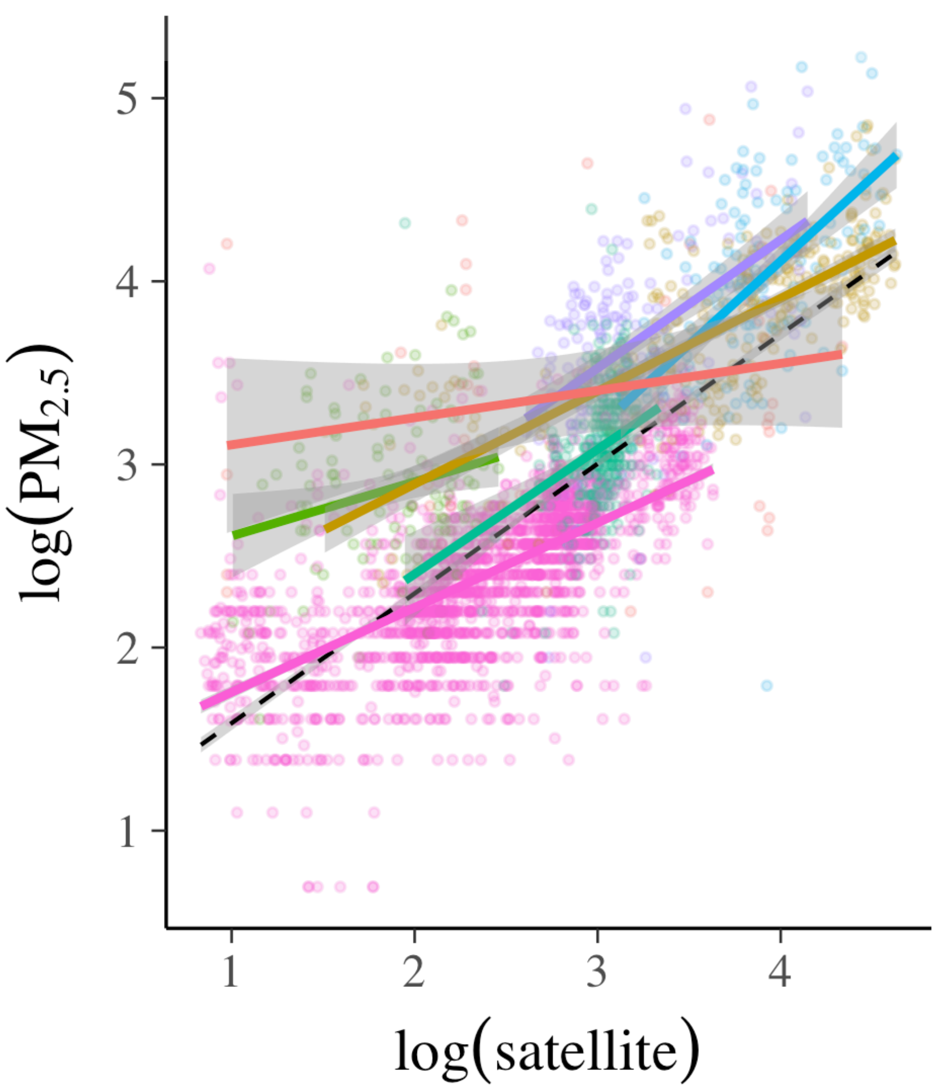

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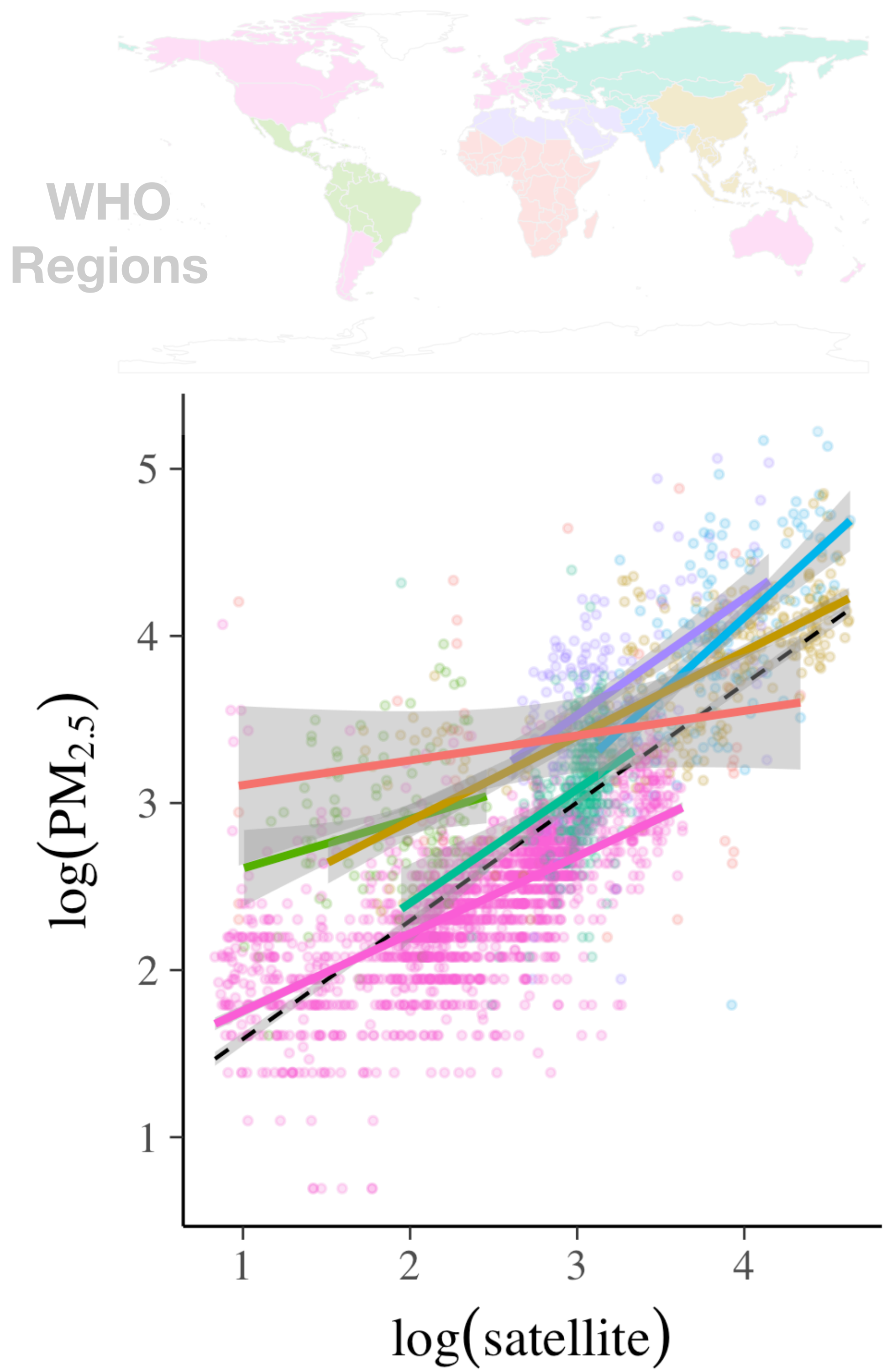

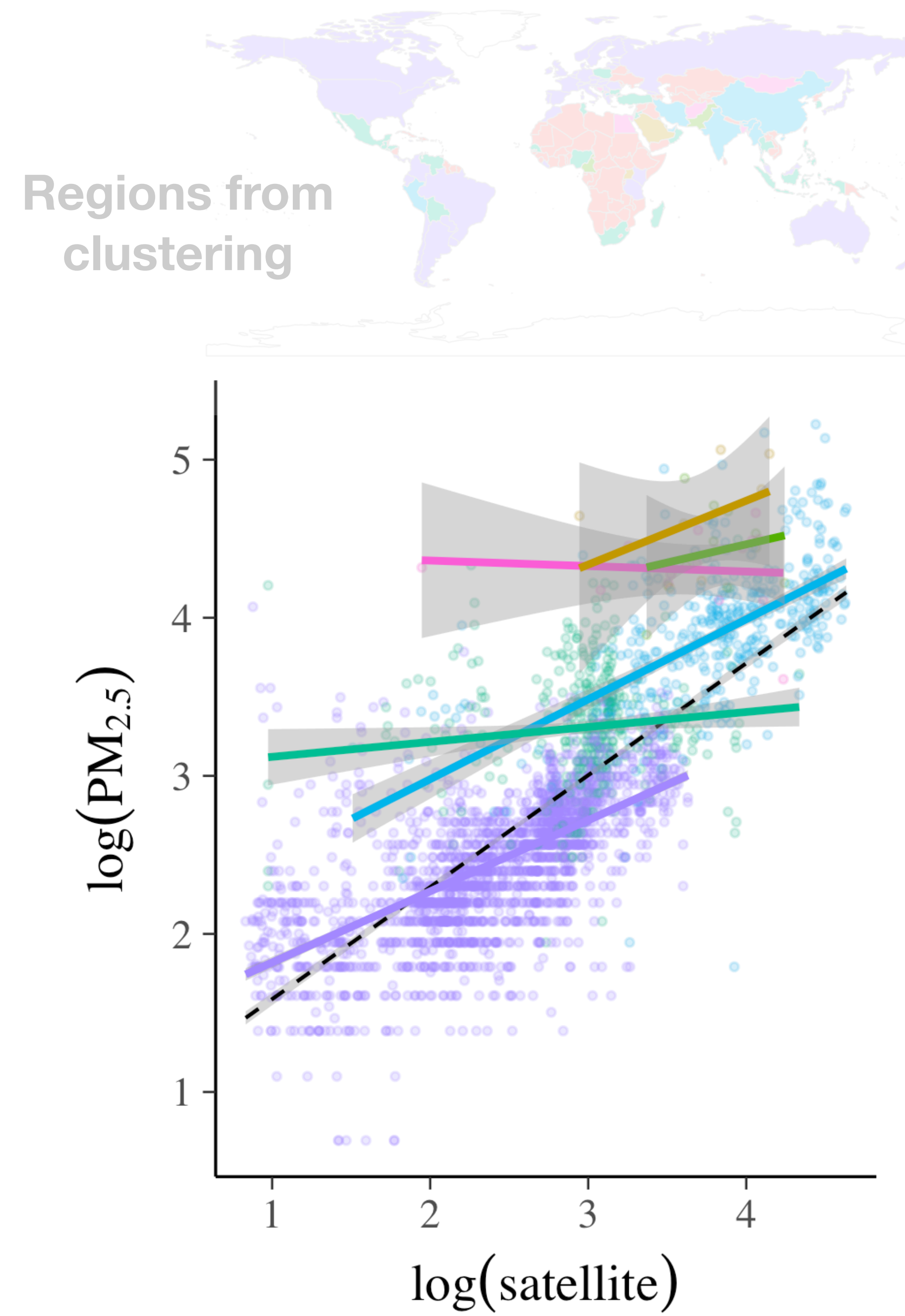

Exploratory data analysis
building a network of models

For measurements $n = 1, \ldots, N$

and regions $j = 1, \ldots, J$

Model 1

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Model 1

$$\log\left(\mathrm{PM}_{2.5,nj}\right) \sim N(\alpha + \beta \log\left(\mathrm{sat}_{nj}\right), \sigma)$$

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Models 2 and 3

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$$\alpha_j \sim N(0, \tau_\alpha) \quad \beta_j \sim N(0, \tau_\beta)$$

Prior predictive checks

Fake data can be almost as valuable as real data

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fake data is almost as useful as real data

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4. Repeat 2 & 3 many times
5. Compute summaries / visualize

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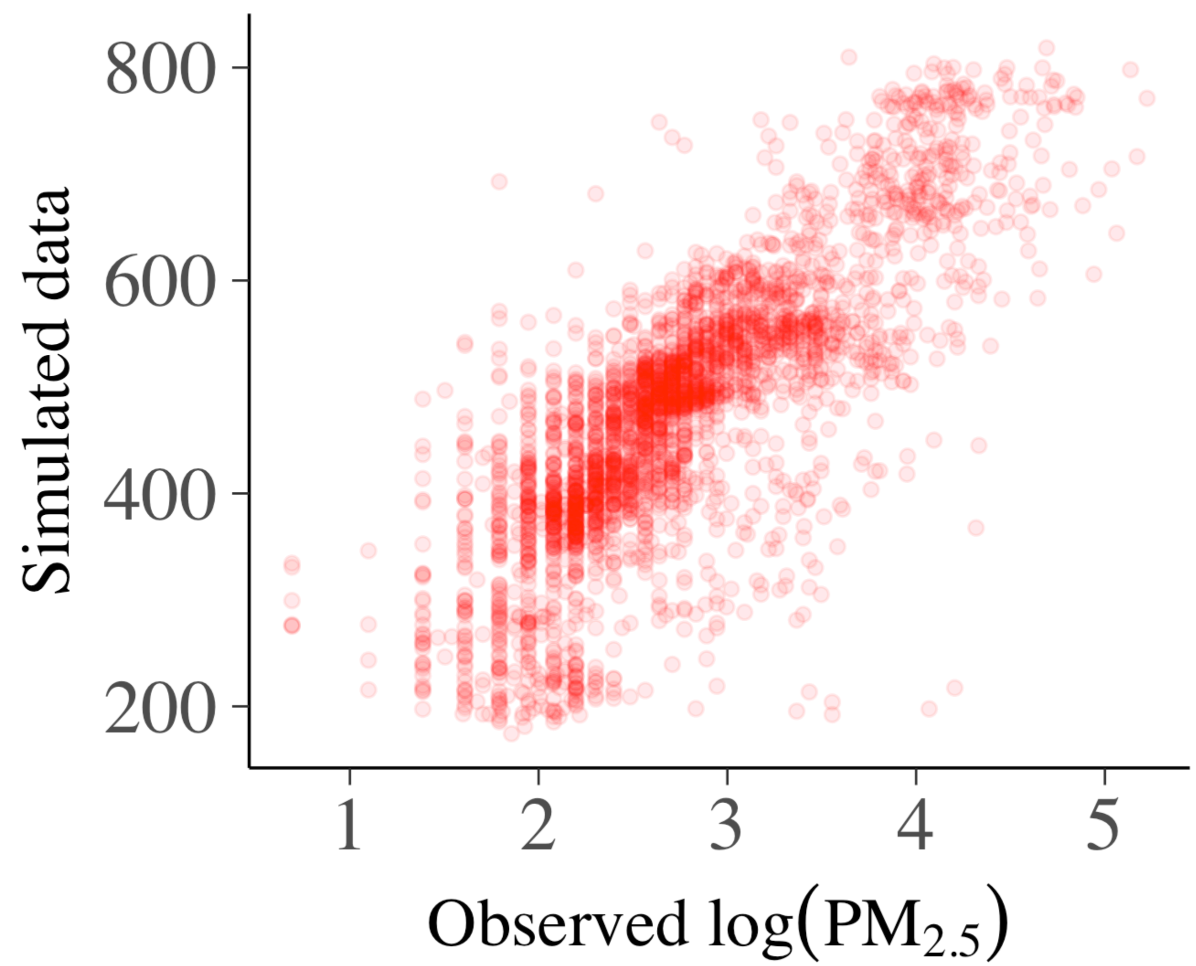

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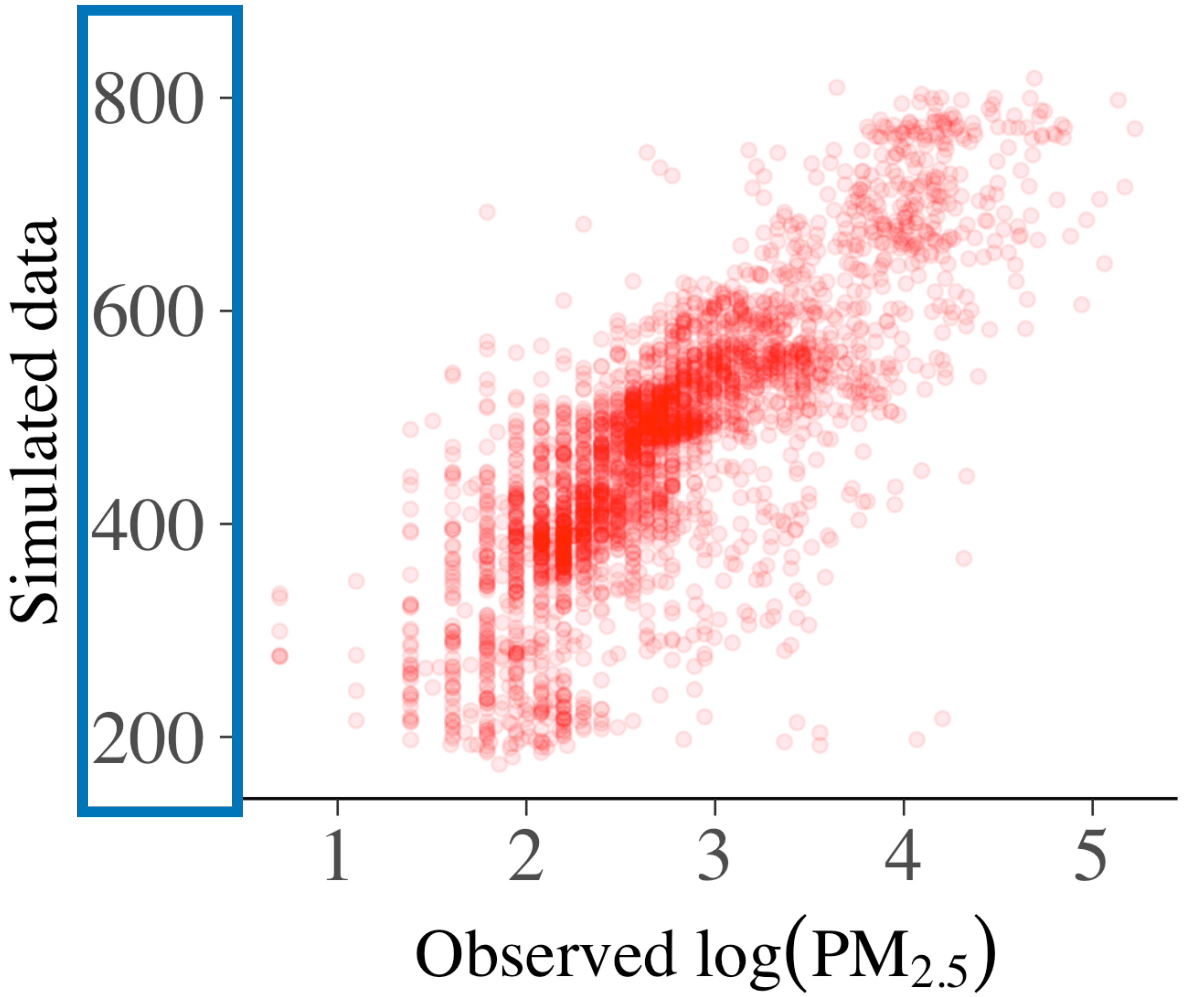

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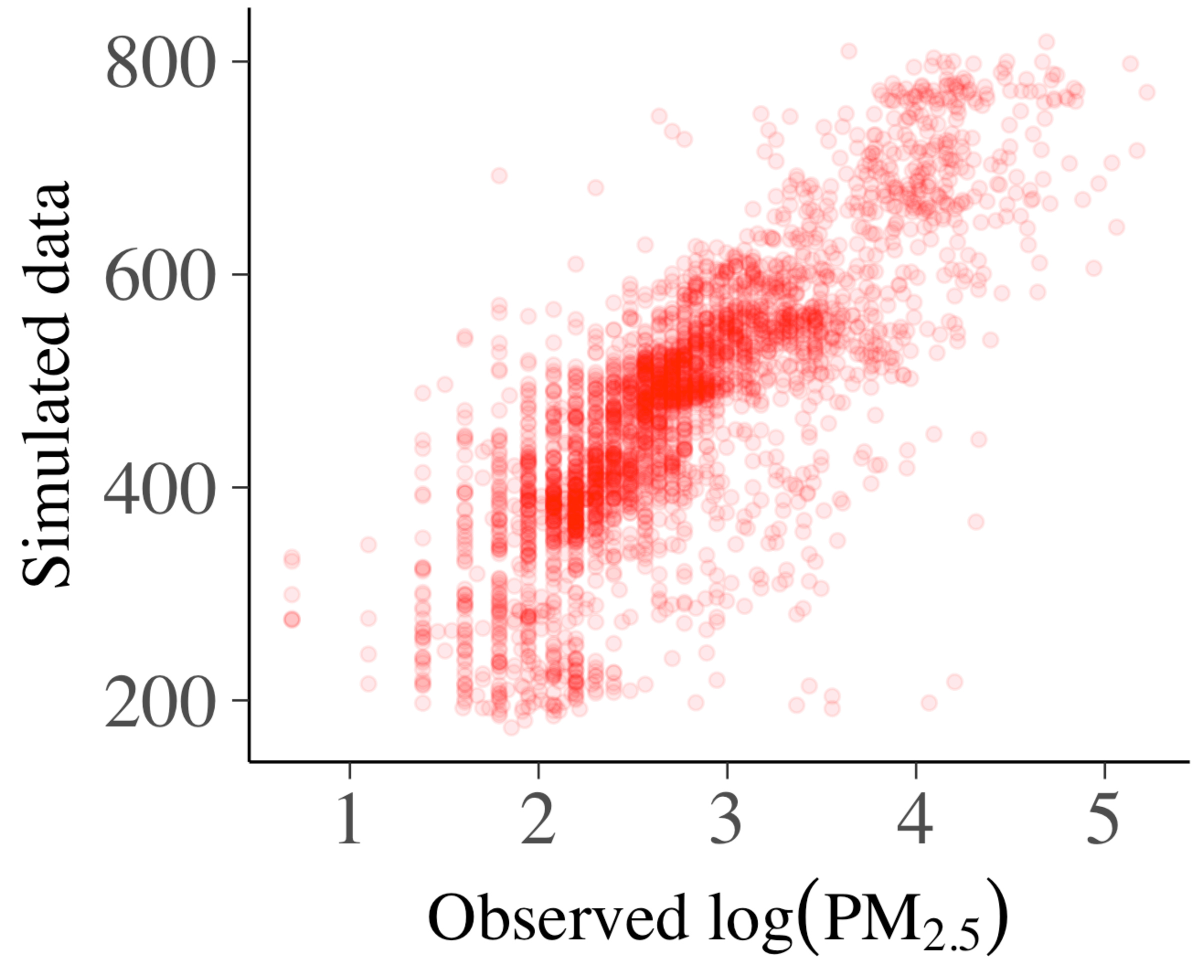

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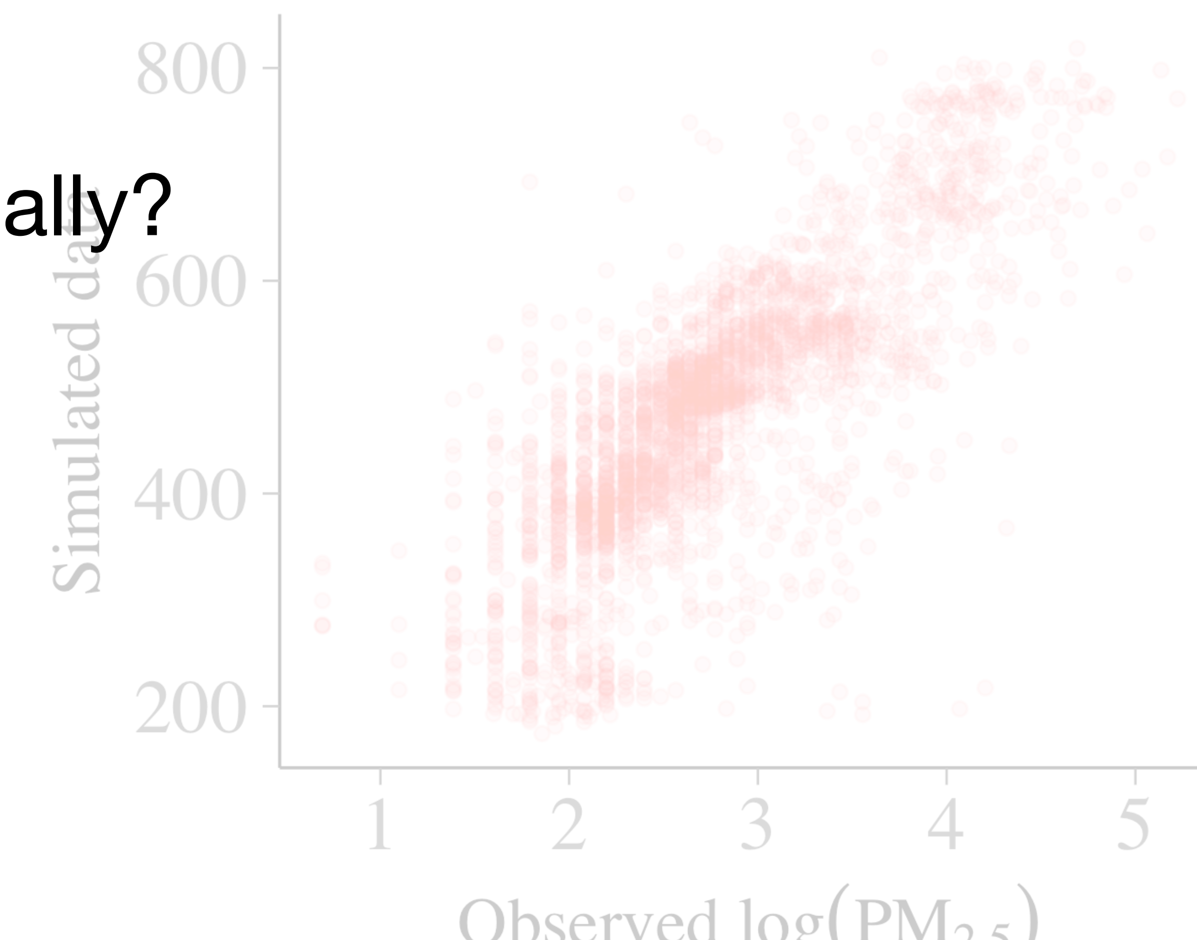

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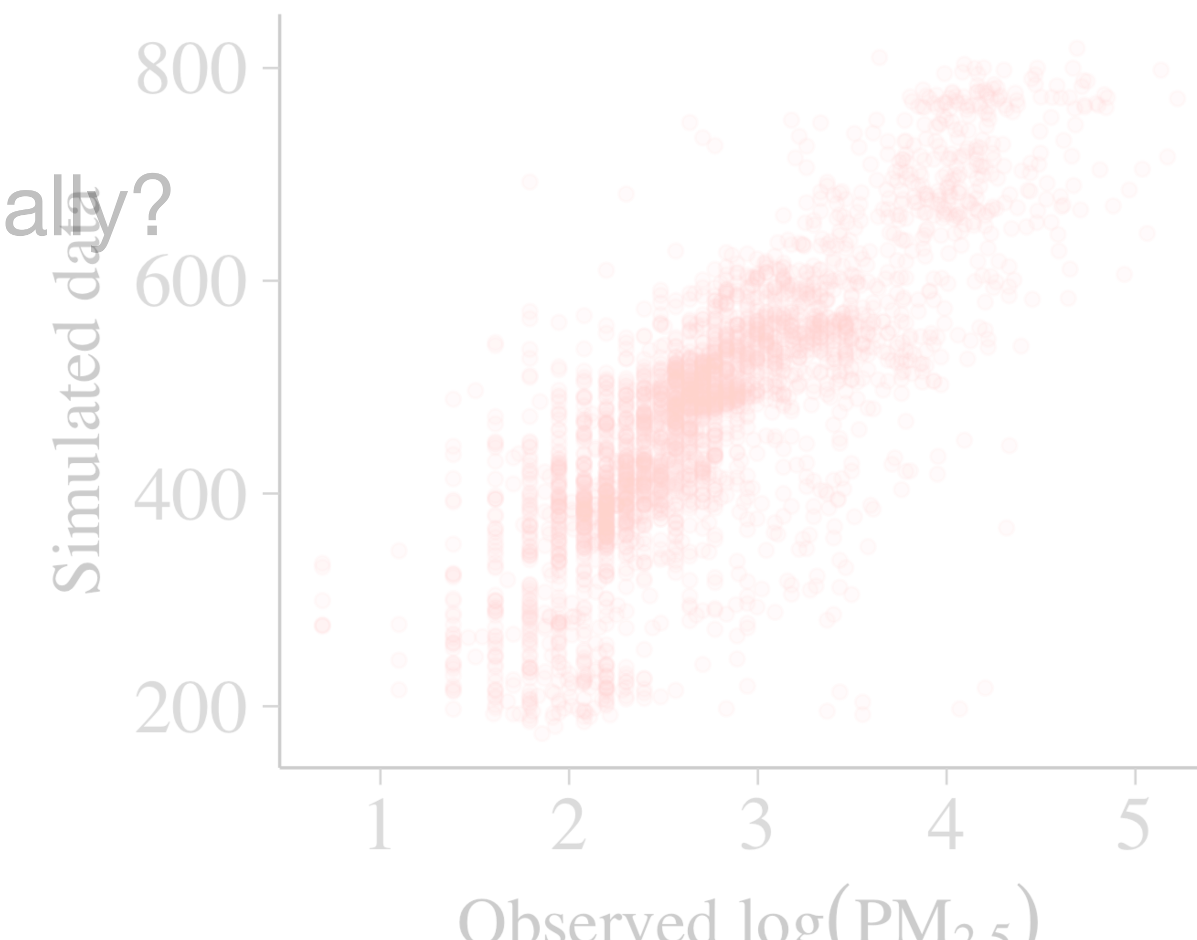
- The data will have to overcome the prior…



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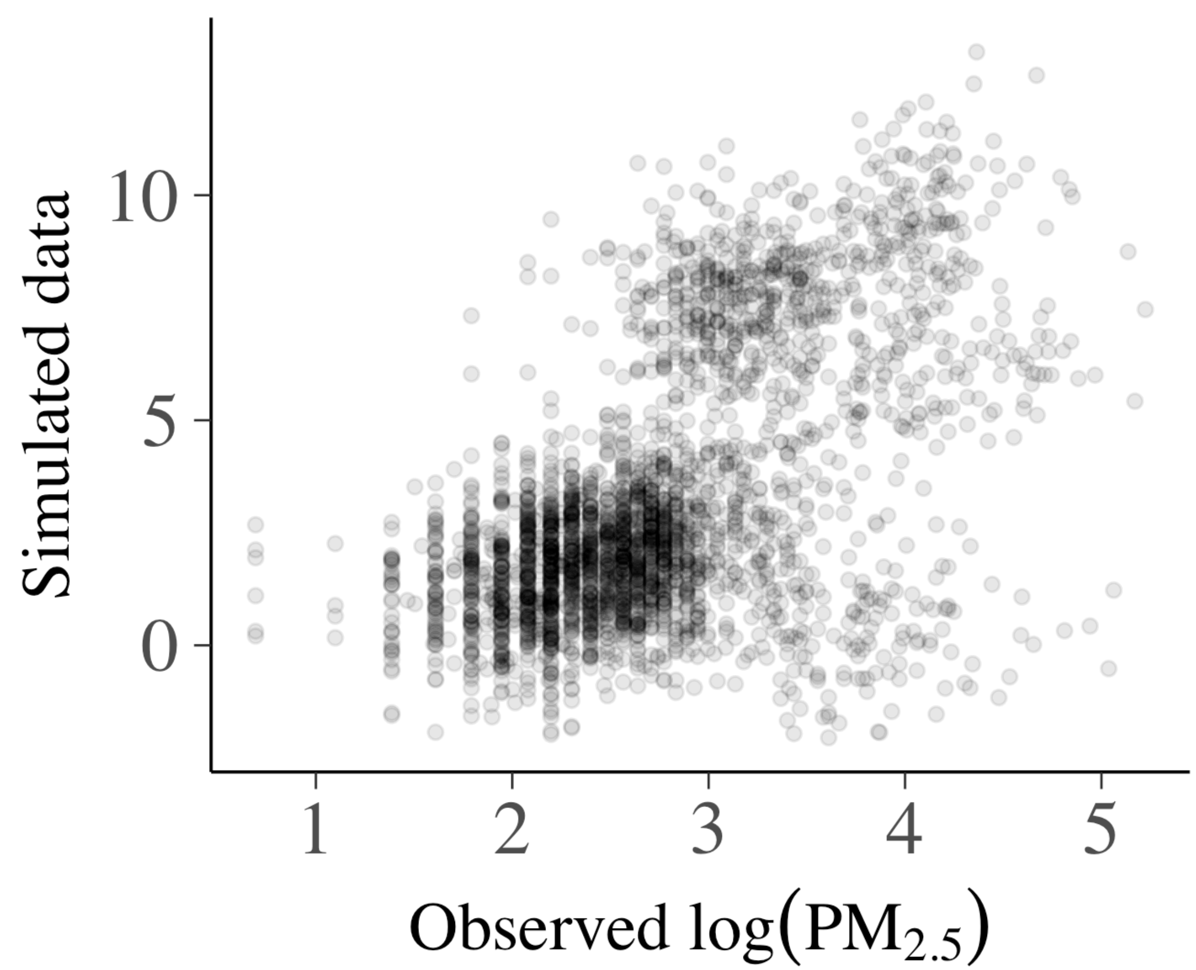

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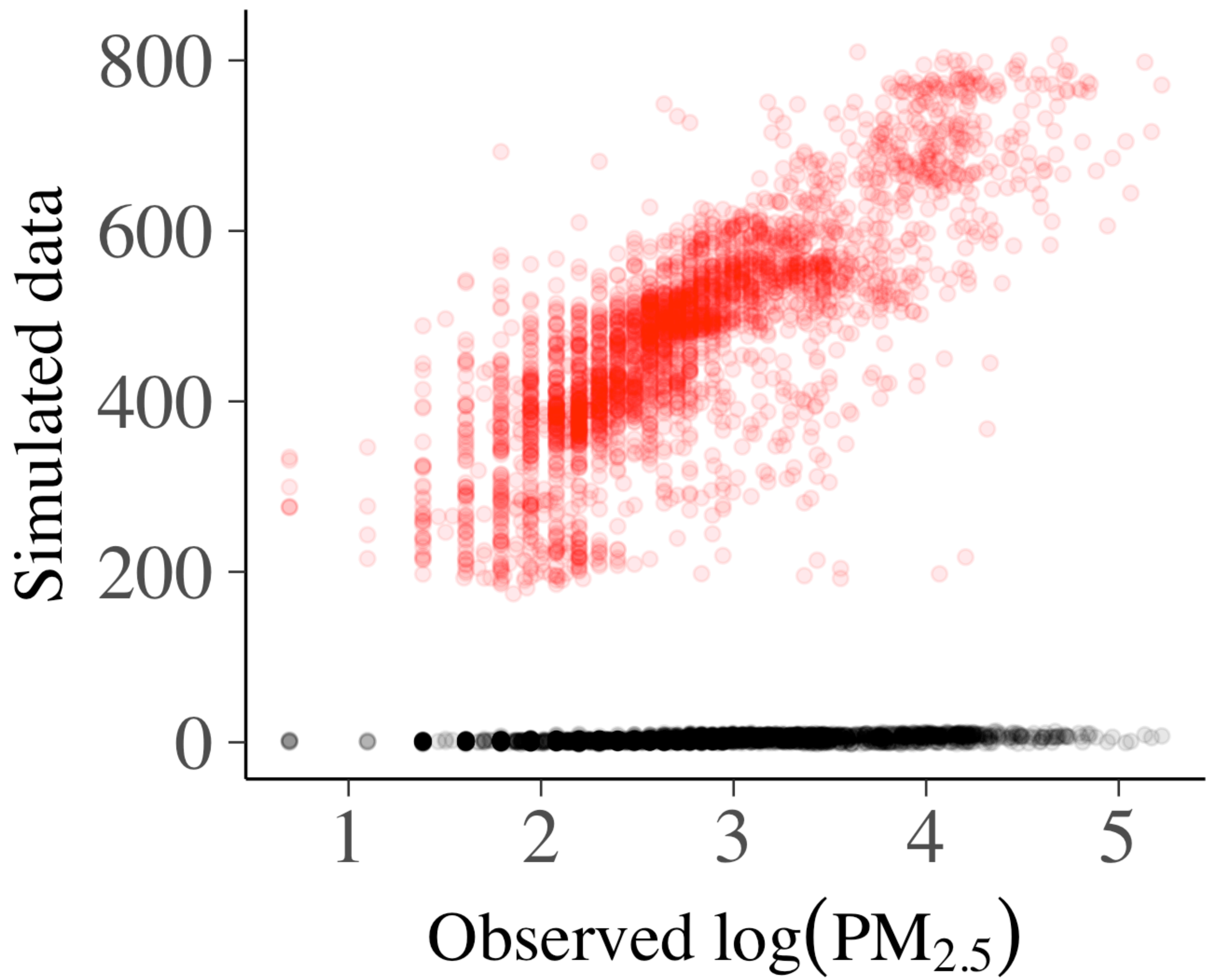

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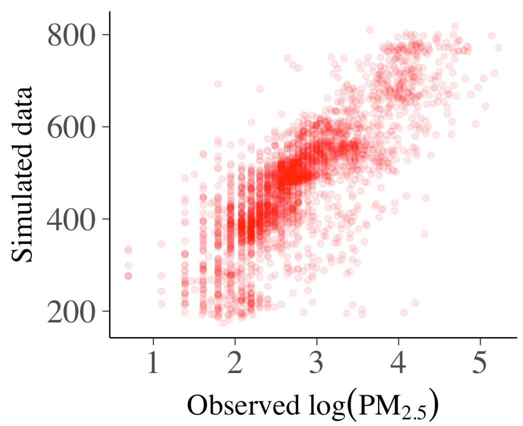

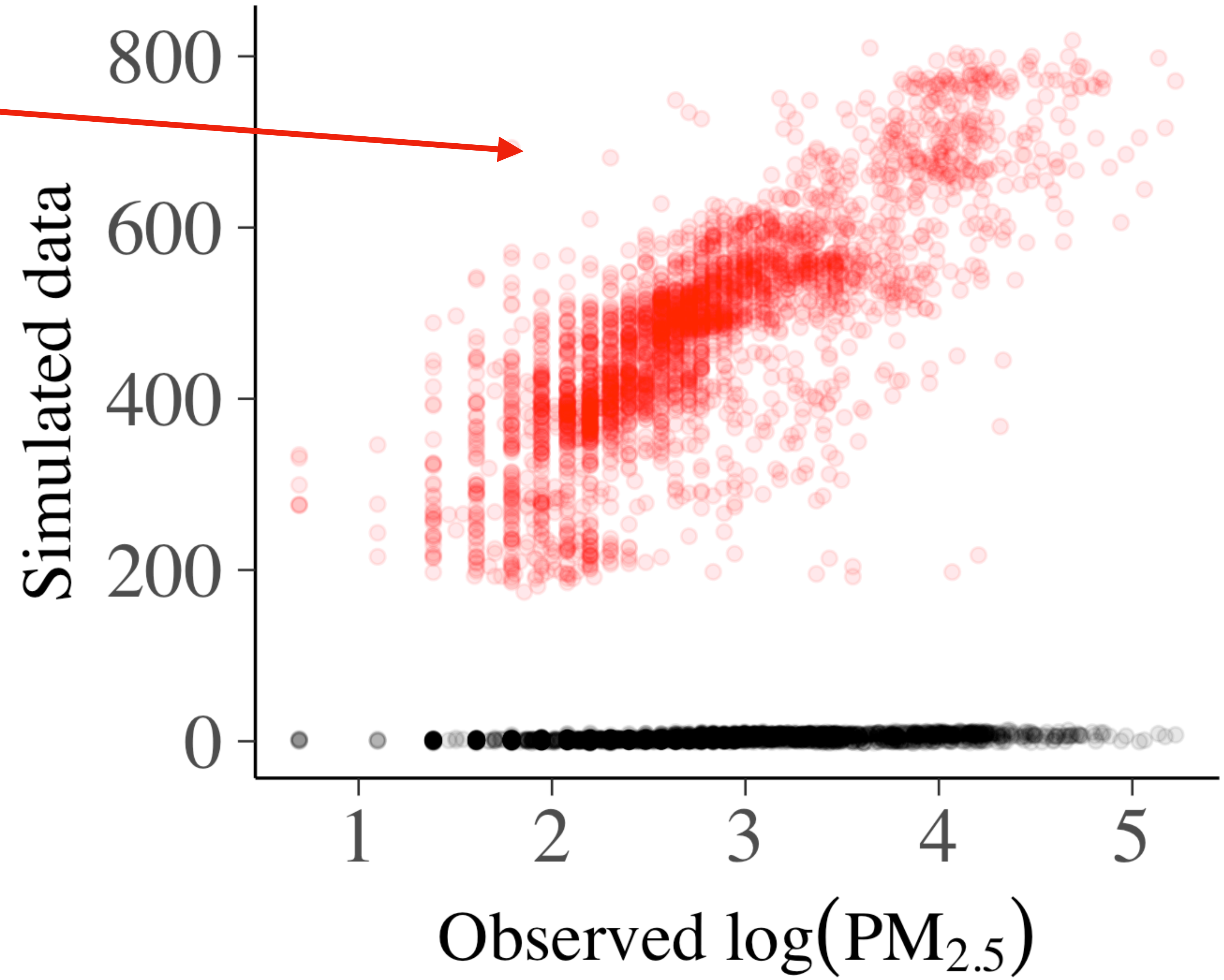

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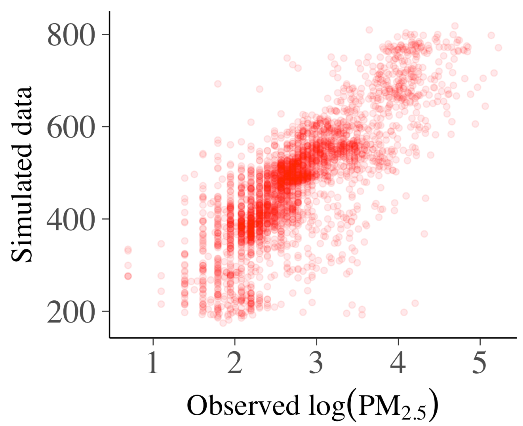
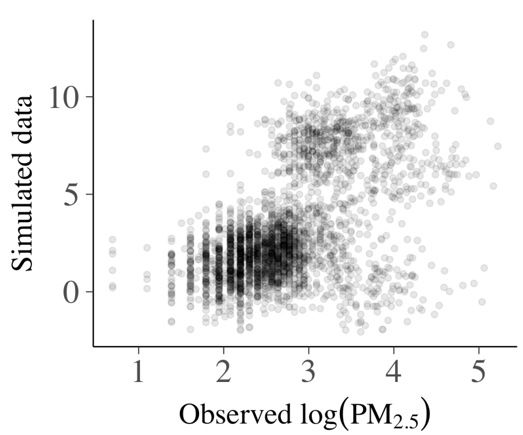
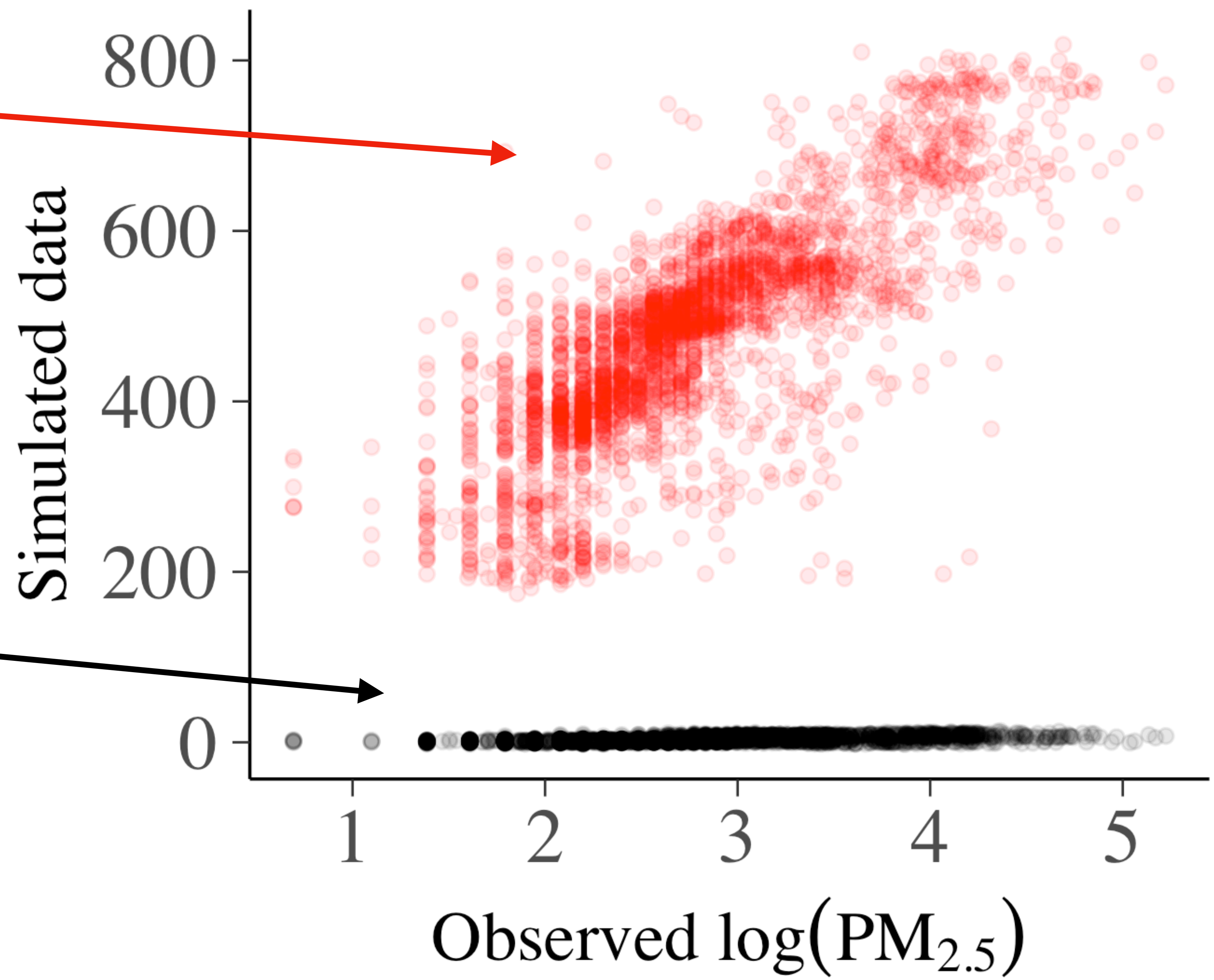


Weakly informative





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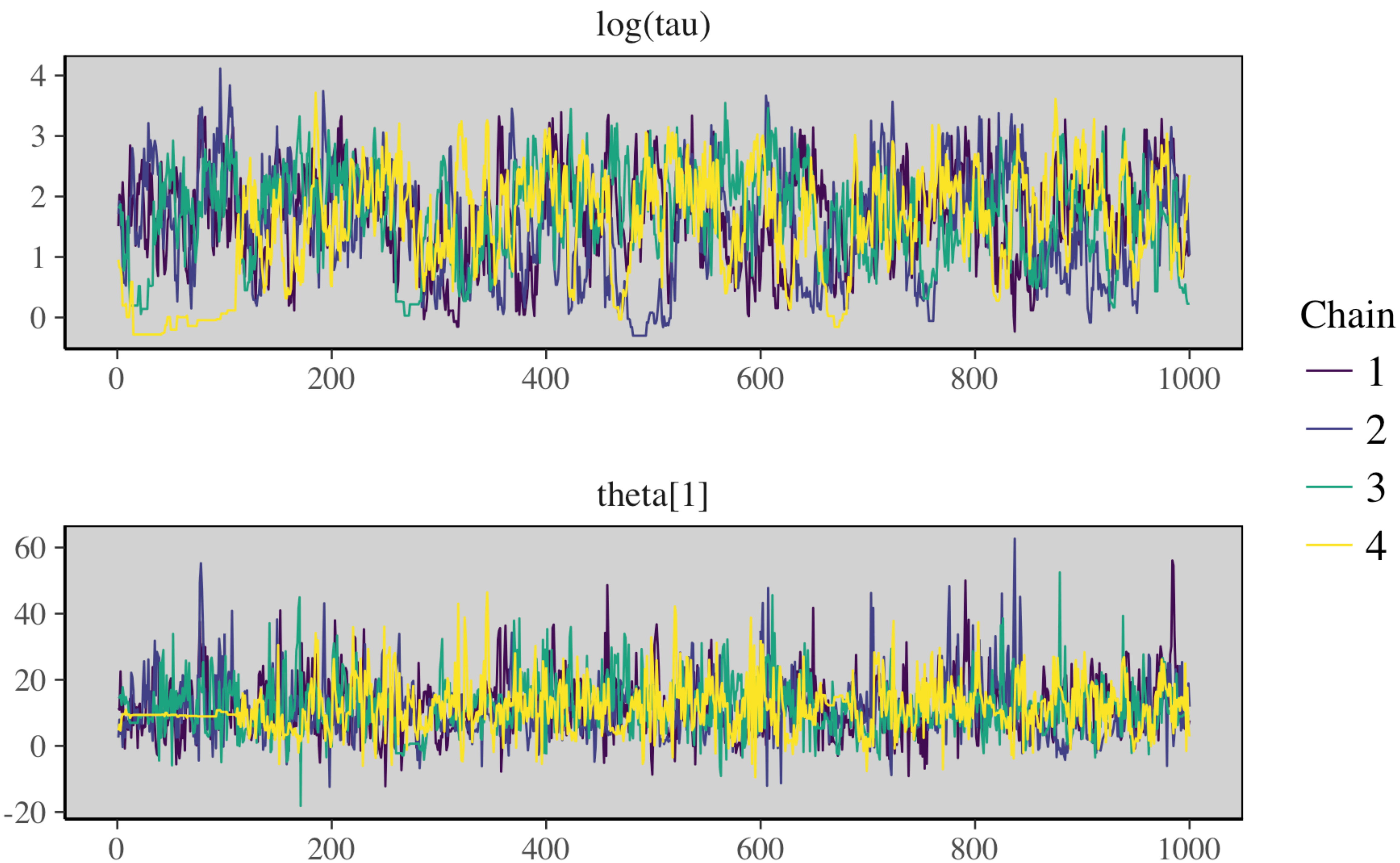
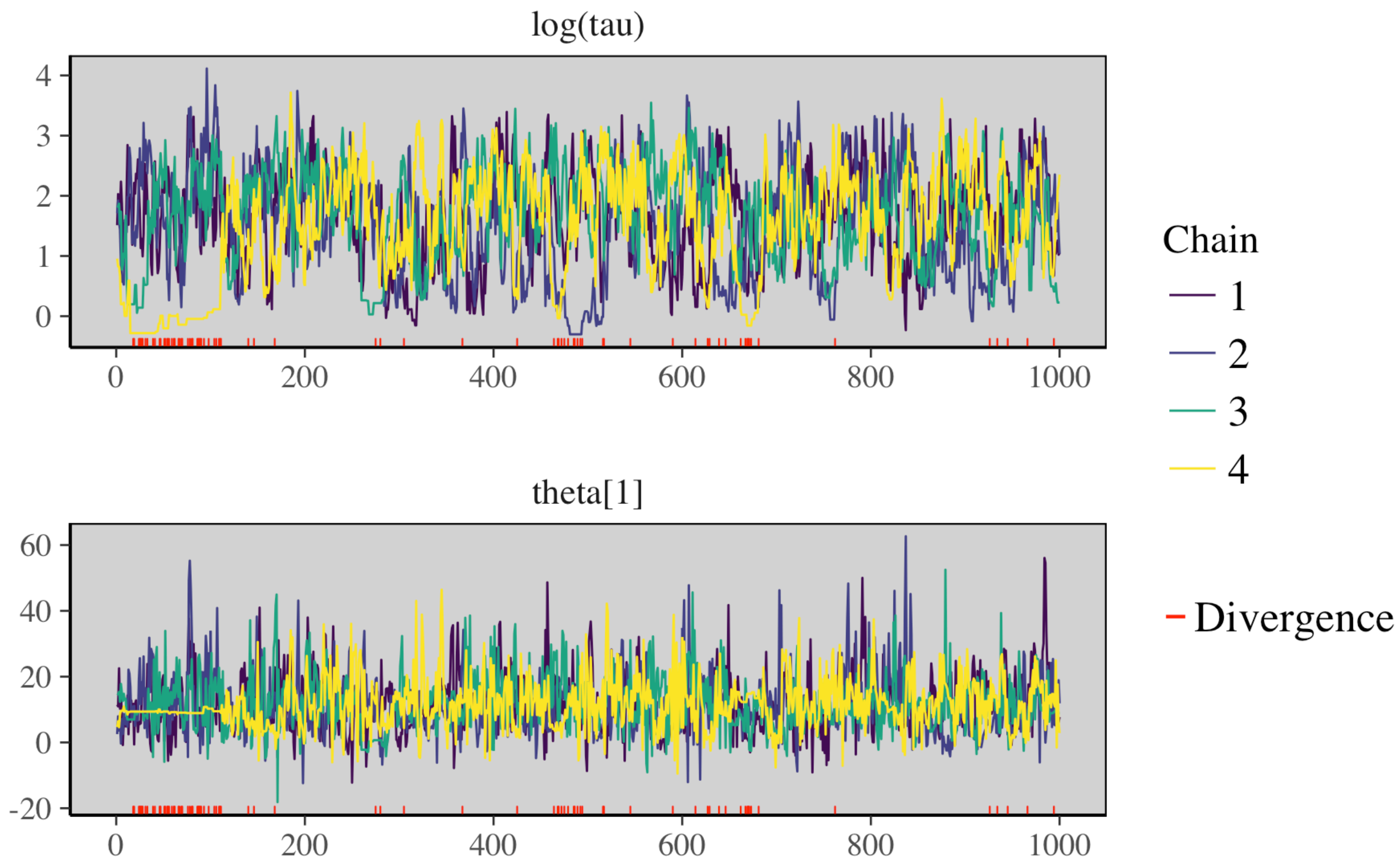
- If we use an *improper* prior, then we do not specify a joint model for our data and parameters
- More importantly, we do not specify a data generating mechanism ***p(y)***
- By construction, these priors do not regularize inferences, which is quite often a bad idea
- Proper but diffuse is better than improper but is still often problematic

MCMC diagnostics

Beyond trace plots

https://chi-feng.github.io/mcmc-demo/


log(tau)
theta[1]
4
3
2
1
0
60
40
20
0
-20
0
200
400
600
800
1000
Chain
1
2
3
4



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Divergence

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MCMC diagnostics

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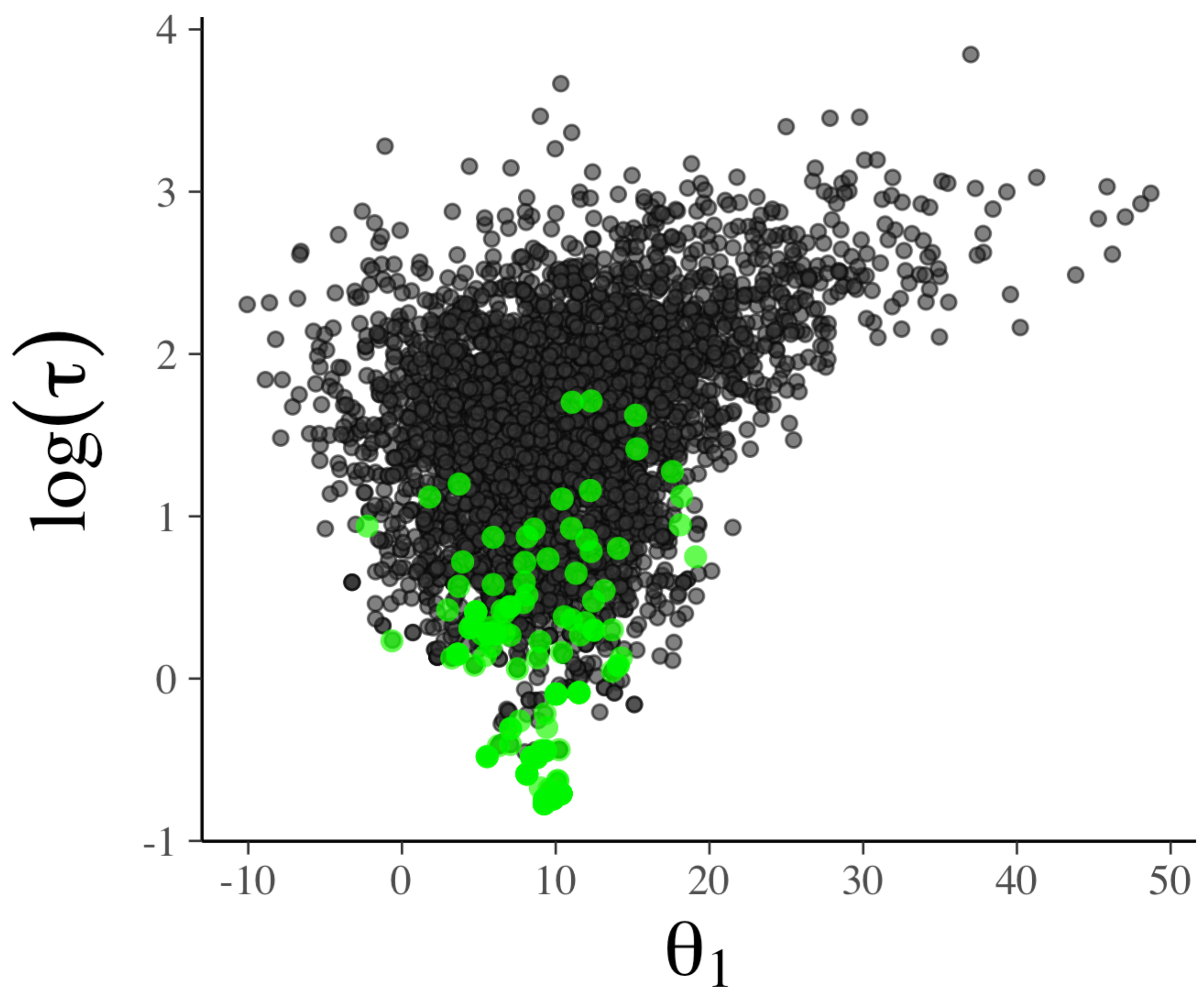

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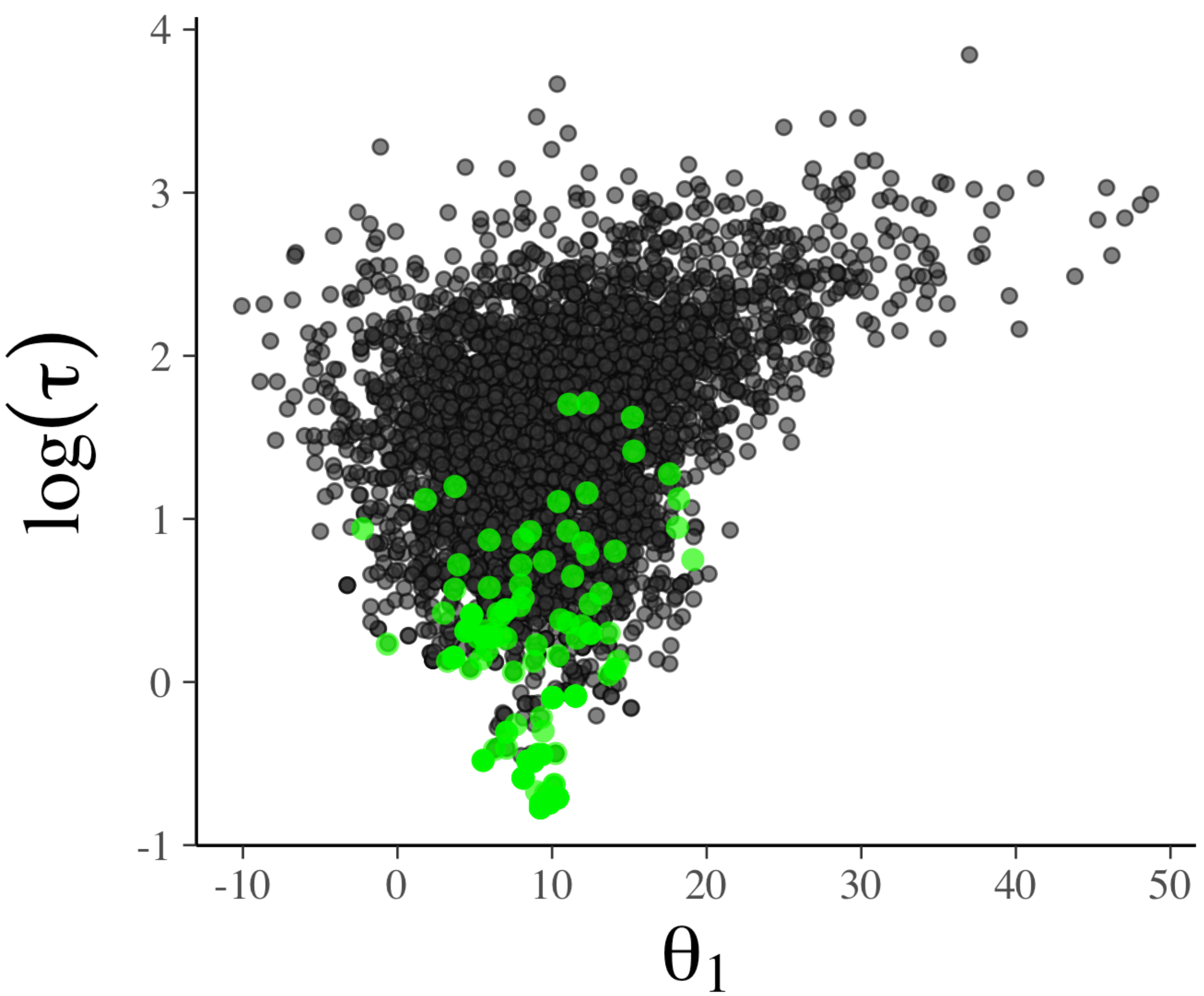

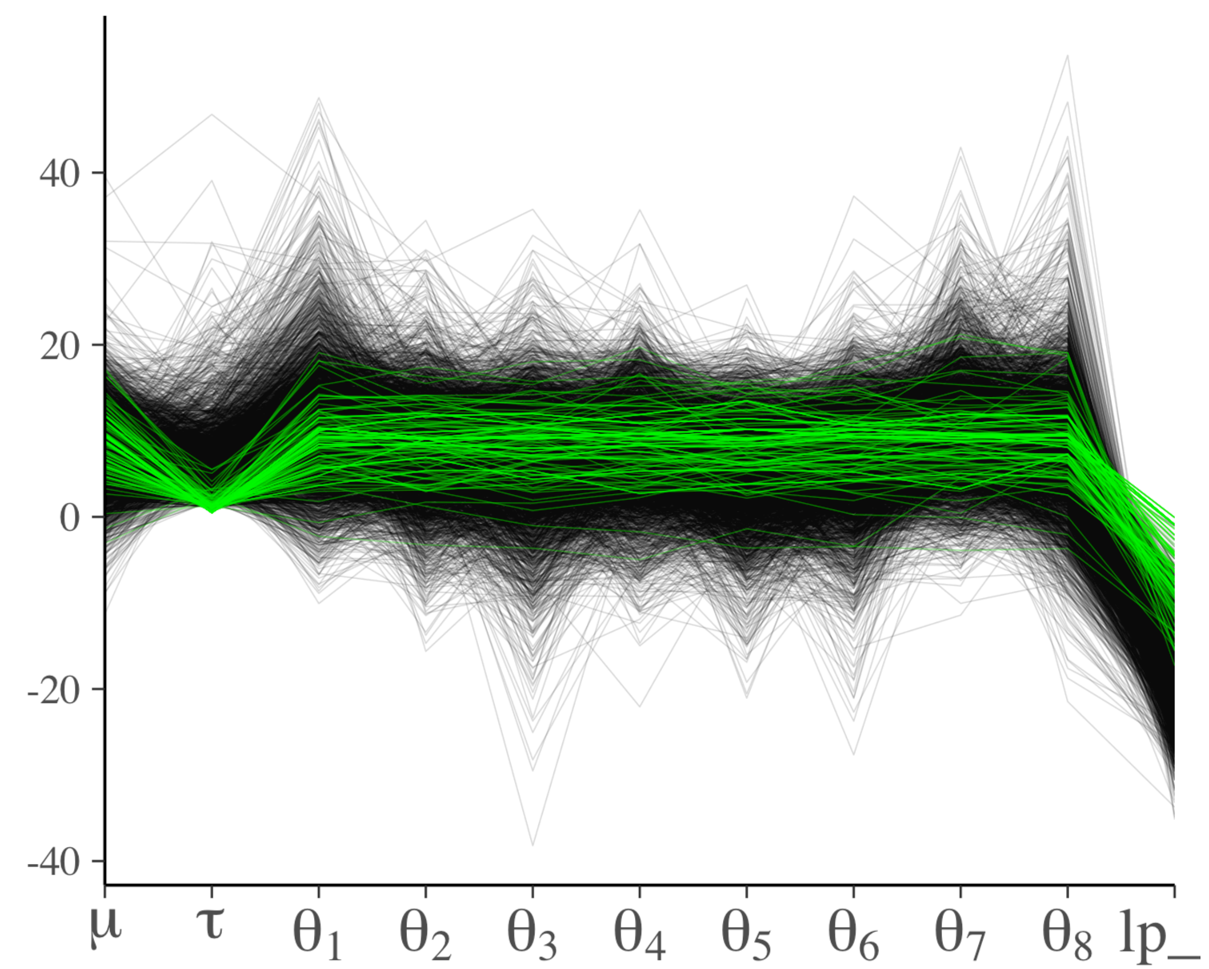

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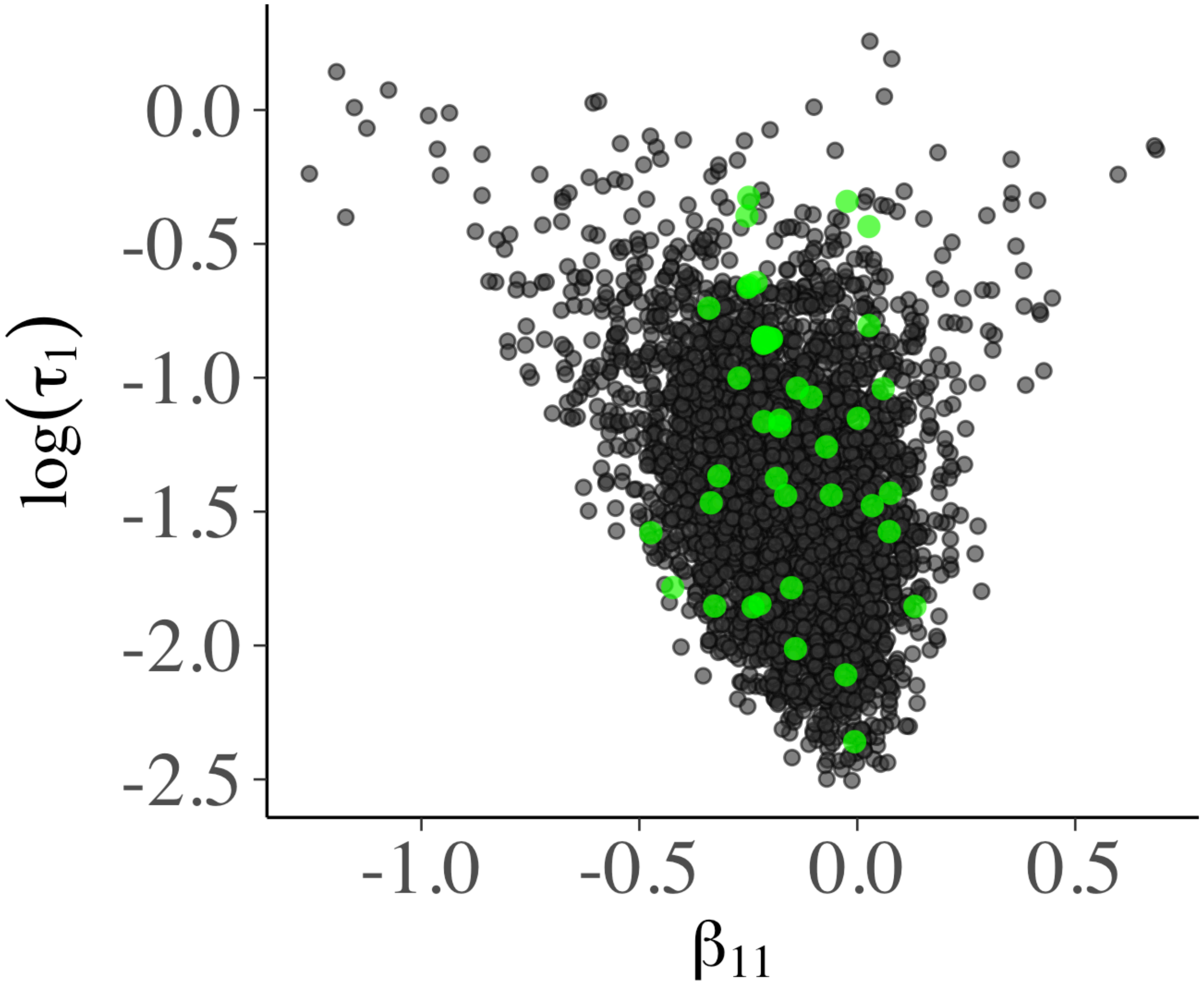

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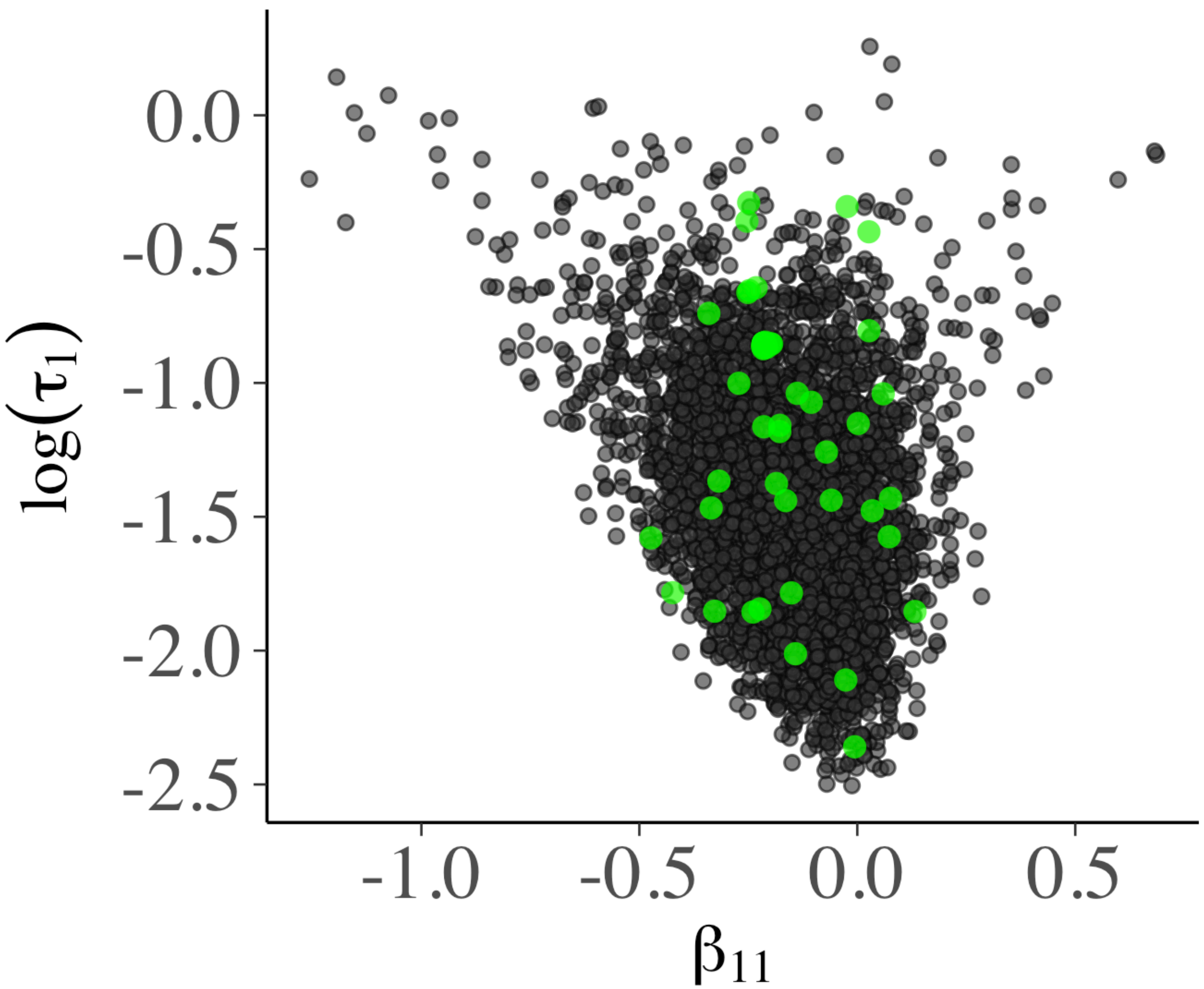
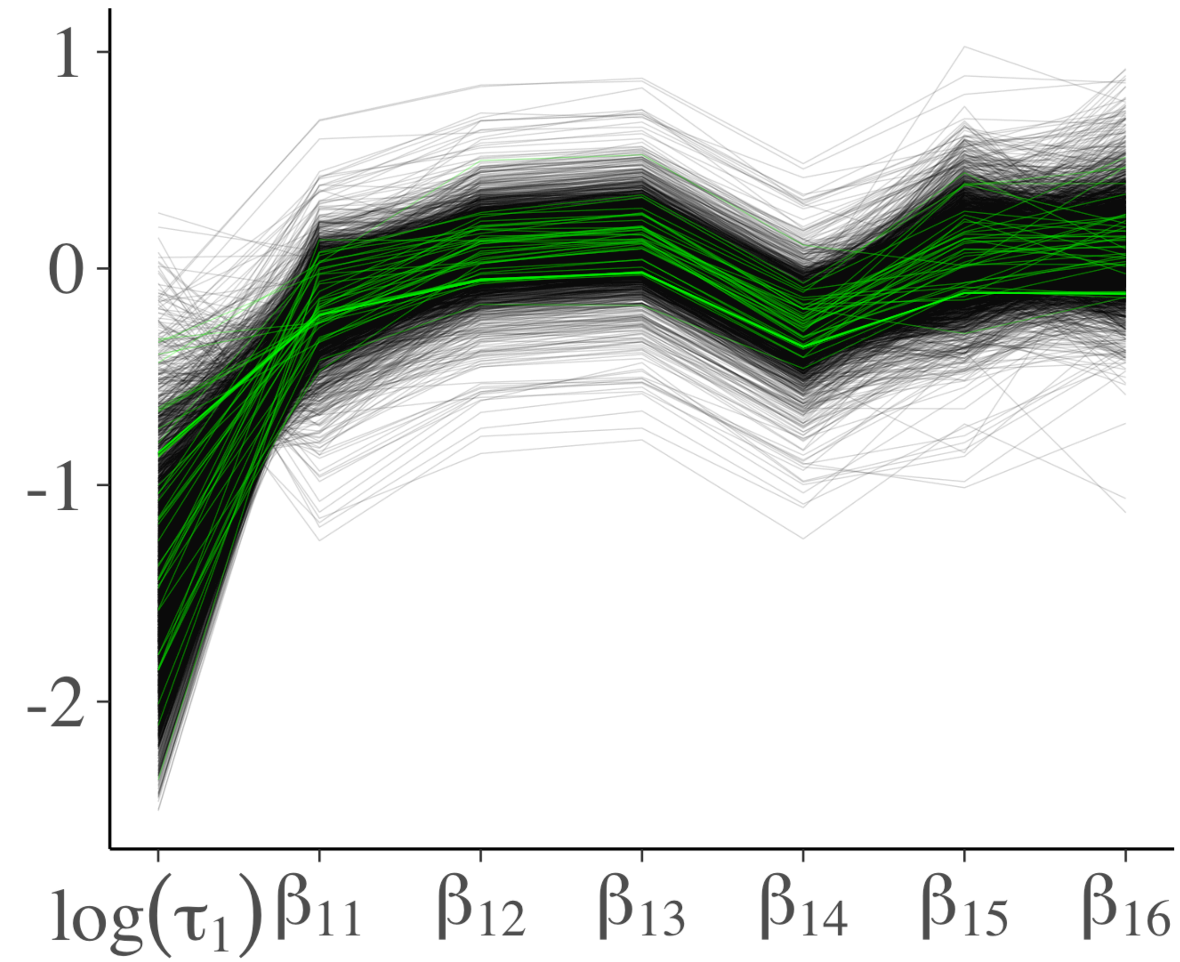
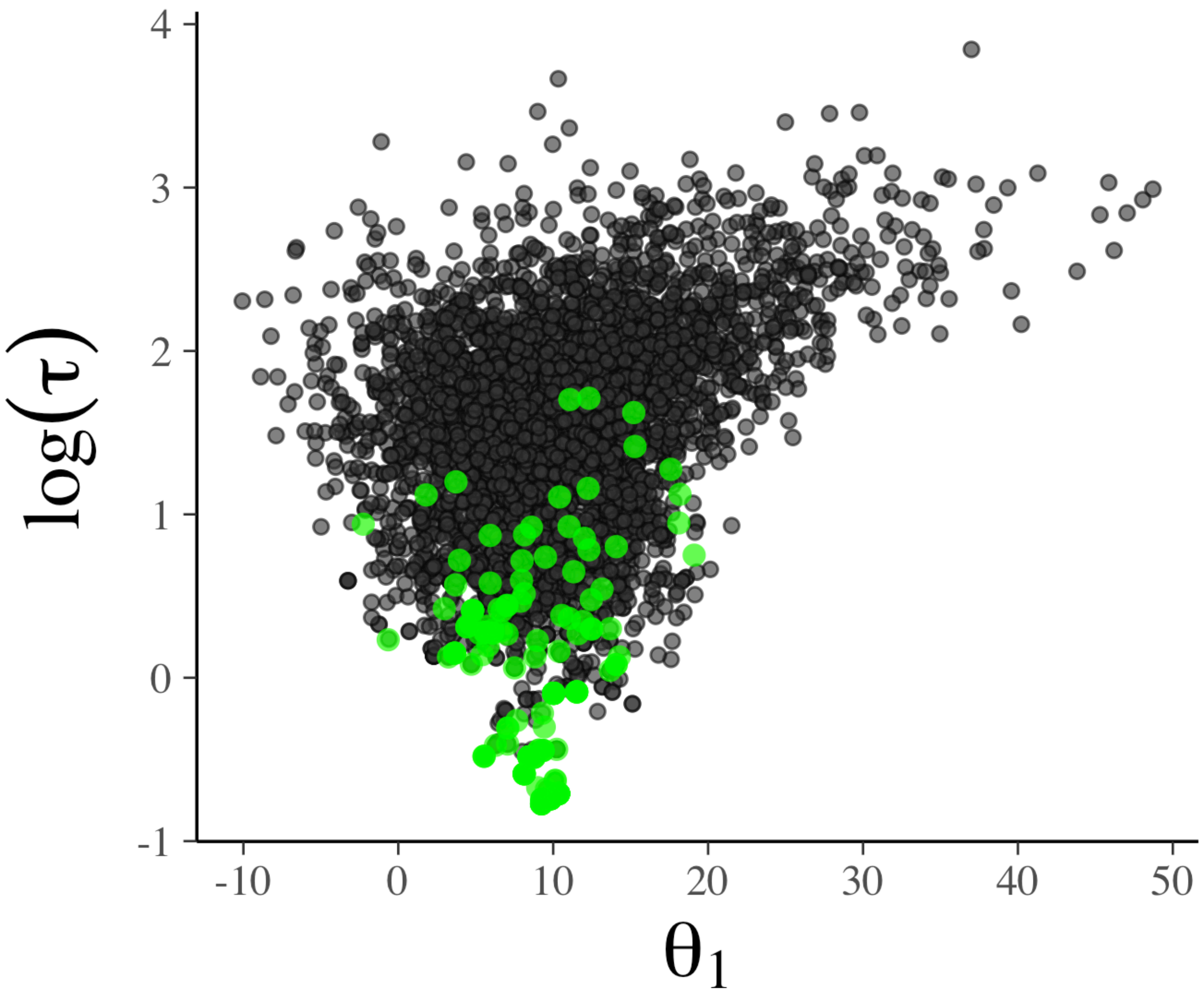
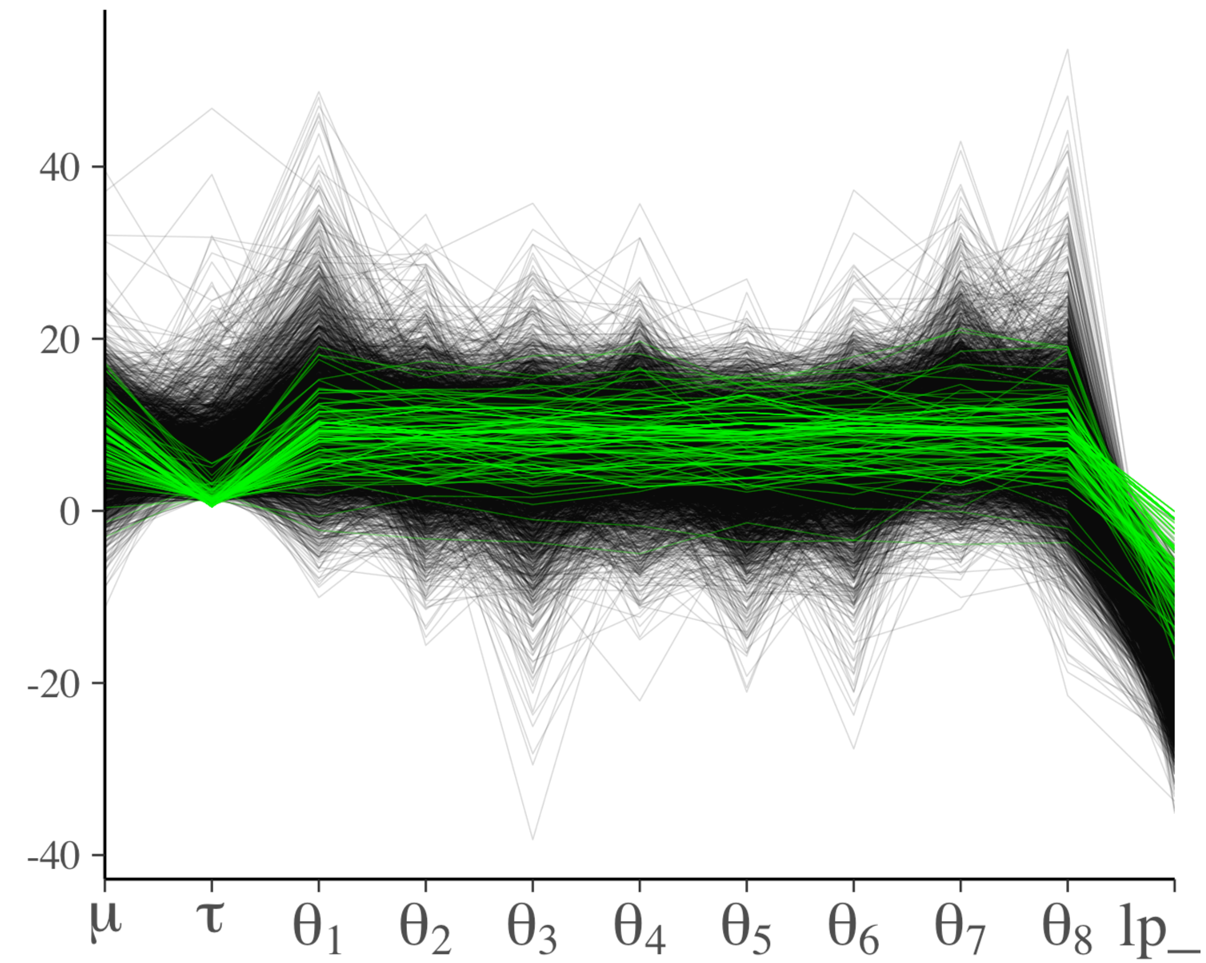


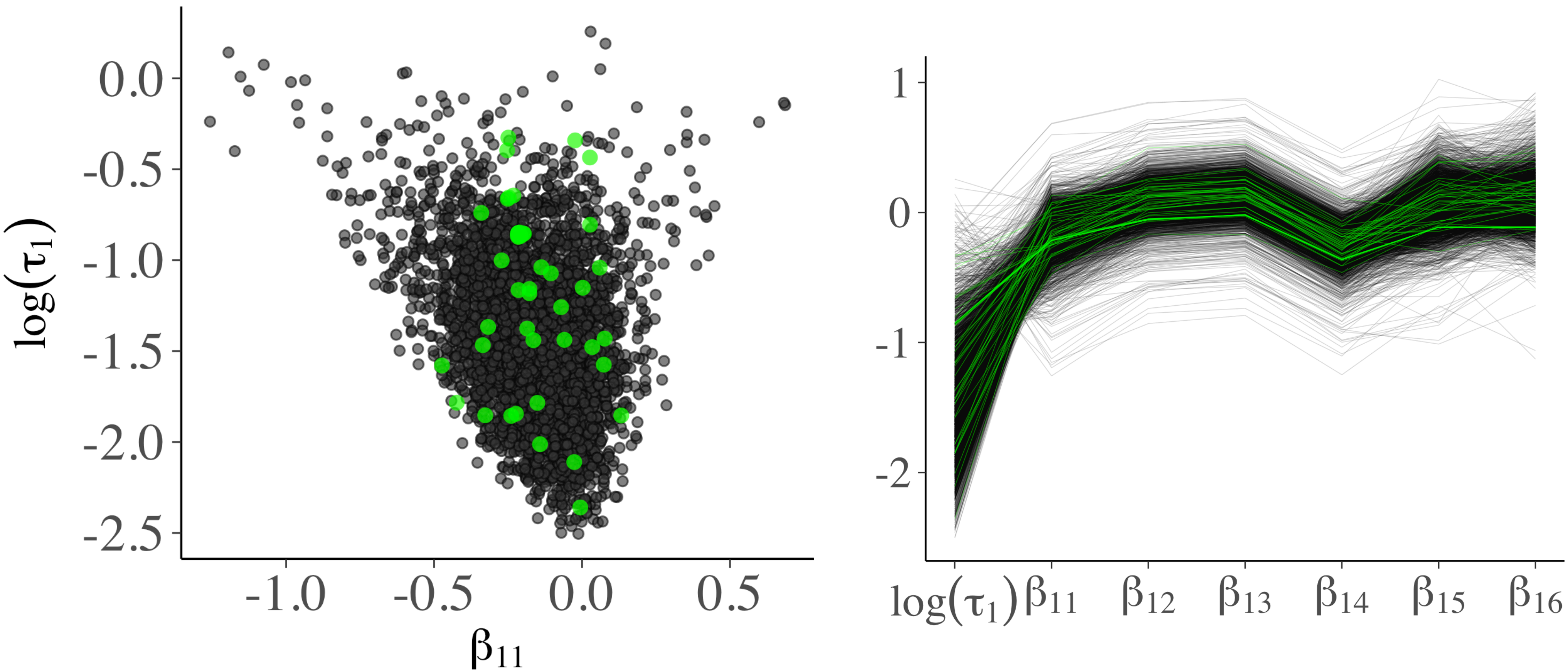


Pathological geometry





“False positives”



Posterior predictive checks

Visual model evaluation

Posterior predictive checking
visual model evaluation

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- Misfitting and overfitting both manifest as tension between measurements and predictive distributions

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$$p(\tilde{y}|y) = \int p(\tilde{y}|\theta)\, p(\theta|y)\, d\theta$$

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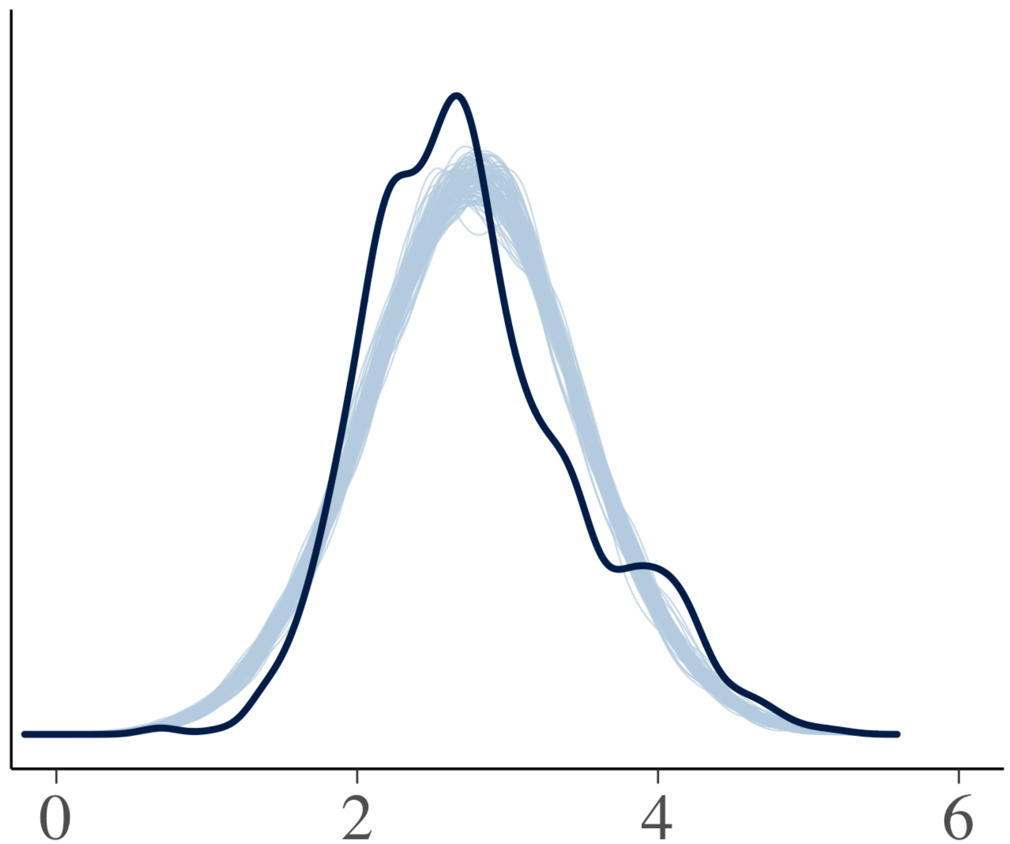
    4. Repeat 2 & 3 many times
    5. Compute summaries / visualize

Posterior predictive checking

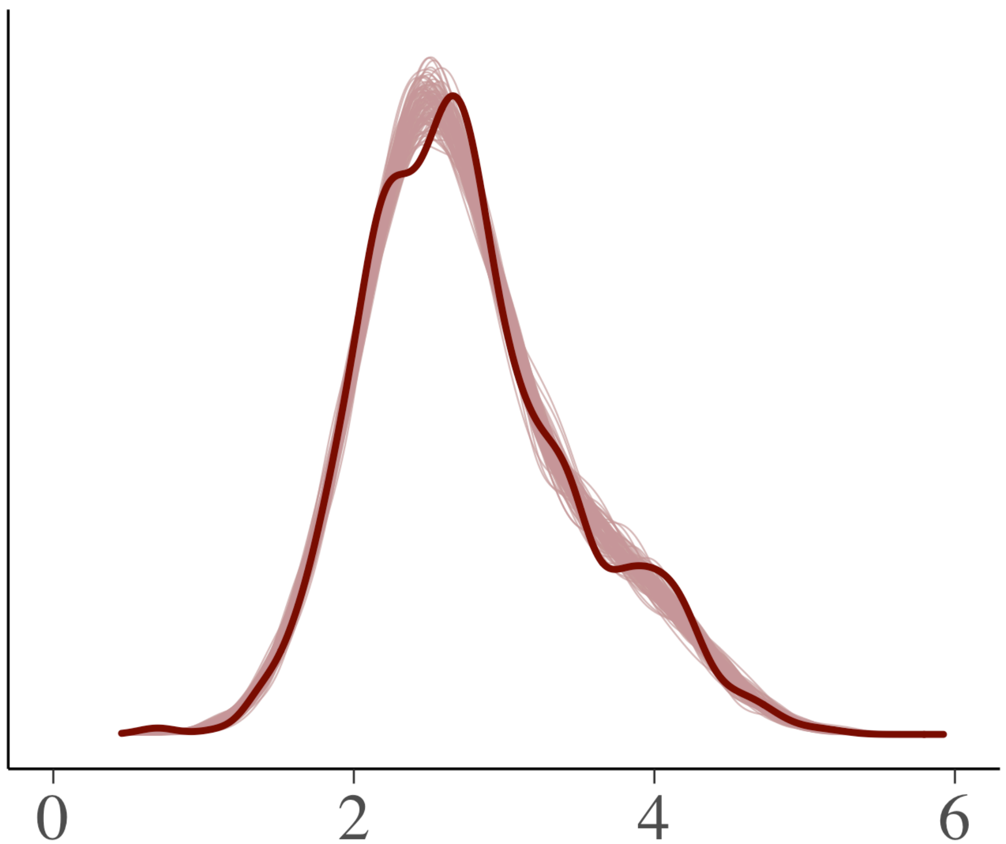
visual model evaluation

Observed data vs posterior predictive simulations

Model 1 (single level)



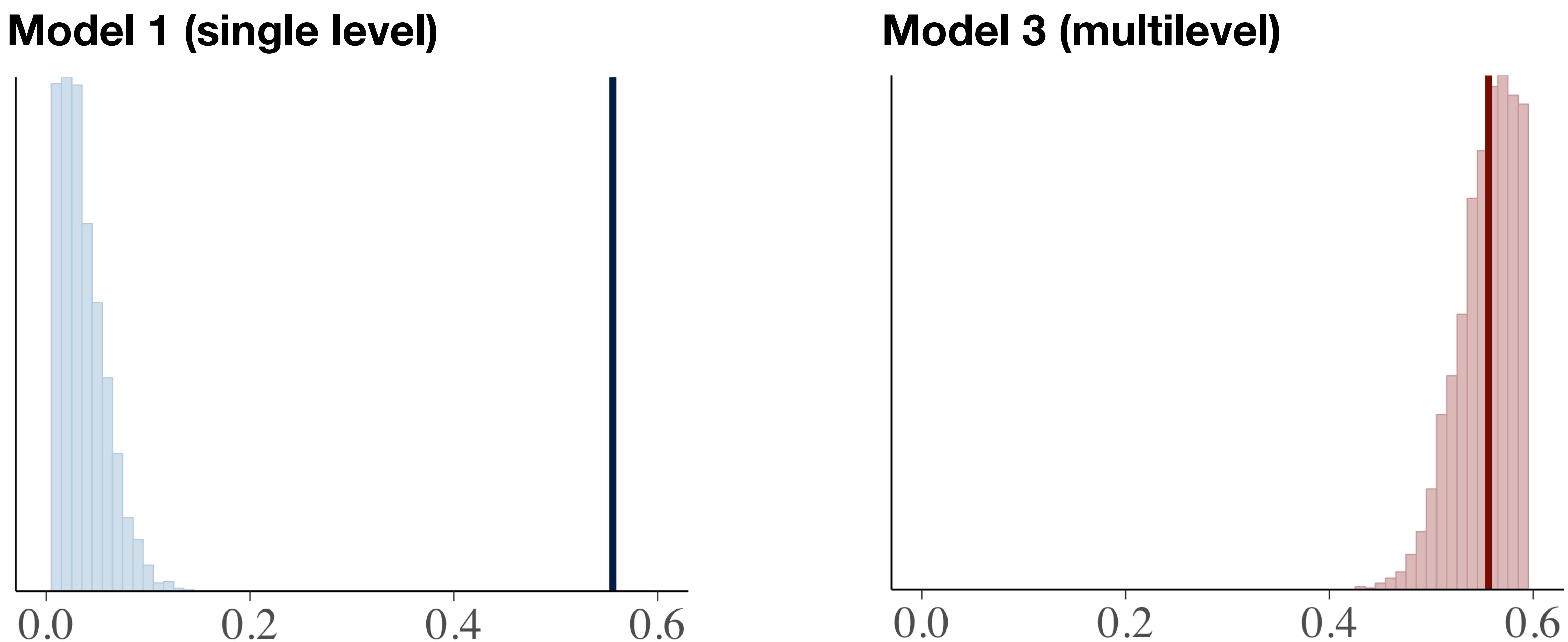
Model 3 (multilevel)



Posterior predictive checking

visual model evaluation

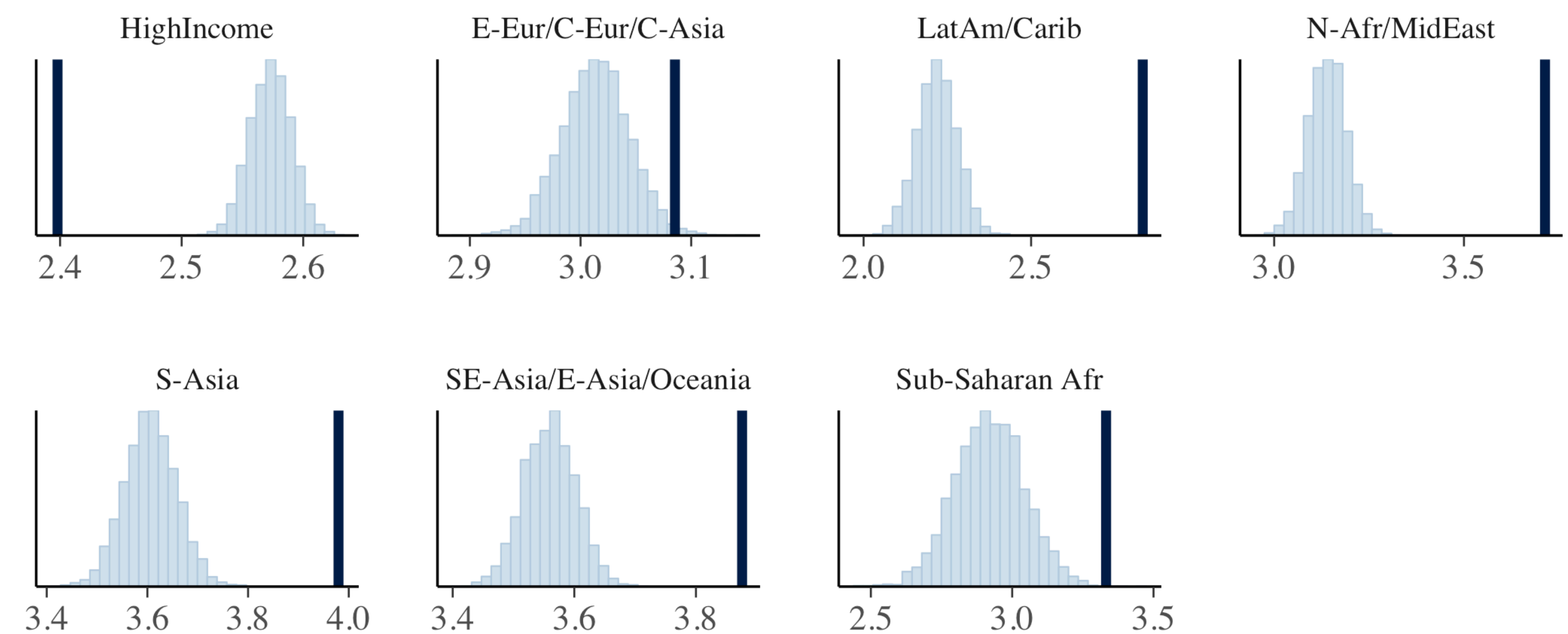
Observed statistics vs posterior predictive statistics



$$T(y) = \mathrm{skew}(y)$$

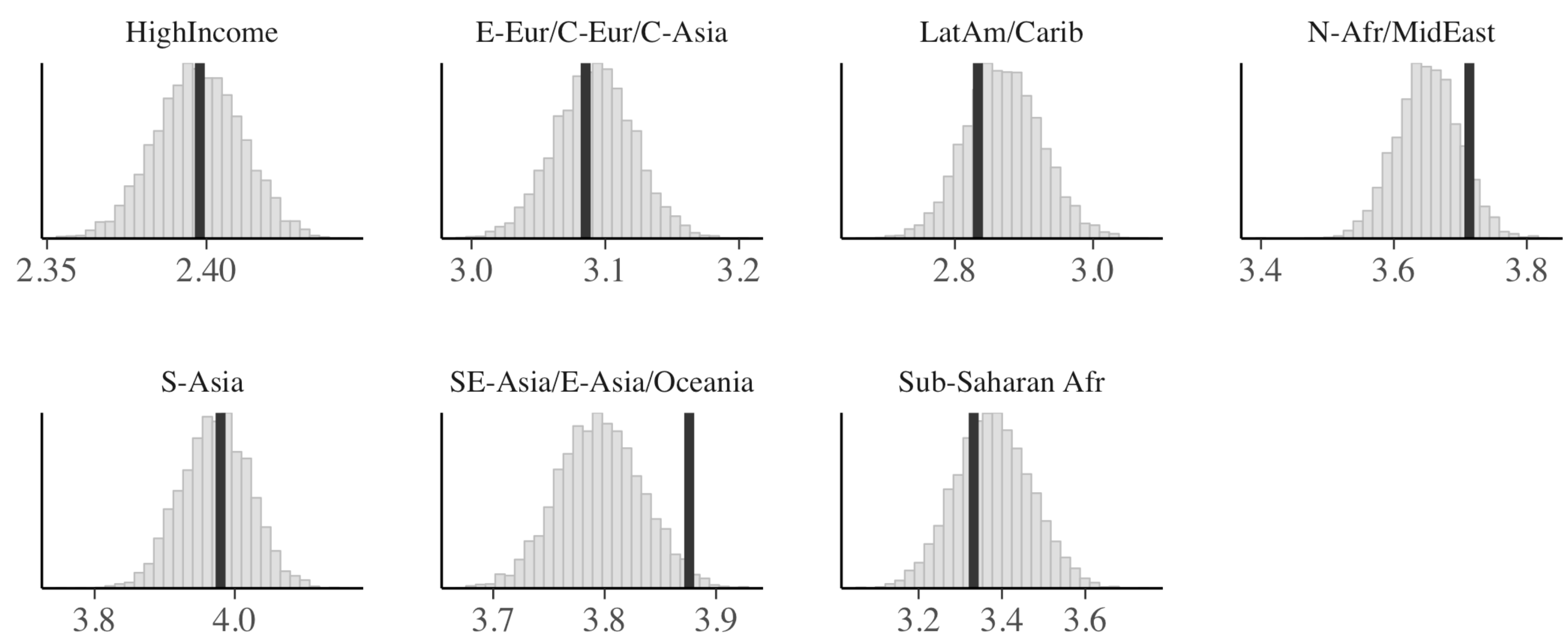
Posterior predictive checking:
visual model evaluation

Model 1 (single level)



$$T(y) = \mathrm{med}(y|\mathrm{region})$$

Model 2 (multilevel)



Model comparison

Pointwise predictive comparisons & LOO-CV

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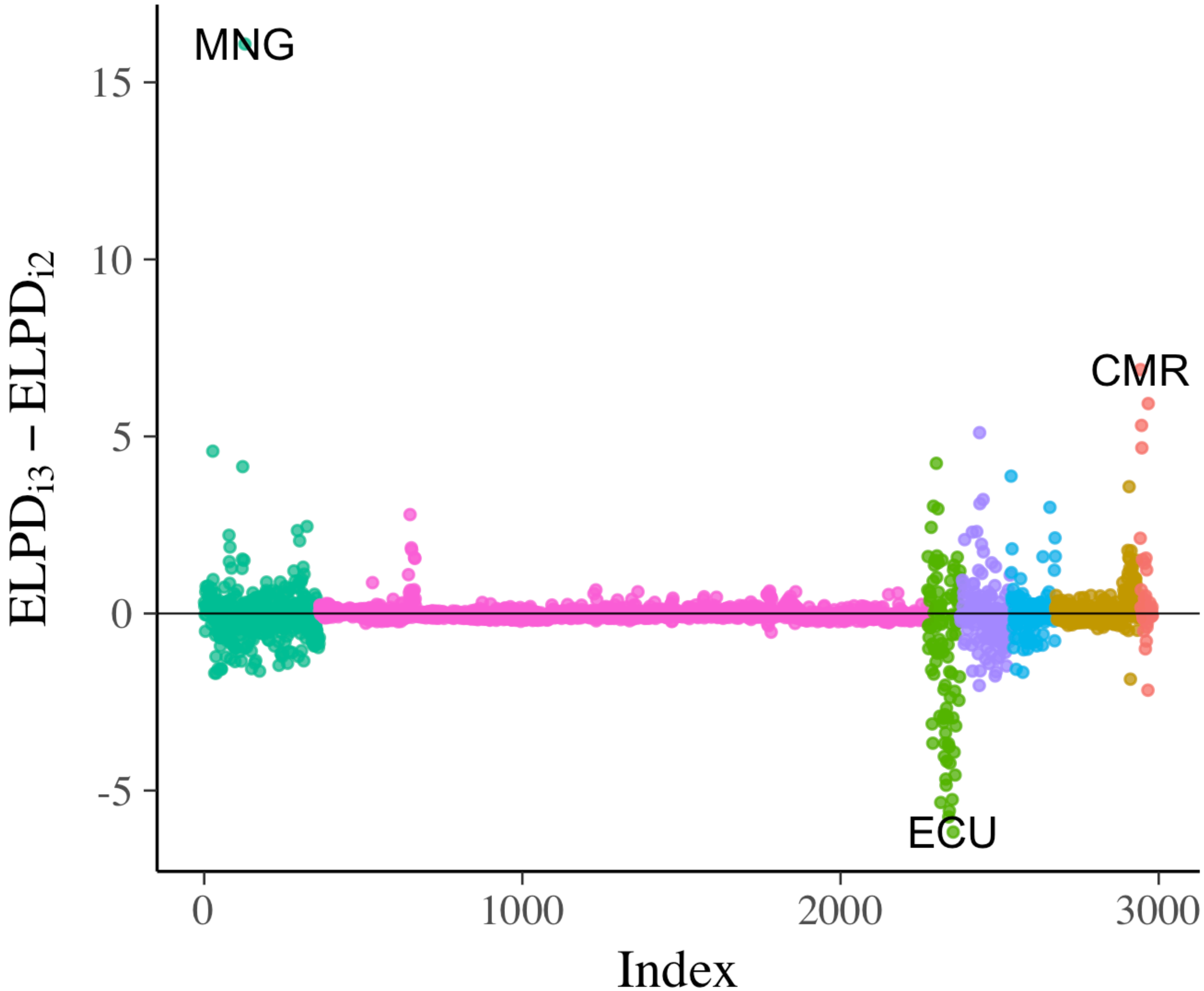
- Which model best predicts each of the data points that is left out?

Model comparison

pointwise predictive comparisons & LOO-CV



Model comparison

Efficient approximate LOO-CV

Vehtari, A., Gelman, A., and Gabry, J. (2017).
Practical Bayesian model evaluation using leave-one-out cross-validation and WAIC.
Statistics and Computing. 27(5), 1413–1432.
doi: [10.1007/s11222-016-9696-4](https://doi.org/10.1007/s11222-016-9696-4)

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- Advantage: PSIS-LOO CV more robust + has diagnostics (check assumptions)

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Diagnostics

Shape parameter of generalized Pareto distribution & influential observations