MNIST Digit Classification

SKIP most for PBDS Meetup 07/18/2019

[Show MNIST digit set image]

In [1]:
```
# Using TensorFlow 1.13.1
import tensorflow as tf
import tensorflow.keras as keras
import numpy as np
import os
import matplotlib.pyplot as plt
from datetime import datetime

# Add tensorflow.keras imports
from keras import models
from keras import layers
from keras.utils import to_categorical
from keras.datasets import mnist

print("TensorFlow:", tf.__version__, " Keras:", keras.__version__)
```

```
TensorFlow: 1.13.1  Keras: 2.2.4-tf

Using TensorFlow backend.
```

MNIST Digits Dataset

In [2]:
```
# Read Keras MNIST dataset
(train_images, train_labels), (test_images, test_labels) = mnist.load_data()

print("train_images.shape:", train_images.shape, "train_labels len:", len(train_labels))
```

```
train_images.shape: (60000, 28, 28) train_labels len: 60000
```

In [3]:
```python
# First three labels
print("MNIST First four labels:\n", train_labels[:4])
# First digit
print("MNIST First digit (partial crop):\n", train_images[0,4:26,6:24])
```

```
MNIST First four labels:
 [5 0 4 1]
MNIST First digit (partial crop):
 [[  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0]
 [  0   0   0   0   0   0   3  18  18  18 126 136 175  26 166 255 247 127]
 [  0   0  30  36  94 154 170 253 253 253 253 253 225 172 253 242 195  64]
 [  0  49 238 253 253 253 253 253 253 253 253 251  93  82  82  56  39   0]
 [  0  18 219 253 253 253 253 253 198 182 247 241   0   0   0   0   0   0]
 [  0   0  80 156 107 253 253 205  11   0  43 154   0   0   0   0   0   0]
 [  0   0   0  14   1 154 253  90   0   0   0   0   0   0   0   0   0   0]
 [  0   0   0   0   0 139 253 190   2   0   0   0   0   0   0   0   0   0]
 [  0   0   0   0   0  11 190 253  70   0   0   0   0   0   0   0   0   0]
 [  0   0   0   0   0   0  35 241 225 160 108   1   0   0   0   0   0   0]
 [  0   0   0   0   0   0   0  81 240 253 253 119  25   0   0   0   0   0]
 [  0   0   0   0   0   0   0   0  45 186 253 253 150  27   0   0   0   0]
 [  0   0   0   0   0   0   0   0   0  16  93 252 253 187   0   0   0   0]
 [  0   0   0   0   0   0   0   0   0   0   0 249 253 249  64   0   0   0]
 [  0   0   0   0   0   0   0   0  46 130 183 253 253 207   2   0   0   0]
 [  0   0   0   0   0   0  39 148 229 253 253 253 250 182   0   0   0   0]
 [  0   0   0   0  24 114 221 253 253 253 253 201  78   0   0   0   0   0]
 [  0   0  23  66 213 253 253 253 253 198  81   2   0   0   0   0   0   0]
 [ 18 171 219 253 253 253 253 195  80   9   0   0   0   0   0   0   0   0]
 [226 253 253 253 253 244 133  11   0   0   0   0   0   0   0   0   0   0]
 [253 253 212 135 132  16   0   0   0   0   0   0   0   0   0   0   0   0]
 [  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0]]
```

Preparing MNIST Digits for Input to MLP

Convert MNIST images for perceptron input

- from 2-D arrays of 28 x 28 pixels,
- to 1-D vector of 784 (28*28) numbers, scaled to range [0, 1].

In [4]:
```
# MNIST images as normalized (_, 28*28) tensors for MLP
train_images_mlp = train_images.reshape((60000, 28 * 28))
train_images_mlp = train_images_mlp.astype('float32') / 255

test_images_mlp = test_images.reshape((10000, 28 * 28))
test_images_mlp = test_images_mlp.astype('float32') / 255
```

In [5]:
```
# First digit
(shape0, mean0) = (train_images_mlp[0].shape[0], np.mean(train_images_mlp[0]))
(min0, max0) = (np.min(train_images_mlp[0]), np.max(train_images_mlp[0]))
print("MNIST First digit shape:", shape0, "mean:", mean0, "- min, max:", min0, max0)
```

```
MNIST First digit shape: 784 mean: 0.13768007 - min, max: 0.0 1.0
```

In [6]:
```
# Convert MNIST labels [0-9] to categorical
train_labels_cat = to_categorical(train_labels)
test_labels_cat = to_categorical(test_labels)
```

In [7]:
```
# First three labels
print("MNIST First three labels:\n", train_labels[:4])
print("First three labels categorical:\n", train_labels_cat[:4])
```

```
MNIST First three labels:
 [5 0 4 1]
First three labels categorical:
 [[0. 0. 0. 0. 0. 1. 0. 0. 0. 0.]
 [1. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 1. 0. 0. 0. 0. 0.]
 [0. 1. 0. 0. 0. 0. 0. 0. 0. 0.]]
```

Multi-Layer Perceptron MNIST Classifier (MLP)

- MLP single-layer (~97% with one hidden layer, 256 nodes; total trainable params: 203,530)

Homework

- MLP two-layer (~98% with two hidden layers) - skip
- Deep MLP with regularization (dropout, data augmentation) - skip

In [9]:
```
# Define MLP: one hidden layer of 256 nodes - 10-class softmax output
mlp = models.Sequential()
mlp.add(layers.Dense(256, activation='relu', input_shape=(28 * 28,)))
mlp.add(layers.Dense(10, activation='softmax'))

mlp.compile(optimizer='rmsprop', loss='categorical_crossentropy', metrics=['accuracy'])
```

In [10]:
```
# Summarize MLP
mlp.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #   
=================================================================
dense_3 (Dense)              (None, 256)               200960    
_________________________________________________________________
dense_4 (Dense)              (None, 10)                2570      
=================================================================
Total params: 203,530
Trainable params: 203,530
Non-trainable params: 0
_________________________________________________________________
```

Train Multi-Layer Perceptron on MNIST

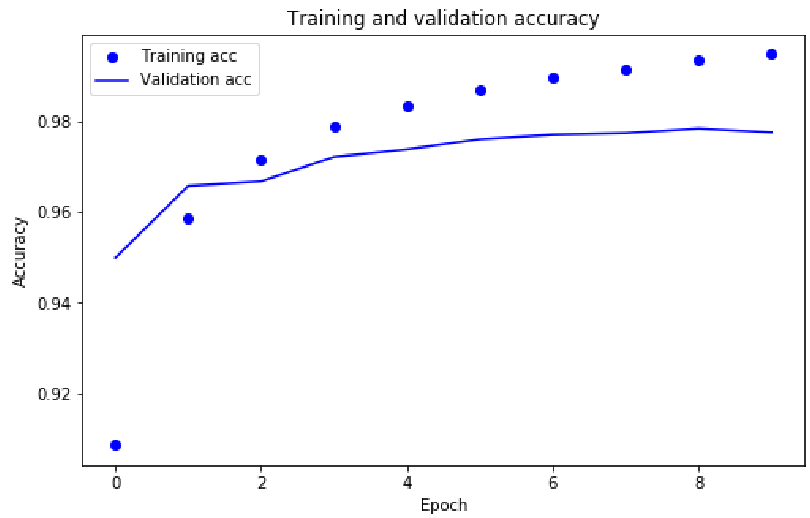
- Use Keras `mlp.fit(train_inputs, train_outputs)`

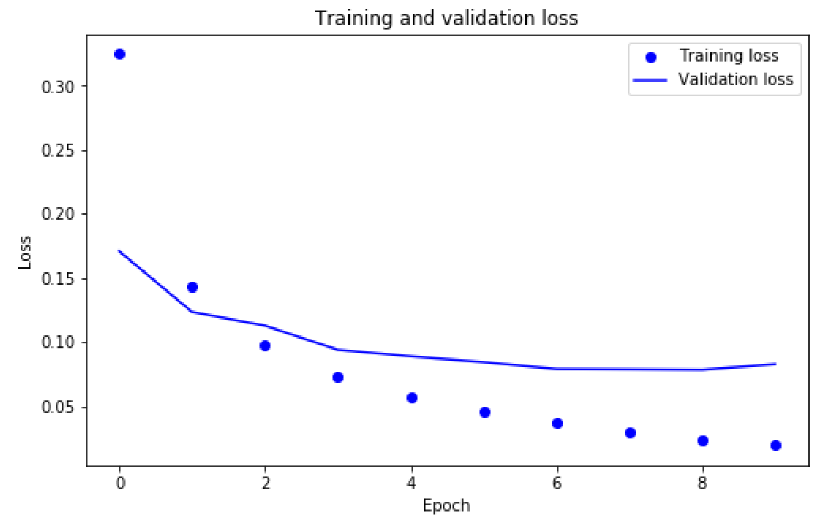
In [11]:
```
# Train MLP; using validation_split = 0.2
history = mlp.fit(train_images_mlp, train_labels_cat,
                  epochs=10, batch_size=128, validation_split=0.2)
```

```
WARNING:tensorflow:From /Users/nelson/dev/anaconda3/lib/python3.7/site-packages/tensorflow/python/op
s/math_ops.py:3066: to_int32 (from tensorflow.python.ops.math_ops) is deprecated and will be removed
in a future version.
Instructions for updating:
Use tf.cast instead.
Train on 48000 samples, validate on 12000 samples
Epoch 1/10
48000/48000 [==============================] - 1s 25us/step - loss: 0.3247 - acc: 0.9085 - val_loss:
0.1709 - val_acc: 0.9499
Epoch 2/10
48000/48000 [==============================] - 1s 21us/step - loss: 0.1432 - acc: 0.9588 - val_loss:
0.1235 - val_acc: 0.9658
Epoch 3/10
48000/48000 [==============================] - 1s 21us/step - loss: 0.0976 - acc: 0.9716 - val_loss:
0.1127 - val_acc: 0.9668
Epoch 4/10
48000/48000 [==============================] - 1s 22us/step - loss: 0.0731 - acc: 0.9789 - val_loss:
0.0939 - val_acc: 0.9723
Epoch 5/10
48000/48000 [==============================] - 1s 21us/step - loss: 0.0568 - acc: 0.9836 - val_loss:
0.0887 - val_acc: 0.9739
Epoch 6/10
48000/48000 [==============================] - 1s 22us/step - loss: 0.0457 - acc: 0.9870 - val_loss:
0.0842 - val_acc: 0.9762
Epoch 7/10
48000/48000 [==============================] - 1s 21us/step - loss: 0.0367 - acc: 0.9897 - val_loss:
0.0789 - val_acc: 0.9772
Epoch 8/10
48000/48000 [==============================] - 1s 22us/step - loss: 0.0298 - acc: 0.9916 - val_loss:
0.0787 - val_acc: 0.9775
Epoch 9/10
48000/48000 [==============================] - 1s 22us/step - loss: 0.0237 - acc: 0.9936 - val_loss:
0.0784 - val_acc: 0.9785
Epoch 10/10
48000/48000 [==============================] - 1s 22us/step - loss: 0.0193 - acc: 0.9949 - val_loss:
0.0826 - val_acc: 0.9777
```

In [12]:
```
# Visualize MLP accuracy, loss on MNIST training and validation data

# Define acc, loss plot function
def plot_acc_loss(history):
    acc = history.history['acc']
    val_acc = history.history['val_acc']
    loss = history.history['loss']
    val_loss = history.history['val_loss']

    epochs = range(len(acc))
    accuracy_template = "Epochs: {}, Best training acc: {}, Best val_acc: {}"
    print(accuracy_template.format(len(epochs), np.max(acc), np.max(val_acc)))

    plt.figure(figsize=(18,5))
    plt.subplot(121)
    plt.plot(epochs, acc, 'bo', label='Training acc')
    plt.plot(epochs, val_acc, 'b', label='Validation acc')
    plt.title('Training and validation accuracy')
    plt.xlabel('Epoch')
    plt.ylabel('Accuracy')
    plt.legend()
    plt.subplot(122)
    plt.plot(epochs, loss, 'bo', label='Training loss')
    plt.plot(epochs, val_loss, 'b', label='Validation loss')
    plt.title('Training and validation loss')
    plt.xlabel('Epoch')
    plt.ylabel('Loss')
    plt.legend()
    plt.show()
    return None

plot_acc_loss(history)
```

```
Epochs: 10, Best training acc: 0.9949375, Best val_acc: 0.9785000001589457
```





Evaluate Multi-Layer Perceptron on MNIST

- Use Keras `mlp.evaluate(test_inputs, test_outputs)`

In [13]:
```
# Evaluate MLP classifier
test_loss, test_acc = mlp.evaluate(test_images_mlp, test_labels_cat)
print('MLP Test accuracy:', test_acc, '- MLP Test loss:', test_loss)
```

```
10000/10000 [==============================] - 0s 19us/step
MLP Test accuracy: 0.98 - MLP Test loss: 0.07149975089433429
```

Convolutional Neural Networks (CNNs)

- LeCun, 1989.

ZIPCODE digit recognizer

CNN Elements: Filters, Layers, Strides and Padding

Figure

Convolutional Neural Network MNIST Classifier (CNN)

- CNN (~98% with one hidden layer) - skip
- CNN (~99.2% with three hidden layers, total trainable params: 130,890)
- CNN Test accuracy: 0.9919

Contrast CNN trainable parameters and accuracy (~99.2%, trainable params: 130,890) to single-layer MLP (~98% with one hidden layer, 256 nodes; total trainable params: 203,530).

In [14]:
```
# Keras CNN MNIST classifier - Three-Layer Convolutional Neural Network
# CNN input (28, 28, 1) images - CNN output (3, 3, 64) tensor
cnn = models.Sequential()
cnn.add(layers.Conv2D(32, (3, 3), activation='relu', input_shape=(28, 28, 1)))
cnn.add(layers.MaxPooling2D((2, 2)))
cnn.add(layers.Conv2D(64, (3, 3), activation='relu'))
cnn.add(layers.MaxPooling2D((2, 2)))
cnn.add(layers.Conv2D(64, (3, 3), activation='relu'))

# Fully-connected (MLP) output classifier - 10-class softmax output
cnn.add(layers.Flatten())
cnn.add(layers.Dense(128, activation='relu'))
cnn.add(layers.Dense(10, activation='softmax'))

cnn.compile(optimizer='rmsprop', loss='categorical_crossentropy', metrics=['accuracy'])
```

In [15]:
```
# Model summary
cnn.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #   
=================================================================
conv2d_1 (Conv2D)            (None, 26, 26, 32)        320       
_________________________________________________________________
max_pooling2d_1 (MaxPooling2 (None, 13, 13, 32)        0         
_________________________________________________________________
conv2d_2 (Conv2D)            (None, 11, 11, 64)        18496     
_________________________________________________________________
max_pooling2d_2 (MaxPooling2 (None, 5, 5, 64)          0         
_________________________________________________________________
conv2d_3 (Conv2D)            (None, 3, 3, 64)          36928     
_________________________________________________________________
flatten_1 (Flatten)          (None, 576)               0         
_________________________________________________________________
dense_5 (Dense)              (None, 128)               73856     
_________________________________________________________________
dense_6 (Dense)              (None, 10)                1290      
=================================================================
Total params: 130,890
Trainable params: 130,890
Non-trainable params: 0
_________________________________________________________________
```

Preparing MNIST Digits for Input to CNN

Convert MNIST images for CNN input

- from 2-D array of 28 x 28 pixels,
- to 3-D tensor of shape (28, 28, 1), scaled to range [0, 1].

In [16]:
```
# MNIST images as (_, 28, 28, 1) tensors for CNN input
train_images_cnn = train_images.reshape((60000, 28, 28, 1))
train_images_cnn = train_images_cnn.astype('float32') / 255

test_images_cnn = test_images.reshape((10000, 28, 28, 1))
test_images_cnn = test_images_cnn.astype('float32') / 255
print("train_images_cnn.shape:", train_images_cnn.shape)
print("train_labels_cat len:", len(train_labels_cat))
```

```
train_images_cnn.shape: (60000, 28, 28, 1)
train_labels_cat len: 60000
```

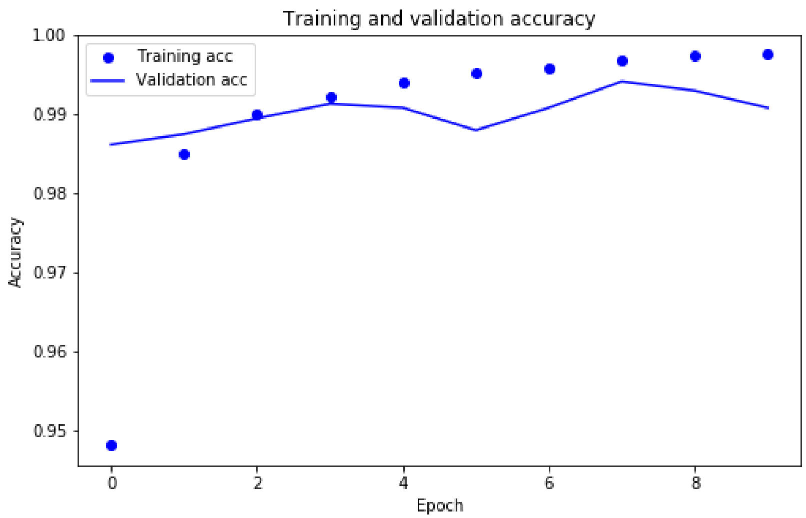
Train CNN with MLP output classifier

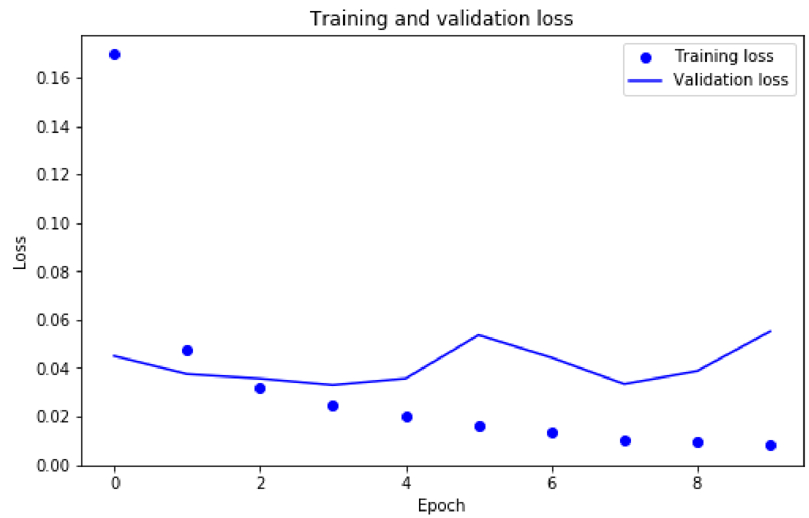
In [17]:
```
# Train CNN with MLP output classifier
history = cnn.fit(train_images_cnn, train_labels_cat,
                  epochs=10, batch_size=64, validation_split=0.1)
```

```
Train on 54000 samples, validate on 6000 samples
Epoch 1/10
54000/54000 [==============================] - 21s 395us/step - loss: 0.1697 - acc: 0.9481 - val_los
s: 0.0450 - val_acc: 0.9862
Epoch 2/10
54000/54000 [==============================] - 22s 406us/step - loss: 0.0478 - acc: 0.9850 - val_los
s: 0.0375 - val_acc: 0.9875
Epoch 3/10
54000/54000 [==============================] - 23s 420us/step - loss: 0.0317 - acc: 0.9900 - val_los
s: 0.0356 - val_acc: 0.9895
Epoch 4/10
54000/54000 [==============================] - 22s 416us/step - loss: 0.0244 - acc: 0.9923 - val_los
s: 0.0329 - val_acc: 0.9913
Epoch 5/10
54000/54000 [==============================] - 22s 414us/step - loss: 0.0198 - acc: 0.9939 - val_los
s: 0.0356 - val_acc: 0.9908
Epoch 6/10
54000/54000 [==============================] - 22s 409us/step - loss: 0.0161 - acc: 0.9952 - val_los
s: 0.0537 - val_acc: 0.9880
Epoch 7/10
54000/54000 [==============================] - 22s 409us/step - loss: 0.0136 - acc: 0.9959 - val_los
s: 0.0443 - val_acc: 0.9908
Epoch 8/10
54000/54000 [==============================] - 22s 411us/step - loss: 0.0103 - acc: 0.9968 - val_los
s: 0.0333 - val_acc: 0.9942
Epoch 9/10
54000/54000 [==============================] - 22s 411us/step - loss: 0.0097 - acc: 0.9975 - val_los
s: 0.0387 - val_acc: 0.9930
Epoch 10/10
54000/54000 [==============================] - 22s 417us/step - loss: 0.0079 - acc: 0.9976 - val_los
s: 0.0552 - val_acc: 0.9908
```

In [18]:
```
# Plot acc, loss
plot_acc_loss(history)
```

```
Epochs: 10, Best training acc: 0.9975740740740741, Best val_acc: 0.9941666666666666
```





Evaluate CNN classifier

In [19]:
```
# Evaluate CNN classifier
test_loss, test_acc = cnn.evaluate(test_images_cnn, test_labels_cat)
print('CNN Test accuracy:', test_acc, 'CNN Test loss:', test_loss)
```

```
10000/10000 [==============================] - 1s 104us/step
CNN Test accuracy: 0.9914 CNN Test loss: 0.04005222867553516
```

MNIST with Traditional Machine Learning

Multinomial MNIST Classifiers

Homework: Naive Bayes (NB), Stochastic Gradient Descent (SGD)

GaussianNB, SGDClassifier from Scikit-Learn

- e.g., sklearn.GaussianNB, sklearn.SGDClassifier (MNIST accuracy of ~85%)

Feature engineering

Feature engineering is important, more so for shallow MLP and traditional statistical classifiers.

- Input scaling (e.g., sklearn.StandardScaler on MNIST improves accuracy to ~90%)
- Deep networks beat hand-engineered features, with optimal, automatically learned features of, for example, CNN filters

Capsules Neural Network MNIST Classifier (CapsNet)

- Capsule Networks (CapsNet, Sabour, X, Hinton, 2017; Zhen, KPU, 2019) - skip

MNIST State-of-the-Art (SOTA)

MNIST Test accuracy stands at 99.83% (in 2019)

Record (03/2019)

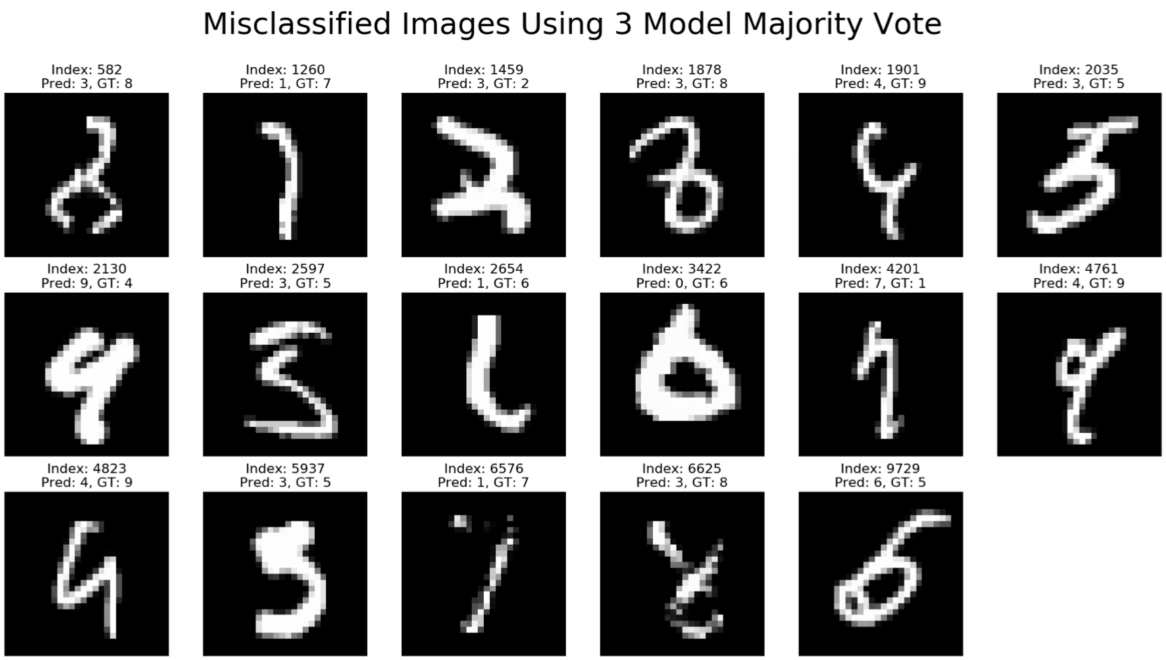
- Zhao et al., 2019, report absolute MNIST error rate reduction from previous best 0.21% (99.79% accuracy) to 0.17% (99.83% accuracy).
- *Capsule Networks with Max-Min Normalization*, Zhen Zhao, Ashley Kleinhans, Gursharan Sandhu, Ishan Patel, K. P. Unnikrishnan, 2019, https://arxiv.org/abs/1903.09662 (https://arxiv.org/abs/1903.09662).

Others (2017 - 2018)

- *Regularization of neural networks using dropconnect*, Li Wan, Matthew D Zeiler, Sixin Zhang, Yann LeCun, and Rob Fergus, ICML 2013. 99.79% accuracy (0.39% error rate; 0.21% with ensembling)
- *Dynamic Routing Between Capsules*, Sara Sabour, Nicholas Frosst, Geoffrey E Hinton, 2017 (https://arxiv.org/abs/1710.09829 (https://arxiv.org/abs/1710.09829)) 99.75% accuracy (0.25% error rate)
- Simple 3-layer CNN above (total params, 130,890) without any training regularization: `acc: 99.15%`

CapsNet with Max-Min: MNIST Test set errors

Misclassified MNIST images using 3-model majority vote from CapsNets trained using Max-Min normalization



- Total of 17 digit errors in 10,000 digit test set (99,83% test accuracy)
- MNIST digit index 6576 - Ground Truth: "7"
- current deep networks recognize as "1"; no human makes such error

Source: *Capsule Networks with Max-Min Normalization*, Zhao et al., 2019 (https://arxiv.org/abs/1903.09662)

After 30 years, MNIST Test must be considered a *validation* set

... it is no longer a *test* set.

MNIST Test can no longer be considered a proper Test set. It is a digit classification Validation set.

The MNIST dataset availability and focus on the MNIST task since 1989 have made the MNIST, 10,000 digit "*Test*" set into a "validation" dataset.

- New *test sets* for MNIST, different from the original 1990 one, are needed!
- A new digit recognition benchmark is needed.

MNIST improvements are near the end (only 20 digits left for improvement)

The MNIST Test set has a total of 60,000 training and 10,000 test digits.

Zhao et al., 2019, use single classifiers with recognition error on only 20 to 23 digits of the 10,000 MNIST test digits.

In [ ]: