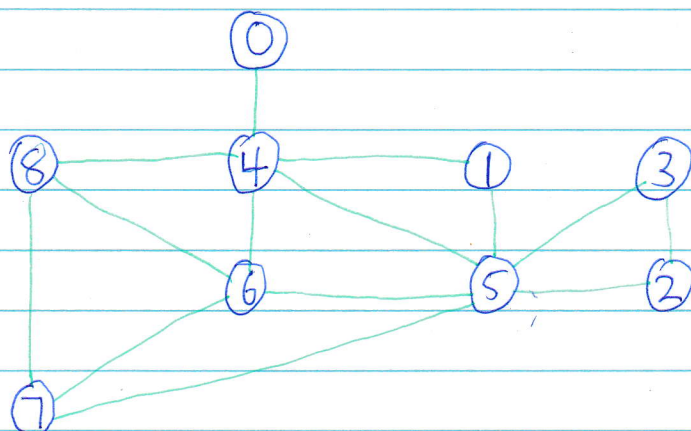


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1.



Adjacency list representation:

0 → 4 ✓

1 → 4 → 5 ✓

2 → 3 → 5 ✓

3 → 2 → 5 ✓

4 → 0 → 1 → 5 → 6 → 8 ✓

5 → 1 → 2 → 3 → 4 → 6 → 7 ✓

6 → 4 → 5 → 7 → 8 ✓

7 → 5 → 6 → 8 ✓

8 → 4 → 6 → 7 ✓

Adjacency matrix representation:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	1	0	0	0	0
1	0	0	0	0	1	1	0	0	0
2	0	0	0	1	0	1	0	0	0
3	0	0	1	0	0	1	0	0	0
4	1	1	0	0	0	1	1	0	1
5	0	1	1	1	1	0	1	1	0
6	0	0	0	0	1	1	0	1	1
7	0	0	0	0	0	1	1	0	1
8	0	0	0	0	1	0	1	1	0

2. DFS:  $0 \rightarrow 4 \rightarrow 1 \rightarrow 5 \rightarrow 2 \rightarrow 3 \rightarrow 6 \rightarrow 7 \rightarrow 8$

BFS:  $0 \rightarrow 4 \rightarrow 1 \rightarrow 5 \rightarrow 6 \rightarrow 8 \rightarrow 2 \rightarrow 3 \rightarrow 7$

3. As we told that a tree  $T$  with  $n$  vertices has  $n-1$  edges, then  $F = \bigcup_{i=1}^k T_i$  is the forest - a union of trees,  $n_i$  is the number of vertices of each tree  $\rightarrow |V(F)| = \sum_{i=1}^k n_i = n$ . Since forest is combined by trees, we have  $n_i - 1$  edges for  $T_i \rightarrow |E(F)| = \sum_{i=1}^k (n_i - 1) = n - k$ .

4. initialize a empty list  $L$

initialize a stack  $S$

push all vertices with no incoming edges into stack

while stack is not empty do  
     $V \leftarrow \text{stack.POP()}$   
    add  $V$  to  $L$   
    if stack has more than one item  
        stop return false (the topologic order is not unique)

for all the vertices  $w$  with an edge  $e$  from  $V$  to  $w$  do

    if there is more than one ' $w$ '

        stop and print "The topological order is not unique."

    remove edge  $e$  from  $G$

    if  $w$  has no other incoming edge then

        push  $w$  into stack

    if  $G$  has edges left then

        return false ( $G$  is not topological ordered)

    else

        return  $L$  (the topologic order is unique)

5. Put all vertices into a list  $L$

color first vertex to white

creat a queue of vertex number and enqueue the first vertex

while queue is not empty

dequeue a vertex from queue

for all non-colored adjacent vertices

if an edge from  $u$  to  $v$  exists and  $v$  is not colored

assign alternate color to  $v$

else if an edge from  $u$  to  $v$  exist and  $v$  is colored same as  $u$

return false

return true

Since we are using BFS here, the running time is  $O(n+m)$