

Method Performance

Worst Case Performance:

$O(\log n)$ for `insert_element` and `remove_element`:

For every insertion or deletion, the performance is $O(\log n)$. The `cur_node_height()`, `get_height()`, creation of the node, and `self.__balance()` are all constant time. The rotations in `self.__balance()` take a constant number of steps depending on the balance factor. The time complexity comes from the actual insertion and removal itself and the traversal of the tree. Since the tree is balanced after every insertion or removal, the insertion/removal is $O(\log n)$ and the traversal of the tree is linear. Therefore, the complexity of the `insert_element` and `remove_element` methods are $\log n + n = O(\log n)$.

$O(n)$ for `to_list`:

The `to_list` method is linear because it traverses the tree in order, so as n increases, `to_list` has to recur n times, hence linear. And since each recursion is $O(1)$, `to_list` runs in linear time $O(n)$.