**CSC 225 SUMMER 2016**
**ALGORITHMS AND DATA STRUCTURES I**
**ASSIGNMENT 4**
**UNIVERSITY OF VICTORIA**

1. Let $G$ be the undirected graph with vertices $V = \{0,1,2,3,4,5,6,7,8\}$ and edges

   $$E = \{\{0,4\},\{1,4\},\{1,5\},\{2,3\},\{2,5\},\{3,5\},\{4,5\},\{4,6\},\{4,8\},\{5,6\},\{5,7\},\{6,7\},\{6,8\},\{7,8\}\}$$

   (a) Draw $G$ in such a way that no two edges cross (i.e. it is a planar graph.)
   (b) Draw adjacency list representation of $G$.
   (c) Draw adjacency matrix representation of $G$.

2. For the graph $G$ in Problem 1 assume that, in a traversal of $G$, the adjacent vertices of a given vertex are returned in their numeric order.

   (a) Order the vertices as they are visited in a DFS traversal starting at vertex 0.
   (b) Order the vertices as they are visited in a BFS traversal starting at vertex 0.

3. Let $F = (V, E)$ be a forest with $n$ vertices, $m$ edges and $k$ connected components. Prove that the number of edges in $F$ is $m = n - k$.

4. Design an algorithm (in pseudocode) to determine whether a digraph has a unique topological ordering. Your algorithm should return the ordering if a unique one exists or indicate that no unique order exists.

5. A *2-colouring* of an undirected graph with $n$ vertices and $m$ edges is the assignment of one of two colours (say, black or white) to each vertex of the graph, so that no two adjacent nodes have the same colour. So, if there is an edge $(u,v)$ in the graph, either node $u$ is black and $v$ is white or vice versa. Give an $O(n + m)$ time algorithm (pseudocode!) to 2-colour a graph or determine that no such colouring exists, and justify the running time. The following shows examples of graphs that are and are not 2-colourable: